SUPERSYMMETRY AND SUPERGRAVITY

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Talk given at the 1985 International Symposium on Lepton and Photon Interactions at High Energies
19 August - 24 August 1985, Kyoto, Japan
1. - MOTIVATIONS

You have heard from previous speakers how successful the standard model continues to be when subjected to experimental tests, even if theorists insist that it is very unsatisfactory and propose all sorts of improvements on the standard model. Even after choosing its gauge group and particle representations, specification of the standard model requires 20 more parameters:

\[ \begin{align*}
\mathcal{O}_{1,2,3} & \quad \mathcal{O}_{2,3} & \quad W_{\mu}, W_{\nu}, W_{\gamma}, \mathcal{O}_{\nu} & \quad \mathcal{O}_{\nu} \\text{gauge sector} & \quad \mathcal{O}_{\nu} \\text{fermion sector} \quad Higgs \text{ sector} \\
\end{align*} \]

arising in the three main sectors indicated. These in turn pose three major problems arising in attempts to go beyond the standard model: the unification, flavour and hierarchy problems.

The goal of unification is to relate the different gauge couplings observed to a single grand unified theory (GUT) gauge coupling \( g \), presumably associated with a simple non-Abelian group \( G \) containing \( SU(3) \times SU(2) \times U(1) \), of which the most attractive examples are \( G = SU(5), SO(10) \) and \( E_6 \). Estimates of gauge coupling renormalization put the grand unification scale above \( 10^{14} \) to \( 10^{15} \) GeV. The most direct test of GUTs is to look for baryon decay, a venture not yet crowned with success. Other tests are hard to find, there being no theoretical guarantee, for example that monopoles are copious enough, or neutrinos heavy enough, to be observable. Therefore, GUTs are in the phenomenological doldrums.

The problem of flavour is to understand the number of fermion generations, their "random"-seeming masses and Kobayashi-Maskawa mixing angles. A favoured approach is to propose that quarks and leptons are composite, and perhaps also the \( W^\pm, Z^0 \) and Higgs boson. The distance \( R_C = 1/\Lambda_C \) at which the preon constituents of the known "elementary" particles are bound is unknown and model-dependent. Lower bounds on \( \Lambda_C \) of order 1 to 100 TeV come from the impressive experimental agreement with the standard model while there is no clear upper bound on \( \Lambda_C \) for the constituents of quarks and leptons. If Higgses are composite, the hierarchy problem to be discussed
shortly suggests that their $\Lambda_c$ should be $O(1)$ TeV, and a similar
though less convincing argument is sometimes applied to the conjectured scale of $W^0 \nu$ and $Z^0$ compositeness. The uncertain scale $\Lambda_c$ makes it difficult to be confident of early experimental evidence for compositeness. Moreover, it is difficult to make composite models which are more than phenomenological confections, but also satisfy the necessary conditions of theoretical consistency. Fully consistent models are often \cite{6} more complicated that the standard model they are supposed to "explain". Moreover, the whole idea of "another layer of the onion" seems to me rather unimaginative.

The hierarchy problem is that of understanding the enormous mass ratios in physics, and in particular why and how $m_t/m_\rho \ll 1$. The favoured approach that I am to discuss in this talk is that of supersymmetry \cite{7} in which offending large corrections to the Higgs and hence the $W$ mass are cancelled \cite{8} by supersymmetric particles $\tilde{X}$ which should have masses $m_\tilde{X} \lesssim 1$ TeV in order for the corrections

$$\sum m_{\tilde{X}}^2 = O\left(\frac{\alpha_s}{\pi}\right) |m_{\rho}^2 - m_{\tilde{X}}^2|$$

(2)

to the Higgs boson mass to be "naturally" small:

$$\sum m_{\tilde{X}}^2 \lesssim m_{\rho}^2 = O(m_{W}^2)$$

(3)

Thus, unlike the other two problems mentioned above, the hierarchy problem sets an accessible scale for the new physics which solves it.

The natural framework \cite{9} for constructing supersymmetric models is $N = 1$, $d = 4$ supergravity \cite{10} which incorporates one local supersymmetry (cf. gauge theories) and Einstein's gravity in four dimensions. As we will see later, the natural next step is $N = 1$ supergravity in $d = 10$ dimensions \cite{11}. This theory by itself is not renormalizable, but may be the low-mass limit of a more complete theory such as the superstring \cite{12}. So far this is the only candidate "Theory of Everything", but others may exist. Fresh approaches to the unification and flavour problems have been suggested \cite{13}-\cite{16} on the basis of the superstring, which actually emerge already from the less ambitious $N = 1$, $d = 10$ supergravity theory. Thus, the solution to the hierarchy problem may lead naturally to solutions of the unification and flavour problems as well.

2. - THE HIERARCHY PROBLEM

We have already said that the hierarchy problem is that of understanding the origin of the minute ratio $m_t/m_\rho \ll 1$. Let us now see in more detail why this is so difficult. If the standard model is to be unitary in perturbation theory there must \cite{17} be at least one scalar Higgs boson with mass similar to that of the $W^+$:
\[ m_H/m_W = (\sqrt{\alpha})^{0^\pm 1} \]  

(4)

The upper bound in (4) is reached when the Higgs quartic self-coupling \( \lambda = 0(1) \) (8), while the lower bound corresponds to the minimal gauge boson contribution to \( \lambda = 0(\alpha^2) \) (9), taking the Higgs vacuum expectation value \( \langle 0 | H | 0 \rangle = (\sqrt{2}/g)m_W \) as given. However, the quadratic Higgs mass squared coefficient

\[ m_H^2 = - \mu^2 = - 2 \lambda \langle 0 | H | 0 \rangle^2 \]  

(5)

is notoriously unstable, acquiring large corrections in many different theoretical frameworks. The Higgs mass \( m_H \) is said\(^{20}\) to be "naturally" small if and only if corrections \( \delta m_H^2 \approx m_H^2 \) as in (3).

Examples of large corrections \( \delta m_H^2 \) are provided by the loops of Fig. 1, which are quadratically divergent in the standard model:

\[ \delta m_H^2 \left| \text{loops} \right| = \pm \frac{g}{2} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} = \pm O\left( \frac{\alpha}{\Lambda} \right)^2 \]  

(6)

We can regard the cut-off \( \Lambda \) as representing the energy at which new physics comes in to modify the standard model. Another example of a large correction \( \delta m_H^2 \) is provided\(^{21}\) by the couplings in Fig. 2a of light Weinberg-Salam Higgses (\( m_H = 0(10^2) \text{GeV} \)) to heavy GUT Higgses (\( m_H = 0(m_X) > 0(10^{15}) \text{GeV} \)). The vacuum expectation values \( \langle 0 | \phi | 0 \rangle = 0(\alpha_X) \) give

\[ \delta m_H^2 \left| \text{GUTs} \right| = O(m_H^2) > O(10^{15} \text{GeV})^2 \]  

(7)

Even if this contribution can be set to zero at the tree level by some miraculous symmetry as yet unknown, radiative corrections to the couplings of Fig. 2a, shown in Fig. 2b, generate\(^{21}\)

![Fig. 1: Quadratically divergent one-loop radiative corrections to scalar mass squared.](image)

(a) ![image](image) (b) ![image](image)

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![Fig. 2: GUT contributions to \( m_H^2 \) (a) without (b) with radiative corrections.](image)

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Fig. 2: GUT contributions to \( m_H^2 \) (a) without (b) with radiative corrections.
\[ \Delta m^2_{\text{Higgs}} \left|_{\text{loops}} \right. = O(\frac{\alpha}{\pi}) \times O(m^2_{\chi}) \geq O(10^{14} \text{GeV})^2 \]  
(8)

Higher order GUT loops to \( O(\alpha/\pi)^{0(13)} \) would have to be cancelled in order to satisfy the naturalness condition (3). Finally, it has been argued[22] that in quantum gravity elementary scalar masses acquire corrections

\[ \Delta m^2_{\text{Higgs}} \left|_{\text{gravity}} \right. = O(m^2_F) = O(10^{17} \text{GeV})^2 \]  
(9)

All the corrections (7) to (9) are "unnatural" in the sense that they give corrections \( \Delta m^2 \gg m^2_F \), as do the standard model loops (6) unless the cut-off

\[ \Lambda \lesssim 1 \text{ TeV} \]  
(10)

representing the appearance of new physics at this scale.

Thus the hierarchy problem sets the scale for its own solution in the near future, much as the absence of flavour-changing neutral currents told us to expect charmed particles with masses \( \lesssim 0(5) \text{ GeV} \). One of the alternative new physics solutions is to make the Higgs, quarks and leptons and/or \( W^\pm \), \( Z^0 \) composite at a scale \( \Lambda \sim \Lambda \lesssim 1 \text{ TeV} \). These ideas are the province of Michael Peskin[5], and my opinions on them were expressed in Section 1. My task is to report attempts to solve the hierarchy problem via cancellations of the diagrams in Figs. 1, 2 exploiting supersymmetry (susy)). This strategy is based on the observation that the boson loops of Fig. 1a and the fermion loops of Fig. 1b have opposite signs:

\[ \Delta m^2_{\text{Higgs}} \left|_{\text{loops}} \right. = \pm O(\frac{\alpha}{\pi}) S \left[ + O(m^2_B) \right] \times \pm O(\frac{\alpha}{\pi}) S \left[ + O(m^2_F) \right] \]  
(11)

directly related to Bose-Einstein versus Fermi-Dirac statistics. Adding together the two contributions (11) we get a residual

\[ \Delta m^2_H = O(\frac{\alpha}{\pi}) |m_B^2 - m_F^2| \]  
(12)

which is "natural" (3) if

\[ |m_B^2 - m_F^2| \lesssim O(m^2_W)/O(\frac{\alpha}{\pi}) = O(1 \text{ TeV})^2 \]  
(13)

Here it is the difference in boson and fermion masses squared which provides the new physics cut-off

\[ \Lambda^2 \propto |m_B^2 - m_F^2| \approx 1 \text{ TeV}^2 \]  
(14)

Supersymmetric theories also have remarkable no-renormalization properties\[24\] which cancel systematically all the GUT radiative corrections (8) and the gravity corrections (9). Thus, the naturalness aspect (3) of the hierarchy problem is solved. What remains is to
explain the origin of the hierarchy $m_u/m_p$ which we now know how to stabilize.

3. - **SUPERSYMMETRIC MODELS**

Cancelling systematically the loops of Figs. 1 and 2 requires, as seen in Table 1, bosons and fermions with identical internal quantum numbers (colour, $Q_{em}$, $B$, $L$) and hence identical gauge and Yukawa couplings:
\[
\begin{align*}
\tilde{q} \tilde{f} V &\rightarrow \tilde{q} \tilde{f} V, \quad \tilde{q} \tilde{f} \tilde{f} V; \quad \tilde{q} \tilde{f} \tilde{f} V \rightarrow \tilde{q} \tilde{f} \tilde{f} V; \quad (\tilde{q}^2, \tilde{q}^2) \times \tilde{f}^4 \tag{15}
\end{align*}
\]

Table 1 shows the sparticle content of the minimal supersymmetric standard model (MSSM): unfortunately no known particle can be the supersymmetric partner of any other known particle. Note that the MSSM actually requires two light Higgs doublets (and their shiggs spartners) in order to provide masses for all the quarks and leptons, and to avoid chiral triangle gauge anomalies\(^8\). The two Higgs doublets yield three neutral Higgs bosons $H^0$, $H^0'$, and $a$, and two charged ones $H^{\pm25}$, which we will discuss later along with possible extensions of the MSSM spectrum suggested by superstring (sorry, $d = 10$ supergravity) models. While the couplings (15) of sparticles are completely fixed, the masses are model-dependent, though they should weigh less than 0(1) TeV if the technical naturalness aspect of the hierarchy problem is to be solved as outlined above. However, we do know that susy must be broken, because particles and their spartners have different masses; e.g.,
\[
\begin{align*}
\tilde{t} \neq t, \quad \tilde{q} \neq q \quad \tilde{e} \neq e \quad \tilde{\mu} \neq \mu \quad \tilde{\tau} \neq \tau \quad \ldots
\end{align*}
\]

Our next job is to see how susy can be broken.

**Table 1:** Spectrum of Sparticles

<table>
<thead>
<tr>
<th>Particle</th>
<th>Spin</th>
<th>Sparticle</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quark $q_{L,R}$</td>
<td>$\frac{1}{2}$, $\frac{1}{2}$</td>
<td>Squark $\tilde{q}_{L,R}$</td>
<td>0, 0</td>
</tr>
<tr>
<td>Lepton $\ell_{L,R}$</td>
<td>$\frac{1}{2}$, $\frac{1}{2}$</td>
<td>Slepton $\tilde{\ell}_{L,R}$</td>
<td>0, 0</td>
</tr>
<tr>
<td>Photon $\gamma$</td>
<td>1</td>
<td>Photino $\tilde{\gamma}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$W^+$</td>
<td>1</td>
<td>$W$-ino $\tilde{W}^+$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$Z^0$</td>
<td>1</td>
<td>Z-ino $\tilde{Z}^0$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Higgs $H$</td>
<td>0</td>
<td>Shiggs $\tilde{H}$</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>
Should it be broken explicitly (i.e., in the Lagrangian), or spontaneously (i.e., in the vacuum)? Explicit susy breaking is ugly, contradicts our experience with gauge theories, and gives rise to violations of unitarity when the gravitino is introduced in locally supersymmetric theories. Far preferable is spontaneous susy breaking, which is achieved in globally supersymmetric theories via a massless Goldstone fermion $\chi$ coupled to the vacuum by the susy charge $Q$:

$$\langle \chi | Q | 0 \rangle = \lambda^2 \neq 0$$  \hspace{1cm} (17)

and necessarily implies positive field energy $V$

$$\langle 0 | V | 0 \rangle = \lambda^4 > 0$$  \hspace{1cm} (18)

The positive piece of the field energy $V$ (18) may be provided either by the gauge sector of the theory (D-breaking) or by the Yukawa couplings $\lambda$ (F-breaking). D-breaking requires a new $U(1)$ gauge group beyond the hypercharge of the standard model, which has couplings of the same sign to all the known particles. This gives rise to problems with chiral triangle anomalies, which can only be cancelled by introducing many new particles. These bring with them the danger that they drive gauge couplings to infinity at some energy $<m_p$ or $m_\chi$, and it is a tricky job to avoid a susy vacuum somewhere in field space which would have zero field energy and hence be energetically preferred over the configuration with broken susy (18). Although it is not impossible to build global D-breaking susy models they have now been largely abandoned. On the other hand, F-breaking models require additional gauge singlet fields which must have rather artificial couplings and are in danger of being technically "unnatural". Thus these have also proved unsatisfactory, and most model-builders have turned to local supersymmetry or supergravity.

There is an elegant mechanism for the spontaneous breakdown of local susy which is a direct analogy of the Higgs mechanism in conventional gauge theories. Just as in that mechanism, a helicity 0 Goldstone boson is eaten by a massless gauge boson with helicity states $\pm 1$, giving it the total of three polarization states appropriate for a massive gauge boson, so in the super-Higgs mechanism the Goldstone fermion $\chi$ with helicities $\pm \frac{1}{2}$ is eaten by the massless gravitino with helicity states $\pm \frac{3}{2}$, giving it the total of four polarization states appropriate for a massive gravitino:

$$m_{\tilde{\chi}} = O(\sqrt{\lambda_m})$$  \hspace{1cm} (19)

Moreover, this spontaneous breakdown of susy can be realized while keeping zero vacuum energy: $\langle 0 | V | 0 \rangle = 0$ and the cosmological constant vanishes.
There are many motivations for making models\textsuperscript{33}) based on \( N = 1, d = 4 \) supergravity\textsuperscript{9}). They make susy local, cf., gauge theories. They are a step towards the unification of gravity with the other particle interactions. They are inevitable, since gravity and susy both (i) exist. They have fewer infinities than conventional gravity coupled to matter. However, the most important feature for the practical purposes of model-building is this super-Higgs mechanism for spontaneous susy breaking. It provides an effective low energy (\( E \ll m_p \)) theory which is supersymmetric apart from soft susy breaking terms of which the most important are scalar masses \( m_0 \) and gaugino masses \( m_{\frac{3}{2}} \). Both of these can be non-zero only if \( m_{\frac{3}{2}} \neq 0 \), but while \( m_0, m_{\frac{1}{2}} = 0(m_{\frac{3}{2}}) \) in many simple models, they can be much smaller, or somewhat larger than \( m_{\frac{3}{2}} \). The observable \((q_1, l, H)\) fields are not by themselves sufficient to get \( m_{\frac{3}{2}} = 0(m_W) \). This is generally achieved by adding in an extra "hidden" sector, linked to ordinary matter through small couplings \( O(1/m_p) \) only. In most models, all the scalar masses \( m_0 \) are equal to each other at tree level, and the gaugino masses are also universal, though this is not an inevitable feature. These universality properties hold when the effective low energy theory is renormalized at a scale \( \mu = O(m_p) \), where the gravitino and hidden sector interactions decouple from the effective low energy theory. At lower renormalization scales \( \mu < m_p \), the different scalar and gaugino masses are renormalized \( \text{differently}\textsuperscript{35}) \), just like the SU(3), SU(2) and U(1) gauge couplings\textsuperscript{36}) and fermion masses\textsuperscript{37}) in GUTs. For financial and technological reasons, we are currently limited to performing experiments at energies \( E = 0(m_H) \), and we hope to measure the physical particle masses renormalized at \( \mu = O(m_0, m_1) = 0(m_W) \). In the MSSM these are related as follows to the input susy breaking parameters \( m_{0,1} \):\textsuperscript{38})

\[
\begin{align*}
M_G^2 &\approx m_0^2 + 7m_{1/2}^2, \\
M_L^2 &\approx m_0^2 + 0.5m_{1/2}^2, \\
M_Q^2 &\approx 3m_{1/2}^2 + 7m_{1/2}^2
\end{align*}
\] (20)

but the renormalizations may be different in models with non-minimal low energy spectra, as suggested for example by the superstring\textsuperscript{39}). The input parameters \( m_0 \) and \( m_{1/2} \) are model-dependent, but should be \( \sim 1 \) TeV if the naturalness aspect of the hierarchy problem is to be solved. Next we must tackle the more fundamental problem of the origin of the hierarchy.

4. - GENERATING THE HIERARCHY

We have seen how susy/supergravity can be used to safeguard \( m_T/m_p \ll 1 \), and our next job is to obtain a formula for this small ratio. It is natural to look for a formula analogous to that for the small ratio \( \Delta_{QCD}/m_X \) in GUTs\textsuperscript{1}):

\[
\Delta_{QCD} = m_X \exp\left(-\frac{\alpha(m_X)}{\alpha(m_p)}\right)
\] (21)
This could perhaps be done using non-perturbative susy breaking effects in susy gauge theories\(^{40}\), but no realistic scenario based on this mechanism has been developed. A more promising approach\(^{39,41}\) is to exploit the logarithmic evolution\(^{42}\) of the soft susy breaking Higgs mass squared \(m_H^2\), which can be driven from the positive values \(m_0^2\) specified by the hidden sector at a renormalization scale \(\mu = 0(m_P)\) to negative values at a renormalization scale \(\mu_0 = m_P \exp(-O(1)/\alpha_t)\), where \(\alpha_t\) is a heavy quark Yukawa coupling, e.g., for the top quark \(\alpha_t \equiv \frac{g^2_2 m_t^2 E}{4\pi}\), as seen in Fig. 3. Once \(m_H^2\) becomes negative at \(\mu_0\), then weak gauge symmetry breaking is possible: 

\[
\langle 0|H|0\rangle = 0(\mu_0) = 0(\mu_H), \quad \text{and hence}
\]

\[
m_w = \frac{3}{2} \langle 0|H|0\rangle = \langle 0(\mu_p) \rangle = m_p \exp\left(\frac{O(1)}{\alpha_t}\right) \tag{22}
\]

It is the linear nature of the renormalization of the soft susy breaking parameters \(m_i^2\):

\[
\mu \frac{\partial}{\partial \mu} m_i^2 = \left(\frac{\alpha}{4\pi}\right) \text{matrix} \cdot m_i^2
\]

(23)

which tells us that \(m_i^2/m_P\) is a pure number, and the slow logarithmic rate of evolution (23) tells us that this number is very small. A suitable hierarchy can easily\(^{33}\) be achieved with

\[
m_t = \int_{\mu_H}^{\mu_0} \langle 0|H|0\rangle = 0(400\text{GeV}) \tag{24}
\]

as suggested by the UAI report\(^{44}\) of last year.

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The above scenario\(^{43}\) works for any value of the susy breaking mass parameters \(m_0, m_1\) fixed by hand to be \(m_0 \ll \mu_H\), but we would like to be able to determine these dynamically also. Indeed, models do exist in which the absolute values of \(m_0, m_1\) and \(m_3/2\) are free at the tree level\(^{43,46}\), and so may be determined by radiative corrections\(^{47,48}\):

\[
m_0, m_1, m_2, m_3, m_5 = m_P \exp\left(\frac{O(1)}{\alpha_t}\right) \tag{25}
\]
also. In these so-called "no-scale" models\(^{49}\), all small mass scales in physics would be dynamically generated from \(m_p\), and the hierarchy problem solved, in principle. The basic idea is sketched in Fig. 4. Starting from a flat tree level potential\(^{45}\) as in Fig. 4a, the radiative corrections of Fig. 1 lead to weak gauge symmetry breaking and hence a negative radiatively corrected potential at scales \(\mu \lesssim \mu_0\). In this region

\[
\mathcal{V} \propto - \frac{|m_{\mu}^2|}{\alpha_\mu^2}
\]

(26)

for small \(m_{\mu}^2\), so that larger values of \(m_{\mu}^2\) and hence \(m_0, \mu_0\) are preferred. But the potential is positive semidefinite for \(m_0, \mu_0 > m_0\), and as seen in Fig. 4b there is a minimum of \(\mathcal{V}\) for some

\[
m_{\omega, \mu_0} = O(m_0) = m_p \exp(-O(1)/\alpha_\mu^2)
\]

(27)

leading\(^{47}\) to the desired result (25). There are uncertainties in the treatment of radiative corrections which could be resolved with a more fundamental understanding of the origin of the no-scale models. They have long been known to have remarkable non-compact group-theoretical structures\(^{45},46\) and to arise in many extended \(N > 1\) supergravity theories\(^{50}\). An exciting recent development has been their emergence\(^{51}\) from the superstring.

Fig. 4: Flat potential (a) without, (b) with radiative corrections.

5. - BEYOND \(N = 1, d = 4\) SUPERGRAVITY?

There have been many theoretical ideas reaching beyond the simple \(N = 1\) susy and minimal \(d = 4\) dimensions discussed so far. A fundamental road-block to most of these ideas has been the existence of parity violation. Theories with \(N > 1\) supersymmetries have left-right symmetric fermion spectra, and there is no way to push the unwanted "mirror" fermions beyond \(O(1)\) TeV in mass\(^{52}\). The same difficulty arises in higher-dimensional theories\(^{53}\) unless one goes to even-dimensional space-times such as \(d = 6\) or 10 where chiral fermion spectra are possible. The largest field theory with susy and parity violation is \(N = 1, d = 10\) supergravity with Yang-Mills\(^{11}\), so it is natural for attention to focus on the phenomenology of this theory. Remarkably, this theory can\(^{54}\) be solved of anomalies for only two possible choices of the gauge group, namely \(SO(32)\) and
E_6 \times E_6'$. Of these, only the latter can give chiral fermions when the six surplus dimensions are compactified\textsuperscript{13}, and it contains an E_6 subgroup suitable for use as a GUT. Thus $N = 1, d = 10$ E_6 \times E_6' supergravity is the unique field theory candidate for a "Theory of Everything".

However, this field theory is flawed. Tree amplitudes grow rapidly with energy and violate unitary bounds close to $m_p$, presumably quantum corrections are unrenormalizably infinite, and the number of terms required to make the theory anomaly-free, while retaining supersymmetry, is very large and may even be infinite\textsuperscript{25}. To cure these problems, one probably needs to go beyond field theory and the currently available extension is the heterotic superstring\textsuperscript{26}. This contains the $N = 1, d = 10$ supergravity particles as its massless sector, plus infinitely many other excited states with masses $O(m_p)$. As it is probably finite and anomaly-free\textsuperscript{12}, it is the only true candidate "Theory of Everything" that we possess, but there is no proof that it is unique. There may well be other consistent theories of extended objects which reduce to $N = 1, d = 10$ supergravity in the massless sector and it is not yet excluded that even the superstring has uncontrollable infinities in higher orders.

Much of the phenomenology done in the name of the superstring actually does not depend in an essential way on the higher massive string degrees of freedom. It does rely on many supplementary features of the massless $d = 10$ supergravity theory which were deduced first by superstrings, though they in fact are required by the consistency of the supergravity theory alone. In addition to the $E_6$ solution to the unification problem mentioned above, which breaks to a low energy gauge group at least as large as $SU(3)_C \times SU(2)_L \times U(1)_Y$, the superstring (sorry, $d = 10$ supergravity) also offers solutions to many aspects of the flavour problem. The number of generations may just be half the Euler number of the manifold in which the six surplus dimensions are compactified\textsuperscript{13}, while Yukawa couplings are in principle calculable as the overlaps of the wave functions of zero modes in the compactification manifold\textsuperscript{15}, and their properties are determined by topological considerations\textsuperscript{15},\textsuperscript{16}. Moreover, $d = 10$ supergravity predicts additional light ($\lesssim 1$ TeV) matter particles and at least one extra neutral U(1) gauge boson, to whose phenomenology\textsuperscript{39} we return in the latter part of this talk.

6. - SIGNATURES OF SUPERSYMMETRY

In most models, sparticles carry a new multiplicatively conserved parity quantum number: $R = +1$ for particles, $R = -1$ for sparticles\textsuperscript{28}). $R$ conservation follows from $B$ and $L$ conservation in the minimal supersymmetric standard model, and $R$ is obviously conserved by the conventional supersymmetric interactions:
\[
\begin{align*}
\overline{f g} \nu & \rightarrow \overline{f g} \bar{\nu}, \  \overline{f g} \nu & \rightarrow \overline{f g} \bar{\nu}; \quad \overline{f g}^2 \rightarrow \overline{f g}^2; \quad \overline{f g}^2
\end{align*}
\] (28)

There are three important phenomenological consequences of R conservation. (I) Particles are always produced in pairs, e.g.,
\[
\ell^+ \ell^- \rightarrow \mu^+ \mu^-, \  \overline{\nu} \nu \rightarrow \overline{\nu} \nu,
\] (29)
as can be seen immediately from the interactions (28). (II) Heavier sparticles decay into lighter sparticles, e.g.,
\[
\overline{\mu} \rightarrow \mu^- \nu, \  \overline{\nu} \rightarrow \overline{\nu} \nu, \  \overline{\nu} \rightarrow \overline{\nu} \nu
\] (30)
(III) The lightest sparticle is absolutely stable, because it has no available decay mode.

This third property means that there should be many of the lightest sparticles present in the Universe today as supersymmetric relics from the Big Bang, and therefore cosmology imposes severe constraints on their properties\(^{39})^{,48}\). Cosmology in fact decrees that the lightest sparticle be electrically neutral and have no strong interactions\(^{38}\). If it had either electromagnetic or strong interactions, it would condense in galaxies and planets, and bind to form anomalous heavy nuclei. Their calculated abundance is \(10^{-10}\) or more\(^{59}\), to be compared with upper limits\(^{60}\) of order \(10^{-20}\) to \(10^{-10}\) for anomalous heavy isotopes weighing \(1\) TeV as expected for sparticles. Therefore, the lightest sparticle should be neutral and only weakly interacting, so that it does not condense in galaxies and planets, and also escapes from experimental apparatus after production at an accelerator. Thus the characteristic signature of susy is missing energy-momentum p\(_{\mu}\). Possible candidates for the lightest sparticle are the \(\tilde{\nu}\) of spin 0, \(\tilde{\nu}/R^0\) of spin \(\frac{1}{2}\) and the gravitino of spin 3/2. More detailed astrophysical, cosmological and particle physics arguments\(^{58},61,62\) favour the photino \(\tilde{\gamma}\), as implicitly assumed in the decay modes (30).

As already mentioned in Section 3, while the absolute scale of the sparticle masses is uncertain in the range 20 GeV (experimental lower limit) to 1 TeV (plausible theoretical upper limit), one can predict mass relations in a range of supergravity models\(^{38}\). Equation (20) gives an example in the MSSM. In the low energy limit of the superstring with a minimal SU(3)\(_C\times\text{SU}(2)_L\times\text{U}(1)_Y\times\text{U}(1)'\) gauge group, the corresponding predictions if \(m_0 < m_{3/2}\) as an initial condition are\(^{39}\):
\[
\begin{align*}
m_{\tilde{\mu}} : m_{\tilde{\nu}} : m_{\tilde{e}} : m_{\tilde{\chi}_0} : m_{\tilde{\chi}_R} : m_{\tilde{\chi}_L} &= \left|1::0.75:0.9:0.38::0.4\right|
\end{align*}
\] (31)
Notice that in both (20) and (31) the photino emerges lighter than the other sparticles, as desired by the previous cosmological argument\(^{58}\).
In addition to sparticles, susy models often require other particles accessible to experiment. For example, all susy models require at least two light Higgs doublets and hence five physical Higgs bosons, two charged $H^\pm$ and three neutral $H^0$, $H'^0$ and $a$ with scalar, scalar and pseudoscalar couplings respectively to quarks and leptons. Although

$$m_{H^\pm} \gg m_{W^\pm}, m_{H^0}, m_{W^0}$$

in general, the lighter scalar $H^0$ and the pseudoscalar $a$ could well be lighter than the $Z^0$, perhaps weighing only a few GeV\(^2\)). Moreover, $E_6$ models obtained by compactification supergravity or the superstring from $d = 10$ contain generations with $27$ matter fields each containing extra 3 and 3 $SU(2)$ singlet "quark" fields $g$ and $g'$ and two $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlets $N$ and $N'$ in each generation\(^4\). Finally, as already mentioned, this $E_6$ is broken down to a residual low energy gauge group containing at least one extra $U(1)'$ factor beyond the standard model\(^1\). We will return later to the phenomenology of these indirect harbingers of susy, after considering direct susy searches.

7. - HADRONS NISSING ENERGY EVENTS

The UA1 Collaboration reported\(^6\)) observing several hadronic events with large missing energy among their 1983 data: mono-, di- and trijet + missing $p_T$ events, where a hadronic jet is counted if its transverse energy $E_T > 12$ GeV. The interpretation of these events has passes through several stages.

Round 1 - Optimism

The fact that relatively few events with $p_T > 4\sigma$ (the standard deviation in $|p_T|$ is $\sigma = 0.7 \sqrt{E_T}$) enabled one to set an upper bound on susy cross-sections. Since these could be calculated "reliably", i.e., to within a factor $O(2)$, as functions of $m_\tilde{g}$ and $m_\tilde{\chi}$ as seen in Fig. 5, one could give lower bounds\(^6\))

$$m_\tilde{\chi} \geq O(4\sigma) \sqrt{E_T}$$

if $m_\tilde{g} \gg m_\tilde{\chi}$, and

$$m_\tilde{\chi} \geq O(4\sigma) \sqrt{E_T}$$

if $m_\tilde{g} \gg m_\tilde{\chi}$. Even more optimistically\(^6\)), one could seek to interpret the observed monojet events as due to susy with either

$$(a) \quad m_\tilde{g} \leq O(4\sigma) \sqrt{E_T} \ \text{or} \quad (b) \quad m_\tilde{\chi} \geq 40 \text{ GeV}$$

Note that according to Fig. 5, the monojet topology dominates for $m_\tilde{g} < 40$ GeV because the trigger and cuts used by UA1, even though one might have expected events with (a) four, or (b) two jets to dominate. The MSSM relations (20) led one to expect\(^6\)) in either
of the two cases (34) that $m_T \sim$ few GeV, and in the second case (34b) that $m_T \lesssim 30$ GeV. However, in this second case, one would also have expected $m_T > m_{\tilde{g}}$, which leads to a large increase in the total susy cross-section, since $\sigma_{88}$ is increased if $m_{\tilde{g}}$ is not $\gg m_T$, as was assumed in deriving (33b), and one must also add $\sigma_{\tilde{g}T}$ (which is quite large) and $\sigma_{\tilde{g}8}$. In this case the bounds (33) become

$$m_{\tilde{g}} \times m_{\tilde{q}} > 0.6 \text{ pb}$$

and one might expect on the basis of Fig. 5 more dijets than monojets. However, it is difficult to estimate the dijet/monojet ratio reliably because of uncertainties in jet energy measurements, etc.

**Round 2 - The backgrounds strike back**

The UA1 Collaboration always said that events with $p_T^2 < 1000$ GeV were compatible with standard model backgrounds such as jet fluctuations, heavy flavours such as $b\bar{b}$ production, $W + t\bar{t}$, etc... Moreover, the 1983 event with $p_T = 34$ GeV was identified as a likely $W + t\bar{t}$ decay. However, there has been discussion whether the standard model backgrounds are negligible for $p_T^2 > 1600$ GeV. Table 2 below lists some background calculations by theorists and approximations to those calculated by the UA1 Collaboration itself and shown by Carlo Rubbia at this meeting.

**Table 2: Backgrounds to monojets (per 100nb$^{-1}$ at $\sqrt{s} = 540$ GeV)**

<table>
<thead>
<tr>
<th>Background</th>
<th>Ref.70)</th>
<th>Ref.71)</th>
<th>Ref.69)</th>
</tr>
</thead>
<tbody>
<tr>
<td>jet + ($Z^0 + \nu\bar{\nu}$)</td>
<td>0.36</td>
<td>0.32</td>
<td>$\sim 1$</td>
</tr>
<tr>
<td>$W + e$ lost</td>
<td>0.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W + \mu$ lost</td>
<td>&lt;0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W + \tau$ lost</td>
<td>0.25</td>
<td></td>
<td>$\sim 0.6$</td>
</tr>
<tr>
<td>$W + \tau$ seen</td>
<td>0.50</td>
<td>$\sim 0.5 \text{ to } 1.6$</td>
<td></td>
</tr>
</tbody>
</table>

[see also Ref. 72]
These estimates of order 1 to 2 events/100nb⁻¹ can be compared with the five events seen. They indicated that the backgrounds were not negligible, and that more data would be needed to establish any monojet signal beyond the standard model.

**Round 3 - Gluino Wars**

Even before the 1983 UA1 data, it had been proposed⁷³) that one look for a monojet signature due to a light gluino present intrinsically in one nucleon fusing with a q or q̄ from the other nucleon to form a heavy squark which subsequently decays into q+γ:

\[ \tilde{g} (m_{\tilde{g}} \sim 500 \text{ GeV}) + q \rightarrow \tilde{q} (m_{\tilde{q}} \sim 100 \text{ GeV}) \rightarrow q + \gamma \]  

(36)

These monojets would be guaranteed and gold-plated, with no need to appeal to UA1 trigger conditions, cuts or inefficiencies to lose secondary jets, unlike the earlier scenarios (34)/⁴. However, the scenario (36) necessarily brings with it other mechanisms for missing energy events, notably \( \tilde{g} + q \rightarrow q + q + \gamma \) and \( gg \) or \( qq \rightarrow gg \). The former probably also gives mainly monojets, with the jet(s) from \( g \rightarrow q + \gamma \) decay being lost. For \( m_{\tilde{g}} = 0(5) \text{ GeV} \), \( \sigma(gg,qq \rightarrow gg) \) is very large, but most events are lost by the UA1 trigger and cuts. Moreover, the \( p_T \) is softened by non-trivial \((\tilde{g} \text{ jet}) + (\tilde{g} \text{ hadron})\) fragmentation, so it was suggested⁷⁴,⁷⁵ that there might be a "window of opportunity" for

\[ 5 \text{ GeV} \leq m_{\tilde{q}} \leq 20 \text{ GeV} \]  

(37)

In this case, most events with \( p_T > 4\sigma \) have \( p_T < 40 \text{ GeV} \), and it is necessary to model carefully susy events in this range of \( p_T \) to see if there really is a "window" (37). Since most of the light \( gg \) cross-section is outside, but close, to the nominal UA1 trigger and cuts, it is important to model them and measurement errors very carefully. It was on the basis of such a detailed study (see Fig. 6)

![Fig. 6: Monojet cross-sections with the following effects included: fragmentation, \( E_T \) from the surrounding event and its fluctuations, the total \( E_T \) trigger and measurement errors in \( p_T \) and in trigger jet \( E_T \). The error bars from \( m_{\tilde{g}} = 5 \text{ GeV} \) represent the Monte Carlo uncertainty in evaluating the cross-sections. The errors are negligible for larger \( m_{\tilde{g}} \).]
that we claimed there was no such "window" (37), and this feeling was reinforced by the large increase in the $p_T$ cross-section found when gluon bremsstrahlung: $gg \rightarrow gg\tilde{g}$, $q\tilde{q} \rightarrow gg\tilde{g}$ was included\(^{78}\). However, the opposing point of view was expressed by other authors\(^{78}\) and although we would question some of the assumptions made elsewhere about perturbative $\tilde{g}$ jet evolution, $E_T$ fluctuations in the rest of the event, and smearing due to errors in $p_T$ and jet $E_T$ measurement, the prosecution was unable to get a conviction against the light $\tilde{g}$ (37). The theoretical jury was divided, and uncertainties should perhaps confer the benefit of the doubt on the accused. In a moment we will discuss the retrial with the benefit of the new 1984 data\(^{69}\).

Before doing so, however, note that the light $\tilde{g}$, heavy $\tilde{q}$ scenario (36) also has potential problems\(^{69}\) with cosmology\(^{38}\). The MSSM, mass relations (20) would suggest in this case that

$$m_{\tilde{g}} \sim 10^3 m_{\tilde{q}} \ll m_{\tilde{q}}, \sim \mathcal{L}$$  \hspace{1cm} (38)

in which case standard cosmological calculations yield a relic $\tilde{\gamma}$ density far above the allowable critical closure density, as seen in Fig. 7. Although this problem can be avoided by modifying the MSSM\(^{80}\) modifications have little else to recommend them.

![Fig. 7: Regions of $(m_{\tilde{g}}, m_{\tilde{q}})$ parameter space consistent with cosmological and terrestrial particle physics constraints. The dashed lines traces isobars of constant cosmological relic photino density equal to the closure density for $H_0 = 25$ km s$^{-1}$ Mpc$^{-1}$, $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$ and $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$.](image-url)
Round 4 - The truth, the whole truth, ...

Earlier in this meeting, Carlo Rubbia has presented the 1984 \( p_T \) data. You have seen that there are no new monojet events with \( p_T > 50 \text{ GeV} \) analogous to event A of 1983. Moreover, despite the increase in integrated luminosity and centre-of-mass energy, the number of 1984 events with \( p_T \approx 40 \) to 50 GeV is the same as in 1983. This corresponds to a reduction in apparent effective rate by a factor 0(3), although the data are clearly consistent, given the statistical errors on small event samples. However, now the gross number of events is completely consistent with the number of standard model background (\( \tau, b\bar{b}+c\bar{c}, t\bar{t}, W, Z, \ldots \)) events suggested by Table 2. As pointed out earlier, this was always true of events with \( p_T < 40 \text{ GeV} \); now it is also true for \( p_T > 40 \text{ GeV} \). However, one can still ask whether detailed differences between the data and the backgrounds are possible. There has been considerable discussion of the narrowness of the 1983 monojets, and one possible measure of this is the jet energy collimation

\[
F \equiv \frac{\text{energy in } \Delta R = \sqrt{\Delta \phi^2 + \Delta \eta^2} < 0.4}{\text{energy in } \Delta R = \sqrt{\Delta \phi^2 + \Delta \eta^2} < 1.0}
\]

around the jet axis azimuthal angle \( \phi \) and pseudorapidity \( \eta \). We find (Fig. 8) that \( \tilde{q} \) and \( \tilde{g} \) monojets would be thinner than conventional QCD jets, but fatter than \( \tau \) monojets. The UA1 Collaboration prefers to present data in terms of a likelihood variable \( L \) which is computed from a combination of \( F \) (39), the \( \Delta R \) of the most energetic charged particle in the monojet, and the observed charged multiplicity. There are 0(10) thin monojets with values of \( L \) typical for the \( \tau \), and other fatter monojets resembling conventional QCD jets. There is also a handful of events with intermediate values of \( L \) and \( E_T(\text{jet}) > 40 \text{ GeV} \). These events may just be some combination of

---

**Fig. 8:** Energy collimation for \( \tilde{q}\tilde{q} \) and \( \tilde{g}\tilde{g} \) monojets.
\( \tau \) monojets and other standard model backgrounds, though the exclusion of more exciting hypotheses requires still more data. If one accepts that the gross number of monojet events with \( p_T > 4 \sigma \) is compatible with backgrounds, the reduced 1984 monojet rate compared with 1983 allows\(^{33},^{84}\) the previous bounds [(33) and (35)] to be improved (see Fig. 9):

\[
\begin{align*}
(a) \quad m_{\tilde{q}} & \gtrsim (45 \text{ to } 50) \text{ GeV} \\
(b) \quad m_{\tilde{q}^*} & \gtrsim (50 \text{ to } 60) \text{ GeV} \\
(c) \quad m_{\tilde{q}} \times m_{\tilde{q}^*} & \gtrsim (60 \text{ to } 70) \text{ GeV}
\end{align*}
\]

(40)

The bound (40a) requires all \( p_T > 4 \sigma \) data, whereas (40b) only needs \( p_T > 40 \text{ GeV} \) data. Note that the light gluino (37) is convicted by the new (lack of) evidence.

8. OTHER \( \bar{p}p \) COLLIDER LIMITS

\[
\begin{align*}
W^+ \to \gamma \nu, \chi^+_L \to \ell^+ \gamma, 85) \\
\text{This process would give an excess of } \ell^+ + p_T \text{ events with } p_T < 30 \text{ GeV}, \cos \theta_{\ell \gamma} < 0, \text{ which could in principle be distinguished from } W + e^+ \nu \text{ and } W + \mu^+ \nu. \text{ Including } UAN \text{ cuts and resolutions, one finds}
\end{align*}
\]

\[
\begin{align*}
\sigma(W \to e^+ \nu) / \sigma(W \to e^-) & \approx 0.1 \\
\sigma(W \to \mu^+ \nu) / \sigma(W \to \mu^-) & \approx 0.15
\end{align*}
\]

(41)

for \( m_{\tilde{e}} \lesssim 30 \text{ GeV} \) as expected in scenario (34b). Note that this reaction only yields \( \chi^+_L \), and not \( \chi^+_R \) which do not couple to the \( W^\pm \). The UA1 Collaboration have used this process to set a lower limit

\[
m_{\tilde{e}^+_L} > 26 \text{ GeV} \quad \forall \quad m_{\tilde{e}^+_L} = m_{\tilde{\chi}^+_R}
\]

(42)

which is somewhat better than direct limits from \( e^+e^- \) annihilation\(^{87}\).

\[
\begin{align*}
Z^0 + \chi^+_L, \chi^+_R \to \ell^+ \ell^- 88) \\
\text{This process would give } \ell^+ \ell^- + p_T \text{ events with } |p_T(\ell^+ \ell^-)| \sim 20 \text{ GeV} \sim |p_T| \text{ and } m(\ell^+ \ell^-) \sim 40 \text{ GeV}. \text{ The missing } p_T \text{ makes the reaction distinguishable from conventional Drell-Yan or } p_T \to \ell^+ \ell^- \text{ production. Including } UAN \text{ cuts and resolutions one finds}
\end{align*}
\]

\[
\begin{align*}
\sigma(Z^0 \to \ell^+ \ell^-) / \sigma(Z^0 \to \ell^+ \ell^-) & \approx 0.1
\end{align*}
\]

(43)

for \( m_{\tilde{\chi}^\pm} \lesssim 30 \text{ GeV} \). Note that in this case both \( \chi^+_L \) and \( \chi^+_R \) contribute. Neither UA1 and UA2 has yet quoted a lower limit on \( m_{\tilde{\chi}^\pm} \) based on the non-observation of this reaction.

\[
W^+ \to W^+ Z^0, \quad W^+ \to \ell^+ \nu \gamma, \quad Z^0 \to \ell^+ \ell^- \gamma 89)
\]

The \( W^\pm \) and \( Z^0 \) decay via \( \chi^\pm \) with \( 0(100)\% \) branching ratios if \( m_{\tilde{\chi}^\pm} > m_{\tilde{\chi}^0} > m_{\tilde{\chi}^\pm} \). This process then gives observable trilepton signatures, e.g.:
Fig. 9: Monojet event rates (a) with $E_T^{\text{miss}} > 15$ GeV, (b) with $E_T^{\text{miss}} > 40$ GeV.
\[ \sigma(3\mathbf{\tau}_+ \mathbf{\tau}_-) \gtrsim 20 \mu \text{b} : \mathbf{\gamma}_t \sim 150 \text{GeV} \] \hfill (44)

if \( m_{\mathbf{\tau}_0} \sim 40 \text{ GeV}, \ m_{\mathbf{\tau}_\pm} < 30 \text{ GeV}. \)

\[ \mathbf{\gamma}_0 \rightarrow \mathbf{\tau}_+ \mathbf{\tau}_-, \mathbf{\tau}_+ \rightarrow \mathbf{\gamma}_0 \mathbf{\gamma}_0 \] \hfill (89)

This process also gives large \( \mathbf{\tau}_+ \mathbf{\tau}_- + p_T \) signatures if \( m_{\mathbf{\tau}_\pm} < m_{\mathbf{\tau}_0}/2, \) e.g.,

\[ \sigma(\mathbf{\tau}_+ \mathbf{\tau}_- + p_T) \gtrsim 40 \mu \text{b} : \mathbf{\gamma}_t \sim 150 \text{GeV} \] \hfill (45)

if \( 40 \text{ GeV} \lesssim m_{\mathbf{\tau}_\pm} \lesssim 30 \text{ GeV}. \) Even in the absence of explicit statements from the UA5 collaborations, it seems safe\(^\S\) to exclude on the basis of (44) and (45) the possibility

\[ m_{\mathbf{\tau}_\pm} \lesssim m_{\mathbf{\tau}_0} \lesssim 40 \text{ GeV} \] \hfill (46)

This bound can be compared with the \( e^+e^- \) bounds quoted earlier by Komamiya\(^\S\), namely

\[ m_{\mathbf{\tau}_0} + m_{\mathbf{\tau}_0} > 32 \text{GeV} \quad \text{if} \quad m_{\mathbf{\tau}_0} < 40 \text{ GeV} \] \hfill (47)

from the unsuccessful hunt for \( e^+e^- \rightarrow \mathbf{\gamma}_0 \mathbf{\gamma}_0 \), and

\[ m_{\mathbf{\tau}_0} > 22 \text{GeV} \] \hfill (48)

from the direct hunt for \( e^+e^- \rightarrow \mathbf{\tau}_+ \mathbf{\tau}_- \).

9. \( e^+e^- \rightarrow \gamma + \text{nothing} \)

Here "nothing" means unidentified escaping weak neutrals which could be \( \nu \bar{\nu}, \nu \bar{\nu}, \nu \bar{\nu} \) or ... The \( e^+e^- \rightarrow \gamma + \text{nothing} \) annihilation process is only observable when one tags it with a bremsstrahlung photon. In the prototype process where nothing = \( \sum_i \nu_i \bar{\nu}_i \)\(^\S\),\(^\S\), one has

\[ \frac{d^2 \sigma}{dx_\mathbf{\gamma}_0 dsin^2 \theta_\mathbf{\gamma}_0} = \frac{E_c^{\mathbf{\gamma}_0}}{x_\mathbf{\gamma}_0^{\mathbf{\gamma}_0} \mathbf{\gamma}_0} \left[ \left( 1 - x_\mathbf{\gamma}_0 \right) \left( 1 - x_\mathbf{\gamma}_0 \right) - \frac{x_\mathbf{\gamma}_0^2 (1 - x_\mathbf{\gamma}_0) \cos \theta_\mathbf{\gamma}_0}{4} \right] \]

\[ \times \left\{ \frac{G_F^2 \alpha}{12 \pi^2} \left( A + N_N \right) \right\} \] \hfill (49)

where the first term in the parenthesis \( \{ \} \) reflects \( W^\pm \) exchange in \( e^+e^- \rightarrow \nu_e \bar{\nu}_e \) annihilation, and the \( N_N \) dependence reflects annihilation by the \( Z^0 \) pole. As reported by Komamiya\(^\S\), the ASP experiment has used this process at \( E_{\text{cm}} = 20 \text{ GeV} \) to set the limit

\[ N_N \gtrsim 14 \] \hfill (50)
The \( \tilde{\nu} \nu \) process will really come into its own at \( E_{\text{cm}} > m_{\nu} \), where the enormous rate for \( e^+ e^- \to \gamma + (\text{on-shell} \ Z^0 + \tilde{\nu} \nu) \) will provide a very accurate means of counting the number of neutrinos \( \gamma_\gamma \). In the case of nothing \( \gamma_\gamma \), the parenthesis \( [\ ] \) in (49) is replaced by

\[
\frac{2}{3} \kappa^3 \left\{ \frac{1}{m_{\tilde{\nu}_L}^2} + \frac{1}{m_{\tilde{\nu}_R}^2} \right\} \tag{51}
\]

with known modifications for arbitrary ratios of \( m_\tilde{\nu}, E \) and \( m_{\tilde{\nu}}^2 \). This process has been used by ASP to set bounds

\[
m_{\tilde{\nu}_L} > m_{\tilde{\nu}_R} > \left\{ \begin{array}{ll} 51 \hbar \text{eV} & \text{if} \quad m_B = 0 \\ 45 \hbar \text{eV} & \text{if} \quad m_B = 5 \hbar \text{eV} \end{array} \right. \tag{52}
\]

with \( m_\tilde{\nu} \) bound at all if \( m_\gamma > 13 \text{ GeV} \). Nothing could also be \( \Sigma_1 \tilde{\nu}_1 \tilde{\nu}_1 \), but in most models one expects \( m_\gamma = m_{\tilde{\nu}} \gamma_\gamma > 20 \text{ GeV} \), in which case this process is suppressed by phase space at present energies. It could be interesting beyond the \( Z^0 \) peak, since each species of \( \tilde{\nu} \) counts as \( \frac{1}{2} \) a neutrino species of \( m_\gamma \ll E_{\text{cm}}/2 \), and one may hope \( \gamma_\gamma \) to count "N_\gamma" with the precision of a fraction of a unit. On the \( Z^0 \) peak itself, in the expected presence of \( \gamma_\gamma \) mixing, the reaction \( e^+ e^- \to Z^0 \to B^0 \gamma_\gamma \) followed by \( B^0 \gamma_\gamma \) becomes observable if \( m_\phi, m_\gamma \lesssim 30 \text{ GeV} \) as in many models.

Thus the reaction \( e^+ e^- \to \gamma \) nothing is not only able to set very competitive limits (52) on sparticle masses at present energies, but may also have an interesting future on the \( Z^0 \) peak and beyond.

10. COMPILATION OF CONSTRAINTS ON SUSY BREAKING

We have seen that in the minimal supersymmetric standard model there are two input susy breaking parameters \( m_0 \) and \( m_1 \) which determine (20) much of the sparticle spectrum. Figure 10 shows a compilation of the lower bounds on these parameters coming from the different unsuccessful searches for sparticles reported in this meeting \( \gamma_\gamma \) and discussed in earlier sections of this talk. It should be remembered that despite the best efforts of their protagonists, bounds from \( \bar{p}p \) collisions and cosmology are necessarily less precise than those from \( e^+ e^- \) annihilation.

In the minimal superstring model where the only global susy breaking parameter is \( m_1 \), the plane of Fig. 10 becomes a line, and the deduced bounds are

\[
m_{\tilde{\nu}} > \left\{ \begin{array}{ll} 37 \hbar \text{eV} & \text{if} \quad (\tilde{\nu} \tilde{\nu}) \gamma_\gamma \\ 45 \hbar \text{eV} & \text{if} \quad (e^+ e^- \tilde{\nu}_L \tilde{\nu}_R) \\ 55 \hbar \text{eV} & \text{if} \quad (e^+ e^- \tilde{\nu}_R \tilde{\nu}_L) \\ 68 \hbar \text{eV} & \text{if} \quad (\tilde{\nu} \tilde{\nu}) \gamma_\gamma \end{array} \right. \tag{53}
\]
Fig. 10: Compilation of experimental constraints of susy breaking parameters.
It is not yet possible to estimate reliably the value of $m_t$, but a value $O(2)m_W$ is not implausible\cite{39,101}, which is safely beyond the reach of the present experiments (53).

11. INDIRECT SUSY SEARCHES

Under this heading are included searches for new particles (with \( R = +1 \)) required in at least some susy models.

Higgses As mentioned earlier, susy models require an even number of light Higgs doublets, with two being the minimal choice\cite{39}. In this case one has the two charged $H^\pm$ and the three neutral Higgs bosons $H^0$, $H^0'$, and a mentioned in Section 6 (25). In many models based on dimensional transmutation\cite{43} as described in Section 4, with relatively light $\tilde{q}$ and/or $\tilde{g}$, one expects one light neutral $H^0$ with a mass of a few GeV, analogous to the historical Coleman-Weinberg Higgs boson\cite{19}. This Higgs also acquires much of its mass from electroweak radiative corrections\cite{63}. Its presence with $m_{H^0} < m_{\tau}$ is not excluded by recent CUSB upper limits on $T \rightarrow H^+\gamma$\cite{102}, since QCD radiative corrections\cite{103} suppress the branching ratio for any $H^+\tau$ final state by a factor $O(2)^{103}$ compared with the tree level prediction\cite{104}. There may also be a pseudoscalar Higgs with mass less than $m_{\omega}^2$\cite{63}, so that the decay $Z^0 \rightarrow H^0 + a$ might occur with an observable branching ratio and an enticing signature:

$$Z^0 \rightarrow (H^0 \rightarrow \tau^+\tau^-) + (a \rightarrow b\ell)$$

As mentioned earlier (32), the other susy Higgs must be heavier than the $W^\pm$ and $Z^0$\cite{25,64}.

Additional Matter Particles As mentioned in Section 6, extra colour triplet "quarks" and $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet fields are expected in models derived from $d = 10$ supergravity or the superstring\cite{13}. Their phenomenology is rather model-dependent and we do not discuss them here.

Additional Gauge Boson(s) The superstring or $d = 10$ supergravity must contain an $E_8 \times E_8'$ gauge group, of which $E_8'$ is hidden, while $E_8$ is broken down to some subgroup of $E_6$ which contains at least one extra $U(1)'$ gauge factor beyond the standard model\cite{14}. Here we concentrate on this minimal possibility. The coupling of the corresponding new $Z'$ boson to known particles are specified\cite{39},\cite{11}, with hypercharges

$$Y' = \sqrt{\frac{3}{5}} \times \left( -\frac{1}{6}, -\frac{1}{3}, +\frac{1}{3}, -\frac{1}{3}, +\frac{1}{6} \right)$$

and the magnitude of the gauge coupling calculable using the renormalization group:
\[ \alpha' = \frac{g^2}{4\pi} = 0.016 \]  

(56)

The only unknown is the mass of the gauge boson, which cannot be much larger than \( m_{Z^0} \), if some sort of no-scale scenario along the lines of Section 4 is to apply. The agreement of conventional low energy neutral current measurements with the standard model tells us already that

\[ m_{Z'} \geq 150 \text{ GeV} \]  

(57)

The production rate in \( \bar{p}p \) collisions is

\[ \sigma(Z')/\sigma(Z^0) \sim \frac{i}{4} \ln \left( \frac{1}{\xi} \right) \]  

(58)

for \( m_{Z'} = m_{Z^0} \), decreasing rapidly for \( m_{Z'} > m_{Z^0} \), while the branching ratio

\[ \mathcal{B}(Z' \rightarrow e^+e^-) = \left( 1 - \xi^3 \right) \xi \]  

(59)

to be compared with the 3% of the standard model \( Z^0 \). Combining (58) and (59) and requiring at most one event in either of UA1 and UA2, we find

\[ m_{Z'} \geq 110 \text{ GeV} \]  

(60)

which is comparable to the indirect bound (57), though somewhat weaker. However, there is a real possibility of discovering this additional \( Z' \) in forthcoming experiments at the CERN \( \bar{p}p \) collider, the FNAL Tevatron collider, or LEPII.

12. CONCLUSIONS

We have seen in this talk how the hierarchy or naturalness problem can be alleviated or solved by introducing simple \( N = 1 \) supersymmetry, broken in such a way that sparticles have masses \( \lesssim 1 \text{ TeV} \). The appropriate framework for accommodating susy is \( N = 1 \), \( d = 4 \) supergravity, and a certain class of these models with flat potentials, called no-scale models, may offer a complete solution to the hierarchy problem. \( N = 1 \), \( d = 4 \) supergravity is by itself incomplete, and the natural next step is \( N = 1 \), \( d = 10 \) supergravity, which may be regarded as the low energy limit of the superstring.

So far there is no experimental evidence for any of this theoretical conjecture, and all we know so far is that sparticles must have masses \( \gtrsim 50 \text{ GeV} \). Somewhere between this lower limit and the flexible upper limit \( \sim 1 \text{ TeV} \) we should find sparticles, and possibly
some of the other particles discussed in this talk. To paraphrase St Augustine: "May God give us susy, but not yet".

By comparison with the Planck scale of order $10^{19}$ GeV which theorists believe to be fundamental, finance and technology condemn experiment to crawl slowly from the far infra-red ($E \lesssim 10^{2}$ GeV) towards the ultra-violet. However, theorists' imaginations are not constrained by finance or technology, and they have already jumped to the ultra-violet range $E \sim 10^{19}$ GeV. Some theorists try to turn back towards the infra-red and try to construct an "axiomatic phenomenology" based on their a priori ideas. However, the contact with experiment has yet to be made. Such an a priori approach has been tried before by Aristotle and by Einstein. Aristotle considered experiment unnecessary, while Einstein's theories were not informed by experiment. Subsequently experiment confirmed the essence of Einstein's ideas, and presumably today's a priori theorists seek to follow in the footsteps of Einstein rather than those of Aristotle. But they are still waiting for their experimental Eddington.
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