Agns Desolneux Lionel Moisan
Jean-Michel Morel

From Gestalt Theory to Image Analysis

A Probabilistic Approach

4: Springer

Contents

Preface	v
1 Introduction	1
1.1 Gestalt Theory and Computer Vision	1
1.2 Basic Principles of Computer Vision	3
2 Gestalt Theory	11
2.1 Before Gestaltism: Optic-Geometric Illusions	11
2.2 Grouping Laws and Gestalt Principles	13
2.2.1 Gestalt Basic Grouping Principles	13
2.2.2 Collaboration of Grouping Laws	17
2.2.3 Global Gestalt Principles	19
2.3 Conflicts of Partial Gestalts and the Masking Phenomenon	21
2.3.1 Conflicts	21
2.3.2 Masking	22
2.4 Quantitative Aspects of Gestalt Theory	25
2.4.1 Quantitative Aspects of the Masking Phenomenon	25
2.4.2 Shannon Theory and the Discrete Nature of Images	27
2.5 Bibliographic Notes	29
2.6 Exercise	29
2.6.1 Gestalt Essay	29
3 The Helmholtz Principle	31
3.1 Introducing the Helmholtz Principle: Three Elementary Examples	31
3.1.1 A Black Square on a White Background	31
3.1.2 Birthdays in a Class and the Role of Expectation	34
3.1.3 Visible and Invisible Alignments	36
3.2 The Helmholtz Principle and E-Meaningful Events	37
3.2.1 A First Illustration: Playing Roulette with Dostoevski	39
3.2.2 A First Application: Dot Alignments	41
3.2.3 The Number of Tests	42

3.3	Bibliographic Notes	...43
3.4	Exercise	44
3.4.1	Birthdays in a Class	...44
4	Estimating the Binomial Tail	47
4.1	Estimates of the Binomial Tail	47
4.1.1	Inequalities for $B(l, k, p)$	49
4.1.2	Asymptotic Theorems for $B(l, k, p) = P[S_i > k]$	50
4.1.3	A Brief Comparison of Estimates for $B(l, k, p)$	50
4.2	Bibliographic Notes	52
4.3	Exercises	52
4.3.1	The Binomial Law	52
4.3.2	Hoeffding's Inequality for a Sum of Random Variables	53
4.3.3	A Second Hoeffding Inequality	55
4.3.4	Generating Function	56
4.3.5	Large Deviations Estimate	57
4.3.6	The Central Limit Theorem	60
4.3.7	The Tail of the Gaussian Law	63
5	Alignments in Digital Images	65
5.1	Definition of Meaningful Segments	65
5.1.1	The Discrete Nature of Applied Geometry	66
5.1.2	The A Contrario Noise Image	67
5.1.3	Meaningful Segments	70
5.1.4	Detectability Weights and Underlying Principles	72
5.2	Number of False Alarms	74
5.2.1	Definition	74
5.2.2	Properties of the Number of False Alarms	75
5.3	Orders of Magnitudes and Asymptotic Estimates	76
5.3.1	Sufficient Condition of Meaningfulness	77
5.3.2	Asymptotics for the Meaningfulness Threshold $k(l)$	78
5.3.3	Lower Bound for the Meaningfulness Threshold $k(l)$	80
5.4	Properties of Meaningful Segments	81
5.4.1	Continuous Extension of the Binomial Tail	81
5.4.2	Density of Aligned Points	83
5.5	About the Precision p	86
5.6	Bibliographic Notes	87
5.7	Exercises	91
5.7.1	Elementary Properties of the Number of False Alarms	91
5.7.2	A Continuous Extension of the Binomial Law	91
5.7.3	A Necessary Condition of Meaningfulness	92

- 6 Maximal Meaningfulness and the Exclusion Principle 95**
 - 6.1 Introduction 95
 - 6.2 The Exclusion Principle 97
 - 6.2.1 Definition 97
 - 6.2.2 Application of the Exclusion Principle to Alignments 98
 - 6.3 Maximal Meaningful Segments 100
 - 6.3.1 A Conjecture About Maximality 102
 - 6.3.2 A Simpler Conjecture 103
 - 6.3.3 Proof of Conjecture 1 Under Conjecture 2 105
 - 6.3.4 Partial Results About Conjecture 2 106
 - 6.4 Experimental Results 109
 - 6.5 Bibliographical Notes 112
 - 6.6 Exercise 113
 - 6.6.1 Straight Contour Completion 113
- 7 Modes of a Histogram 115**
 - 7.1 Introduction 115
 - 7.2 Meaningful Intervals 115
 - 7.3 Maximal Meaningful Intervals 119
 - 7.4 Meaningful Gaps and Modes 122
 - 7.5 Structure Properties of Meaningful Intervals 123
 - 7.5.1 Mean Value of an Interval 123
 - 7.5.2 Structure of Maximal Meaningful Intervals 124
 - 7.5.3 The Reference Interval 126
 - 7.6 Applications and Experimental Results 127
 - 7.7 Bibliographic Notes 129
 - 7.8 Exercises 129
 - 7.8.1 Kullback-Leibler Distance 129
 - 7.8.2 A Qualitative *a Contrario* Hypothesis 130
- 8 Vanishing Points 133**
 - 8.1 Introduction 133
 - 8.2 Detection of Vanishing Points 133
 - 8.2.1 Meaningful Vanishing Regions 134
 - 8.2.2 Probability of a Line Meeting a Vanishing Region 135
 - 8.2.3 Partition of the Image Plane into Vanishing Regions 137
 - 8.2.4 Final Remarks 141
 - 8.3 Experimental Results 144
 - 8.4 Bibliographic Notes 145
 - 8.5 Exercises 150
 - 8.5.1 Poincaré-Invariant Measure on the Set of Lines 150
 - 8.5.2 Perimeter of a Convex Set 150
 - 8.5.3 Crofton's Formula 150

9 Contrasted Boundaries	153
9.1 Introduction	153
9.2 Level Lines and the Color Constancy Principle	153
9.3 A <i>Contrario</i> Definition of Contrasted Boundaries	159
9.3.1 Meaningful Boundaries and Edges	159
9.3.2 Thresholds	162
9.3.3 Maximality	163
9.4 Experiments	164
9.5 Twelve Objections and Questions	168
9.6 Bibliographic Notes	174
9.7 Exercise	175
9.7.1 The Bilinear Interpolation of an Image	175
10 Variational or Meaningful Boundaries ⁷	177
10.1 Introduction	177
10.2 The "Snakes" Models	177
10.3 Choice of the Contrast Function g	180
10.4 Snakes Versus Meaningful Boundaries	185
10.5 Bibliographic Notes	188
10.6 Exercise	188
10.6.1 Numerical Scheme	188
11 Clusters	191
11.1 Model	191
11.1.1 Low-Resolution Curves	191
11.1.2 Meaningful Clusters	193
11.1.3 Meaningful Isolated Clusters	193
11.2 Finding the Clusters	194
11.2.1 Spanning Tree	194
11.2.2 Construction of a Curve Enclosing a Given Cluster	194
11.2.3 Maximal Clusters	196
11.3 Algorithm	196
11.3.1 Computation of the Minimal Spanning Tree	196
11.3.2 Detection of Meaningful Isolated Clusters	197
11.4 Experiments	198
11.4.1 Hand-Made Examples	198
11.4.2 Experiment an a Real Image	198
11.5 Bibliographic Notes	198
11.6 Exercise	201
11.6.1 Poisson Point Process	201
12 Binocular Grouping	203
12.1 Introduction	203
12.2 Epipolar Geometry	204
12.2.1 The Epipolar Constraint	204
12.2.2 The Seven-Point Algorithm	204

12.3 Measuring Rigidity	205
12.3.1 F-rigidity	205
12.3.2 A Computational Definition of Rigidity	206
12.4 Meaningful Rigid Sets	207
12.4.1 The Ideal Case (Checking Rigidity)	207
12.4.2 The Case of Outliers	208
12.4.3 The Case of Nonmatched Points	210
12.4.4 A Few Remarks	214
12.5 Algorithms	215
12.5.1 Combinatorial Search	215
12.5.2 Random Sampling Algorithm	216
12.5.3 Optimized Random Sampling Algorithm (ORSA)	217
12.6 Experiments	217
12.6.1 Checking All Matchings	217
12.6.2 Detecting Outliers	219
12.6.3 Evaluation of the Optimized Random Sampling Algorithm	219
12.7 Bibliographie Notes	222
12.7.1 Stereovision	222
12.7.2 Estimating the Fundamental Matrix from Point Matches	223
12.7.3 Robust Methods	224
12.7.4 Binocular Grouping	224
12.7.5 Applications of Binocular Grouping	225
12.8 Exercise	225
12.8.1 Epipolar Geometry	225
13 A Psychophysical Study of the Helmholtz Principle	227
13.1 Introduction	227
13.2 Detection of Squares	227
13.2.1 Protocol	227
13.2.2 Prediction	228
13.2.3 Results	230
13.2.4 Discussion	231
13.3 Detection of Alignments	231
13.3.1 Protocol	232
13.3.2 Prediction	233
13.3.3 Results	233
13.4 Conclusion	234
13.5 Bibliographie Notes	235
14 Back to the Gestalt Programme	237
14.1 Partial Gestalts Computed So Far	237
14.2 Study of an Example	240
14.3 The Limits of Every Partial Gestalt Detector	242
14.3.1 Conflicts Between Gestalt Detectors	242

14.3.2 Several Straight Lines or Several Circular Arcs?	244
14.3.3 Influence of the A-contrario Model	246
14.4 Bibliographic Notes	247
15 Other Theories, Discussion	249
15.1 Lindenbaum's Theory	249
15.2 Compositional Model and Image Parsing	250
15.3 Statistical Framework	252
15.3.1 Hypothesis Testing	252
15.3.2 Various False Alarms or Error Rates Compared to NFA ...	253
15.3.3 Comparison with Signal Detection Theory	254
15.4 Asymptotic Thresholds	255
15.5 Should Probability Be Maximized or Minimized?	256
References	261
Index	271