Report

Design Study: ELENA bending magnet prototype

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Abstract
The ELENA bending magnet prototype shall prove that the proposed design meets the requirements set by the ELENA beam dynamics. The following points will be discussed in detail: (i) production process of a magnetic yoke diluted with stainless steel plates, (ii) the stability and repeatability of the field homogeneity of such a yoke over the full working range, (iii) choice of soft magnetic steel, (iv) hysteresis effects, (v) mechanical deformations, (vi) thermal insulation to intercept heat load from baking for activation of NEG coating in the vacuum chamber, (vii) end shim design. In order to verify these points the following measurements will be performed: (i) Hall probe scanning, (ii) integrated field homogeneity measurement (DC), (iii) integrated field homogeneity measurement (AC).
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1. Introduction

ELENA (Extra Low ENergy Antiproton Ring) is a compact ring for cooling and further deceleration of 5.3 MeV antiprotons delivered by the CERN Antiproton Decelerator (AD) to 100 keV. The AD physics program is focused on trapping antiprotons in Penning traps where antihydrogen is formed after recombination with positrons. The ultimate physics goal is to perform spectroscopy on antihydrogen atoms at rest and to investigate the effect of the gravitational force on matter and antimatter. Figure 1 presents a layout of the ELENA ring and shows the ELENA ring bending magnet. For the project a total of eight ELENA ring bending magnets will be required, six magnets in the ELENA ring, one magnet for the so-called B-train control system, and one spare magnet. Further information on the project can be found in [1].

![Figure 1 - ELENA ring layout. In grey-blue the six ELENA ring bending magnets (BM) are shown.](image1)

The baseline design of the ELENA ring bending magnets is shown in Figure 2. In the prototype phase a simplified, straight magnet will be built in order to answer all open questions listed in the next section. The design of this magnet is described in detail in this report. The preliminary parameters of the ELENA ring bending magnets as assumed for this prototype are summarized in Table 1.

![Figure 2 - Preliminary design of ELENA ring bending magnet. Left: 3D-model of a tentative design of the ELENA magnet. The magnetic yoke and the coils (red coloured) are shown. Right: Sketch of ELENA bending magnet.](image2)

<table>
<thead>
<tr>
<th>Table 1 – Main parameters of ELENA bending magnet prototype</th>
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<tbody>
<tr>
<td>Units to be produced</td>
</tr>
<tr>
<td>Coil cooling</td>
</tr>
<tr>
<td>Ratio Magnetic Steel: Stainless Steel</td>
</tr>
<tr>
<td>Minimum field in aperture</td>
</tr>
<tr>
<td>Maximum field in aperture</td>
</tr>
</tbody>
</table>
Magnetic length | 666 mm
Overall magnet length | 700 mm
Ramp rate | 0.4 T/s
Available field stabilisation time | 500 ms
Horizontal good field region | 66 mm
Vertical good field region | 48 mm
Field homogeneity in the GFR in the centre of the magnet | $\pm 8 \cdot 10^{-4}$ $1-B/B_{ref}$
Integrated field homogeneity in the GFR | $\pm 2 \cdot 10^{-4}$ $1-|B_{d1}|-B_{d1}$
Maximum gap height | 76 mm
Minimum gap height | 75.2 mm
Pole width | 260 mm
Maximum temperature of iron yoke *) | 50 °C

*) Assuming DC operation at maximum current or bake-out of vacuum chamber

2. Scope and purpose of the prototype

The following open questions and manufacturing methods shall be studied with the help of the prototype:

a. To increase the magnetic induction in the iron the magnetic yoke will consist of stainless steel plates and magnetic steel. By doing so, the magnetic flux density will be concentrated in the magnetic steel ($\mu_r > 2000$), as the electrical steel has a much higher permeability than the stainless steel ($\mu_r < 2$). A thickness ratio of around 1:2 between iron and stainless steel is proposed, which will increase the magnetic induction in the magnetic steel by a factor of approximately three and subsequently shift the working range of the iron from the range of 0.05 T - 0.42 T to 0.15 T - 1.26 T. By doing so, the average relative permeability can be maximized, and particularly the lowest permeability part of the magnetization curve will be avoided.

b. The field homogeneity of the diluted, laminated yoke composed of stainless steel and magnetic steel has to be investigated. Simulations presented in this report have shown that the lamination is a far-field effect with no influence on the magnetic field homogeneity. Also the effect of electrical steel sheets with varying thickness (nominal thickness of the electrical steel varies by up to $\pm 8\%$ according to EN 10106:2007 for a thickness of 0.5 mm and in a direction perpendicular to the direction of rolling shall not exceed 0.020 mm for a thicknesses of 0.5 mm); can be considered a far field effect. The prototype shall prove the results from these simulations.

c. Soft magnetic steel with very high permeability over the operation range and small hysteresis loops, i.e., with small coercive force was selected. The relevant magnetic performance (constant field homogeneity, hysteresis cycle, mechanical deformations, eddy currents) should be evaluated.

d. The regular cycle of ELENA consists of three flat tops, injection at a magnetic field of 0.37 T, cooling by using the electron cooling system at around 0.2 T, and ejection at a magnetic field of 0.05 T. The cycle has to be well repeatable. However, due to the low field in conjunction with a dynamic range of 7 and the stringent field homogeneity requirements in ELENA, the effect of hysteresis depending on the history of powering has to be investigated. As already mentioned, the magnetic measurement results with the two yokes, discussed under point c) will also be assessed according to the hysteresis effects in the magnet.

e. The mechanical deformation and its influence on the field homogeneity can be measured and compared to mechanical simulations.

f. End shims were designed in order to correct the integrated field homogeneity. End shims have to be mountable in order to be able to easily alter their shape. The design of the end shims will be tested. For the prototype small solid ARMCO end shims are proposed, for mechanical stability and improved magnetic performance.

g. For ELENA a vacuum in the order of $10^{-12}$ Torr is required. This level of vacuum can only be reached by coating the majority of the vacuum chambers with NEG coating, which is required to be baked out at several hundred degrees over several days. The insulation system will be tested at the prototype, mainly
to ensure that the vacuum chamber can be heated up. For glued yokes the temperature limit is 130°C (see visit report EDMS 1220958) to avoid ageing of the bonding varnish. However, the ELENA dipole magnets will be also welded, therefore the limit of 130°C is less critical.

The following magnetic and mechanical measurements should be performed:

a. Field homogeneity in the center of the magnet over a transversal cross-section over the full dynamic range and different cycles by using Hall probes
b. Integrated field homogeneity by using rotating coils or flux meters
c. Transient field during ramping and the time to reach the specified flat top after ramping, probably with search coils
d. Optional: Mechanical measurements (deformation of the magnet).

3. Magnet design & manufacturing methods

To answer the questions outlined before, a simplified, straight magnet can be manufactured and measured. A straight design simplifies the manufacturing process (coil and yoke) largely.

3.1 General design considerations

The following different designs were analysed:

<table>
<thead>
<tr>
<th>Design</th>
<th>Advantages</th>
<th>Disadvantages</th>
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<tbody>
<tr>
<td>C-Shape</td>
<td>• Easily accessible</td>
<td>• Asymmetric</td>
</tr>
<tr>
<td></td>
<td>• Manufacturing: one part</td>
<td>• Field homogeneity more difficult to achieve at small fields</td>
</tr>
<tr>
<td></td>
<td>• Asymmetric</td>
<td>• Mechanical stiffness</td>
</tr>
<tr>
<td></td>
<td>• Field homogeneity more difficult to achieve at small fields</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Mechanical stiffness</td>
<td></td>
</tr>
<tr>
<td>H-Shape</td>
<td>• Symmetry (only broken by edge angles)</td>
<td>• No easy access</td>
</tr>
<tr>
<td></td>
<td>• Mechanical stiffness</td>
<td>• Hole has to be drilled for vacuum pump access</td>
</tr>
<tr>
<td></td>
<td>• Field homogeneity easier achievable</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Manufacturing: two parts</td>
<td></td>
</tr>
<tr>
<td>Window Frame</td>
<td>• With wider aperture and shims very good field homogeneity achievable</td>
<td>• Complicated bedstead coils or double number of Ampere turns with potentially high stray fields.</td>
</tr>
<tr>
<td></td>
<td>• No access for vacuum pump</td>
<td>• No access for vacuum pump</td>
</tr>
<tr>
<td></td>
<td>• Field homogeneity difficult to achieve</td>
<td></td>
</tr>
<tr>
<td>Coil dominated magnets</td>
<td>• Coil-dominated dipole magnets would have no non-linearities, i.e., no hysteresis effects</td>
<td>• Air cooled cos $\theta$ coil with a maximum engineering current density of 2 A/mm$^2$ would require a maximum thickness of around 0.3 m</td>
</tr>
<tr>
<td></td>
<td>• No access for vacuum pump</td>
<td>• Field homogeneity difficult to achieve</td>
</tr>
</tbody>
</table>

From the arguments listed all magnet types but the C-shape magnet types were excluded. C-magnet types allow for easy access to the vacuum chamber.
3.2 Material selection

The ELENA bending magnet prototype is a normal-conducting, iron-dominated electro-magnet. For the prototype different magnetic yoke materials (steel grades and steel thickness) were investigated. The advantages and disadvantages were gathered and harmonized with the technical and commercial requirements of the ELENA project.

3.2.1 Non-magnetic steel

Austenitic steel was selected as non-magnetic filler material for the dilution of the magnetic yoke because it shows superior mechanical properties and relatively low electrical conductivity compared to other non-magnetic materials which are easily commercially available in small sized sheets (1000 x 2000 mm$^2$) such as plastic or aluminium. Three main requirements of the steel were identified: (i) Small relative permeability, (ii) easy to weld, i.e., low carbon austenitic steel is preferred, (iii) low cost. The last point was identified as especially important because high-alloy materials (for example high Manganese steels, or the CERN standard for cryogenic applications: 316 LN, etc.) may cost up to 50 times more than electrical steel and would become the main cost factor of the magnet manufacturing for ELENA. The low-alloy steels (for example 304 L) have a price very similar to aluminium sheets (around 2-4 CHF/kg) but hold the risk of martensitic transformations [2] which may result in a relative magnetic permeability in the range of 1 to 2. This permeability range is acceptable. The choice of a low-alloy stainless steel (304 L) will be checked with this prototype. The steel for ELENA preferably should:

- have a composition which avoids the formation of any $\delta$-ferrite [3]. The stability of austenite vs. $\delta$-ferrite precipitation can be quantified based on the composition (Ni, C, N, Cr, Mn and Si). 1% of $\delta$-ferrite increases the relative permeability by around 0.08.
- be alloyed enough to avoid $\alpha'$-martensitic formation during cold-work (for example stamping).

In the meantime discussions and negotiations with steel manufacturers are on-going to find a better suited grade for the same cost (see travel reports EDMS 1240830, 1240832).

3.2.2 Semi-finished electrical steel

Semi-finished electrical steel has the advantage that the annealing is performed after finishing the cold work, so the influence of the processing is erased [4]. However, the Backlack varnish for gluing the yoke has to be applied either after the final annealing on the electrical steel or on the austenitic steel, which are both non-standard procedures and would require a manufacturer to apply the coating on the steel.

The influence of cold work on magnetic properties was studied in detail by Schoppa [4]. He found that for small fields (the most critical region for ELENA) the influence of cold work can be neglected. The influence of laser cutting on the magnetic properties depends on the cutting speed, the faster the cutting is performed the smaller the influence on the magnetic properties but also the mechanical tolerances are larger for faster cutting speeds. However, the influence of laser cut steel is more pronounced as with other production methods.

Figure 3 shows the measurement results performed by [4]. The thickness of the steel band (standard thickness: 0.35 mm, 0.5 mm, 0.65 mm, 1.0 mm) has no influence on the magnetic properties for the same manufacturing process of the laminations. One has to note that for laser cutting the cutting speed will be lower for thicker steel bands, and therefore, the magnetic properties are likely to be altered negatively. The influence for the prototype (cut with laser cutting) is larger as for the series magnet (produced by stamping).
Figures 3 - AC measurements of magnetic properties (magnetic polarization versus magnetic field) for a magnetic steel with high silicon content, thickness 0.5 mm measured with single stripes. Left: Influence of cutting on magnetic materials. The single strip with an initial width of 30 mm was cut into widths of 15 mm, 10 mm, 7.5 mm, 6 mm and 5 mm. Right: Influence of used cutting tool on magnetic properties [4].

Moreover, the influence of stamping is believed to be homogenously distributed over the whole pole, which means, that the field is homogenously disturbed with no negative influence on the relative field homogeneity. Therefore, the use of semi-finished material was not pursued further.

### 3.2.3 Fully-finished electrical steel

Fully-finished electrical steel was intensively investigated. To evaluate the DC magnetic characteristics of standard electrical steel DC Epstein frame measurements of the electrical steels listed below were performed in rolling direction, perpendicular to the rolling direction and mixed. All steels are available with a bonding varnish (DuPont Backlack Voltatex E 1175) in B-B state, i.e., both sides can be glued to non-coated material such as stainless steel. The gluing of electrical steel (M800-50 A) coated with Rembrandtin Backlack to standard stainless steel 1.4301 was tested with the roller peel test (according to EN 1464); a bonding force of 4.6 N/mm was measured.

The following electrical steel grades were considered:
- NO30
- M270-50A
- M330-50A
- M330-50A HP
- M400-65A
- M530-65A
- M600-65A
- M700-65A
- M800-50A
- M800-65A

The measurements were analysed according to the following criteria and will serve as a library for accurate FEM calculations:

- High permeability in the working area to minimize the influence of the permeability’s variation over the operation area on the total reluctance of the magnet.
- Alternatively a rather constant permeability over the working area would be desirable, which could be even very small, i.e. below 1000.
- Low coercive force to minimize the effect of residual fields.

High-silicon content electrical steel (low-loss electrical steel, for example M270 or M330) has a lower coercive force, higher permeability and higher electrical resistivity but lower saturation induction than electrical steels with low-silicon content, although they are highly anisotropic (up to 2.5 times higher permeability in rolling direction). The properties of high-silicon electrical steel match the main ELENA
magnets requirements. The electrical steels containing high amounts of silicon (around 3%-3.5%) are only available in small thickness (0.65 mm and smaller).

As an example Figure 4 shows for some different grades the relative permeability versus polarization. These measurements were performed by using a single sheet AC measurement device and performing the measurement at 50 Hz. DC measurements show even larger permeability values than AC measurements because of decreased strength of eddy currents. Figure 4 shows, that a grade with lower losses, meaning higher silicon content, has larger permeability in the required working range of the ELENA dipole. To further optimize for larger permeability the best working area can be selected by adjusting the dilution factor between magnetic steel and austenitic steel (non-magnetic) by maximizing the average relative permeability over the working range.

According to the flux conversation law (neglecting stray fields):

\[ \Phi_{\text{gap}} = \Phi_{\text{yoke}}, \]
\[ B_{\text{gap}} \cdot A_{\text{gap}} = B_{\text{yoke}} \cdot A_{\text{yoke}}. \]

To maximize the relative permeability (see Figure 5) over the whole working range of the material, the magnetic flux density in the yoke \( B_{\text{yoke}} \) has to be preferably within the range of constant high-permeability. If we assume that the field in the gap is constant, the effective surface area of the yoke \( A_{\text{yoke}} \) has to be decreased. This decrease of the effective surface area \( A_{\text{yoke}} \) can be achieved by spacing the laminations in the yoke with low-permeability material. Then the flux will be concentrated in the ferromagnetic material, and the effective surface of the yoke is smaller.

The mean value of the relative permeability over the working range is:

\[ \bar{\mu}_r = \frac{1}{J_{\text{max}} - J_{\text{min}}} \int_{J_{\text{min}}}^{J_{\text{max}}} \mu_r(J) \, dJ \]

For an undiluted yoke the polarization in the electrical steel will be \( J_{\text{min}} = B_{\text{min}} - \mu_0 H = 0.05 \, \text{T} \) and \( J_{\text{max}} = 0.36 \, \text{T} \). The maximum polarization can be expressed in terms of the minimum polarization and the dynamic factor as \( J_{\text{max}} = \frac{0.36}{0.05} \cdot J_{\text{min}} = 7.2 \cdot J_{\text{min}} \). Therefore, we can write for the calculation of the mean value of the permeability over the working range:

\[ \bar{\mu}_r = \frac{1}{6.2 \cdot J_{\text{min}}} \int_{J_{\text{min}}}^{7.2 J_{\text{min}}} \mu_r(J) \, dJ \]

Searching for the maximum of the relative mean permeability one finds \( J_{\text{min}} \approx 0.15 \, \text{T} \). Therefore, the value of the field in the yoke has to be increased by a factor of \( \frac{0.15}{0.05} \approx 3 \), which yields a dilution factor 1:2.

Figure 5 shows the DC measurement of the electrical steel M270-50 A HP in rolling direction, perpendicular to rolling direction and mixed. The blue box indicates the working area without dilution and the red box the working area with dilution. The effect of the dilution on the flux lines in the yoke and gap is illustrated in Figure 6.

Another strategy would be to find a magnetic material with a relative permeability as constant as possible over the operation area. Such a material cannot be easily found and procured.
3.2.4 Grain-oriented electrical steel

Grain-oriented electrical steel (potentially by modifying the texture with laser scribing) shows highly anisotropic properties with highly favourable properties in the direction of grain orientation. The coercive force $H_c$ is low, the magnetic saturation induction and permeability are high. However, the chosen steel M270-50 A HP already shows similar favourable properties. Therefore, grain-oriented electrical steel was not considered.
3.2.5 Amorphous metal

Amorphous metal is an alloy with a non-crystalline structure produced by ultra-rapid quenching (about 1 million °C per second) of molten alloy. Because amorphous metal has isotropic properties, which originate from a crystalline structure, and there are no crystalline grain boundaries to prevent motion of magnetic domain walls, it shows high permeability (μ > 35000) and low loss (low expected Hc), while having still acceptable saturation magnetic flux density (B = 1.41 T). The only manufacturer known by the author is Hitachi [5]. The produced sheets are not suitable for stamping and have weak mechanical properties (the material is “glass-like”), also the maximum widths available are only around 5 cm with a thickness of 0.02 mm, which is not practical for accelerator magnets. Therefore, this product was not chosen.

3.2.6 Very high-silicon electrical steel

High-silicon electrical steel (6.5% silicon content) is commercially available from JFE steel corporation [6]. It is a hard, brittle and difficult to punch material. The thickness available are 0.1 mm and 0.2 mm, which makes precise staking required in accelerator magnets challenging. Also the sheet width is limited with 600 mm, which would set a difficult to achieve limitation on the bending magnet. However, the magnetic properties are excellent: high permeability, low magnetostriction and low coercive force. The material was due to the mentioned limitation excluded despite its very good magnetic properties.

3.2.7 NiFe steels

NiFe steels with low coercive force can be used as a reference material for low-field magnets, because of this extremely low coercive force. In [7] it was shown that for a standard cycle the field homogeneity and repeatability was not significantly improved compared to the standard electrical steel by using NiFe alloys. Moreover, NiFe has a prohibitive high cost for the series production of ELENA magnets and therefore, it was not further considered.

3.2.8 Soft magnetic composite materials

Soft magnetic composite materials have the advantage that almost no eddy currents can be generated [8]. Therefore, this material is of special interest for the end shims of a magnet, where eddy currents are especially generated. Transient simulations for investigating the use as end shim material have not shown major improvements in the design (see Appendix 0). Therefore, this material will not be used.

3.3 Determination of lamination thickness

In the following paragraphs the decision on the lamination thickness is briefly discussed. The choice of the lamination thickness is usually determined by:

- The ramp rate of the magnet. Thinner laminations reduce the amount of induced eddy currents and allow for higher ramp rates for a given magnet that means the delay of the magnetic field following the current is reduced by thinner laminations.
- Eddy currents may damp the higher harmonics of the power converter (colloquially called “ripples”). For the ELENA dipole magnets the higher harmonics are multiplies of 6.5 kHz, i.e., thicker laminations increase the damping capabilities of the magnet and therefore, reduce the visibility of the power converter’s ripples in the magnetic field.
- Tolerances of stacking are easier to meet when the single laminations are thicker.
- Economic considerations. Using thinner laminations tend to be more expensive because the number of required laminations for a given magnet length increases and machining and handling time is lengthened and more inter-laminar insulation is required.

The magnetic field should follow the low-frequency current (typical frequencies well below 10 Hz) but not the higher harmonics ≥6.5 kHz from the switch-mode power converter. According to Faraday’s law eddy currents will be induced which will oppose the field excited from the main current. Our design goal is that the low-frequencies are not damped but the high frequencies are completely damped and are not visible in the magnetic field. The transfer function f(I) = B of the magnet should be a low-pass filter. We will investigate in the following paragraphs if this design goal can be achieved by choosing an appropriate thickness of the electrical steel laminations.
We start studying the low frequency damping. To estimate the largest time constant of decay of eddy currents after a sudden change in the current in a DC magnet we start from the field diffusion equation:

$$\nabla^2 H = \sigma \mu_0 \frac{\partial H}{\partial t} = \frac{1}{\kappa} \frac{\partial H}{\partial t},$$

where the diffusion coefficient is defined as $\kappa = \frac{1}{\sigma \mu}$ with $\sigma$ being the conductivity. In [9] an updated diffusion coefficient is derived which can be used with all standard solutions of the diffusion equation. Starting from the magnetic diffusion equation, they found a special differential equation for the C-dipole:

$$\nabla^2 H = \sigma_0 \frac{l}{g + \frac{l}{\mu_r}} \frac{\partial H}{\partial t} = \frac{1}{\kappa_1} \frac{\partial H}{\partial t},$$

where $l$ is the average magnetic path length and $g$ the gap height. For the following calculation we limit ourselves to $H = H_y$ to simplify the treatment. The time constant for this differential equation is a standard solution [10]:

$$\tau_n = \frac{4 \pi^2}{\kappa n^2 \sigma a^2},$$

where $2a$ is the lamination thickness. For a C-shaped dipole we use the diffusion coefficient $\kappa_1$, the parameter $n$ has to be odd, and for the smallest odd number ($n = 1$) we find the largest time constant:

$$\tau_1 = \frac{4 \pi^2 \sigma_0}{\kappa n^2 \sigma a^2} \frac{l}{g + \frac{l}{\mu_r}}.$$  

Most references on the topic refer to Reistad (1968) [11], who finds by neglecting the existence of a gap in a magnet:

$$\tau_1 = \frac{\mu_r \sigma a^2}{\pi^2},$$

Reistad (1968) [11] finds the same time constant as proposed above, with the exception that he defines the effective thickness of the lamination as half the lamination thickness. Therefore, he finds a four times smaller time constant. He assumes that in one slab “two sets” of eddy currents are flowing which are influenced from the corresponding outer fields. This assumption can be imagined to be correct if the slab thickness is thick compared to the skin depth. He searched for eddy currents in a DC magnet with 60 mm thick laminations after a sudden change of current. With such thick laminations for a DC magnet the eddy currents induced by the relatively high frequency magnetic field cannot penetrate fully into the material. However, if the skin depth is large compared to the lamination thickness the effective thickness is the whole lamination thickness, and then the factor of 4 introduced above is justified. In accelerator magnets the applied frequencies are usually small (for example 1 Hz or smaller for ELENA), so we find for the penetration depth:

$$\delta = \frac{1}{\sqrt{\pi f \mu_r \mu_0 \sigma}} = \frac{1}{\sqrt{\pi \cdot 1 \cdot 3000 \cdot 4 \cdot \pi \cdot 10^{-7} \cdot 1.9 \cdot 10^6}} \text{m} = 6.6 \text{ mm}$$

A typical lamination thickness is 0.5-1 mm thick, that means the total lamination will be used to build up eddy currents. Therefore, we propose to use the larger time constant, in any case it is a conservative assumption [9].

We calculate the longest time constant of eddy currents in the ELENA magnets with considering the air gap:

$$\tau_1 = \frac{4 \pi^2 \sigma_0}{\kappa n^2 \sigma a^2} \frac{l}{g + \frac{l}{\mu_r}} = \frac{4 \cdot (0.25 \cdot 10^{-3})^2}{\pi^2} \cdot \frac{1.9 \cdot 10^6 \cdot 4 \cdot \pi \cdot 10^{-7} \cdot 2}{0.1 + \frac{2}{2000}} = 1.2 \mu\text{s}$$

If the air gap is neglected the reluctance of the slab is determined only by the iron magnetic circuit and one finds:
\[
\tau_1 = 4 \frac{\mu_0 \mu_0 \sigma a^2}{\pi^2} = 4 \frac{2000 \cdot 4 \pi \cdot 10^{-7} \cdot 1.9 \cdot 10^6 \cdot (0.25 \cdot 10^{-3})^2}{\pi^2} = 0.12 \text{ ms}
\]

To reach \(10^{-5}\) precession one has to wait approximately \(\ln 10^{-5} = 12\) times the time constant. Even taking 0.12 ms \(\times 12 = 1.44\) ms, leaves a safety factor of almost 350, if 0.5 s is the allowed maximum time to reach \(10^{-5}\) precession after ramping.

Usually the excitation in accelerator magnets is not sinusoidal but they are linearly ramped. In [10] they find a time lag between the power converter ramp and the magnetic field ramp of

\[
\tau_d(x) = \frac{x^2}{2\kappa}
\]

by using \(x = a\) one finds a similar result as for sinusoidal excitation, the corresponding factor for sinusoidal excitation is \(4/\pi^2 \approx 0.4\) and \(\frac{1}{2} = 0.5\) for a ramp.

We continue now with calculating the response of the magnet for high frequencies. The transfer function of the magnet excited with a sinusoidal current \(I\) with frequency \(f\) can be derived from the diffusion equation [11]

\[
B_y(\frac{a}{\delta}) = \frac{\mu_0 N I}{g + \frac{1}{\mu_r \rho (\frac{a}{\delta})}}
\]

\((B_y):\) dipolar field in gap, \(NI:\) Ampere turns of dipole, \(g: \) gap, \(l: \) length of magnet iron circuit, \(F(\frac{a}{\delta}) = \tan h \frac{a}{\delta} (1 + jf)\), \(a: \) half the lamination thickness, \(j: \) imaginary unit, \(\delta = \frac{1}{\omega \mu_0 \mu_0 \sigma}, \) \(\omega: \) angular frequency \((\omega = 2\pi f), 1/\rho = \sigma\) conductivity of the iron.

The complex part has a maximum at around \(a/\delta \approx 1\) (see Figure 7), this is sometimes called cut-off frequency by magnet designers, because for frequencies \(a/\delta \geq 1\) not the full material can be used to develop eddy currents due to the skin effect. This frequency can be calculated from the skin depth equation

\[
f = \frac{1}{\pi a^2 \mu_0 \mu_0 \sigma} = \frac{1}{\pi (0.25 \cdot 10^{-3})^2 \cdot 2000 \cdot 4 \pi \cdot 10^{-7} \cdot 1.6 \cdot 10^6} \text{ Hz} = 1267 \text{ Hz.}
\]

Figure 7 - Real and imaginary part of the lag function \(F(a/\delta)\).

The switching mode frequency of the power converter for the ELENA dipole magnets is 6.5 kHz. Therefore, frequencies with integer multiples of 6.5 kHz will be present. Using Equation (1) with the above presented material parameters results only in a damping of 52 mdB and a phase shift of 0.65 deg for a lamination thickness of \(2a = 0.5\) mm. Therefore, higher harmonics of the power converter (multiples of 6.5 kHz) are not sufficiently damped by the magnet yoke and they have to be filtered with appropriate filters installed in the power converter; if the ripples are despite the damping due to the impedance of the magnet circuit larger than
desired. The exemplary calculation has shown that analysing just the skin depth yields to an underestimation of the cut-off frequency.

We have reviewed two equations one which takes a slab with gap into account and another one which neglects the existence of a gap, which is widely used in the community and gives conservative estimations for infinite long magnets. Both equations can be used for an estimation of the approximate range of the eddy currents in the centre of the magnet. If problems during operation are expected due to too large time constants, one should perform more precise 3D FEM-calculation of the transient effects to take the dominant end effects into account. The transient effects depend strongly on the end design, which is not taken into account in the simple expressions reviewed above. Saturation effects are not expected because the end shims are fabricated by using solid ARMCO iron, which will not saturate due to the low magnetic field of around 0.4 T.

From these considerations the lamination thickness can be selected arbitrary and was chosen with 0.5 mm due to material availability and reduction of eddy currents in the magnet. Ripples will not sufficiently damped by eddy currents induced in the magnet.

### 3.4 Magnetostriction

Magnetostriction causes ferromagnetic materials to change their shape or dimensions during the process of magnetization or to change their magnetization curve depending on the applied stress. The effect is well known from the core noise in electrical power transformers, reducing these noises is an active field of research.

In the LEP dipoles a major decrease of the relative permeability due to internally induced compressive stress in the order of 30 MPa was observed caused by hydration of the mortar used as spacer material, which had to be released to maintain the required field homogeneity at low field [12]. In the ELENA prototype dipole we expect a length contraction due to magnetostriction in the electrical steel. It can be estimated for M270-50 A electrical steel at magnetic saturation induction that the total length change of the magnet is [13]:

\[
\Delta l = \lambda_{\perp} l_0 = -4 \cdot 10^{-6} \cdot \frac{1}{3} \cdot 600 \text{ mm} = -0.8 \mu\text{m},
\]

Stainless steel will not change its shape when exposed to a magnetic field; due to the different contraction factors stress will be induced. Assuming that the stainless steel has much higher rigidity because of its superior mechanical properties and the double thickness compared to the electrical steel this contraction will be translated into stress in the electrical steel. We assume the following maximum stress in a lamination due to the effect described above:

\[
|\sigma| = |\varepsilon|E = \frac{\Delta l}{l_0} E = \max(|\lambda_{\parallel}|, |\lambda_{\perp}|) E = 4 \cdot 10^{-6} \cdot 210 \text{ GPa} = 0.84 \text{ MPa},
\]

where \(\max(|\lambda_{\parallel}|, |\lambda_{\perp}|)\) is the maximum magnetostriction factor and \(E\) is the Young’s modulus of the M270-50 A electrical steel.

Stress will also be induced from the stress induced by the deformation due to the magnetic forces. The stress level will be calculated by using the Maxwell Stress Tensor in Section 5. Comparing the stress levels with the results presented in Figure 8 into account no major change of the permeability is expected.
3.5 Manufacturing of iron yoke

The magnets’ yoke will be laminated with magnetic steel and austenitic, non-magnetic, steel cut by using a laser cutting machine. Laser cutting was selected despite the large tolerances in the order of $\pm 15 \, \mu m$ for one head laser cutting machines and $\pm 20 \, \mu m$ for two head laser cutting machines, see visit report EDMS 1225966; because of (i) the very flexible manufacturing, which allows for minor “last-minute” changes of the geometry; (ii) because of the rapidness (production time for laminated yokes around 8 weeks) and (iii) the cheapness of the manufacturing compared to stamping for one prototype.

The production process will be performed as follows: After cutting the stainless steel and electrical steel laminations will be stacked alternating in a custom-made stacking tool. Then a pressure (around 3 MPa, at least 2 MPa minimum at end of production process) has to be applied by using a press. This pressure has to be maintained by using temperature stable springs maintaining the pressure over the full manufacturing cycle. The curing process has to be performed in a furnace to activate the bonding between the laminations, typically at 205°C. The yoke will shrink due to the curing process by about 1.2%. This shrinkage has to be taken into account by the manufacturer both to ensure a minimum pressure of at least 2 MPa during the full manufacturing process in the stacked yoke and to meet the total length requirements. After bonding the stacking structure will be opened to adapt the length of the magnet and then it will be re-installed in the stacking tool for welding to prevent deformation of the yoke during welding.

3.6 End shim design

The end shims of the magnet will consist out of 5 pieces which can be individually trimmed to enhance the integrated field. The detailed design of the end shims is presented in Section 4.2. In the prototype solid, non-laminated, end shims produced by using ARMCO iron will be installed. We will try to reduce the required time for shimming by introducing springs between the shims and the yoke allowing for changing the longitudinal position of the individual end shim blocks, taking advantage of the limited magnetic pressure due to the low field. If this approach is successful it may reduce the required cost, time and manpower for the shimming procedure after manufacturing.

3.7 Coil design of prototype

In the next sub-sections the design and manufacturing method of the coil is presented. Important aspects for the series magnets are discussed as well, which may not be followed due to economic aspects for this prototype magnet.

3.7.1 Ampere turns per pole

We assume that (i) the stray field around the air gap is small and that (ii) the relative permeability of the yoke is very high ($\mu_r \gg 1$). These two effects can be taken into account by assuming an efficiency $\eta = 0.95$. The magnet will be operated at three field levels with ramps in between. The required ampere turns per pole for the dipole can then be calculated for the injection, cooling and extraction field levels:
Top field, injection (0.37 T): \[ NI = \frac{Bg}{2\mu_0\eta} = \frac{0.42 \cdot 10^{-3} m}{2 \cdot 4 \pi \cdot 10^{-7} \frac{m}{m^2} \cdot 0.95} = 13400 \text{ A} \]

Medium field for cooling (0.21 T): \[ NI = \frac{Bg}{2\mu_0\eta} = \frac{0.21 \cdot 10^{-3} m}{2 \cdot 4 \pi \cdot 10^{-7} \frac{m}{m^2} \cdot 0.95} = 6700 \text{ A} \]

Low field, extraction (0.05 T): \[ NI = \frac{Bg}{2\mu_0\eta} = \frac{0.05 \cdot 10^{-3} m}{2 \cdot 4 \pi \cdot 10^{-7} \frac{m}{m^2} \cdot 0.95} = 1600 \text{ A} \]

\( NI \): Ampere turns, \( B \): magnetic field in the gap, \( g \): gap between magnetic poles, \( \mu_0 \): relative permeability, \( \eta \): magnet’s efficiency.

The cycle time will be in the order of 10 s or longer, no definitive cycle is defined for the magnet. Therefore, the RMS current concept will not be applied and the coil design will be done (for the series magnet) for injection energy. This approach will also reduce the overall resistance of the coil and therefore reduce the power consumption.

### 3.7.2 Selection of current density in conductor

To limit ageing of the insulation system of the coils in the radioactive environment of accelerators the permitted temperature gradient is set to only around 25 K. Moreover, decreasing the current density in the conductor helps to save on energy during operation. Typical current densities used in accelerator magnets are in the order of:

- Air cooled coils: 1 A/mm\(^2\)
- Water cooled coils: 5 A/mm\(^2\)

To get a rough estimate on costs we estimate the production and running costs of this magnet’s coil.

We estimate the running costs by assuming a life-time of 30 years (8 months/year in operation), that means the machine will run around 180,000 h. We assume that all other operating costs are independent of the current density (including cooling) and can be therefore neglected. We assume the electricity cost to be 6.5 cts. CHF/kWh, averaged and discounted to 2012 values from the EPEX spot market for peak load in Europe (excluding transmission costs to CERN). We decide to discount the cost of electricity although history has not shown a major increase in cost for electricity: the price at the spot market is almost unchanged compared to the retail price of electricity paid by CERN in 1976 [14] (4.2 cts. CHF/kWh – 6.15 cts. CHF/kWh). We estimate the cost with:

\[ C_{\text{run}} = 2\rho l_{\text{Turn}} J_{\text{RMS}} (NI_{\text{RMS}}) t_{\text{C kWh}}, \]

where \( \rho_{\text{Cu}} = 18 \cdot 10^{-9} \Omega \text{m}, l_{\text{Turn}} = 2.65 \text{ m}, J \) is the optimal current density in the conductor, \( NI_{\text{RMS}} = 9818 \text{ A}, t = 180000 \text{ h}, C_{\text{kWh}} = 0.065 \text{ CHF/kWh}, \) averaged and discounted to 2012 values.

The production costs in kCHF for the two coils produced for one magnet is calculated according to [15]. The equation is composed out of three parts which are summing up to the total production costs per coil: (1) manufacturing costs, (2) fixed costs and (3) material costs:

\[ C_{\text{prod}} = 2 \left[ (2m_{\text{coil}})^{0.5} - 0.945 \cdot m_{\text{coil}} \right] + \frac{500 \cdot V_{\text{coil}}}{8} + 2 \cdot 0.02 \cdot m_{\text{coil}} \]

Where the mass \( m_{\text{coil}} \) and the volume \( V_{\text{coil}} \) of one coil can be roughly estimated from

\[ m_{\text{coil}} = \frac{d_{\text{Cu}} l_{\text{Turn}} N_{\text{RMS}}}{J_{\text{RMS}}}, \]

\[ V_{\text{coil}} = \frac{m_{\text{coil}}}{d_{\text{Cu}}^3}, \]

where the density of the copper is \( d_{\text{Cu}} = 8920 \text{ kg/m}^3 \).

This simple and rough cost comparison shows that the cost function has a minimum at around 3-4 A/mm\(^2\) (see Figure 9). Therefore, a slightly lower current density is favorable and reduces costs over the life-time. For the prototype the production costs are the driving factor. Therefore, a higher current density is accepted.
3.7.3 Selection of copper conductor

A hollow conductor has to be selected for water cooling; to ease winding a rectangular conductor is chosen. The total cross-section of the conductor has to be around

\[ A = \frac{13400 \text{ A}}{5 \text{ A/mm}^2} = 2680 \text{ mm}^2 \]

We select a conductor 13 x 13 mm\(^2\) with a hole with a diameter of 6 mm (cross-section 139.9 mm\(^2\)/conductor) because of availability and to achieve a reasonable balance of inductance and resistance values for the magnet. This conductor dimension is also well below the maximum conductor dimension which can be easily wound at CERN, which is 15 x 15 mm\(^2\). Therefore, a total of 2680 mm\(^2\)/ (139.9 mm\(^2\)/conductor) = 20 turns are needed.

For geometrical reasons and ease of manufacturing it is chosen to produce 4 coils with 2 x 4 turns, which results in a total of 16 turns (4 x 4 turns) per pole. This choice allows for manufacturing double layer pancake coils (see Figure 10). The resulting maximum current density will be

\[ J_{\text{RMS}} = \frac{13400 \text{ A}}{16 \cdot 139.9 \text{ mm}^2} = 6 \frac{\text{A}}{\text{mm}^2}. \]

Each conductor will be insulated with 0.5 millimeter thick polyimide insulation and the coil will be insulated with around 2 mm thick insulation to ground. Therefore, the geometrical size of the coils will be around 60 x 60 mm\(^2\). The minimum bending radius of the conductor is around four times the size of the conductor, i.e., 4 x 13 mm = 52 mm. The total required length for the prototype is 32 x 2 m (average length of one turn) = 64 m. The total weight of the required conductor is around 85 kg.
3.7.4 Cooling requirements

One coil (refers to the coil(s) around one pole) will have an approximate length of

\[ L = 16 \cdot 2 \text{ m} = 32 \text{ m}. \]

Therefore, the resistance of the coil will be

\[ R = \rho \frac{l}{A_c} = 18 \cdot 10^{-9} \Omega \text{m} \frac{32 \text{ m}}{139.9 \text{ mm}^2} = 4.1 \text{ m}\Omega. \]

\( R \): resistance of one coil, \( \rho \): resistivity of copper, \( l \): length of coil, \( A_c \): cross-section of conductor

The maximum voltage at steady operation will be therefore

\[ U = \frac{RI}{N} = \frac{4.1 \text{ m}\Omega \cdot 13400 \text{ A}}{16} = 3.4 \text{ V}. \]

\( U \): Voltage over one coil, \( R \): Resistance of one coil, \( I \): total current in one coil, \( N \): number of turns

The maximum power we find with

\[ P = UI = 3.4 \text{ V} \cdot 13400/16 \text{ A} = 2.8 \text{ kW}, \]

and a cooling flow per coil

\[ Q = 14.3 \frac{P}{\Delta T} = 14.3 \frac{2.8 \text{ kW}}{25 \text{ K}} = 1.6 \text{ l/min}, \]

\( Q \): water flow in litre/minutes, \( P \): power in kW, \( \Delta T \): allowed temperature increase of the water

In practice the two coils can be hydraulically connected in series, i.e., a flow of 3.2 l/min will be required.

With a tube of inner diameter of 6 mm one finds a velocity of

\[ v = 21.2 \frac{Q}{d^2} = 21.2 \frac{3.2 \text{ m}}{6^2 \text{ s}} = 1.9 \text{ m/s}. \]

\( v \): velocity in m/s, \( Q \): required flow in l/min, \( d \): diameter in mm

In practice, for efficient cooling, turbulent flow has to be established, which is the case if

\[ v > \frac{1.4}{d} = \frac{1.4}{6} = 0.2 \text{ m/s}. \]

\( v \): velocity, \( d \): diameter in mm), which is here clearly the case.

The Reynolds Number is

\[ \text{Re} = \frac{v_d D}{v} = \frac{1.9 \frac{\text{m}}{\text{s}}}{6 \cdot 10^{-3} \text{m}} \frac{6.982 \cdot 10^{-7} \text{m}^2}{\text{s}} = 16300, \]

which means that turbulent flow is established.

The pressure drop over the two coils is:

\[ \Delta p = 60l \frac{0.175}{d^{4.75}} = 60 \cdot 54 \cdot \frac{3.2^{1.75}}{6^{4.75}} = 5 \text{ bar}. \]

Table 1 summarizes the relevant parameters for cooling.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooling circuits</td>
<td>1</td>
</tr>
<tr>
<td>Power per circuit</td>
<td>5.6 kW</td>
</tr>
<tr>
<td>Coolant velocity</td>
<td>1.9 m/s</td>
</tr>
<tr>
<td>Cooling flow per circuit</td>
<td>3.2 l/min</td>
</tr>
<tr>
<td>Pressure drop per circuit</td>
<td>5 bar</td>
</tr>
</tbody>
</table>
3.7.5 Power converter requirements

The two coils will be powered in series. The inductance is approximately [16]:

\[ L = \frac{\eta \mu_0 N^2 A}{g} = \frac{\eta \mu_0 N^2 (w+1.2g)(l+g)}{g} = \frac{0.95 \cdot 4 \cdot \pi \cdot 10^{-7} \cdot 32^2 \cdot (0.26+1.2-0.076)(0.6+0.076)}{0.076} = 3.8 \text{ mH}. \]

The 3D analysis around peak current yields an inductance of 3.6 mH (total stored energy 1100 J at 784 A/conductor).

Therefore, the maximum inductive voltage, if the magnet is ramped up in one second, is

\[ U = L \frac{di}{dt} = 3.6 \text{ mH} \times 840 \text{ A} = 3.0 \text{ V}. \]

Table 2 summarizes the relevant information for the power converter operation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Powering circuits</td>
<td>1</td>
</tr>
<tr>
<td>Peak power (including L)</td>
<td>8 kW</td>
</tr>
<tr>
<td>Peak voltage (including L)</td>
<td>9.4 V</td>
</tr>
<tr>
<td>Maximum current</td>
<td>840 A</td>
</tr>
<tr>
<td>Total resistance</td>
<td>8.2 mΩ</td>
</tr>
<tr>
<td>Total inductance</td>
<td>3.8 mH</td>
</tr>
</tbody>
</table>

4. Magnetic field simulations

The design was optimized by using finite-element methods in 2D and 3D. For the calculations the Cobham Opera software package and ANSYS (only for structural simulations) were used. In the following sections the results are briefly presented. The design was started by performing 2D simulations. The goal was to find the optimum design for the cross-section of the dipole magnet. This cross-section was studied in detail including a sensitivity analysis to analyse the sensitivity of different BH curves and the sensitivity of geometrical errors.

The integrated \((\int B_y \text{d}z/\int B_{ref} \text{d}z - 1)\) field homogeneity is considered more important than the local \((B_y/B_{ref} - 1)\) field homogeneity, because the phase space advance shows only a small variation in the bending magnets. Therefore, the local field homogeneity must only be in the \(8 \times 10^{-4}\) level, if the global integrated field homogeneity for all field levels remains in the \(2 \times 10^{-4}\) homogeneity level. Furthermore, it can be pointed out that the \(2 \times 10^{-4}\) homogeneity level mainly refers to higher order multipoles (beyond quadrupolar components). A quadrupolar component can be easily compensated with the installed quadrupoles. In the following sections this optimization process is briefly presented. Due to the smaller ratio between gap size and good-field region (GFR) it was challenging to compensate for these effects in the 3D design.

4.1 2D magnetic simulations

The field homogeneity was evaluated around the contour of half the GFR beginning on the left bottom corner (coordinate 0) going counter-clockwise around the contour with respect to the centre (0, 0), see Figure 11 for the coordinates of the corresponding corners.
The pole width was selected with 260 mm (including rounding off the ends), a bit wider as the standard,

\[ w = 1.92g + \text{GFR}_{x} = 1.92 \cdot 76 \text{ mm} + 66 \text{ mm} = 212 \text{ mm} \]

\( w \): standard pole width, \( g \): gap height, \( \text{GFR}_{x} \): horizontal good-field region), because there is no real space limitation perpendicular to the beam direction and only eight magnets will be manufactured. A wider pole width was chosen to gain some additional margin.

To simulate the dilution of the iron in the 2D simulations not the BH curve was altered but the current in the conductors was multiplied by the dilution factor plus one. For the 2D simulations the anisotropic BH curve of M270-50 A HP was used (see also Figure 12). A better field homogeneity for all shim designs was found when orienting the rolling direction perpendicular to the pole face because the increased relative permeability in this direction helps to equally distribute the flux over the pole width (see Figure 13).

The main negative impact of the dilution is that the back leg and the top leg of the magnet have to be as thick as the pole to not saturate. A magnet without dilution could be much more compact because the back leg
could be reduced by a factor of $1.2 \, T / 0.4 \, T = 3$, if the highest allowed field level is $1.2 \, T$ to avoid any kind of saturation (compare Figure 12). Nevertheless, working in the highly non-linear regime of very low field is undesirable because the permeability is small, that means, the overall influence on the magnetic field is more visible, as evident from the transfer function:

$$B_y = \frac{\mu_0 NI}{g + \frac{l}{\mu_r}}.$$  

($NI$: Ampere turns, $B_y$: magnetic field in the gap, $g$: gap between magnetic poles, $\mu_0$: relative permeability, $\eta$: magnet’s efficiency, $l$: iron length, $\mu_r$: relative permeability).

The shim presented in Figure 16 left side allowed the best compensation for field reduction in the vertical direction. The design was optimized and the field homogeneity around the contour of the good-field region with largely increased sextupole component (smaller sextupole enhancement is possible as well) is presented in Figure 16. For this shim version $a = 0.4 \, \text{mm}$ and $n = 1.4$. Similar field homogeneity values can be achieved by using the shim version presented in the center and increasing the values of $w_3$ to $w_5$. The conductor position in $y$ direction has little influence on the field homogeneity. The introduction of holes or slits results in no major improvement of the field homogeneity.

The magnetic field homogeneity calculated with the anisotropic BH curve of M270-50 A is presented in Figure 16, left. Most challenging was to achieve the field homogeneity around the contour of the 3D designs. Especially the field drop off along the vertical outer boundary was remarkably strong. To compensate for this drop off the 2D design was optimized such that no drop off on the vertical outer boundary took place. This design could only be achieved with a hyperbolic pole shape. The resulting field homogeneity is presented in Figure 16, here a result with large sextupole component in the mid-plane is presented because it helps to compensate for the field drop off from the ends and may allow for simpler end chamfer design. However, even this design showed a large field drop off over the vertical good field region, by using a simple 45 degree chamfer or a Rogowski profile. The 2D design was iterated in combination with the 3D design with the goal to enhance the integrated field homogeneity.

The design was optimized such that the difference between the different field levels is small and that there is no negative field drop off in vertical direction. To analyse the influence of the laser-cutting tolerances the maximum tolerances were added to the pole profile and the field homogeneity was investigated, the change is below 2 units and may be adjusted with an appropriate end chamfer design (see next section). The chosen parameters for the final design are presented in Table 3 and Figure 15.
The 2D design was performed iteratively by checking the field homogeneity in 3D as well. For example, the 2D version with field homogeneity better than one unit showed in the 3D model equipped with a 45 degree end chamfer a field homogeneity of more than ±10 units. The final 2D design is presented in Figure 17. In the median plane of the 2D calculation a negative sextupole component was allowed, to have a field increase in vertical direction which compensated for the field drop of found in the 3D design. The detailed 3D design process is described in the next section.
4.2 3D static magnetic simulations

The 3D design was performed to study the integrated field homogeneity. In a first attempt a parameter study both for 45 degree chamfers and Rogowski profiles (approximated with a circle with a radius of 0.83 times the aperture and a tangent) was pursued. The field homogeneity could not be met; in a parameter study usually two solutions are searched in which the sign of the field homogeneity changes. By finding such two solutions one can quickly find the minimum according to the principle of nested intervals. This method usually allows reducing the number of models which have to be solved. No solution meeting the requirements could be found with the simple end chamfer designs, also not by iterating between 2D and 3D designs as described in the proceeding section.

Therefore, the two dominant multipoles $b_3$ and $b_5$ were optimized separately. A hyperbolic and a circular curved (Rogowski type) end chamfer were introduced for compensating the multipolar field components ($b_3$ and $b_5$). Figure 18 shows the results of a study with hyperbolic curved Rogowski end chamfer design. In the median plane easily field qualities in the order of $10^{-4}$ can be reached. However, going once around the contour the field homogeneity even with this complicated end shim stays in the order of $6 \times 10^{-4}$. This end chamfer design is very complicated and costly to machine and more of academic interest. The study was pursued further to study the effect of different end chamfer designs single, one-step and two-step chamfers with different widths and heights of the chamfer, but no solution meeting the field homogeneity requirements with the above mentioned method could be found.
Figure 18 - Integrated field homogeneity around the contour for a Rogowski end chamfer with different hyperbolic shapes $z = a/2\left(\sqrt{x/polewidth}\right)^2$, where $a$ is given in the legend of the graph (units: mm).

To look in more detail and with higher precession in the field distribution along the magnet the multipolar components were analysed with Fourier analysis along the $z$-axis. The analysis was performed on a circle with a diameter of 48 mm (the vertical width of the good-field region). Then, the circle was displaced by -9 mm and +9 mm in $x$-direction to evaluate also the outer parts of the good-field region. Additionally a large circle with a diameter of 66 mm (the horizontal height of the good-field region) was evaluated (see Figure 19). To take the full GFR into account a reference radius of $\sqrt{24^2 + 33^2}$ mm = 40.8 mm would be required, which cannot be evaluated because the gap size is only 76 mm.

![Diagram](image)

Figure 19 - Analysis of good-field-region in prototype magnet

Then, with a number of 3D models it was tried to understand how $b_3$ and $b_5$ can be independently controlled with the end chamfer design presented in Figure 20. It was found that this design has enough free parameters to independently control the two different multipolar components $b_3$ and $b_5$. Therefore, choosing five independent shims gives enough degrees of freedom during the shimming process to correct the multipole components.

![Diagram](image)

Figure 20 - End chamfer design.

By systematically researching different parameters $h_1$, $h_2$, $h_3$ and $w_1$, $w_2$, $w_3$ (see again Figure 20) for a two-step chamfer and looking both at the local and the integrated Fourier coefficients it was found that around the chosen favourable solution:

- The quadrupole ($b_2$) coefficient depends on the field level, and may not be fully compensated for all field levels. However, the quadrupole component is not critical in the ELENA machine and can be compensated by adjusting the field gradient in the quadrupoles. It has also to be noted that the dilution is not taken into account in this calculation. The dilution will reduce the field level dependence of the quadrupole component.
• The sextupole \( (b_3) \) coefficient can be regulated for the chosen configuration by adjusting the coefficients of the height \( h_1, h_2, \) and \( h_3 \). It is not very dependent on the total field level. Therefore, the integrated shim may work over the full dynamic range of the magnet.
• The octupole \( (b_4) \) coefficient was negligible.
• The decapole \( (b_5) \) coefficient can be regulated for the chosen configuration by changing the coefficients of the width \( w_1, w_2, \) and \( w_3 \). It is not very dependent on the total field level. Therefore, the integrated shim may work over the full dynamic range of the magnet.

The field homogeneity \( (2 \times 10^{-4}) \) could be met by putting a lot of effort in the end shim design on the smaller circles (diameter 48 mm) as well as on the larger circle (diameter 66 mm). The parameters of the optimized end chamfer are presented in Table 4. The magnet will be measured with flipped or rotating coil measurement systems.

Table 4 - End chamfer design

<table>
<thead>
<tr>
<th>( w )</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>65 mm</td>
<td>10</td>
</tr>
<tr>
<td>35 mm</td>
<td>13</td>
</tr>
<tr>
<td>60 mm</td>
<td>14</td>
</tr>
</tbody>
</table>

The local multipole coefficients over the \( z \)-axis are presented in Figure 21 for a reference radius of 24 mm starting from the centre of the magnet (\( z \)-coordinate is 0 at centre of magnet). The total iron length of the magnet is 600 mm. Table 5 lists the integrated multipole coefficients for a coarse mesh and Table 6 for a fine mesh at a reference radius of 24 mm. The difference is small and for the optimization process the coarse mesh is very handy and allows for quick answers. The running time was optimized such that the model solved in around 1 h (1,628,150 elements) on a dedicated server, to check the validity of the final model a very fine mesh was applied (running time 2.5 d, 28,007,587 elements).

Table 7 lists the multipole coefficients for a radius of 33 mm. The chosen end chamfer design improves the field homogeneity by one order of magnitude compared to a magnet without any chamfer (hard edge). The multipole coefficients for a design without any chamfer (hard edge) are listed in Table 8.

The multipolar components (except \( b_2 \)) do not change much from injection to extraction energy, because of the low field levels, that means, no saturation is present and the relative permeability of the material is very large. Even the quadrupole component is expected to be smaller in the final model, because of the dilution which was not taken into account in the above simulation. Also the anisotropic BH curve was not taken into account for the 3D design, which slightly increases the quadrupolar component.

The overall influence on the field homogeneity will be studied with the prototype. It has to be noted that this choice complicates the 3D simulations because the iron field level in the main yoke is due to the dilution three times higher than the one in the end shim, which cannot be simulated easily. However, the field

![Figure 21 - Multipolar coefficients along z-axis on a reference radius of 24 mm.](image)
homogeneity in the 2D design (simulation) for a field level at 0.05 T is similar to the other field levels. Therefore, the overall field homogeneity in the yoke is investigated in the 3D simulation at the actual field levels between 0.05 T and 0.40 T.

The field homogeneity around the contour of the good-field region is shown in Figure 22. The required field homogeneity cannot be met, even with this sophisticated end chamfer design. In the upper left and right corner the field homogeneity is in the order of $6 \times 10^{-4}$. Further, improvement with an updated 2D design with reduced field enhancement in the upper corners and repeating the steps described in this section may slightly improve further the field homogeneity and will be repeated for the final curved ELENA dipole magnet. Also, the final field homogeneity requirements for the final ELENA dipole magnet will be re-discussed and adjusted to the requirements of beam dynamics, which is currently checked by performing tracking studies.

Table 5 - Integrated multipole coefficients of prototype magnet at injection energy (top) and extraction energy (bottom), coarse mesh, reference radius of 24 mm.

<table>
<thead>
<tr>
<th>Injection</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
<th>$b_7$</th>
<th>$b_8$</th>
<th>incl. $b_2$</th>
<th>without $b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-9 mm</td>
<td>5.68E-05</td>
<td>1.19E-05</td>
<td>-3.40E-06</td>
<td>4.59E-05</td>
<td>1.35E-05</td>
<td>3.06E-05</td>
<td>-1.38E-05</td>
<td>1.76E-04</td>
<td>1.19E-04</td>
</tr>
<tr>
<td>0 mm</td>
<td>1.27E-07</td>
<td>1.65E-05</td>
<td>9.77E-06</td>
<td>6.19E-05</td>
<td>-5.54E-06</td>
<td>1.14E-05</td>
<td>-1.04E-06</td>
<td>1.06E-04</td>
<td>1.06E-04</td>
</tr>
<tr>
<td>9 mm</td>
<td>-2.46E-05</td>
<td>1.09E-05</td>
<td>-1.44E-05</td>
<td>1.79E-05</td>
<td>-1.16E-05</td>
<td>2.55E-05</td>
<td>-3.23E-06</td>
<td>1.08E-04</td>
<td>8.36E-05</td>
</tr>
</tbody>
</table>

Extraction

<table>
<thead>
<tr>
<th>Injection</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
<th>$b_7$</th>
<th>$b_8$</th>
<th>incl. $b_2$</th>
<th>without $b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-9 mm</td>
<td>5.79E-04</td>
<td>2.99E-05</td>
<td>-5.85E-06</td>
<td>4.50E-05</td>
<td>1.35E-05</td>
<td>3.07E-05</td>
<td>-1.39E-05</td>
<td>7.18E-04</td>
<td>1.39E-04</td>
</tr>
<tr>
<td>0 mm</td>
<td>5.34E-04</td>
<td>3.08E-05</td>
<td>5.76E-06</td>
<td>6.09E-05</td>
<td>-5.53E-06</td>
<td>1.13E-05</td>
<td>-1.08E-06</td>
<td>6.50E-04</td>
<td>1.15E-04</td>
</tr>
<tr>
<td>9 mm</td>
<td>5.18E-04</td>
<td>1.96E-05</td>
<td>-2.00E-05</td>
<td>1.68E-05</td>
<td>-1.17E-05</td>
<td>2.57E-05</td>
<td>-3.00E-06</td>
<td>6.15E-04</td>
<td>9.68E-05</td>
</tr>
</tbody>
</table>

Table 6 - Integrated multipole coefficients of prototype magnet at injection energy (top) and extraction energy (bottom), fine mesh, reference radius of 24 mm.

<table>
<thead>
<tr>
<th>Injection</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
<th>$b_7$</th>
<th>$b_8$</th>
<th>incl. $b_2$</th>
<th>without $b_2$</th>
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<tr>
<td>-9 mm</td>
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<td>-2.87E-05</td>
<td>4.34E-05</td>
<td>-3.21E-05</td>
<td>-1.42E-06</td>
<td>9.47E-06</td>
<td>-1.05E-05</td>
<td>1.32E-04</td>
<td>1.26E-04</td>
</tr>
<tr>
<td>0 mm</td>
<td>-3.22E-06</td>
<td>-6.04E-06</td>
<td>-2.37E-06</td>
<td>-2.89E-05</td>
<td>-8.36E-08</td>
<td>6.97E-06</td>
<td>-1.08E-06</td>
<td>6.50E-04</td>
<td>1.15E-04</td>
</tr>
<tr>
<td>9 mm</td>
<td>-1.47E-05</td>
<td>-3.42E-05</td>
<td>-4.86E-05</td>
<td>-3.24E-05</td>
<td>1.24E-06</td>
<td>9.46E-06</td>
<td>1.04E-05</td>
<td>1.51E-04</td>
<td>1.36E-04</td>
</tr>
</tbody>
</table>

Extraction

<table>
<thead>
<tr>
<th>Injection</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
<th>$b_7$</th>
<th>$b_8$</th>
<th>incl. $b_2$</th>
<th>without $b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-9 mm</td>
<td>5.32E-04</td>
<td>-1.13E-05</td>
<td>4.00E-05</td>
<td>-3.25E-05</td>
<td>-1.45E-06</td>
<td>9.44E-06</td>
<td>-1.05E-05</td>
<td>6.37E-04</td>
<td>1.05E-04</td>
</tr>
<tr>
<td>0 mm</td>
<td>5.34E-04</td>
<td>7.11E-06</td>
<td>-6.55E-06</td>
<td>-2.95E-05</td>
<td>-1.97E-07</td>
<td>4.47E-09</td>
<td>5.81E-04</td>
<td>4.72E-05</td>
<td></td>
</tr>
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<td>9 mm</td>
<td>5.30E-04</td>
<td>-2.63E-05</td>
<td>-5.38E-05</td>
<td>-3.32E-05</td>
<td>1.16E-06</td>
<td>9.57E-06</td>
<td>1.05E-05</td>
<td>6.65E-04</td>
<td>1.35E-04</td>
</tr>
</tbody>
</table>

Table 7 - Integrated multipole coefficients of prototype magnet at injection energy (top) and extraction energy (bottom), fine mesh, reference radius of 33 mm.

<table>
<thead>
<tr>
<th>Injection</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
<th>$b_7$</th>
<th>$b_8$</th>
<th>incl. $b_2$</th>
<th>without $b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 mm</td>
<td>-4.03E-06</td>
<td>-1.74E-05</td>
<td>-6.80E-06</td>
<td>-1.00E-04</td>
<td>-1.83E-07</td>
<td>-2.68E-05</td>
<td>3.27E-07</td>
<td>1.56E-04</td>
<td>1.52E-04</td>
</tr>
</tbody>
</table>

Extraction

<table>
<thead>
<tr>
<th>Injection</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
<th>$b_7$</th>
<th>$b_8$</th>
<th>incl. $b_2$</th>
<th>without $b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 mm</td>
<td>7.35E-04</td>
<td>7.50E-06</td>
<td>-1.76E-05</td>
<td>-1.02E-04</td>
<td>-7.66E-07</td>
<td>-2.71E-05</td>
<td>4.94E-07</td>
<td>8.90E-04</td>
<td>1.56E-04</td>
</tr>
</tbody>
</table>

Table 8 - Integrated multipole coefficients of prototype magnet with hard edge at the end, coarse mesh, at a reference radius of 33 mm.

<table>
<thead>
<tr>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
<th>$b_7$</th>
<th>$b_8$</th>
<th>incl. $b_2$</th>
<th>without $b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 mm</td>
<td>-1.58E-04</td>
<td>-1.64E-03</td>
<td>-4.88E-05</td>
<td>-1.14E-04</td>
<td>1.69E-05</td>
<td>5.09E-05</td>
<td>-1.78E-05</td>
<td>2.04E-03</td>
</tr>
</tbody>
</table>
4.3 3D dilution simulation

A simulation of each lamination in the magnet would require large calculation times and is not needed. To investigate the effect of the dilution different approaches were tried, which are described in the section.

The dilution was studied by simulating the models mentioned earlier with currents up to \((1+r)\) times the nominal current, where \(r\) is the dilution factor. This approach was outlined also for the 2D simulations. The drawback of this calculation method is that it does not account for the anisotropy in the model. Because of the stainless steel lamination the permeability in \(z\)-direction is reduced to [17]:

\[
\mu_z = \frac{\mu_t \mu_0^2}{f \mu_0 + (1-f) \mu_t \mu_0}, \quad \text{where } f = 0.33.
\]

This reduction of the permeability was taken into account by performing one final check with the final model described above by using the pre-implemented packing factor option for the diluted part of the yoke in Opera. The end shims were simulated with solid material. The result showed no significant difference to the above outlined model. Even with high permeability in \(z\)-direction the magnetic flux density does not use paths with \(z\)-direction components because due to the optimization and symmetries the paths laying in the \(xy\)-plane have lower reluctance and are therefore energetically preferred.

Due to the non-homogenous distribution of iron (thickness 0.5 mm) and stainless steel (thickness 1 mm) we studied the re-distribution of the magnetic flux density in the gap. Therefore, a model was built up consisting of half an iron and half a stainless steel lamination, as shown in Figure 23.

The results of this analysis can be summarized as follows:

- The dilution (1 mm non-magnetic material) is a far-field effect.
- As expected the magnetic induction in the iron is concentrated in the iron and therefore \((1+r)\) times higher as in the aperture as long as the iron is not saturated. The parameter \(r\) indicates the amount of non-magnetic material compared to one part iron.

For illustration the field along the \(z\)-axis of the magnet is illustrated in Figure 24 at different heights: in the centre \((y = 0 \text{ mm})\), 2 mm below the pole face \((y = 36 \text{ mm})\), on the pole face \((y = 38 \text{ mm})\) and 2 mm above the pole face \((y = 40 \text{ mm})\). From the plot it can be seen that even two mm below the pole the magnetic flux...
density is equally re-distributed in the yoke, 2 mm above the pole face the magnetic flux it is completely
concentrated in the electrical steel.

Figure 24- Magnetic flux density in the centre of the gap at different heights

4.4 Remanent field calculation

The goal of the remanent field calculation is to find out whether the relative field homogeneity and the
absolute field values at the same currents are sensitive on the history of powering. The goal is to produce a
magnet which follows the typical ELENA cycle, i.e., \( B = f(I) \).

In a closed magnetic circuit the residual field is entirely determined by the remanent polarization \( J_r \). In an
open magnetic circuit with soft magnetic material, where the highest reluctance appears in the gap, the
residual field is mainly determined by the coercive field \( H_c \) [18]. Therefore, we write Ampere’s law taking
the dilution factor into account as:

\[
NI = \int \vec{H} \, dl = \frac{B_{\text{res}} g}{\mu_0} + H_c l = 0. \]

Now, the residual field in the iron for the diluted magnet is:

\[
3B_{\text{gap}} = B_{\text{res}} = -\frac{\mu_0 H_c \lambda}{g} = 1.2 \text{ mT},
\]

for a coercive force of \( H_c = 35 \text{ A/m} \), an iron path length \( \lambda = 2 \text{ m} \) and a gap height of \( g = 76 \text{ mm} \). The
remanent field in the gap is \( 0.15/1.2 \cdot 10^3 = 125 \) times larger than the residual field. Therefore, no negative
impact on the field homogeneity is expected. The coercive force for M270-50 A HP excited at 1.1 T
(maximum excitation with dilution) compared to electrical steel excited at 0.4 T (maximum excitation
without dilution) is around 2 times larger. Using this information as input for the above presented equation,
one sees that the residual field of the diluted magnet will reach to around \( 2/3 \) compared to the non-diluted
yoke.

To put the analysis on a better quantitative basis finite element simulation with Opera Version 15R1 were
performed. Opera provides two tools for the calculation of hysteresis effects: (i) Demagnetization (DM)
solver for hard magnetic properties, (ii) hysteretic material properties solved with standard transient solver
for soft magnetic materials, which was used in the analysis for the dipole magnet. As an input the upper
branch of the hysteresis loops with a maximum of 50 data points is needed. The inner hysteresis loops are
modelled by Opera, for more information on this topic consult [19]. The program can only calculate isotropic
material properties.

This solver has never been used at CERN before. To verify the simulation results, the simulation was also
performed for the synchrotron quadrupole of MedAustron (MQZ-C) and compared with simulation results
performed with the Opera DM (demagnetization solver) and measurements of the remanent field. Both simulation results showed similar results, but unfortunately neither DEMAG nor TR with hysteretic material properties match the rotating coil or local measurement results well, not even the functional behavior could be predicted. Therefore, these results were not used.

5. Mechanical calculations and simulations

To evaluate the influence of mechanical deformations on the magnetic field an ANSYS multi-physics model was set up, which re-calculates the magnetic field with the deformed shape. With the currently released Opera version (15.0) this multi-physics calculation is not straight forward.

For the analysis it was assumed that the magnet is fixed at the bottom. Therefore, no symmetry is present and the full magnet had to be simulated.

Figure 25 shows the deformed shape of the iron and the mechanical stress in the iron up to 5 MPa. For this calculation both plane strain and plane stress option was evaluated, the difference for this two methods is negligible. For the calculation an E-modulus of 152 GPa was assumed, as given in [20] for laminated yokes. The magnetic flux will be concentrated in the iron and subsequently higher, therefore, also the force on the electrical steel laminations will be higher (as if the magnet would be operated at a three times higher field).

Figure 27 compares the calculated field homogeneity of ANSYS magnetic, ANSYS structural + magnetic (takes the Maxwell and virtual work forces into account and recalculates the magnetic field of the deformed magnet), and Opera ST 2D. The difference in field homogeneity is small. The same calculation was performed for the PS main magnets and a comparison with mechanical measurement data showed very good agreement.

The force on the end shims was calculated by using the Maxwell Stress Tensor option within Opera. The end shims will have a thickness of 30 mm. The total force \( F = (F_x, F_y, F_z) = (0.1, 1.7, -2.2) \) kN on the end-chamfer is calculated for around 3 times the nominal field (1.18 T). The actual forces will be smaller. For the following calculation we assume that the force in \( z \)-direction will be transferred through the laminations, where the shim is screwed in, to the end-plate and will be taken by it. For simplicity we assume that the end plate is fixed on top and with steel belts at the sides, and that it can move freely like a beam. To estimate the maximum deformation, we assume that the end-plate is a beam with the dimension \((x, y, z) = (260, 120, 30)\) which yields

\[
I_z = \frac{bh^3}{12} = \frac{260 \cdot 30^3}{12} = 585,000 \text{ mm}^4
\]

The bending line yields a deflection of

\[
f = \frac{Fl^3}{2EI_z} = \frac{2200 \text{ N} \cdot (120 \text{ mm})^3}{2 \cdot 210 \cdot 10^3 \text{ MPa} \cdot 585,000 \text{ mm}^4} = 15.5 \mu\text{m},
\]

which can be accepted in this design. The Lorentz force on the conductor was evaluated and is shown in Figure 26.
6. Conclusions

The design study tackles all relevant engineering issues of the ELENA bending magnet. A separate design report was issued with the details of the magnetic design of the final curved ELENA bending magnet [21]. The purpose of the prototype is that the (i) production process of a magnetic yoke diluted with stainless steel plates, (ii) the stability and repeatability of the field homogeneity of such a yoke over the full working range, (iii) choice of soft magnetic steel, (iv) hysteresis effects, (v) mechanical deformations, (vi) thermal insulation to intercept heat load from baking for activation of NEG coating in the vacuum chamber, (vii) end shim design. In order to verify these points the following measurements will be performed: (i) Hall probe scanning, (ii) integrated field homogeneity measurement (DC), (iii) integrated field homogeneity
measurement (AC). The results will be reported and archived in CERN’s Electronic Document Management System (EDMS).

For the final ELENA design the magnetic design can be re-optimized and all parameters can be easily changed within reasonable limits of some millimetres and re-calculated with finite-element methods. The results can therefore be used to scale to the finally required design of the ELENA bending magnet. The prototype will give sound answers in terms of performance and costs for the final ELENA bending magnets. All potential design problems can be already spotted and solved in an early stage of the project. The field homogeneity requirements can be re-discussed for the series magnet.

7. Acknowledgements

I would like to express my gratitude to my colleagues from the TE-MSC-MNC section for many interesting discussions. A special thanks goes to D. Tommasini and L. Bottura for their valuable comments on this document.

8. Bibliography

Appendix A: 3D transient simulations

All ramp rates in ELENA are slow, several seconds are usually available to ramp magnets from the lowest field to the maximum operation field (no beam in machine) and moreover, electrical steel with a 5 times higher specific electrical resistivity was selected (see Table 9).

Table 9 - Specific electrical resistivity of electrical steel and stainless steel

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity [Ωm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stainless Steel 304 L</td>
<td>$7.2 \times 10^{-7}$</td>
</tr>
<tr>
<td>M270-50 A HP</td>
<td>$5.2 \times 10^{-7}$</td>
</tr>
<tr>
<td>Armco iron</td>
<td>$1.1 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

For ramping down a ramp-speed of around 0.04T/s is foreseen, which will limit the eddy currents to very low values especially with the foreseen high-silicon steel, which has a high resistivity.

If eddy currents in the end shims cause a visible distortion of the magnetic field the use of end shims produced out of soft magnetic composites (SMCs) were studied. Several hundred tonnes of SMCs are produced each year for high frequency applications such as speed controlled electrical motors, fast switching actuators, inductors and power electronics. SMCs are used because of their very high electrical resistivity largely hindering the formation of eddy currents up to kHz-frequencies. A broad overview on the development and different types of SMCs under development is given in [8]. The two products dominating the SMC market are Ancorlam from Hoeganaes Corporation, USA and Somaloy from Höganäs AB, Sweden (see also EDMS 1240824). Both companies offer iron powders with inorganically insulated surfaces. Compaction is achieved for both products by using a pressure of around 800 MPa. No glue or sintering is required for adhesion of the material. Due to the high required pressure of 800 MPa it is not straightforward to build a magnetic yoke because of the huge required forces. Therefore, in this study the use of SMC material for the poles was investigated.

A small H-type dipole was designed to study the influence of the eddy currents on the field homogeneity after ramping with different ramp rates and field levels. The H-dipole design was the preferred dipole design for this study because it allows to limit the simulation to one eight of the magnet by taking into account its symmetry, which is a large advantage for transient simulations in 3D as it limits the calculation time. However, the influence on the transient behaviour is similar for H- and C-dipoles. Figure 28 shows the model with the optimized mesh (running time: 11h).
To study the influence on the magnetic field the change of the integrated field \( \int_0^\infty B_y(z,t)dz \) in the center of the gap with respect to the integrated field in steady state \( \int_0^\infty B_y(z,\infty)dz \)

\[
B_{\text{Hom}}(t) = 1 - \frac{\int_0^\infty B_y(z,t)dz}{\int_0^\infty B_y(z,\infty)dz}
\]

is plotted in Figure 29. As expected the improvement is largest for the largest ramp rate.

The difference of the local field distribution at steady state \( B_y(z,\infty) \) and after the ramp is finished \( B_y(z, 0.05 \text{ s}) \) (for the example with a ramp rate of 24 T/s) was plotted normalized to the maximum magnetic field at steady state

\[
B_{\text{diff}} = \frac{B_y(z, \infty) - B_y(z, 0.05 \text{ s})}{\max(B_y(z, \infty))}
\]

The results for regular shims and SMC shims (thickness 50 mm) for a ramp rate of 24 T/s, a maximum magnetic field of 1.25 T and a magnetic yoke length of 2 x 300 mm are shown in Figure 30. This plot shows that the difference is especially large at the position of the end shims, which was expected. For the yoke with SMC shims the maximum field distortion is smaller and it is shifted to the centre with a maximum at around \( z = 250 \text{ mm} \), the position at which the laminated yoke starts.

A study with a coarse mesh was performed to obtain estimations on the effect of end shims produced with SMCs. This study gives a good indication on the effect of the shims. However, a mesh study was performed that showed that the effect of the mesh on the results is not completely negligible. The boundary condition that only tangential magnetic fields exist in the x-y-plane may not be correct for this calculation and introduces some additional errors. The use of SMC poles may only provide limited improvements because by using SMC poles, the eddy currents still occur in the rest of the magnetic circuit. It was shown that these eddy currents have a similar influence on the magnetic field homogeneity as those in the pole meaning the end shims not allowing eddy currents for compensating for the field change cause the induction of eddy currents in the main part of the yoke.
Figure 29 - Relative field change after end of ramp at time 0 after a ramp rate of 4 T/s (top), 12 T/s (middle) and 24 T/s (bottom). The maximum field in the gap is 1.25 T.
Figure 30 - Difference between field directly after ramp and at steady state for a ramp rate of 24 T/s, a maximum field of 1.25 T and a yoke length of 2 x 300 mm.