CERN-TH.4286/85

NO FOURTH GENERATION FOR NO-SCALE MODELS?

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A B S T R A C T

We show that unless $m_t \lesssim 35$ GeV (with possible exceptions in the unlikely case $m_b \gg m_t$), there is a stable charged s-particle in the simplest four-generation $SU(n,1)$ supergravity model, which may be the low-energy limit of superstrings. A stable charged s-particle contradicts astrophysical data and therefore the existence of a fourth generation can be decided experimentally at the present accelerators.

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CERN-TH.4286/85

October 1985

At present, our theoretical understanding of the number of fermion generations is almost non-existent. There is nothing in the low-energy standard model nor in grand unified theories that would fix this number. Yet there exist several indications that it should be small. In grand unified theories successful predictions like m_b/m_τ would be destroyed if the number of generations were larger than four¹⁾. Likewise, the primordial nucleosynthesis gives a similar limit provided each generation has its associated light neutrino²⁾. These can also be counted in Z^0 decays, which can be used to set an upper limit of 5.6 (at 90% c.l.) on light neutrinos³⁾. If one believes in a grand desert above $O(100 \text{ GeV})$, one finds that perturbation theory will break down at scales much less than the Planck scale M_P if too many generations are allowed, especially in supersymmetric models.

What makes the whole issue of the number of generations very interesting is that experimentally we must be already relatively near to the final answer. This is because the masses of the extra generation fermions break $SU(2) \times U(1)$, and therefore they cannot be heavier than $O(1 \text{ TeV})$. Here contact can also be made with the recent theoretical developments in superstring models⁴⁾. In the simplest cases they predict an even number of generations⁵⁾ (although in more complicated situations also odd numbers may be allowed⁶⁾), and this question is therefore one of the few instances where superstring models can be tested directly in the low-energy regime. Apart from its obvious phenomenological interest, the settling of the number of generations would also be very valuable for theoretical development in superstring theories, because there this number is connected with the topological properties of the internal six-dimensional compact manifold, which manifest also in other parts of these theories⁷⁾.

The four-dimensional pointlike field theory limit of the superstring theories may be⁸⁾ $N = 1$ supergravity with an $SU(n,1)/SU(n) \times U(1)$ structure of the scalar fields⁹⁾. This is exactly the same situation that was encountered in the phenomenological no-scale models^{10), 9)}. There the only seed of global supersymmetry breaking is a universal gaugino mass. At $Q = M_X$ all other soft supersymmetry breaking parameters such as the scalar masses are absent⁹⁾, but they will naturally arise through loop corrections¹¹⁾ at lower scales. Of course, the no-scale models deserve a special attention independently of their possible stringy origins because of their many phenomenologically interesting consequences, and in a previous paper¹²⁾ we have studied general features of four-generation $SU(n,1)$ models [minimal supergravity models have been discussed in Ref. 13)].

In the present paper we perform a systematic search for the lightest supersymmetric particle of the four-generation supergravity models with a universal gaugino mass $m_{\frac{1}{2}}$ as the only source of supersymmetry breaking. We find that almost invariably the lightest charged sparticle is stable, which is in contrast with astrophysical observations: in fact, charged supersymmetric relics from the Big Bang would tend to condense into galaxies, stars and planets along with ordinary matter, with an abundance which is many orders of magnitude above the severe limits on the abundances of exotic isotopes¹⁴⁾. These limits are not violated only if the fourth generation lepton τ' has a mass $m_{\tau'} \lesssim 35$ GeV (this bound can be stretched in the unlikely case in which $m_{b'} \gg m_{\tau'}$), a range that can possibly be excluded in the very near future¹⁵⁾. This result has important consequences for both the existence of a fourth generation in supergravity and the fate of $SU(n,1)$ no-scale models.

For the details of the model, including the notation, we refer to our previous paper¹²⁾. Let us simply recall here the soft supersymmetry breaking part of the Lagrangian:

$$\begin{aligned}
 -\mathcal{L}_{\text{soft}} = & \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 + m_{Q_a}^2 |Q_a|^2 + m_{U_a}^2 |U_a^c|^2 + m_{D_a}^2 |D_a^c|^2 + m_{L_a}^2 |L_a|^2 \\
 & + m_{E_a}^2 |E_a^c|^2 + (B\mu H_1 H_2 + \text{h.c.}) + [(A_{ab}^U h_{ab}^U H_2 Q_a U_b^c + \\
 & + A_{ab}^D h_{ab}^D H_1 Q_a D_b^c + A_{ab}^E h_{ab}^E H_1 L_a E_b^c) + \text{h.c.}] + \frac{1}{2} M_i (\lambda_i \lambda_i + \text{h.c.}), \quad (1)
 \end{aligned}$$

where $a, b = 1, \dots, 4$ are generation indices and $i = 1, 2, 3$ labels the factors of the standard gauge group. At $Q = M_X$, where M_X is the GUT scale obtained from the one-loop renormalization group equations, we impose the no-scale boundary conditions:

$$\mu_1^2 = \mu_2^2 = m_{Q_a}^2 = m_{U_a}^2 = m_{D_a}^2 = m_{L_a}^2 = m_{E_a}^2 = 0 ; \quad (2a)$$

$$A_{ab}^U = A_{ab}^D = A_{ab}^E = 0 ; \quad B = 0 ; \quad (2b)$$

$$M_1 = M_2 = M_3 = m_{1/2} ; \quad \mu = \mu_0 . \quad (2c)$$

For a range of the Yukawa couplings $h_t, (M_X)$ and $h_b, (M_X) \equiv h_\tau, (M_X)$ of the fourth generation, the neutral Higgs potential

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2 (H_1 H_2 + \text{h.c.}) + (\text{quartic terms}) \quad (3)$$

(where $m_1^2 \equiv \mu^2 + \mu^2$, $m_2^2 \equiv \mu^2 + \mu^2$, $m_3^2 \equiv -B\mu$) develops a $SU(2)_L \times U(1)_Y$ breaking minimum at scales $Q \approx m_W$. Since we are interested in the mass spectrum only, we have not assumed any specific type of electroweak symmetry breaking (i.e., dimensional transmutation¹⁶⁾ or proper no-scale^{9),10)}). This leaves us with the additional parameter $\omega \equiv v_2/v_1$, which is determined to be

$$\omega = \text{sign}(m_3^2) \cdot \left(\frac{m_1^2 + m_2^2/2}{m_2^2 + m_2^2/2} \right)^{1/2} \quad (4)$$

and, in contrast to three generation models, can be different from ± 1 , although being restricted in the range¹²⁾ $0.5 \lesssim |\omega| \lesssim 2$.

We now impose on the supergravity model (1)+(2) the following phenomenologically motivated constraints. All non-standard charged particles must naturally weigh more than $O(20 \text{ GeV})$. In particular¹⁷⁾:

$$m_{\tau'} \gtrsim 22 \text{ GeV}. \quad (5)$$

The possible improvement of the above bound in the near future would simply strengthen our conclusions. The Yukawa couplings must stay in the perturbative regime at all scales Q between M_X and m_W . Following Ref. 18), we require:

$$\frac{h_a^2}{4\pi} \lesssim 1. \quad (6)$$

We also require that at $Q = m_W$ the third generation masses should lie within the generous bounds (which take also into account the uncertainties of our calculational scheme):

$$1.70 \text{ GeV} \lesssim m_{\tau'} \lesssim 1.85 \text{ GeV}, \quad 3 \text{ GeV} \lesssim m_b \lesssim 6 \text{ GeV}, \quad 30 \text{ GeV} \lesssim m_t \lesssim 50 \text{ GeV}. \quad (7)$$

Our results are not very sensitive to the assumed third generation masses, in particular the top quark mass. Furthermore, we make the sensible restriction, motivated by the results of the heavy-quark searches at the collider¹⁹⁾:

$$m_b', m_t' \gtrsim 40 \text{ GeV}, \quad (8)$$

where m_b' and m_t' refer to the fourth generation quarks. Finally, we assume a minimal Higgs content and impose conditions to have $SU(2)_L \times U(1)_Y$ breaking at the proper scale and avoid $SU(3)_C \times U(1)_{em}$ -breaking minima and/or potentials unbounded from below [for a discussion of these, see Ref. 12)].

We have integrated numerically the renormalization group equations to obtain the mass spectrum at $Q = m_W$. Fixing the set of Yukawa couplings in the allowed range, the requirement of electroweak symmetry breaking associates to every value of $m_{\frac{1}{2}}$ a corresponding value of the free parameter μ_0 and determines ω : typically, one finds $\mu_0 \sim O(m_{\frac{1}{2}})$. As was pointed out in our previous paper¹²⁾, the lightest scalar s-particle is invariably the fourth generation $\tilde{\tau}_1'$ (which is not purely right-handed because of the off-diagonal entries in the corresponding mass matrix¹²⁾). To find the lightest sparticles, we have also diagonalized numerically the chargino and the neutralino mass matrices of the supersymmetric fermionic sector:

$$\begin{pmatrix} \tilde{W}^- \\ \tilde{H}_1^- \end{pmatrix} \begin{pmatrix} \tilde{W}^+ & \tilde{H}_2^+ \\ M_2 & g_2 v_2 \\ g_2 v_1 & \mu \end{pmatrix}, \quad (9)$$

$$\begin{pmatrix} \tilde{W}^0 \\ \tilde{B}^0 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix} \begin{pmatrix} \tilde{W}^0 & \tilde{B}^0 & \tilde{H}_1^0 & \tilde{H}_2^0 \\ M_2 & 0 & -g_2 v_1/\sqrt{2} & g_2 v_2/\sqrt{2} \\ 0 & M_1 & g_1 v_1/\sqrt{2} & -g_1 v_2/\sqrt{2} \\ -g_2 v_1/\sqrt{2} & g_1 v_1/\sqrt{2} & 0 & \mu \\ g_2 v_2/\sqrt{2} & -g_1 v_2/\sqrt{2} & \mu & 0 \end{pmatrix}. \quad (10)$$

Representative examples of the results are shown in Fig. 1 for $\omega^2 \approx 1$, in Fig. 2 for $\omega^2 \approx 0.5$ and $\omega^2 \approx 2$. In the mass matrices (9) and (10) the relative sign of $m_{\frac{1}{2}}$ and μ_0 is important and leads to two different sets of masses, which are both taken into account in the Figures. Another thing to notice is that the value of ω^2 is determined essentially by the Yukawa couplings h_b and h_t : higher values of the ratio h_t/h_b , at M_x give rise to higher values of ω^2 . In particular, the case $\omega^2 \approx 1$ corresponds to $m_b' \approx m_t' + O(12) \text{ GeV}$: therefore, unless the mass pattern of the known generations is completely upset in the case of the fourth

one, we naturally expect $\omega^2 > 1$. From Fig. 1 (corresponding to the case $\omega^2 \approx 1$), one can see that the lightest charged s-particle is indeed $\tilde{\tau}_1'$ (only in the extreme case $m_{\tilde{\tau}_1'} \approx 22$ GeV it is a chargino $\tilde{\chi}^\pm$). If $\tilde{\tau}_1'$ cannot decay into $\tau + \tilde{\chi}^0$, where $\tilde{\chi}^0$ is the lightest neutralino, it is stable, and therefore not allowed on astrophysical grounds, as explained before¹⁴⁾. In Fig. 3 we show the kinematic limit $m_{\tilde{\tau}_1'} > m_{\tilde{\chi}^0} + m_{\tau}$. This naturally gives a much more severe constraint than the PETRA limit $m_{\tilde{\tau}_1'} > 0(20)$ GeV. If $m_{\tilde{\tau}_1'} \approx 22$ GeV, $\tilde{\tau}_1'$ can decay into $\tilde{\chi}^\pm + \nu_\tau$. However, in this case one finds that $m_{\tilde{\chi}^\pm} < m_{\tilde{\chi}^0}$, and therefore we are faced with the same problem as before.

Our results were obtained by restricting μ_0 and $m_{\frac{1}{2}}$ to be less than 1 TeV. Moreover, as the renormalization group equations for soft SUSY breaking parameters effectively freeze at scales $Q < m_{\frac{1}{2}}$, our results are not compatible with electroweak symmetry breaking if $\ln|m_{\frac{1}{2}}/m_W| \gg 1$. The exact limit depends on the mechanism of symmetry breaking. For instance, in no-scale models one dynamically determines $m_{\frac{1}{2}} \approx 0(1) m_W$. Therefore we can choose with some arbitrariness $m_{\frac{1}{2}} \lesssim 500$ GeV. From Fig. 3 we see that these requirements correspond to:

$$22 \text{ GeV} \lesssim m_{\tilde{\tau}_1'} \lesssim 35 \text{ GeV} \quad (11)$$

a region which should be experimentally accessible already at the CERN $p\bar{p}$ collider. The above constraints also imply for the gluino that $250 \text{ GeV} \lesssim m_{\tilde{g}} \lesssim 850 \text{ GeV}$. Strictly speaking, the given limits on $m_{\tilde{\tau}_1'}$ and $m_{\tilde{g}}$ apply only to the case $\omega^2 \approx 1$. As can be seen from Fig. 2, they become even more stringent in the case $\omega^2 > 1$, which we believe is the most natural situation in the present framework; on the other hand, they can be relaxed if $\omega^2 < 1$, which, however, occurs only in the case $m_b \gg m_{\tilde{\tau}_1'}$.

Our lower bound on $m_{\frac{1}{2}}$ should not come as a total surprise if one looks at the structure of the $\tilde{\tau}'$ and neutralino mass matrices. The lightest neutralino $\tilde{\chi}^0$ turns out to be essentially $\tilde{B}^0$, associated with the $U(1)_Y$ gauge factor, and its mass is close to $M_1 \sim 0.23 m_{\frac{1}{2}}$. On the other hand, concentrating for simplicity in the case $\omega^2 \approx 1$, and recalling that in the model under consideration we do not have primordial soft scalar masses, the diagonal entries in the $\tilde{\tau}'$ mass matrix are of the form $(\text{const} \times m_{\frac{1}{2}}^2) + m_{\tilde{\tau}}^2$, while the off-diagonal ones have the form $m_{\tilde{\tau}}(A_{\tilde{\tau}}, \pm \mu)$. Since the off-diagonal entries give a negative contribution to the

lightest mass eigenvalue $m_{\tilde{\tau}_1}^2$, it becomes apparent that to avoid negative eigenvalues, $m_{\frac{1}{2}}$ has to be sufficiently large: in that case, however, it may easily happen that $m_{\tilde{\chi}^0} \gtrsim m_{\tilde{\tau}_1}^2$.

If experiments find no τ' in the interval (11), will the four generation superstring models be ruled out? Unfortunately in these theories the low energy gauge group will most likely contain extra pieces⁷⁾, and there can also be additional light particles. In the simplest cases²⁰⁾ with an extra $U(1)$ group we do not expect any qualitative change in the limit (11), but intermediate thresholds may lead to more drastic changes. Of course, if one abandons²¹⁾ the natural and stringy supported assumption^{8),20)} of universal gaugino masses at M_X , one could hope to find models in which neutralino masses are highly suppressed in order to alleviate this problem. Barring more baroque possibilities, however, it is striking how well constrained the simplest four-generation $SU(n,1)$ supergravity models are, and it is now a matter of experiment to provide the final verdict.

ACKNOWLEDGEMENTS

One of us (F.Z.) acknowledges the financial support of "Fondazione Angelo Della Riccia". Another one (K.E.) thanks the Academy of Finland for financial support.

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FIGURE CAPTIONS

Fig. 1 The spectrum of the lightest supersymmetric particles, as a function of the primordial gaugino mass $m_{\frac{1}{2}}$, for three typical cases with $\omega^2 \approx 1$ (assuming $m_t \approx 40$ GeV):

a) $m_{t'} = 64$ GeV, $m_{b'} = 78$ GeV, $m_{\tau'} = 24$ GeV;

b) $m_{t'} = 84$ GeV, $m_{b'} = 96$ GeV, $m_{\tau'} = 32$ GeV;

c) $m_{t'} = 100$ GeV, $m_{b'} = 112$ GeV, $m_{\tau'} = 40$ GeV.

The solid line represents the lightest scalar partner of the fourth generation lepton ($\tilde{\tau}_1'$); the dotted line represents the lightest neutralino ($\tilde{\chi}^0$); the dashed line represents the lightest chargino ($\tilde{\chi}^{\pm}$). Since the masses of charginos and neutralinos depend on the relative sign of $m_{\frac{1}{2}}$ and μ_0 (which is irrelevant for the discussion of symmetry breaking), the two different possibilities are denoted by $(++)$ and $(+-)$, respectively.

Fig. 2 The spectrum of the lightest supersymmetric particles, as a function of the gaugino mass $m_{\frac{1}{2}}$, for two typical cases with $\omega^2 \neq 1$ (assuming $m_t \approx 40$ GeV):

a) $m_{t'} \approx 51$ GeV, $m_{b'} \approx 103$ GeV, $m_{\tau'} \approx 33$ GeV ($\omega^2 \approx 0.5$);

b) $m_{t'} \approx 114$ GeV, $m_{b'} \approx 71$ GeV, $m_{\tau'} \approx 23$ GeV ($\omega^2 \approx 2$).

The notation is the same as in Fig. 1.

Fig. 3 The allowed region in the $(m_{\tau'}, m_{\frac{1}{2}})$ plane for the case $\omega^2 \approx 1$ (assuming $m_t = 40$ GeV). The values of the gluino mass $m_{\tilde{g}}$ corresponding to the different values of $m_{\frac{1}{2}}$ are displayed on the second vertical axis. The vertical line at $m_{\tau'} = 22$ GeV is the PETRA limit on new charged leptons. The region below the solid line is excluded by the experimental limit $m_{\tau'} \gtrsim 20$ GeV, while the region below the dotted lines can be excluded by cosmological and astrophysical considerations (the decay $\tilde{\tau}_1' \rightarrow \tau' + \tilde{\chi}^0$ is kinematically forbidden, so that $\tilde{\tau}_1'$ is stable): the two different possibilities $(++)$ and $(+-)$ for the relative sign of $m_{\frac{1}{2}}$ and μ_0 are considered. The region above the dashed lines is excluded by the theoretical limits $|\mu_0| \lesssim 1$ TeV and $|m_{\frac{1}{2}}| \lesssim 500$ GeV.

- Figure 1a -

- Figure 1b -

- Figure 1c -

- Figure 2a -

- Figure 2b -

- Figure 3 -