SCALE-INVARIANT $d = 10$ SUPERSPACE AND THE HETEROTIC STRING

R.E. Kallosh

Lebedev Physical Institute, 117 924 Moscow, USSR

and

B.E.W. Nilsson^{*)}

CERN -- Geneva

ABSTRACT

The superspace formulation of the $d = 10$ Einstein-Yang-Mills system is presented. Our torsion constraints imply that the solution of the Bianchi identities will be expressed in terms of fields appearing naturally in the heterotic string. The scaling properties of superspace allow a formulation with all superfields dimensionless as well as one with superfields of canonical dimensions. The dimensionless formulation gives rise to a set of very simple supersymmetry transformation rules. This approach may facilitate the investigation of the effects of introducing string-induced higher derivative terms.

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^{*)} On leave of absence from the Institute of Theoretical Physics, S-412 96 Göteborg, Sweden.

Superspace is a very useful concept when trying to understand supersymmetric field theories. Since supersymmetry is manifest in superspace, it also provides an extremely powerful tool e.g. in the construction of new field theories having this symmetry. With the recent discovery of the anomaly cancellation of the type I open superstring [1,2] and the subsequent construction of the heterotic strings [3], the problem of building new supersymmetric field theories has acquired a new dimension. It may be the case that once any of the string-induced higher derivative terms [1,4,5,6] are added to the supersymmetric Lagrangian of the massless sector of these strings, i.e. the Chapline-Manton theory [7], supersymmetry is broken and cannot be restored unless an infinite set of terms are included (see [6] for further comments). The theory would then presumably correspond to the full string theory. In a situation like this it seems crucial to work in an as technically powerful and economical setting as possible. Superspace does provide such a setting. We therefore need to develop further our understanding of the superspace formulation of $d=10$ supergravity [8] and its coupling to the supersymmetric Yang-Mills theory. (We will return to the question of higher-derivative terms at the end of the paper.) Although the component formulation of $d = 10$ supergravity coupled to Abelian [9] and non-Abelian [7] supersymmetric gauge theories have previously been discussed in the literature, we think that the superspace approach described below will turn out to enhance our understanding not only of the field theories in question, but in fact also of the string theories

themselves. Indeed, the idea of merging superspace and string theories has already proved useful in e.g. the recent work on superstrings propagating in an arbitrary supergravity background [10,11].

In this paper we present the superspace Bianchi identities for the Einstein-Yang-Mills system and propose a set of constraints from which to derive the on-shell theory. The basis for this is Ref. [8], where the supergravity Bianchi identities were analyzed and the superspace to be used here introduced. Surprisingly enough, in the supergravity sector of the coupled system the constraints coincide exactly with the ones given in [8] (apart from one trivial modification). We will also discuss a phenomenon, appearing already in [8] but not fully appreciated at the time, related to this particular set of constraints (compare e.g. [10]), namely the manifest scale invariance of the field equations. This invariance has attracted a lot of attention recently, due to its appearance also in the heterotic string [12,13,14]. In this connection, it is interesting to note that the fields in x-space emerging from the analysis of [8] are exactly the ones appearing naturally in the effective Lagrangian approach to strings, as proposed by Polyakov [15] and further developed by Fradkin and Tseytlin [16], and Callan et al. [17]. (The relation between the metric in this effective Lagrangian and in the manifestly scale-invariant field equations in x-space was noticed in [16,17,13,14].) This has prompted us to investigate further the dependence on coupling constants and Weyl-

rescalings, and to clarify the relation between the component and superspace versions of the coupled Einstein-Yang-Mills theory in ten dimensions.

We now turn to the presentation of the superspace equations which define the Einstein-Yang-Mills theory. As we know from previous work in x-space [9,7] the supersymmetry of this coupled theory implies the presence of a Yang-Mills Chern-Simons term in the definition of the field strength for the two-form potential in the supergravity multiplet. One also finds that the supersymmetry transformation rules need to be augmented by a number of terms involving the different Yang-Mills fields [9,7,18,19]. Although these terms appear in a variety of places, they are easily seen to be of a form indicating that they all stem from one term in superspace, namely the right-hand side of the Bianchi identity for the two-form superpotential given below. This analysis is also a strong indication of the correctness of the constraints.

We will use the chiral superspace defined in [8]. It is parametrized locally by co-ordinates

$$z^M = (x^\mu, \theta^m) \quad , \quad (1)$$

where the world super index M is split into a bosonic index μ and a fermionic index m . In this superspace we introduce the vielbein E^A , the spin connection ω_A^B , valued in the Lorentz algebra in the standard fashion [8], the two-form potential B and the Yang-Mills potential A . The tangent space indices $A, B, \dots$ split into bosonic indices $\alpha, \beta, \dots$ and fermionic Weyl-projected indices $a, b, \dots$. The fermionic quantities also satisfy the Majorana condition. Next, we define an exterior derivative covariant with respect to general superspace co-ordinate transformations as well as local Lorentz and local Yang-Mills transformations:

$$D = d + \omega + A \quad , \quad (2)$$

where the generators of the Lorentz group and the gauge group have been absorbed into ω and A , respectively. The field strengths are

$$T^A = DE^A \quad , \quad (3)$$

$$R_A^B = d\omega_A^B - \omega_A^C \wedge \omega_C^B \quad , \quad (4)$$

$$H = dB + \text{tr}(A \wedge F + \frac{1}{3} A \wedge A \wedge A) \quad , \quad (5)$$

$$F = dA - A \wedge A \quad . \quad (6)$$

Thus the new terms appearing in the coupled theory in x -space

correspond in superspace to the trace term on the right-hand side of (5). Interestingly enough, this term can be derived without (directly) requiring supersymmetry by using the approach of [20]. There the bosonic part of this term is obtained by a Kaluza-Klein procedure starting from a field theory without Yang-Mills fields in $10+k$ dimensions, and compactifying the k internal dimensions on a group manifold.

The Bianchi identities take the form

$$DT^A = -E^B \wedge R_B^A, \quad (7)$$

$$DR_A^B = 0, \quad (8)$$

$$DH = \text{tr } F \wedge F, \quad (9)$$

$$DF = 0. \quad (10)$$

As usual we need not examine (8) explicitly since it is a consequence of (7) [21].

In order to obtain the field equations in this approach, one must impose constraints on some of the components of the tensors T_{AB}^C , H_{ABC} and F_{AB} , defined by

$$T^C = \frac{1}{2} E^B \wedge E^A T_{AB}^C \quad (11)$$

etc. It is in the process of determining these constraints

that one usually employs dimensional arguments [8]. (One can also get information about the constraints by coupling a superstring to an arbitrary supergravity background [10,11].) Naively, however, there seems to be a problem with dimensions in e.g. (9). Namely, if one insists on using fields with dimensions that make them fit naturally into a superfield, as done in [8], (9) would seem to conflict with any natural choice of dimensions assigned to the Yang-Mills field. To explain how such problems are avoided, we will now describe two different sets of field variables in which the Chapline-Manton theory may be formulated in x-space [16,22,13,14], and then relate these to the superspace formulation used here.

Let us first consider this theory in terms of field variables to which we can assign canonical dimensions. We will refer to this Lagrangian as L^{can} and present it in the form given by Kallosh [14]:

$$L^{\text{can}}(\kappa^2, g^2, \varphi, e, \psi, \dots) = \frac{e}{\kappa^2} [L_0 + \varphi^{1/2} L_{1/2} + \varphi^1 L_1 + \varphi^2 L_2] , \quad (12)$$

where (using the heterotic string relation between κ, g and α' [3])

$$\varphi = \frac{\kappa^2}{g^2} e^\phi = \alpha' e^\phi . \quad (13)$$

The fields of this theory are e_μ^α , ϕ_μ , $B_{\mu\nu}$, λ and φ in the

supergravity multiplet and A_μ and χ in the Yang-Mills supermultiplet. In (12) the kinetic terms for e_μ^α , ϕ_μ , λ , ϕ and χ are to be found in L_0 , the ones for $B_{\mu\nu}$ and A_μ in L_2 and L_1 , respectively. Higher order terms appear in all L_n 's except for L_2 . This formulation was suggested by Witten's observation [23] that in the superstring the Yang-Mills coupling constant can be absorbed into the scalar φ , which is also true in the field theory considered here. The significance of (13) was discussed in [14], where the observation was made that the α' that appears naturally in (12) is the α' of the heterotic string and not of the open type I superstring. The quantities in L^{can} have the following dimensions [14]:

$$\begin{aligned} \dim[\kappa^2] &= 8 \quad , \quad \dim[g^2] = 6 \quad , \quad \dim[\varphi] = 2 \quad , \\ \dim[e_\mu^\alpha] &= 0 \quad , \quad \dim[\phi_\mu, \lambda, \chi] = -1/2 \quad , \\ \dim[A_\mu] &= -1 \quad , \quad \dim[B_{\mu\nu}] = -2 \quad . \end{aligned} \tag{14}$$

From (12) it is trivial to see that L^{can} transforms homogeneously under $g_{\mu\nu} \rightarrow t^2 g_{\mu\nu}$, $\varphi \rightarrow t^2 \varphi$, $\phi_\mu \rightarrow t^{1/2} \phi_\mu$, $\lambda \rightarrow t^{-1/2} \lambda$, $\chi \rightarrow t^{-1/2} \chi$ and with the rest of the fields inert [12,13,14].

A second set of field variables is obtained by multiplying the fields in L^{can} by appropriate powers of φ to make them all (except φ) invariant under the scale transformations given above [13,14]; i.e. by defining $\tilde{e}_\mu^\alpha = \varphi^{-1/2} e_\mu^\alpha$ etc., one arrives at the following form of the Lagrangian (see note

added in [14])

$$\tilde{L}(\lambda^2, \phi, \tilde{e}_\mu^\alpha, \tilde{\psi}_\mu, \dots) = \frac{\tilde{e}}{\lambda^2} e^{4\phi} [\tilde{L}_0 + \tilde{L}_{1/2} + \tilde{L}_1 + \tilde{L}_2] \quad , \quad (15)$$

where none of the $\tilde{L}_n$'s in the bracket contains any non-derivative ϕ -dependence. λ is defined by

$$\lambda = \frac{\kappa^3}{g^4} \quad (16)$$

and is dimensionless [14]. It is now consistent to assign zero dimension to all fields including ϕ (as opposed to φ). It is clear that in this form of the theory, one has removed all non-derivative φ -dependence from the supersymmetry transformation rules. Furthermore this will continue to be true also when higher σ -model loop corrections are calculated and the corresponding terms are added to the effective field theory Lagrangian ($(R_{\mu\nu\rho\sigma})^2$ is an example of such a term [4]).

In order to relate these two formulations of the Chapline-Manton theory to the theory defined in superspace by (7)-(10) together with the constraints below, we start by observing that in terms of the variables used in L^{can} , all space-time differential forms will have dimension zero. Extending this to superspace, we get

$$\dim d = \dim \omega = \dim B = \dim A = 0 \quad . \quad (17)$$

(9) is now dimensionally correct. Furthermore, we can define dimensionless fields with tangent space indices by using the fields $\tilde{e}_\mu^\alpha$ and $\tilde{\psi}_\mu$ in constructing the 1-form vielbein in superspace. Thus

$$\dim E^A = \dim d_A = \dim \omega_A = \dim A_A = \dim B_{AB} = 0 \quad . \quad (18)$$

Therefore, if φ does not appear explicitly (i.e. without derivatives) in the constraints either (we have already seen that φ does not appear explicitly in the Bianchi identities), then the on-shell theory so obtained will be the Chapline-Manton theory formulated in terms of the scale-invariant and dimensionless fields of $\tilde{L}$ in (15). In the supergravity sector of the theory this was already seen to be the case in [8]. Of course, the argument can be turned around; since in x-space the theory is known to be scale-invariant, it should be possible to write down a consistent set of Bianchi identities and constraints with no explicit φ -dependence. (The dimensionful fields of [8] can be arrived at by performing constant rescalings with appropriate factors of $\langle\varphi\rangle=\kappa^2/g^2$.)

In view of the above arguments we propose the constraints for the coupled Einstein-Yang-Mills system given in the Table. The constraints in the Yang-Mills sector are

$T_{ab}{}^\gamma = -i(\gamma^\gamma)_{ab}$	$H_{abc} = 0$	$F_{ab} = 0$
$T_{ab}{}^c = i \delta_{(a}{}^c T_{b)}$	$H_{ab\gamma} = i(\gamma_\gamma)_{ab}$	$F_{a\beta} = i(\gamma_\beta)_a{}^b \chi_b$
$T_{a\beta}{}^\gamma = i \delta_\beta{}^\gamma \hat{T}_a$	$H_{a\beta\gamma} = -2i(\gamma_{\beta\gamma})_a{}^b \Lambda_b$	
$T_{\alpha\beta}{}^\gamma = 0$		

Table: On-shell constraints for the coupled Einstein-Yang-Mills system in superspace.

the standard ones in ten dimensions $[24,10]^+$. As a result of solving the Bianchi identities, one finds that the Einstein-Yang-Mills system in superspace is described on-shell by the two superfields $\phi(z)$ [8] and $\chi_a(z)$ [24,10]. The first few levels in these superfields are

$$\phi(z)|_{\theta=0} = \phi(x) \quad , \quad D_a \phi(z)|_{\theta=0} = i \lambda_a(x) \quad , \quad (19)$$

$$\begin{aligned} D_a D_b \phi(z)|_{\theta=0} = & \frac{i}{2}(\gamma^\alpha)_{ab} \partial_\alpha \phi(x) + i(\gamma^{\alpha\beta\gamma})_{ab} \times \\ & \times \left(-\frac{1}{72} H_{\alpha\beta\gamma}(x) - \frac{5}{2} i \Lambda_{\alpha\beta\gamma}(x) + \frac{1}{3} i \chi_{\alpha\beta\gamma}(x) \right) , \end{aligned} \quad (20)$$

$$\chi_a(z)|_{\theta=0} = \chi_a(x) ,$$

$$D_a \chi_b(z)|_{\theta=0} = -\frac{1}{4}(\gamma^{\alpha\beta})_{ab} F_{\alpha\beta}(x) - \frac{3}{2} i \lambda_a(x) \chi_b(x) , \quad (21)$$

⁺The arguments given here are not conclusive. Only a complete analysis of the Bianchi identities and constraints would prove the consistency of our equations. However this has now been carried out and will be presented elsewhere [25,26].

where $\Lambda_{\alpha\beta\gamma}$ is defined by $\Lambda_a \Lambda_b = (\gamma^{\alpha\beta\gamma})_{ab} \Lambda_{\alpha\beta\gamma}$ and similarly for $\chi_{\alpha\beta\gamma}$ in terms of the Yang-Mills spinor χ . In these transformation rules we see, as asserted above, that ϕ does not appear explicitly as, of course, is the case also in the field equations and the remaining transformation rules. For instance from

$$\delta E_M^A(z) = D_M \xi^A + E_M^C \xi_{BC}^A \quad (22)$$

we obtain, with $\xi^A(z) = (0, \xi^a(x))$ and using the above constraints and some information from the Bianchi identities, for $\theta=0$

$$\delta E_\mu^\alpha = i \bar{\xi} \gamma^\alpha E_\mu + i E_\mu^\alpha (\bar{\xi} \lambda) \quad . \quad (23)$$

In (23) we can either put $E_\mu^\alpha|_{\theta=0} = \tilde{e}_\mu^\alpha$ and $E_\mu^a|_{\theta=0} = \tilde{\phi}_\mu^a$, and arrive at the rather unconventional-looking spin 2 transformation rule of the manifestly scale-invariant form of the theory, or define the $\theta=0$ components so that (23) takes the conventional form and obtain the field equations in terms of canonical variables. In the latter case one will be forced to introduce explicit ϕ dependence. Thus from a superspace point of view, at least with the constraints given here (compare [10]), it seems natural to work with the manifestly scale-invariant fields. Interestingly enough, this seems to be the case also in the effective Lagrangian approach to string theories [15,16,17,13]. In, for example, the generating

functional Γ of [16],

$$\begin{aligned} \Gamma[\phi(x), \tilde{g}_{\mu\nu}(x), \tilde{B}_{\mu\nu}(x), \dots] = \\ = \sum_{\chi=2,0,-2} e^{\sigma\chi} \int [d\gamma_{ij}^{(\chi)}][dx^\mu] e^{-S}, \end{aligned} \quad (24)$$

where σ is a dimensionless parameter, γ^{ij} the metric on M_2 and

$$\begin{aligned} S = \int_{M_2} \frac{d\tau d\sigma}{4\pi\alpha'} \{ \alpha' \sqrt{\gamma} R(\gamma) \phi(x) + \sqrt{\gamma} \gamma^{ij} \partial_i x^\mu \partial_j x^\nu \tilde{g}_{\mu\nu}(x) \\ + \epsilon^{ij} \partial_i x^\mu \partial_j x^\nu \tilde{B}_{\mu\nu}(x) + \dots \} \quad , \quad (25) \end{aligned}$$

the metric is exactly the scale-invariant metric discussed previously. This is easily seen by noting that if $\phi = \text{const.}$, the evaluation of Γ for $\chi=2$ (the case we are interested in here) will not contain this constant since $\int_{M_2} \sqrt{\gamma} R(\gamma) \phi d\tau d\sigma$ is then a number (the Euler characteristic) and becomes just an overall factor [16,13].

Finally, we return to the problem of string-induced higher derivative terms. To get some understanding of whether there exists an intermediate supersymmetric field theory obtained by adding a finite number of such terms to the Chapline-Manton theory, one could try to add also the $\text{tr} R \wedge R$ term [1] to the right-hand side of the superspace Bianchi identity for H . If these equations are still consistent, either with the constraints presented here or with a relaxed version of them, supersymmetry is manifest and the question

is answered in the affirmative. It is, of course, hard to judge at this stage how likely this is to be the case. If, on the other hand, there does not exist an intermediate theory, it seems clear that by using superspace techniques rather than components, one is in a much better situation when trying to remedy the inconsistency e.g. by adding more terms on the right-hand side of (9). One should also address the question of which additions, if any, have to be made to the supersymmetry transformation rules in either of these cases. This question is, of course, crucial for the analysis carried out by Candelas et. al [4].

To summarize, we have presented a set of Bianchi identities and torsion constraints which will give rise to the on-shell Chapline-Manton theory. The particular choices made will lead to a very simple version of this theory as seen by comparing previous component results with e.g. the transformation rules given above. In trying to extend our results to the full string theory it seems crucial to have such a superspace formulation.

To construct the superspace version of the R^2 -terms [4], one presumably needs to be off-shell at least in the supergravity sector. Off-shell formulations have been discussed previously by Howe et al. [27] in the linear case without trying to put the off-shell fields into geometrical quantities. An attempt at a more geometrical description was recently

made by Kallosh [28] in terms of harmonic superspace⁺. (The main obstacle to constructing the whole Chapline-Manton theory off-shell seems to reside in the Yang-Mills sector⁺⁺, however.)

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⁺ A geometrical off-shell formulation of supergravity has now been found [25].

⁺⁺ A new way to go off-shell in d=10 super Yang-Mills was proposed in [29].

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