THE GEOMETRICAL OFF-SHELL STRUCTURE OF PURE
 $N = 1$ $d = 10$ SUPERGRAVITY IN SUPERSPACE

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ABSTRACT

We solve the $N = 1$ pure supergravity Bianchi identities in $d = 10$ superspace off-shell. The solutions contains the $d = 10$ conformal supermultiplet of currents, a necessary condition for consistent coupling to the supersymmetric Yang-Mills theory.

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With the recent exciting developments in supersymmetric string theories there has followed a renewed interest in their low energy limits, in particular $d=10$ supergravity [1,2] and its coupling to the supersymmetric Yang-Mills theory [3,4]. The anomaly cancellations of the closed (supersymmetric) heterotic string [5,6] with an $E_8 \times E_8$ or $SO(32)$ Yang-Mills gauge group in its low energy field theory make this theory seem especially promising. When dealing with supersymmetric field theories of such complexity, it is of course desirable to construct them in superspace. In this paper we will describe a highly non-trivial and, in fact, unexpected property (see below) of pure $d=10$ superspace supergravity as it was presented in Ref. [2]. This property is essential for a consistent coupling to the supersymmetric Yang-Mills theory [7].

Pure $N=1$ supergravity in superspace is quite well understood, and is known to contain an anti-symmetric third rank tensor in one of its superspace torsion components [2]. In [2], this tensor was constrained to be the field strength of a second rank tensor potential. This was achieved by an explicit introduction of superspace Bianchi identities ensuring this property. When trying to couple pure supergravity to the $d=10$ supersymmetric Yang-Mills theory in superspace, one is immediately faced with the following problem. As can be seen from the work of [3] and [4], the coupled Einstein-Yang-Mills theory formulated in x -space does not contain a Yang-Mills gauge invariant exact three-form.

The three-forms that occur are either exact, i.e. $H' = dB$ and non-invariant since B transforms under gauge transformations [3,4], or not exact, in which case $H = H' + \omega_{YM}$ (where ω_{YM} is the Chern-Simons three-form), and hence gauge-invariant [3,4].

It is of course the gauge-invariant one of these that ultimately has to be related to the three-form occurring in the torsion component mentioned previously. Thus the three-form in the supergravity sector cannot be closed. Now, the (non-zero) four-form constructed from this three-form is part of the conformal supermultiplet of currents [3,8], which also contains a symmetric traceless second-rank tensor and a γ -traceless vector-spinor. These must of course also emerge when solving the superspace Bianchi identities of pure supergravity .

The coupling of $d=10$ supergravity to Yang-Mills in superspace has been described in a previous paper by Kallosh and Nilsson [9]. There the Bianchi identities and the constraints of the coupled theory were presented, without however explicitly demonstrating the consistency of these equations. The existence of this coupling, using the constraints of [2,9], will be guaranteed once the multiplet of currents is shown to be present in the super torsions of pure supergravity. This will now be shown.

The chiral superspace introduced in [2] is spanned locally by

$$Z^M = (x^\mu, \theta^m) \quad , \quad (1)$$

where x^μ are bosonic and θ^m are fermionic co-ordinates. By introducing the vielbein E^A , which splits into the bosonic zehnbein E^α and the fermionic 16-bein E^a satisfying the Weyl condition $E^a = E^b (\gamma_{11})_b^a$ as well as the Majorana condition $\bar{E}^a C_{ab} = E_b$, and the Lie-algebra valued spin connection ω_A^B into superspace, we can construct the torsion tensor

$$T^A = DE^A \equiv dE^A - E^B \wedge \omega_B^A \quad , \quad (2)$$

and the curvature tensor

$$R_A^B = d\omega_A^B - \omega_A^C \wedge \omega_C^B \quad . \quad (3)$$

These tensors satisfy the Bianchi identities

$$DT^A = - E^B \wedge R_B^A \quad , \quad (4)$$

and

$$DR_A^B = 0 \quad . \quad (5)$$

As is well known, (5) follows from (4) [10] and will not be discussed any further.

The next step in this approach is to subject T^A to constraints and extract the information now contained in (4).

The constraints to be used here are the ones introduced in [2], except for one trivial modification. As asserted in [9], these constraints will also work for the coupled Einstein-Yang-Mills system. This may seem rather surprising but is proved correct by the results found here (see, however, the discussion at the end of the paper). The constraints are

$$T_{ab}^{\gamma} = -i(\gamma^{\gamma})_{ab} \quad , \quad (6)$$

$$T_{ab}^c = i \delta_{(a}^c T_{b)} \quad , \quad (7)$$

$$T_{a\beta}^{\gamma} = i \delta_{\beta}^{\gamma} \hat{T}_a \quad , \quad (8)$$

$$T_{\alpha\beta}^{\gamma} = 0 \quad . \quad (9)$$

Note that (7) is related to the corresponding expression in [2] by a Fierz transformation. This new form leads to some useful simplifications.

The analysis of (4) proceeds as follows. We will denote by (ABC,D) the tangent space version of (4):

$$\sum_{(ABC)} (R_{ABC}^D + D_A T_{BC}^D + T_{AB}^E T_{EC}^D) = 0 \quad , \quad (10)$$

where the sum contains all cyclic terms. It is convenient to solve the (ABC,D)'s in the following order [2]: (abc,δ); (abγ,δ) and (abc,d); (aβγ,δ) and (αbc,d); (αβγ,δ) and

$(\alpha\beta c, d)$; and finally $(\alpha\beta\gamma, d)$.

i) (abc, δ) .

Since R_{ABC}^D is valued in the $SO(1,9)$ Lie algebra it follows immediately that

$$T_a = \hat{T}_a . \quad (11)$$

ii) $(ab\gamma, \delta)$ and (abc, d) .

Using (11), these read

$$(ab\gamma, \delta): R_{ab\gamma\delta} + 2i\eta_{\gamma\delta} D_{(a} T_{b)} - 2i(T_{\gamma} \gamma_{\delta})_{(ab)} = 0 , \quad (12)$$

$$(abc, d): R_{(abc)}^d + i\delta_{(a}^d D_{b} T_{c)} - i(\gamma^{\alpha})_{(ab} T_{| \alpha | c)}^d = 0 . \quad (13)$$

From (12) and (13) we obtain the following two equations:

$$\eta_{\gamma\delta} D_{(a} T_{b)} = (T_{(\gamma} \gamma_{\delta)})_{(ab)} , \quad (14)$$

$$\frac{1}{2}(T_{\gamma} \gamma_{\delta})_{(ab} (\gamma^{\gamma\delta})_c)^d + (\gamma^{\alpha})_{(ab} T_{| \alpha | c)}^d = \delta_{(a}^d D_{b} T_{c)} . \quad (15)$$

(14) is just the symmetric part (in γ and δ) of (12), while (15) is arrived at by eliminating the curvature from (12) and (13). In order to proceed we expand T_{ab}^c as in [2] and find that (14) and (15) imply

$$T_{\alpha b}^c = -(\gamma^\beta \gamma_\alpha)_b^c Y_\beta + (\gamma_\alpha^{\beta\gamma\delta} - 6 \delta_\alpha^\beta \gamma^{\gamma\delta})_b^c Y_{\beta\gamma\delta} , \quad (16)$$

where $Y_{\alpha\beta\gamma}$ is an arbitrary totally skew-symmetric tensor and

$$Y_\alpha = -\frac{1}{16}(\gamma_\alpha)^{ab} D_a T_b . \quad (17)$$

Furthermore,

$$(\gamma^{\alpha_1 \dots \alpha_5})^{ab} D_a T_b = 0 , \quad (18)$$

which we will solve by putting [2]

$$i T_a = D_a \phi , \quad (19)$$

where ϕ is a scalar superfield. We use the same notation for this field as in [2] since they are put equal on-shell [8].

(17) now implies that

$$Y_\alpha = \frac{1}{2} D_\alpha \phi . \quad (20)$$

Using these results in (12) we find

$$\begin{aligned} R_{ab\gamma\delta} &= 2i(T[\gamma^\gamma \delta])_{(ab)} = \\ &= -2i[(\gamma[\gamma]_{ab}]^{D\delta}] \phi + 12(\gamma^\alpha)_{ab} Y_{\gamma\delta\alpha} + (\gamma_{\gamma\delta}^{\mu\nu\rho})_{ab} Y_{\mu\nu\rho}] . \end{aligned} \quad (21)$$

It also follows that

$$D_a D_b \phi = \frac{i}{2} (\gamma^\alpha)_{ab} D_\alpha \phi + i (\gamma^{\alpha\beta\gamma})_{ab} K_{\alpha\beta\gamma} \quad . \quad (22)$$

Note that the three-form $K_{\alpha\beta\gamma}$ defined by (22) is unrelated to $Y_{\alpha\beta\gamma}$.

iii) $(a\beta\gamma, \delta)$ and (abc, d) .

Here we solve for $R_{a\beta\gamma\delta}$ from $(a\beta\gamma, \delta)$ and plug it into (abc, d) . We thus obtain

$$\begin{aligned} \frac{i}{8} [2T_{\alpha\gamma}{}^e(\gamma_\delta)_{e(b} - T_{\gamma\delta}{}^e(\gamma_\alpha)_{e(b)}] (\gamma^{\gamma\delta})_{c)}^d + \frac{i}{2} (\gamma^\beta)_{bc} T_{\alpha\beta}{}^d = \\ = \hat{D}_{(b} Y_{\beta\gamma\delta} (\gamma_\alpha^{\beta\gamma\delta} - 6 \delta_\alpha^\beta \gamma^{\gamma\delta})_{c)}^d \quad , \end{aligned} \quad (23)$$

where we have defined

$$\hat{D}_a \equiv D_a + i T_a \quad . \quad (24)$$

It is important to note that the superfield ϕ does not enter into (23) (except in $\hat{D}_a$), and consequently the equation of motion for T_a is not derivable (recall that $T_a|_{\theta=0}$ is the physical spin-1/2 field in the on-shell theory [2]).

By decomposing the Rarita-Schwinger field strength [2],

$$T_{\alpha\beta}{}^c = G_{\alpha\beta}{}^c + 2G[\alpha^d(\gamma_\beta)]_d{}^c + G^d(\gamma_{\alpha\beta})_d{}^c \quad , \quad (25)$$

where $G_{\alpha\beta}\gamma^\beta = G_\alpha\gamma^\alpha = 0$, and $\hat{D}_a Y_{\beta\gamma\delta}$,

$$\begin{aligned} \hat{D}_a Y_{\beta\gamma\delta} = & W_{a\beta\gamma\delta} + 3(\gamma_{[\beta})_a{}^b W_{b|\gamma\delta]} + 3(\gamma_{[\beta\gamma})_a{}^b W_{b|\delta]} \\ & + (\gamma_{\beta\gamma\delta})_a{}^b W_b \quad , \end{aligned} \quad (26)$$

where

$$W_{\alpha\beta\gamma}\gamma^\alpha = W_{\alpha\beta}\gamma^\alpha = W_\alpha\gamma^\alpha = 0 \quad , \quad (27)$$

the solution of (23) can be expressed as follows:

$$W^a = G^a = 0 \quad , \quad (28)$$

$$W^a_\alpha = \frac{i}{12} G_\alpha{}^a = \frac{1}{56} (\hat{D}_b Y_{\alpha\beta\gamma}) (\gamma^{\beta\gamma})^{ba} \quad , \quad (29)$$

$$\begin{aligned} W^a_{\alpha\beta} = & -\frac{i}{72} G_{\alpha\beta}{}^a = \\ = & \frac{1}{12} [(\hat{D}_b Y_{\alpha\beta\gamma}) (\gamma^\gamma)^{ba} - \frac{1}{2} (\hat{D}_b Y_{\gamma\delta[\alpha}) (\gamma_{\beta]}{}^{\gamma\delta})^{ba}] , \end{aligned} \quad (30)$$

$$W^a_{\alpha\beta\gamma} = 0 \quad . \quad (31)$$

The curvature components $R_{a\beta\gamma\delta}$ are

$$R_{a\beta\gamma\delta} = \frac{i}{2} [2(T_{\beta[\gamma\delta]})_a - (T_{\gamma\delta}\gamma_\beta)_a] + 2i\eta_{\beta[\gamma} (\hat{D}_{\delta]})_a{}^b T_b \quad , \quad (32)$$

where

$$(\hat{D}_\alpha)_a^b = D_\alpha \delta_a^b + T_{\alpha a}^b \quad . \quad (33)$$

iv) $(\alpha\beta\gamma, \delta)$ and $(\alpha\beta c, d)$.

$(\alpha\beta\gamma, \delta)$ reads

$$R_{[\alpha\beta\gamma]}^\delta + i \delta_{[\alpha}^\delta T_{\beta\gamma]}^d T_d = 0 \quad , \quad (34)$$

and by contracting $(\alpha\beta c, d)$ with δ_d^c , $(\gamma_\gamma\delta)_d^c$ and $(\gamma_{\gamma\delta\epsilon\eta})_d^c$ we find, respectively,

$$\hat{D}_a T_{\alpha\beta}^a = 0 \quad , \quad (35)$$

$$\begin{aligned} \hat{D}_a (T_{\alpha\beta} \gamma_{\gamma\delta})^a &= \\ &= -8 [R_{\alpha\beta\gamma\delta} + \frac{1}{4} D_{[\alpha} \text{tr}(T_{\beta]} \gamma_{\gamma\delta}) + \frac{1}{4} \text{tr}(T_{[\alpha} T_{\beta]} \gamma_{\gamma\delta})] \quad , \end{aligned} \quad (36)$$

$$\hat{D}_a (T_{\alpha\beta} \gamma_{\gamma\delta\epsilon\eta})^a = -2 [D_{[\alpha} \text{tr}(T_{\beta]} \gamma_{\gamma\delta\epsilon\eta}) + \text{tr}(T_{[\alpha} T_{\beta]} \gamma_{\gamma\delta\epsilon\eta})] \quad , \quad (37)$$

where now $\hat{D}_a = D_a + \frac{3}{2} i T_a$.

To facilitate the analysis of (34) to (37) we expand the two spinor indices in $\hat{D}_a \hat{D}_b Y_{\alpha\beta\gamma}$ in terms of γ -matrices and obtain

$$\begin{aligned}
 \hat{D}_a \hat{D}_b Y_{\alpha_1 \alpha_2 \alpha_3} &= \\
 &= i Q_{\alpha_1 \alpha_2 \alpha_3, \beta} (\gamma^\beta)_{ab} + i Q_{\alpha_1 \alpha_2 \alpha_3, \beta_1 \beta_2 \beta_3} (\gamma^{\beta_1 \beta_2 \beta_3})_{ab} + \\
 &+ i Q_{\alpha_1 \alpha_2 \alpha_3, \beta_1 \dots \beta_5} (\gamma^{\beta_1 \dots \beta_5})_{ab} . \quad (38)
 \end{aligned}$$

The Ricci identity then immediately provides us with explicit expressions for $Q_{\alpha_1 \alpha_2 \alpha_3, \beta}$ and $Q_{\alpha_1 \alpha_2 \alpha_3, \beta_1 \dots \beta_5}$ in terms of D_α -derivatives on ϕ and $Y_{\alpha\beta\gamma}$ and/or products of $Y_{\alpha\beta\gamma}$'s:

$$\begin{aligned}
 Q_{\alpha\beta\gamma, \delta} &= \frac{1}{2} (D_\delta + D_\delta \phi) Y_{\alpha\beta\gamma} + \frac{3}{2} ((D_{[\alpha} \phi) Y_{\beta\gamma]}_\delta - (D_\epsilon \phi) Y_{[\alpha\beta}^\epsilon \eta_{\gamma]}_\delta) \\
 &- 36 Y_{\delta[\alpha}^\epsilon Y_{\beta\gamma]}_\epsilon , \quad (39)
 \end{aligned}$$

$$\begin{aligned}
 Q_{\alpha\beta\gamma, \delta_1 \dots \delta_5} &= 3 (\delta_{[\alpha}^\delta Y_{\beta\gamma]}^{\delta_1 \dots \delta_5} + \\
 &+ \frac{1}{5!} \epsilon^{\delta_1 \dots \delta_5} \epsilon_{\epsilon_1 \dots \epsilon_4} [Y_{\beta\gamma}]_{\epsilon_1} Y_{\epsilon_2 \dots \epsilon_4}^{\epsilon_1}) , \quad (40)
 \end{aligned}$$

which is in accord with the Weyl-projected indices appearing in (38). The objective now is to derive from (34) to (37) as much information as possible about $Q_{\alpha_1 \alpha_2 \alpha_3, \beta_1 \beta_2 \beta_3}$. In particular, we must verify that the two bosonic components of the conformal supermultiplet of currents are present. (The fermionic partner of these is G_α^a ; see (25) and (29)).

$Q_{\alpha_1 \alpha_2 \alpha_3, \beta_1 \beta_2 \beta_3}$ split into irreducible tensors under

SO(1,9) as follows [11]:

$$120 \otimes_s 120 = 1+54+210+770+1050+\overline{1050}+4125 \quad , \quad (41)$$

$$120 \otimes_a 120 = 45+210+945+5940 \quad . \quad (42)$$

We will now discuss each of these irreducible tensors in turn. First we note that the 1 and 4125 (with Young tableau $\begin{smallmatrix} \sim \\ \boxplus \end{smallmatrix}$) in (41) are zero due to (28) and (31) respectively.

Next we consider 5940 in (42) which appears on the left-hand side of (37). However, there is no such tensor on the right-hand side and it must therefore vanish. 1050 and $\overline{1050}$, on the other hand, will be determined by (37) in terms of Y^2 while (36) and (37) imply the same expression for the 945.

We now come to the more interesting terms on the right-hand sides of (41) and (42), namely 54, 210, 770 and 45. First, $Q(\underline{770})$ is found to be related to the Weyl tensor. Secondly, when deriving that $Q(45)$ vanishes, we also discover that

$$D^\alpha Y_{\alpha\beta\gamma} - 6(D^\alpha \phi)Y_{\alpha\beta\gamma} = 0 \quad . \quad (43)$$

(43) can be derived in several ways from the Bianchi identities (35) to (37) and therefore provides an important check of the consistency of this approach. Note that (43) must also be true in the coupled Einstein-Yang-Mills theory and there

fore should be viewed, not as a field equation for a two-form potential, but rather as a Bianchi identity for a six-form potential. It is of course only in an off-shell theory that this statement is meaningful and indeed this six-form potential played a crucial role also in the discussion of the linearized off-shell theory in [8].

Finally, we come to the most important terms on the right-hand sides of (41) and (42), namely the representations 54 and 210. As is known from [3,8] the conformal supermultiplet of currents contains a 54($\tilde{\square}$) and 210($\begin{smallmatrix} \square \\ \square \end{smallmatrix}$) and these must be non-zero in order to make a coupling to Yang-Mills possible. $Q[\alpha_1\alpha_2\alpha_3, \beta_1\beta_2\beta_3]$ does also appear but is not independent and indeed (37) relates it to the dual of $Q^\gamma[\alpha_1\alpha_2, \beta_1\beta_2]_\gamma$:

$$\begin{aligned} Q[\alpha_1 \dots \alpha_6] &= \\ &= \frac{1}{72} \epsilon_{\alpha_1 \dots \alpha_6}^{\beta_1 \dots \beta_4} (Q^\gamma_{\beta_1\beta_2, \beta_3\beta_4\gamma} - 6 Y^\gamma_{\beta_1\beta_2} Y_{\beta_3\beta_4\gamma}). \end{aligned} \quad (44)$$

Then from both (36) and (37) we obtain

$$Q_\epsilon[\alpha\beta, \gamma\delta]^\epsilon = -\frac{1}{2}(D[\alpha Y_{\beta\gamma\delta}] - 2(D[\alpha\phi]Y_{\beta\gamma\delta}) - 12 Y_\epsilon[\alpha\beta Y_{\gamma\delta}]^\epsilon). \quad (45)$$

Similarly, from (36) we find

$$\begin{aligned}
 -\frac{144}{7} Q_{\gamma\delta(\alpha,\beta)}^{\gamma\delta} &= R_{(\alpha\beta)} + \eta_{\alpha\beta} \square \phi + 8D_{(\alpha} D_{\beta)} \phi - 8\eta_{\alpha\beta} (D_{\gamma} \phi)^2 \\
 &+ 8D_{\alpha} \phi D_{\beta} \phi + 144(\eta_{\alpha\beta} Y^{\gamma\delta\epsilon} Y_{\gamma\delta\epsilon} - 7 Y_{\alpha}^{\gamma\delta} Y_{\beta\gamma\delta}), \quad (46)
 \end{aligned}$$

and if we recall that in (41) the singlet vanishes, the trace of (46) becomes

$$R + 18 \square \phi - 72(D_{\alpha} \phi)^2 + 432 Y^{\alpha\beta\gamma} Y_{\alpha\beta\gamma} = 0 \quad . \quad (47)$$

It is important to note that (47) does not imply the rest of the field equations, but does constitute a restriction on possible couplings to matter (see below).

This completes the analysis of the Bianchi identities (4) and the constraints (6), (7) and (8). The results are easily summarized as follows. The system has been shown to be off-shell with all torsion and curvature components expressible in terms of one unconstrained scalar superfield ϕ together with a constrained skew-symmetric third rank tensor superfield $Y_{\alpha\beta\gamma}$. The scalar superfield starts with the physical fields $\phi(x)$ and $\lambda_a(x)$ while all components from θ^2 and up are auxiliary. The superfield $Y_{\alpha\beta\gamma}$ has as its lowest component an x-space three-form which is the dual of the field strength of a six-form potential[†]. This follows from (43). At the next two levels in $Y_{\alpha\beta\gamma}$ we find the Weyl tensor

[†]The formulation of the on-shell theory in terms of a six-form potential is discussed in [12,13].

$C_{\alpha\beta\gamma\delta}$ and its on-shell superpartner $G_{\alpha\beta}^a$ together with the conformal supermultiplet of currents consisting of the vector-spinor W_{α}^a , the traceless energy-momentum tensor $Q^{\gamma\delta}(\widetilde{\alpha},\beta)_{\gamma\delta}$ and the four-form current $Q^{\varepsilon}[\alpha\beta,\gamma\delta]_{\varepsilon}$. These currents are related to physical fields by (28), (29) (recall that $T_{\alpha\beta\gamma}^{\alpha\beta\gamma} = -72 G^{\gamma} + 16 G_{\gamma}^{\gamma}$), (45) and (46).

This should now be compared to the linearized off-shell theory obtained by Howe et. al [8]. These authors found the off-shell fields to be a scalar superfield ϕ (identical to the one introduced in (19) above) and the three-form prepotential $V_{\alpha\beta\gamma}$. $V_{\alpha\beta\gamma}$ satisfies some constraints with the consequence that it is essentially equivalent to an unconstrained scalar superfield T plus the conformal supermultiplet [8]. The auxiliary superfield T does not enter into the theory as presented here. However, this just corresponds to a situation where one of the superspace field equations has been implemented, namely

$$D_{\gamma}^{\alpha\beta\gamma} D V_{\alpha\beta\gamma} = 0 \quad (48)$$

in the linearized version of [8]. (48) could have made impossible the coupling to Yang-Mills, but fortunately this is not the case as we will now explain. In $N=1$ supergravity in four dimensions one encounters a similar situation. The $d=4$ superspace field equations are two in number, $G_{\alpha\beta}=0$ and $R=0$, and if $R=0$ (but not $G_{\alpha\beta}=0$) is imposed, one can only couple to

matter systems which are consistent with zero curvature scalar and zero γ -trace of the Rarita-Schwinger field equation [14]. Similarly in our case, (48) should be interpreted as the reason for the constraints (47) and (28). However, these constraints are consistent with the coupling to Yang-Mills in $d=10$ as was asserted in [9]. This will be fully demonstrated in [7].

Finally we want to emphasize again that the fact that the torsion constraints used here (and in [2,9]) do not lead to the on-shell theory is quite remarkable. This is not the case in for example the chiral $N=2$, $d=10$ theory analyzed in [15].

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