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THE UNITARY DUAL LOOP REVISITED

I. Bengtsson
CERN -- Geneva

ABSTRACT

Using the BRS-invariant tachyon vertex, we recalculate the single planar loop for the bosonic string in a manifestly covariant way, avoiding both the sewing procedure and the Brink-Olive projection operator.

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Covariant quantization of free strings was imperfectly understood in the "old days" of dual models. As a consequence, one did not really know how to compute loop graphs in a manifestly covariant way. The calculation of the single planar loop with external tachyons (Fig. 1) was carried out within the operator formalism in Ref. 1), using Feynman's tree theorem and the projection operator of Brink and Olive, which was non-covariant.

The purpose of this letter is to point out that the diagram of Fig. 1 can be computed trivially today, thanks to the work of Ref. 2), where BRS-quantization of the free string was carried out [Ref. 3) provides a review].

The BRS charge of the free string is

$$Q = c_0 H + \sum_{n=1}^{\infty} (c_n L_{-n} + c_n^\dagger L_n) + \text{pure ghost terms} \quad (1)$$

where

$$H = L_0 - 1 + \sum_{n=1}^{\infty} n (c_n^\dagger \bar{c}_n + \bar{c}_n^\dagger c_n) \quad (2)$$

Here the c 's are ghost oscillators obeying

$$\{c_m, \bar{c}_n^\dagger\} = \{\bar{c}_m, c_n^\dagger\} = \delta_{n,m} \quad (3)$$

For our calculation we need two things, the propagator and the vertex. The propagator used to be

$$D = \frac{1}{L_0 - 1} = \int_0^1 \frac{dx}{x} x^{L_0 - 1} = \int_0^1 \frac{dx}{x} x^{\frac{1}{2} p^2 - 1 + \sum_{n=1}^{\infty} n a_n^\dagger a_n} \quad (4)$$

A modern field theorist immediately notices the absence of any ghost propagators from Eq. (4). The correct BRS-invariant propagator is

$$D' = \frac{1}{H} \quad (5)$$

The vertex for tachyon emission used to be

$$V_0(k) = \exp k \cdot \sum_{n=1}^{\infty} \frac{a_n^\dagger}{\sqrt{n}} e^{ik \cdot x} \exp -k \cdot \sum_{n=1}^{\infty} \frac{a_n}{\sqrt{n}} \quad (6)$$

Actually, this is the complete vertex. To check its BRS-invariance we go to the free mass shell, i.e., we check to first order in the coupling constant only⁴⁾. In a sense, we are doing a string field theory calculation along the

lines of Ref. 5):

$$\begin{aligned}
 \delta(\varphi V \varphi) &= \epsilon \varphi [Q, V] \varphi = \\
 &= \epsilon \sum_{n=-\infty}^{\infty} \varphi (c_n^\dagger [L_n, V] + c_n [L_{-n}, V]) \varphi = \\
 &= \epsilon \sum_{n=-\infty}^{\infty} \varphi (c_n^\dagger [H, V] + n c_n^\dagger V + c_n [H, V] - n c_n V) \varphi = \\
 &= \epsilon \sum_{n=-\infty}^{\infty} \varphi ([c_n^\dagger, H] + n c_n^\dagger + [c_n, H] - n c_n) V \varphi = 0.
 \end{aligned} \tag{7}$$

Now we can start to calculate the diagram. According to the rules, we get

$$M = \int \frac{i d^{26} k}{(2\pi)^{26}} \text{Tr } D'V_1 D'V_2 \dots D'V_N \tag{8}$$

Our modified propagator gives us

$$M = \int \frac{i d^{26} k}{(2\pi)^{26}} \prod_{i=1}^N \int_0^1 dx_i \text{Tr } DV_1 DV_2 \dots DV_N \cdot \text{Tr} (x_1 x_2 \dots x_N)^{\sum_{n=1}^{\infty} n (\bar{c}_n^\dagger c_n + c_n^\dagger \bar{c}_n)} \tag{9}$$

So the ghosts contribute with just a factor, which one can compute straightforwardly. Then

$$M = \int \frac{i d^{26} k}{(2\pi)^{26}} \prod_{i=1}^N \int_0^1 dx_i \text{Tr } DV_1 DV_2 \dots DV_N \prod_{n=1}^{\infty} (1 - (x_1 x_2 \dots x_N)^n)^2 \tag{10}$$

This agrees with Ref. 1). The factor, incidentally, is the inverse of the partition function squared.

We believe that we have justified our claim that the calculation is trivial to perform when one uses the result of Ref. 2). This is encouraging, now that Green and Schwarz have made strings interesting again.

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