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Axial anomaly suppression (and axial $U(1)$ symmetry restoration) at high temperatures: a lattice Monte Carlo study

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Abstract

We calculate the topological charge density in the non-abelian $SU(2)$ vacuum as a function of temperature. We find that the topological susceptibility is strongly suppressed for temperatures above the deconfining (chiral) phase transition.

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QCD with N_f flavours is, classically, invariant under a $U_A(1) \times SU(N_f) \times SU(N_f)$ global flavour symmetry. In the real world the chiral symmetry is spontaneously broken. However instead of a full nonet ($N_f = 3$) of (near-) Goldstone bosons one observes an octet plus a heavy singlet η' . The resolution of this $U(1)$ problem involves the observation that the $U_A(1)$ classical symmetry is broken by quantum fluctuations^(1,2): the conservation of the $U_A(1)$ current $J_\mu^5 = \bar{q} \gamma_\mu \gamma_5 q$ is spoiled by an anomaly:

$$\partial_\mu J_\mu^5 = \frac{g^2 N_f}{16\pi^2} \text{tr} (\tilde{F}_{\mu\nu} F^{\mu\nu}) \quad (1)$$

and in a model where the topological charge density, $\tilde{F}\tilde{F}$, is carried by a dilute instanton gas one can explicitly show how the η' becomes heavy⁽¹⁾. Arguments based on the large number of colours limit and effective Lagrangian techniques lead to a sum rule⁽³⁾

$$m_{\eta'}^2 = \frac{4N_f}{F^2 \pi} \chi_t \quad (2)$$

relating the pseudoscalar masses to the topological susceptibility

$$\chi_t = \frac{1}{\text{vol. space-time}} \langle (\int \tilde{F}\tilde{F} d^4x)^2 \rangle \quad (3)$$

in the pure gauge vacuum (In eqn. (3), $\int \tilde{F}\tilde{F}$ is normalised to be one for one instanton). Putting in experimental values leads to the expectation⁽³⁾

$$\chi_t = 150 - 200 \text{ MeV} \quad (4)$$

All the above is at zero temperature. What happens at non-zero temperature is less certain. The physics implications of some scenarios have been explored in ref (4), where it was emphasised that one interesting question concerns the relation between the temperature at which the chiral symmetry is restored, and the temperature at which χ_t begins to be suppressed. (At high enough temperatures analytic calculations suggest such a suppression - at least when the topological charge is carried by instantons⁽⁵⁾).

The purpose of this note is to present the first results of a lattice Monte Carlo calculation of the temperature dependence of χ_t in the pure $SU(2)$

gauge theory, and to provide evidence that there is indeed a strong high temperature suppression of χ_t and that this suppression begins very close to the deconfining transition.

To calculate χ_t we generate typical vacuum configurations by the lattice⁽⁶⁾ Monte Carlo⁽⁷⁾ technique (using the standard Wilson action⁽⁶⁾), measure the total topological charge, and then use eqn. (3) to extract χ_t . Since the lattice, strictly speaking, renders the topology of the gauge fields trivial, and it is only in the continuum limit—where the dynamics makes the variation of the fields between neighbouring sites as smooth as we like — that we recover topology in the continuum sense, equivalent (in the continuum) ways of measuring $F \cdot \tilde{F}$ may yield different results at finite lattice spacing. Recently we proposed⁽⁸⁾ a method for calculating the local topological charge density which overcomes the known difficulties of constructing such measures, and we used it to gain information about the role of instantons in the SU(2) vacuum⁽⁹⁾. The most naive lattice operator that has $F \cdot \tilde{F}$ for its continuum limit is⁽¹⁰⁾

$$Q(n) = - \frac{1}{2^4 \cdot 32\pi^2} \sum_{(\mu\nu\rho\sigma)=\pm 1}^{\pm 4} \tilde{\epsilon}_{\mu\nu\rho\sigma} \text{tr} \{U_{\mu\nu}(n) U_{\rho\sigma}(n)\} \quad (5)$$

(using the notation of ref (11)) where $U_{\mu\nu}(n)$ is the plaquette in the $(\mu\nu)$ plane and $\tilde{\epsilon}$ is an extension of the usual antisymmetric tensor. An immediate problem with this composite operator is that it receives ultraviolet perturbative contributions in addition to those from the non-perturbative topological objects of interest. Indeed, attempts to extract the latter came up with very small susceptibilities^(11,12) and the distribution of $Q(n)$ seems to have no integer valued contributions⁽¹²⁾. The algorithm we have constructed^(8,9) is specifically designed to remove these offending and uninteresting high frequency fluctuations. To achieve this end we sweep through the gauge field configuration of interest, systematically minimising the local action. This can be done with a Metropolis updating routine⁽⁷⁾ where one rejects changes that increase the action. In the special case of SU(2) one has a faster alternative procedure where one always resets each link variable to equal the inverse of the (normalised) matrix that multiplies it in the action. After an adequate number of such action minimisation sweeps we use the definition, eqn (5), to measure the topological charge on the final "frozen" gauge field configuration. We⁽¹³⁾ have used this method for calculating χ_t at zero

temperature, with an emphasis on uncovering the systematic errors that need to be brought under control. Note that extended topological objects are stable under this action minimisation procedure.

Before moving to our results we remark that numerically minimising the action to reveal extended topological structures has been used in the context of the $O(3)$ σ model⁽¹⁴⁾ and is being used to measure the $SU(3)$ susceptibility⁽¹⁵⁾. From the technical point of view, $\tilde{FF}$ converges to its frozen value about 10 times as fast as the action itself⁽¹³⁾ (which is the method previously used).

We begin with our results for the susceptibility on a 6^4 lattice since this enables us to compare with recent "geometric" calculations⁽¹⁶⁻¹⁹⁾ of χ_t that measure the winding around hypercubes containing gauge singularities and assume that this net winding offsets (on a periodic lattice) that of the interesting extended topological charge carriers.

We plot in fig 1 our results, together with those of ref (17) and ref (18), taken on a 6^4 lattice for β ($\equiv 4/g^2$) values from 2.2 to 2.5. We also show the g^2 dependence expected deep in the continuum limit (asymptotic scaling). We observe that the results of ref (17) and ref (18) conform remarkably well to this test of continuum physics: however since $(6a)^{-1}$ is deconfined for $\beta = 2.5$, we also observe that these data show no temperature dependence. In contrast our values show a rapid suppression at larger β , indicating increasing finite volume effects at larger β . Although we do not possess a definite understanding of the cause of this difference, we remark that in the $O(3)$ σ model⁽¹⁴⁾ the geometric measurements are dominated by unphysical ultraviolet "dislocations" which do not contribute in our measure.

Is this in fact a finite volume effect or is it just that asymptotic scaling is being violated?

To address this question we first convert our measured values of $a^4 \chi_t$ into the physical mass ratio $\chi_t/\Lambda_{\text{mom}}^4$ by assuming that the lattice spacing, $a(\beta)$, is small enough for the 2 loop asymptotic freedom relation

$$a(\beta) = \frac{57.5}{\Lambda_{\text{mom}}} e^{-\frac{3\pi^2}{11}\beta} \left(\frac{6\pi^2}{11}\beta \right)^{\frac{51}{121}} \quad (6)$$

to be valid. Then our test of continuum physics for $\beta > \beta_0$, say, is

$$\chi_t / \Lambda_{\text{mom}}^4 = \text{ind of } \beta ; \beta > \beta_0 \quad (7)$$

(This is just what one means by asymptotic scaling). We now manipulate our measurements of χ_t on 6^4 , 8^4 and 10^4 lattices in this way and present the results in fig 2. We see that the suppression of χ_t moves to larger β as the lattice size is increased. This indeed indicates that we have a finite physical size effect.

An L^4 lattice does not possess a well-defined temperature. To approximate a thermodynamic system we need an $L_s^3 L_t$ lattice with $L_s \gg L_t$: the temperature is then

$$T = \frac{1}{L_t \cdot a} \quad (8)$$

So we have performed measurements on a 10^{34} lattice in a β range which includes the deconfining transition. In fig 3 we show our measurements of $a^4 \cdot \chi_t$ on this lattice together with the values obtained on a much "cooler" 8^4 lattice, which has a volume nearly equal to that of the 10^{34} lattice. We see that for $\beta > 2.3$ the susceptibility on the 10^{34} lattice is severely suppressed with respect to that on the 8^4 lattice, confirming that this is a high temperature effect.

In fig 4 we present our results on the 10^{34} lattice in the form of the physical mass ratio $\chi_t / \Lambda_{\text{mom}}^4$. We also show the thermal loop average from our data and elsewhere⁽²⁰⁾, and the β value corresponding to the deconfining temperature, $T_c^{(20)}$. The topological susceptibility begins to be strongly suppressed near T_c . There is some evidence of a peak at T_c (see also fig 2) with the suppression occurring for temperatures higher than T_c . This would suggest that m_η increases with temperature to a maximum at $T = T_c$. This interesting possibility needs further more accurate work.

Concluding remarks. We have shown that the topological susceptibility in the SU(2) vacuum is strongly suppressed for temperatures significantly above the deconfining transition. There is a hint of a peak in χ_t , and

hence $m_{\eta'}^2$, at T_c . The suppression of the vacuum quantum fluctuations of $\int \tilde{F} F d^4x$ at high temperatures implies the effective restoration of the axial $U(1)$ symmetry at such temperatures. (Note that with fermion loops one expects the light quarks above the chiral transition to suppress topological effects). Our results appear to exclude the interesting possibility⁽⁴⁾ of anomaly suppression before the chiral symmetry restoring temperature, and the resulting appearance of a nonet of (near-) Goldstone bosons. The fact that the topological susceptibility of the pure gauge theory undergoes a transition at a temperature close to the deconfining/chiral symmetry restoring temperature was not expected and emphasizes the interest of the dynamics behind these phenomena.

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References

- 1) G. t'Hooft, Phys. Rev. Lett. 37, 8 (1976); Phys. Rev. D14, 3432 (1976).
- 2) S. Coleman: Uses of Instantons. 1977 Erice Lectures.
- 3) E. Witten: Nucl. Phys. B156, 269 (1979)
G. Veneziano: Nucl. Phys. B159, 213 (1979).
P. Di Vecchia: 1980 Schlading Lectures.
- 4) R.D. Pisarski, F. Wilczek: Phys. Rev. D29, 338 (1984).
- 5) D. Gross, R. Pisarski, L Yaffe: Rev. Mod. Phys. 53, 43 (1981).
- 6) K.G. Wilson: Phys. Rev. D10, 2445 (1974).
- 7) M. Creutz: Phys. Rev. Lett. 43, 553 (1979).
K.G. Wilson: Cargese Lectures (1979).
M. Creutz, L. Jacobs, C. Rebbi: Phys. Rev. D20, 1915 (1979).
- 8) M. Teper: CERN-TH.4215/85 and Proceedings of the LBL Conference on "Quark Confinement" May, 1985.
- 9) M. Teper: CERN-TH.4208/85 (Phys. Lett. B in press).
- 10) M. Peskin: 1978 Thesis.
- 11) P. Di Vecchia, K. Fabricius, G. Rossi, G. Veneziano: Nucl. Phys. B192, 392 (1981).
- 12) K. Ishikawa, G. Schierholz, H. Schneider, M. Teper: Phys. Lett. 128B, 309 (1983).
- 13) M. Teper: Preprint RAL-85-089.
- 14) B. Berg: Phys. Lett. 104B, 475 (1981).
- 15) J. Hoek: RAL-85-042 and in preparation.
- 16) P. Woit: Phys. Rev. Lett. 51, 638 (1982).
- 17) I. Fox, J. Gilchrist, M. Laursen, G. Schierholz: Phys. Rev. Lett. 54, 749 (1985).
- 18) Y. Arian, P. Woit: ITP-SB-85-60.
- 19) G. Lasher, A. Phillips, D. Stone: Proceedings of the LBL Conference "Quark Confinement", May 1985.
- 20) R. Gai, H. Satz: Phys. Lett. 145, 248 (1984).

Figure Captions

- Fig 1 The topological susceptibility, χ_t , versus β on 6^4 lattices: our results ($\bullet$) and those based on geometric measures of the winding $(0,\Delta)$.
- Fig 2 Our values of $\chi_t/\Lambda_{\text{mom}}^4$ for several lattice sizes, versus β .
- Fig 3 χ_t versus β for 8^4 and $10^3 4$ lattices.
- Fig 4 $\chi_t/\Lambda_{\text{mom}}^4$ on a $10^3 4$ lattice for various β ; also shown is the thermal loop average and the location of the deconfining temperature.

Fig 1

Fig 2