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WARD IDENTITIES IN DUAL STRING THEORIES

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ABSTRACT

Ward identities for "internal", hidden symmetries of dual string theories are obtained from the corresponding canonical transformations of the string phase space path integral. The example of general co-ordinate transformations is given in detail.

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1. INTRODUCTION

The daring proposal by Scherk and Schwarz [1], that Dual Resonance Models [2] (DRM) should be taken seriously as candidate theories of all fundamental particles and interactions, rested on the basic fact that all ghost free DRM inevitably lead to massless $J = 1, 3/2$, and 2 particles which couple precisely [1,3] as the gauge bosons of the corresponding local symmetries.

The Scherk-Schwarz conjecture was not taken seriously until, ten years later, Green and Schwarz [4] showed that the above gauge symmetries are anomaly free for a very restricted class of supersymmetric DRM (Superstrings). Such models are likely to be finite or, at least, renormalizable theories which include gravity and some of them [5] offer the hope of yielding an interesting phenomenology.

A crucial, still unanswered question is the origin of these 'internal' hidden symmetries, which are not manifest invariances of the string action. They should be related somehow to the much better understood, explicit symmetry under reparametrization of σ_α ($\sigma_1 = \sigma$, $\sigma_2 = \tau$), the string surface coordinates. A related problem is that of working out the full internal symmetry group, suspected to be much larger than that of General Coordinate Transformations (GCT) $X^\mu(\sigma) \rightarrow \xi^\mu[X(\sigma)]$.

An ambitious approach to this problem tries to construct a second quantized version of the theory in terms of functionals of $X^\mu(\sigma)$, to be later quantized (integrated over). The hope is that such second quantized theory will exhibit a huge invariance group encompassing GCT. Good advances along this line have recently been made [6], at least for the free theory.

The above program is not an easy one. A different approach, that we shall follow here, consists of remaining at the **first quantized level**. After all, this is sufficient in order to construct a unitary quantum theory, string pair creation being included through the sum over topologically inequivalent string-swept surfaces. Of course, if successful, this approach will also shed light on a second quantized formulation.

In the first quantized approach we shall not isolate vertices and propagators, but rather look directly at the connected (non local) string Green's functions generated by W . Writing W as a path integral over string phase space $X^\mu(\sigma_\alpha)$, $P_\mu(\sigma_\alpha)$, we are lead to consider canonical transformations on these variables^{*}). We shall see that these generate the expected Ward Identities (WIs) for Green's functions. An even more suggestive form is then obtained by using an S-matrix functional *a la* Fradkin and Tseytlyn [7].

In this note we shall outline the general procedure and then work out in detail the case of GCT for the closed bosonic string, leaving to further work the case of other space-time or internal symmetries for bosonic, super, or heterotic strings.

The crucial point of our derivation is the fact that the string Hamiltonian H is just a constraint, the canonical one being zero. H plays the role of a gauge fixing term, breaking both σ_α reparametrization and GCT (more generally canonical transformations). This leads to the desired WIs by a procedure analogous to the standard field theoretical one.

2. PHASE-SPACE PATH INTEGRAL FOR THE BOSONIC STRING

Let us recall first some known facts about the covariant, first quantization of the bosonic string in $D = 26$, following Rebbi's notations [2] (in units $2\alpha' = 1$).

^{*}) I am grateful to S. Fubini for drawing my attention on the possible relevance of canonical transformations in a somewhat different framework.

In the Nambu-Goto formulation the canonical Hamiltonian vanishes since the Lagrangian is homogeneous of first degree in $\dot{X}$. H is to be chosen proportional to a constraint and can be regarded as a gauge fixing term for reparametrization invariance. The orthonormal gauge corresponds to the choice

$$H = \frac{1}{2} \int d\sigma \left(\pi P^2 + \frac{1}{\pi} X'^2 \right) \quad (1)$$

Quantization leads in general to the phase-space path integral:

$$Z = \int dX(\sigma_\alpha) dP(\sigma_\alpha) \exp \left(i \int d^2\sigma \{ P \dot{X} - H \} \right) \quad (2)$$

With the choice (1) P can be integrated out giving

$$Z = \int dX(\sigma_\alpha) \exp \left(\frac{i}{2\pi} \int d^2\sigma \{ \dot{X}^2 - X'^2 \} \right) \quad (3)$$

which after going to Euclidean (two-dimensional) metric becomes

$$Z_E = \int dX(z) \exp \left(-\frac{1}{2\pi} \int d^2z \partial X / \partial z \partial X / \partial z^* \right) \quad (4)$$

with $z = \sigma + i\tau_E = \sigma + \tau$. Equation (4) is the usual starting point for amplitude calculations by path-integral methods [8]. Alternatively, one can start from Polyakov's treatment [9] of the Brink-Di Vecchia-Howe action [10] and choose a coordinate system in which:

$$g_{\alpha\beta} d\sigma_\alpha d\sigma_\beta = e^\phi dz dz^* \quad (5)$$

In this gauge (and at $D = 26$) ϕ drops out of the action and one recovers immediately eq. (4).

A more elegant approach to gauge fixing has been given some time ago by Kato and Ogawa [11], who add to the X^μ (and $g_{\alpha\beta}$) coordinate ghost coordinates $C, \bar{C}$. This results in a BRS invariance of the gauge fixed action. The Hamiltonian phase-space formulation of this approach, has been given by Hwang [12]. We have checked that our results also follow in this more rigorous approach but, for the sake of simplicity, we shall base our analysis on the Hamiltonian formulation (2) in the orthonormal gauge given by (1).

In order to compute Green's functions and, ultimately, S-matrix elements among physical states, we introduce sources and arrive at the generating functional W:

$$\exp(iW(J, \tilde{J})) = Z^{-1} \int dX dP \exp(i \int d\sigma^2 \{ P \cdot \dot{X} - H + J \cdot X + \tilde{J} \cdot P \}) \quad (6)$$

Green's functions will be here auxiliary, non-physical objects. For physical S-matrix elements the original constraints^{*)}

$$(P \pm X')^2 = 0 \quad (7)$$

have to be taken into account. This can be done by suitably restricting the admissible sources $J, \tilde{J}$. In Polyakov's language, sources must be chosen in such a way that dependence on ϕ [eq. (5)] does not creep back in [9,13]. From here on the derivation of WIs follows the standard field theoretical one, if we regard $P \cdot \dot{X}$ in (6) as the invariant part of the action and $-H$ as a gauge fixing.

The natural group of transformations to be considered is that of canonical transformations which, at least classically, leave the volume element invariant (Liouville theorem). In their infinitesimal form the canonical transformations are generated by an infinitesimal functional $\Phi(X, P)$ via:

$$\delta X^\mu(\sigma_\alpha) = \frac{\delta \Phi}{\delta P_\mu(\sigma_\alpha)} \quad ; \quad \delta P_\mu(\sigma_\alpha) = - \frac{\delta \Phi}{\delta X^\mu(\sigma_\alpha)} \quad (8)$$

and we shall assume Φ not to depend explicitly on σ_α . It is then easy to see that $P \cdot \dot{X}$ changes by a total τ derivative while:

$$\delta \int H d\sigma^2 = \int d\sigma^2 \left[X'^\mu \left(\frac{\delta \Phi}{\delta P_\mu} \right)' - P_\mu \frac{\delta \Phi}{\delta X^\mu} \right] \eta_{\mu\nu} \quad (9)$$

$$\delta \int (J \cdot X + \tilde{J} \cdot P) d\sigma^2 = \int d\sigma^2 \left[J \frac{\delta \Phi}{\delta P} - \tilde{J} \frac{\delta \Phi}{\delta X} \right] \quad (10)$$

^{*)} In order to simplify the formulae $X(P)$ are rescaled from now on by a trivial factor $1/\pi$ (π) with respect to Rebhi's [2].

Ward Identities now follow from the standard procedure of changing variables from X, P to $X + \delta X, P + \delta P$. Collecting all the terms linear in Φ one immediately gets:

$$\eta_{\mu\nu} \int d^2\sigma \left\langle X'^{\mu} \left(\frac{\delta \Phi}{\delta P_{\nu}} \right)' - P_{\mu} \frac{\delta \Phi}{\delta X^{\nu}} \right\rangle_{J, \tilde{J}} = \int d^2\sigma \left\langle J \frac{\delta \Phi}{\delta P} - \tilde{J} \frac{\delta \Phi}{\delta X} \right\rangle_{J, \tilde{J}} \quad (11)$$

where the $\langle \rangle_{J, \tilde{J}}$ symbol mean the obvious functional integral for connected Green's functions in the presence of $J, \tilde{J}$.

Potentially eq. (11) contains a huge number of constraints. We shall now specify Φ to be the generator of GCT in order to obtain the corresponding Ward Identity.

3. GENERAL COORDINATE TRANSFORMATIONS

Consider the particular generator

$$\Phi = \int d^2\sigma P_{\mu}(\sigma) \xi^{\mu}(X(\sigma)) \quad (12)$$

where ξ^{μ} are infinitesimal functions of the X^{ν} . Φ induces

$$\begin{aligned} \delta X^{\mu} &= \xi^{\mu}(X) \quad ; \quad \delta P_{\mu} = -P_{\nu} \frac{\partial}{\partial X^{\mu}} \xi^{\nu} \equiv -P_{\nu} \xi^{\nu}_{, \mu} \\ \delta(\dot{X} P) &= 0 \quad ; \quad \delta H = \int d^2\sigma [X'_{\mu} X'_{\nu} - P_{\mu} P_{\nu}] \xi^{\nu, \mu} \end{aligned} \quad (13)$$

where indices are raised and lowered by $\eta^{\mu\nu} = \eta_{\mu\nu} = (-1, 1, 1, \dots, 1)$. We get a full set of WIs either by functional differentiation with respect to ξ^{μ} or, more simply, by taking:

$$\xi^{\mu}(X) = \epsilon^{\mu} \cdot \exp(i k X) \quad (14)$$

where μ is an infinitesimal constant vector. This gives through (11):

$$\left\langle \int d^2\sigma e^{i k X(\sigma)} [i k^{\mu} G_{\mu\nu}(\sigma) - (J_{\nu} - \tilde{J}_{\mu} P_{\nu} i k^{\mu})] \right\rangle_{J, \tilde{J}} = 0 \quad (15)$$

where

$$G_{\mu\nu}(\sigma) = (X'_\mu X'_\nu - P_\mu P_\nu) = \frac{1}{2} (X'_\mu + P_\mu)(X'_\nu - P_\nu) + (\mu \leftrightarrow \nu) \quad (16)$$

is the (prefactor of the) graviton vertex [14] of the closed string up to contact terms (equations of motion).

The first term in eq. (15) has the form typical of a WI. In order to see the rest explicitly we can expand (15) around $J = \tilde{J} = 0$. It is better however, to take sources from which we can easily go to the S-matrix (i.e. by a procedure similar to the field theoretical one of removing external propagators and going on shell). For this purpose the simplest choice is

$$\tilde{J} = 0 \quad ; \quad J_\mu(\sigma) = \sum_i k_{i,\mu} \delta^{(2)}(\sigma - \sigma_i) \quad (17)$$

giving:

$$\begin{aligned} & \left\langle \int d^2\sigma \, k^\mu G_{\mu\nu} e^{iKX(\sigma)} \prod_{i=1}^n \exp(iK_i X(\sigma_i)) \right\rangle_0 = \\ & = \sum_{i=1}^n k_{i,\nu} \left\langle \exp(i(K+K_i)X(\sigma_i)) \prod_{j \neq i} \exp(iK_j X(\sigma_j)) \right\rangle_0 \end{aligned} \quad (18)$$

This has precisely the form of a WI for Green's functions and is schematically represented in fig. 1.

If one dimension (say X^{25}) is compactified on a torus one gets the typical U(1) (QED) WI by taking $\epsilon^\mu = e\delta^{\mu 25}$ in eq. (14):

$$\begin{aligned} & \left\langle \int d^2\sigma \, K^\mu G_{\mu 25} \exp(iKX) \prod_i \exp(iK_i X(\sigma_i)) \right\rangle = \\ & = \sum_i q_i \left\langle \exp(i(K_i + K) \cdot X(\sigma_i)) \prod_{j \neq i} \exp(iK_j X(\sigma_j)) \right\rangle \end{aligned} \quad (19)$$

where $q_i = P_{25,i}$ are the (Kaluza-Klein quantized) U(1) charges of the external scalars.

In order to go over to the S-Matrix we have to perform suitable integrations over the σ_i variables, fix the external momenta to be on shell, and divide by an infinite volume factor coming from Möbius invariance [13]. In the case at hand one has to take $k^2 = 0$ for the photon or graviton and $k_i^2 = 4$ for the scalars (tachyons). Furthermore, certain infinities, which come from the self interactions of X 's and P 's at the same point, have to be removed (the analog of normal ordering the vertex in the operator approach [15]). This can be done by changing slightly the source J_μ in eq. (17) so as to separate the σ_i appearing in the positive frequencies of X^μ from those appearing in the negative ones. The limit of coincident σ 's is then taken after multiplying in a suitable (divergent) factor [15].

We have checked in specific examples that:

- i) the off-shell Ward Identity (18) is indeed satisfied and
- ii) the above procedure for going to the S-matrix puts to zero the r.h.s. of eq. (18).

What happens is again very similar to the field theoretic case, where the operation of taking out the external propagators and going on shell eliminates all the terms in the Σ_i since each one of them misses one external particle pole.

We thus obtain:

$$K^\mu T_{\mu\nu}(k, k_i) = K^\mu T_\mu(k, k_i) = 0 \quad (20)$$

for physical one photon or one graviton amplitudes, the same being true for an arbitrary multiparticle process involving physical states.

4. WARD IDENTITIES FOR THE S-MATRIX FUNCTIONAL

The above results take a neat, suggestive form if written in terms of a Fradkin-Tseytlyn functional [7]:

$$S(\hat{A}) ; \hat{A} = \sum_{(n), (\mu)} \hat{A}_{(\mu)}^{(n)}(x_0) X_{(n), (\mu)}^{(\mu_1)} \tilde{X}_{(n_2)}^{(\mu_2)} \quad (21)$$

where $\hat{A}$ is the usual closed string (of coordinates $X_0, X_n, \tilde{X}_n$) functional. S is a functional of a functional and generates full (not IPI), generally off-shell amplitudes upon differentiation with respect to the fields $A_{(\mu)}^{(n)}$.

Ward Identities for S follow from a first quantization expression for S . In the orthonormal gauge this reads:

$$S(\hat{A}) = \int dX^\mu dP_\mu \dots \exp(i(I_0 + \hat{B})) \quad (22)$$

where I_0 is given in eqs. (1) and (2), and $\hat{B}$ is related to $\hat{A}$ by:

$$\begin{aligned} \hat{B} &= \int d\sigma \hat{A}(x_n \rightarrow \sigma X(\sigma)) \equiv \int d\sigma \sum_{(n), (\mu)} \hat{A}_{(\mu)}^{(n)}(X(\sigma)) \frac{\partial^{(n_1)} X_{(\sigma)}^{(\mu_1)}}{\partial \sigma_+^{(n_1)}} \frac{\partial^{(n_2)} X_{(\sigma)}^{(\mu_2)}}{\partial \sigma_-^{(n_2)}} \dots \equiv \\ &\equiv \Omega \hat{A} \end{aligned} \quad (23)$$

Here $\sigma_\pm = (\sigma_1 \pm \sigma_2)/\sqrt{2}$ and the dots in (22) and (23), correspond to ghost coordinate integration and dependence respectively.

The above formula can be obtained from Ref. 7 upon (orthonormal) gauge fixing. It can also be checked for ‘physical’ $\hat{A}$ ’s (i.e. for on-shell matrix elements) using the explicit construction of Ref. 13 (this is actually how we obtained it).

Ward identities for S follow again from the canonical transformation (8) (labelled by Φ) on the integration variables in (22). They are neatly expressed as invariances of S (up to contact terms) under the transformation:

$$A \equiv \hat{A} + \eta_{\mu\nu} X_i^\mu \tilde{X}_i^\nu \rightarrow A + \delta'_\Phi A \quad (24a)$$

defined by

$$\delta'_\Phi A = \Omega^{-1} \delta_\Phi (\Omega A) \equiv \Omega^{-1} \delta_\Phi B = \Omega^{-1} \left[B\left(x + \frac{\delta\Phi}{\delta p}\right) - B(x) \right] \Big|_{p=\dot{x}} \quad (24b)$$

The invariance of S under this large group of transformations is very suggestive, hinting at the non-invariance of the original action (1) as being due to the expectation value of the dilaton field. We shall come back to this reformulation elsewhere.

We have obtained generalizations of the above results for open strings and for the case of independent left and right moving closed string coordinates (which are crucially employed in the heterotic string [4]). In the latter case, one expects to derive Ward Identities for the extra gauge symmetries (obtained upon compactification on particular tori [16]) from associated canonical transformations which mix left to right moving coordinates. Details will be reported elsewhere [17].

The important question to be understood at this point is how the group of canonical transformations is actually realized at the quantum level. Higher quantum corrections can lead to divergences that, through regularization, introduce anomalies. This is probably the fate of most potential symmetries of the classical action. On the other hand, the situation could turn out to be more interesting for symmetries which, though not anomalous, cannot leave the quantum vacuum state invariant in which case a Higgs-type realization of the symmetry with massive gauge bosons may result.

Hopefully, the simple method we have presented will help digging out the hidden symmetry that must lie at the basis of the many wonderful properties of dual string theories.

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Fig. 1 Graphical form of the off-shell Ward Identity for GCT