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PRIMORDIAL NUCLEOSYNTHESIS, ADDITIONAL NEUTRINOS AND NEUTRAL CURRENTS
FROM THE SUPERSTRING

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ABSTRACT

We review critically the constraint on the number of neutrino species N_ν imposed by Big Bang nucleosynthesis. We argue that uncertainties in the neutron lifetime, systematic errors in the measurement of ^4He abundance, and new astrophysical data on ^3He which suggest extensive astration of D, permit $N_\nu = 5$ or even 6. Some phenomenological models based on the superstring expect three additional right-handed neutrinos of low mass which are weakly coupled to ordinary matter through a new heavy Z' gauge boson. Their compatibility with the relaxed nucleosynthesis limits quoted above imposes an interesting lower limit on $m_{Z'}$. As a byproduct of our analysis, we present the corrections to low energy $\nu_\mu e$ scattering due to Z' exchange.

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Just how many light neutrino species are permitted¹⁾ by the successes of conventional Big Bang nucleosynthesis calculations²⁾ of light element abundances? Reappraisal of the classic cosmological calculations²⁾ of D, ³He, ⁴He and ⁷Li abundances is topical now for two new particle physics reasons. One is that finally the accelerator limits on the number of light neutrinos coupled to the Z⁰ have come within hailing distance of the nucleosynthesis limits. Recent UA1 and UA2 pp̄ collider data³⁾ suggest that $N_\nu < 5.4 \pm 1.0$, while a recent upper limit⁴⁾ on $e^+e^- \rightarrow \gamma + \text{invisibles}$ implies that $N_\nu < 14$. The other reason is that phenomenological models motivated by the superstring⁵⁾ contain additional neutral particles in each fermion generation⁶⁾, notably right-handed neutrinos ν_R . It is not clear⁷⁾ how to give these ν_R masses much larger than 1 MeV, so they may have been present during nucleosynthesis, with a density large enough to affect the abundance calculations²⁾. However, the ν_R do not couple to ordinary matter predominantly through the known Z⁰, but through a new neutral gauge boson Z' ⁸⁾. Therefore, their primordial density will be suppressed if the Z' is heavy enough to move the ν_R decoupling back earlier than $\mu^+\mu^-$ or hadron annihilation, in which case each ν_R species will only count as a fraction of a conventional neutrino species⁹⁾. One of the main objectives of this paper is to ascertain how heavy the Z' must be, given a plausible cosmological upper bound on N_ν . Reason to re-discuss this bound is also provided by new astrophysical data which emphasize the uncertainties in observational determinations of the primordial abundances of ⁴He ¹⁰⁾ and ³He ¹¹⁾.

In the first part of this paper we review the standard upper limit²⁾ on N_ν . We show how uncertainties in the neutron half-life¹²⁻¹⁵⁾, systematic errors in the abundance of ⁴He inferred from observations¹⁶⁾ subject to complex corrections¹⁰⁾, and new data¹¹⁾ suggesting destruction of ³He in stars all tend to increase the cosmological upper bound on N_ν to

$$N_\nu < 5.5 \quad (1)$$

Then we calculate the minimum decoupling temperature $T_d = O(m_\mu)$ for which three species of ν_R are compatible with the bound (1), and derive the corresponding upper limit on the $e^+e^- \rightarrow \nu_R \bar{\nu}_R$ annihilation cross-section. We then present a calculation of $\sigma(e^+e^- \rightarrow \nu_R \bar{\nu}_R)$ in the minimal phenomenological model^{17,18,7)} motivated by the superstring, and derive the bounds on its parameters implied by eq. (1). These lead to a lower bound

$$M_{Z'} > 330 \text{ GeV} \quad (2)$$

on the mass of the additional neutral gauge boson Z'. As a byproduct, we also calculate the superstring model corrections to $\sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-)$ and $\sigma(\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-)$,

which can be probed in forthcoming experiments¹⁹⁾. The bound (2) suggests that these corrections are small, but in our view the cosmological bound (1) could be stretched to allow $N_\nu = 6$, in which case the Z' might be lighter than (2) and superstring corrections to $(\bar{\nu}_\mu e)$ scattering would then be observable.

As was first noted by Shvartsman¹⁾, the abundance of primordial synthesized helium constrains the possible existence of new weakly interacting particles which were relativistic during nucleosynthesis in the early universe at $t \sim 1$ sec ($T \sim 1$ MeV). The expansion rate, $H = (8\pi g/3m_p^2)^{1/2} \simeq 1.66 g_*^{1/2} T^2/m_p$, of the radiation-dominated universe would have been speeded up if new particles contributed to g_* , the effective number of relativistic degrees of freedom. This would cause the neutron-proton interaction rate ($\Gamma_{\text{weak}} \propto \tau_n^{-1}$) to freeze out earlier at a higher n/p equilibrium ratio and thus lead to the production of more ${}^4\text{He}$ through two-body nuclear reactions ($\Gamma_{\text{strong}} \propto n_N^2$). The recent numerical calculations of Yang et al.²⁾ show that the primordial mass abundance of ${}^4\text{He}$, Y_p (${}^4\text{He}$) is given by²⁰⁾

$$Y_p({}^4\text{He}) = 0.230 + 0.011 \ln \eta_{10} + 0.013 (N_\nu - 3) \\ + 0.014 (\tau_{1/2} - 10.6) \quad , \quad \text{for } 1.5 \lesssim \eta_{10} \lesssim 10 \quad , \quad (3)$$

where $\eta \equiv 10^{-10} \eta_{10} = n_N/n_\gamma$ is the nucleon-to-photon number ratio, $\tau_{1/2}$ is the neutron half-life in minutes and N_ν is the 'effective' number of 2-component neutrinos with full-strength neutral current interactions. The dependence of Y_p (3) on the observationally uncertain parameter η ($\sim 5 \times 10^{-11} \rightarrow 3 \times 10^{-8}$)²¹⁾ can be eliminated by noting that the residual abundances of D and ${}^3\text{He}$ when the strong nuclear reactions freeze out is a sensitive decreasing function of η , so that²⁰⁾

$$Y_p({}^4\text{He}) = 0.261 - 0.018 \log \left[10^5 \{ (D + {}^3\text{He})/H \}_p \right] \\ + 0.014 (N_\nu - 3) + 0.014 (\tau_{1/2} - 10.6) \quad (4)$$

In order to use eq. (4) to set a limit on N_ν , Yang et al.²⁾ used three key inputs:

- i) the neutron half-life $\tau_{1/2} > 10.4$ min.
- ii) The primordial ${}^4\text{He}$ abundance derived from observations of 12 'metal-poor' galaxies by Kunth and Sargent¹⁶⁾, $Y_p = 0.245 \pm 0.003$, which implies $Y_p < 0.254$ at the 3 standard deviation level.

iii) An argument²⁾ that all primordial D which is burnt in stars creates ^3He , a fraction $g < 1$ of which survives stellar processing. Thus the sum of the primordial abundances is related to that at time t by

$$\left(\frac{D+^3\text{He}}{H}\right)_p \leq \left(\frac{D}{H}\right)_t + \frac{1}{g} \left(\frac{^3\text{He}}{H}\right)_t \quad (5)$$

In the absence of direct observations of ^3He in astrophysical objects, Yang et al.²⁾ appealed to studies of meteorites and the solar wind which suggest that the presolar abundances were

$$\left(\frac{D+^3\text{He}}{H}\right)_{\text{pre } \odot} \lesssim 4.3 \times 10^{-5}; \quad \left(\frac{^3\text{He}}{H}\right)_{\text{pre } \odot} \lesssim 1.9 \times 10^{-5} \quad (6)$$

so that eq. (5) can be rewritten as

$$\left(\frac{D+^3\text{He}}{H}\right)_p \leq \left(\frac{D+^3\text{He}}{H}\right)_{\text{pre } \odot} + \left(\frac{1}{g} - 1\right) \left(\frac{^3\text{He}}{H}\right)_{\text{pre } \odot} \quad (7)$$

which yields

$$\left(\frac{D+^3\text{He}}{H}\right)_p < 10^{-4} \quad \text{for } g > 1/4 \quad (8)$$

where the limit on g comes from stellar evolution models. Using these limits in eq. (4), Yang et al.²⁾ deduce

$$\Delta N_\nu = (N_\nu - 3) < 1 \quad (9)$$

the oft-quoted limit on additional neutrino species.

In view of the importance of this limit, we question whether the assumptions (i), (ii), and (iii) above are sufficiently conservative.

(i) A recent study of the beta decay of polarized neutrons has yielded $\lambda = |g_A/g_V| = 1.262 \pm 0.005^{12)}$ which implies¹³⁾ $\tau_{1/2} = 10.37 \pm 0.07$ min, in support of an earlier experiment which had yielded $\tau_{1/2} = 10.13 \pm 0.09$ min¹⁴⁾, and consistent with preliminary indications from a recent experiment using ultracold neutrons at the Grenoble reactor¹⁵⁾. Thus a conservative lower limit to the neutron half-life is $\tau_{1/2} > 10.2$ min^{*}).

*) An SU(3) Cabibbo fit to hyperon decays can be used to predict the range of λ yielding $\tau_{1/2} = 11.6 \pm 0.5$ min²²⁾, which disagrees by 2σ with the central value used by ref. 2, and by 3σ with our more conservative lower limit. We do not regard this discrepancy as much more serious that it already was previously²³⁾, particularly since SU(3) breaking effects are not yet under theoretical control.

ii) Davidson and Kinman¹⁰⁾ have warned that the quoted errors in the helium abundance estimate of Kunth and Sargent¹⁶⁾ are too optimistic, because certain potentially serious systematic errors were not included. They illustrate their point by describing new extensive observations of the dwarf galaxy I Zwicky 18, the most primordial object known, in which they find a helium mass fraction $Y(^4\text{He}) = 0.212 \pm 0.015$, to be multiplied by a complicated (and uncertain) correction factor which is *perhaps* ~ 1.1 ¹⁰⁾. In general they stress that a realistic error estimate for Y_p cannot be less than ± 0.01 , and that this cannot be improved simply by observing more objects, because of uncertain systematics.

Kunth and Sargent¹⁶⁾ plotted Y as measured in 12 galaxies (1 observed twice) versus the measured oxygen abundance, O/H , which is an indicator of stellar processing (i.e. non-primordial ^4He production). A fit to the (naively) expected linear relationship, $Y = a(O/H) + Y_p$, then yields $Y = (17 \pm 93) O/H + (0.243 \pm 0.010)$ with $\chi^2 = 11.40$ for 11 d.o.f. Kunth and Sargent¹⁶⁾ claimed that this implies no correlation between Y and O/H and took the straight mean of the 13 measurements of Y , $\langle Y \rangle = 0.245 \pm 0.003$ ($\chi^2 = 11.44$ for 12 d.o.f.). We note that this last step is formally unjustifiable and that the intercept at $O/H \approx 0$, viz. $Y_p = 0.243 \pm 0.010$ is the correct representation of their data.

Thus $Y_p < 0.254$ as used by Yang et al.²⁾ is closer to a 1σ than a 3σ upper limit. Moreover, the issues raised by Davidson and Kinman¹⁰⁾ illustrate that such formal limits cannot really be established by the present observations. All the high quality data on Y_p lie in the range $0.22-0.26$ ²⁰⁾, so we adopt $Y_p < 0.26$ as a reasonable bound.

iii) Recently Rood et al.¹¹⁾ have detected ^3He in galactic HII regions and they derive $^3\text{He}/H$ abundance ratios ranging from 4×10^{-5} in W43 to 5×10^{-4} in W3. The size of the ^3He enhancements over the possible presolar value appears to be anticorrelated with indicators of stellar processing, suggesting that there is net destruction rather than creation of ^3He in stars. The observed ^3He is then presumably from primordial D which has been burnt in stars, and Delbourgo-Salvador et al.²⁴⁾ have recently studied models of chemical evolution of the galaxy in which such large astration of D can occur. In view of the present uncertainties the limit $[(D + ^3\text{He})/H]_p < 5 \times 10^{-4}$ would seem more conservative than the value adopted by Yang et al.²⁾.

Using these numbers^{*)} in eq. (4), we obtain the bound (1) on N_V which seems to us more conservative than the bound $N_V < 4$ of ref. 2. However, the sceptic might not choose to believe that astrophysical observations, which have

*) Note that the ranges of ^4He , D , and ^3He abundances which we allow are all compatible with the observed ^7Li abundance, according to the calculations of ref. 2. Also, the actual value of Y_p may differ from the fits in eqs. (3) and (4) if $\Delta N_V > 1$ and $\Delta \tau_{1/2} > 0.2$ ²⁰⁾, but these errors are negligible in the present case.

to be corrected for many uncertain systematic effects, can ever yield reliable upper bounds on the primordial abundances, in which case $N_\nu = 6$ or more could perhaps be tolerated. Though $N_\nu = 3$ is probably the best fit to astrophysical data, we do not believe that values of N_ν which are twice as large can be rigorously excluded.

New massless fermions F such as right-handed neutrinos contribute to the cosmological energy density an amount equivalent to an effective number ΔN_ν of additional 2-component left-handed neutrinos given by

$$\Delta N_\nu = \sum (g_F/2) (T_F/T_\nu)^4 \quad (10)$$

so that three new 2-component ν_R 's may satisfy the limit (1) on N_ν for sufficiently small values of the ratio T_F/T_ν of effective F and ν temperatures. As discussed by Olive et al.⁹⁾ T_F/T_ν is decreased below unity if the new particles decouple while ν_L 's are still in thermal equilibrium, so that subsequent annihilations of non-relativistic particles heat the ν_L 's but not the decoupled particles^{*}). The 'decoupling' temperature T_d can be estimated by equating the annihilation rate $\Gamma_{\text{ann}}^{\text{eq}} = n_F \langle \sigma v \rangle_{F\bar{F} \rightarrow l\bar{l}}$ to the expansion rate;

$$T_d \simeq 7.2 \times 10^{-7} (\text{GeV}) g_*^{1/6} \alpha^{-1/3} \quad (11)$$

where we have parametrized the annihilation rate

$$\langle \sigma v \rangle_{l\bar{l} \rightarrow F\bar{F}} = \left(\frac{n_x^{\text{eq}}}{n_l^{\text{eq}}} \right)^2 \langle \sigma v \rangle_{F\bar{F} \rightarrow l\bar{l}} \equiv \alpha T^2 \quad (12)$$

For example, requiring $T_d > m_\mu$ yields

$$\alpha < 1.4 \times 10^{-15} \text{ GeV}^{-4} \quad (13)$$

which would imply $T_F/T_\nu < 0.91$ ⁹⁾, and thus enable the bound (1) to be satisfied even if there were three ν_R 's. To satisfy the stricter bound (9) would require $T_d > T_c$, the deconfinement transition temperature, in which case $T_F/T_\nu < 0.59$ ⁹⁾. Taking into account that now $g_* = 226/4$, the constraint on α in this case would be

^{*}) Another way of suppressing the ν_R density relative to the ν_L density would be if some heavy particle such as the gravitino decayed between T_d and 1 MeV, producing entropy.

$$\alpha < 2.1 \times 10^{-16} \left(T_c / 0.3 \text{ GeV} \right)^{-3} \text{ GeV}^{-4} \quad (14)$$

where we expect $T_c = 0(0.2 \text{ to } 0.3) \text{ GeV}$.

In order to evaluate more precisely the bound on α implied by eq. (1) it is necessary to consider ν_R decoupling in more detail. If we introduce a notional relativistic fermionic species x which is truly decoupled during $\mu^+ \mu^-$ annihilation, so that

$$\dot{n}_x + (3\dot{R}/R) n_x \equiv 0 \quad (15)$$

then the transport equation governing the number density per helicity state of F is given²⁵⁾ by

$$\frac{d}{dt} \left(\frac{n_F}{n_x} \right) = n_x \langle \sigma v \rangle_{\mu^+ \mu^- \rightarrow F\bar{F}} \left[\left(\frac{n_\mu}{n_x} \right)^2 - f \left(\frac{n_F}{n_x} \right)^2 \right] \quad (16a)$$

where

$$f(T_F) \equiv \left[n_\mu(T_F) / n_F(T_F) \right]_{\text{equilibrium}}^2 \quad (16b)$$

We wish to determine an upper limit on $\langle \sigma v \rangle_{\mu^+ \mu^- \rightarrow F\bar{F}}$ such that T_F is sufficiently close to T_x for the bound (1) to be satisfied with three ν_R species. Defining

$$(T_F - T_x) / T_x \equiv \delta, \quad \delta \ll 1 \quad (17a)$$

$$(T_\gamma - T_x) / T_x \equiv \epsilon \quad (17b)$$

eq. (16) can be written as²⁵⁾

$$\frac{d\delta}{dy} \simeq a y^{-4} (\epsilon - \delta) \quad (18a)$$

where

$$a = 6.33 \times 10^{15} g_*^{-1/2} (\alpha / \text{GeV}^{-4}) \quad (18b)$$

and

$$y \equiv m_\mu / T_\gamma \quad (18c)$$

The evolution of T_x relative to the photon temperature T_γ is governed by entropy conservation,

$$T_x/T_\gamma = (43/57)^{1/3} [\zeta(m_\mu/T_\gamma)]^{1/3} \quad (19a)$$

where

$$\zeta(y) = 1 + \frac{180}{43\pi^4} \int_0^\infty x^2 dx \left[\sqrt{x^2 + y^2} + \frac{x^2}{3\sqrt{x^2 + y^2}} \right] \frac{1}{e^{\sqrt{x^2 + y^2}} + 1} \quad (19b)$$

Numerical evaluation of (19) shows that ϵ is well fitted by

$$\epsilon \approx A y^2 : A \approx 8.5 \times 10^{-3} \quad \text{for } 0.1 \leq y \leq 1 \quad (20)$$

Then the solution to (18) is²⁵⁾

$$\delta(y) = A (a/3)^{2/3} \exp(ay^{-3}/3) \int_{ay^{-3}/3}^\infty d\nu \nu^{-2/3} e^{-\nu} \quad (21)$$

Note that $\lim_{y \rightarrow \infty} \delta \simeq A(a/3)^{2/3} \Gamma(1/3)$ is an upper limit to the value of δ at any finite temperature. Thus we can ensure $\delta < A$ by imposing

$$a < 3\Delta^{3/2} / [A \Gamma(1/3)]^{3/2} \quad (22)$$

Using equation (18), we see that this inequality holds if

$$\alpha < 1.1 \times 10^{-16} g_*^{1/2} A^{-3/2} \Delta^{3/2} \quad (23)$$

In the case of three species of ν_R , eq. (1) requires

$$\Delta N_\nu = 3 (T_F/T_\nu)^4 \simeq 2.06 (1+\delta)^4 < 2.51 \quad (24)$$

implying $\delta < 5.1 \times 10^{-2}$ and hence

$$\alpha < 7.0 \times 10^{-15} \text{ (GeV}^{-4}\text{)} \quad (25)$$

The realistic constraint (24) on α is a factor of about 5 less stringent than the approximate estimate (13)*).

We now apply the bound (24) to the model^{17,18,7)} of interest to us with its three ν_R and a heavy Z' .

We consider first the general case of $e^+e^- \rightarrow \nu_R \bar{\nu}_R$ annihilation through neutral currents mediated by a new gauge boson Z_E which mixes with the Standard Model Z^0 to give physical mass eigenstates Z_1, Z_2 :

$$Z_1 = \cos \theta Z^0 + \sin \theta Z_E; \quad Z_2 = \cos \theta Z_E - \sin \theta Z^0 \quad (26)$$

The couplings of these mass eigenstates to the ν_L and ν_R are:

$$\begin{aligned} Z_1 \nu_L \nu_L: & \cos \theta Y_L^\nu g + \sin \theta Y_{E_L}^\nu g_E; \quad Z_2 \nu_L \nu_L: -\sin \theta Y_L^\nu g + \cos \theta Y_{E_L}^\nu g_E \\ Z_1 \nu_R \nu_R: & \sin \theta Y_{E_R}^\nu g_E; \quad Z_2 \nu_R \nu_R: \cos \theta Y_{E_R}^\nu g_E \\ Z_1 e_{L,R} e_{L,R}: & \cos \theta Y_{L,R}^e g + \sin \theta Y_{E_{L,R}}^e g_E; \quad Z_2 e_{L,R} e_{L,R}: -\sin \theta Y_{L,R}^e g + \cos \theta Y_{E_{L,R}}^e g_E \end{aligned} \quad (27)$$

where g and g_E are the corresponding gauge coupling constants. Introducing the notation

$$r \equiv m_1^2/m_2^2; \quad \rho = \frac{r}{1-r}; \quad f(r) \equiv \cos^2 \theta + \rho; \quad \bar{f}(r) \equiv \sin^2 \theta + \rho \quad (28)$$

and

$$E_{L,R} \equiv g_E Y_{E_{L,R}}^e / g Y_{L,R}^e, \quad N \equiv g_E Y_{E_L}^\nu / g Y_L^\nu \quad (29)$$

we find

$$\begin{aligned} \frac{\sigma(e^+e^- \rightarrow \nu_R \bar{\nu}_R)}{\sigma(e^+e^- \rightarrow \nu_L \bar{\nu}_L)} &= \left(\frac{g_E Y_{E_R}^\nu}{g Y_L^\nu} \right)^2 \times \\ &\times \frac{(Y_R^e)^2 [\sin 2\theta + 2\bar{f} E_R]^2 + (Y_L^e)^2 [\sin 2\theta + 2\bar{f} E_L]^2}{(Y_R^e)^2 [2f + (E_R + N) \sin 2\theta + 2\bar{f} E_R N]^2 + (Y_L^e)^2 [2f + (E_L + N) \sin 2\theta + 2\bar{f} E_L N]^2} \end{aligned} \quad (30)$$

) We have assumed above that the value of g_ is $g_*(\gamma, \mu^\pm, e^\pm, 3\nu_L, 3\nu_R) = 19.5$, and thus not accounted for the decrease in g_* as the muons turn non-relativistic and annihilate, and the ν_R decouple so that finally, $g_*(\gamma, e^\pm, 3\nu_L, 3\nu_R [T_E = (43/57) T_\mu]) \simeq 14.4$. Thus we overestimate the bound on α by a factor $\simeq (19.5/14.4)^{1/2} \simeq 1.2$.

The Standard Model cross-section for $e^+e^- \rightarrow \nu_L \bar{\nu}_L$ is obtained by taking the limit $\theta \rightarrow 0$, $r \rightarrow 0$ in the denominator of eq. (30). In the minimal extension^{17,18,7)} of the Standard Model to include⁸⁾ one new neutral gauge boson Z_E with the couplings specified¹⁸⁾ by the superstring we have

$$g_E = g \sin \theta_W = g_2 \tan \theta_W; Y_{E_L}^e = Y_{E_L}^\nu = -\frac{1}{6}, Y_{E_R}^e = -\frac{1}{3}, Y_{E_R}^\nu = -\frac{5}{6} \quad (31)$$

while

$$Y_L^e = \sin^2 \theta_W - \frac{1}{2}, Y_R^e = -\sin^2 \theta_W, Y_L^\nu = \frac{1}{2} \quad (32)$$

as usual. The Z^0 - Z_E mixing is provided¹⁸⁾ by a mass squared matrix of the form

$$(Z^0, Z_E) \begin{pmatrix} 1 & a \\ a & b \end{pmatrix} \begin{pmatrix} Z^0 \\ Z_E \end{pmatrix} \quad (33)$$

yielding

$$m_{1,2}^2 = \frac{1}{2} m_{Z^0}^2 \left[b+1 \pm \sqrt{(1-b)^2 + 4a^2} \right], \tan 2\theta = 2a/(1-b) \quad (34)$$

where in this case

$$a = \sin \theta_W \frac{4v^2 - \bar{v}^2}{3(v^2 + \bar{v}^2)}, \quad b = \sin^2 \theta_W \frac{25x^2 + 16v^2 + \bar{v}^2}{9(v^2 + \bar{v}^2)} \quad (35)$$

The quantities x , v , and $\bar{v}$ are Higgs vacuum expectation values^{18,7)}, two of which provide the $W^\pm$ and Z^0 masses in the conventional way:

$$m_{W^\pm}^2 = \frac{1}{2} g_2^2 (v^2 + \bar{v}^2), \quad m_{Z^0} = m_{W^\pm} / \cos \theta_W \quad (36)$$

The model contains two unknown parameters x/v and $\bar{v}/v$ which we now constrain using the bound (24) on the temperature averaged $\langle \sigma(e^+e^- \rightarrow \nu_R \bar{\nu}_R) v \rangle$ of eq. (12) recalling that in the Standard Model $\langle \sigma(e^+e^- \rightarrow \nu_L \bar{\nu}_L) v \rangle \approx 0.27 G_F^2 T^2$.

The constraint (25) which results from our conservative bound (1) on N_ν is shown as the solid line in the figure. Also shown as a dashed line is the tighter constraint (14) which would apply if one adopted the less conservative²⁾ cosmological bound (9) on N_ν . As mentioned earlier, one might even take the point of view that cosmological nucleosynthesis allows $N_\nu = 6$, in which case there is no useful bound on the superstring model. The dotted lines are contours of constant values of x/v . We see that the minimum value of the new Z_E gauge boson mass is attained when $\bar{v}/v \sim 0.4$, which is coincidentally a favoured value

in a class of dynamical calculations⁷⁾ in this minimal superstring model. If one accepts the conservative upper bound (1) of 5.5 on the possible number of equivalent light neutrino species, this value of $\bar{v}/v$ gives the lower bound (2) of 330 GeV for the Z_E mass. This bound is considerably more stringent than those obtained from the non-observation of the Z_E in $p\bar{p}$ collisions ($m_{Z_E} \gtrsim 110$ GeV)^{18,26)}, or from the agreement of low-energy neutrino-hadron and e^+e^- scattering experiments with Standard Model predictions ($m_{Z_E} \gtrsim 150$ GeV)^{18,26)}, but is of course subject to more uncertainties.

A new round of $\nu_\mu e^-$ and $\bar{\nu}_\mu e^-$ scattering experiments is now being prepared¹⁹⁾ so as a by-product of our analysis, we also present the modifications to low energy $\nu_\mu e^-$ and $\bar{\nu}_\mu e^-$ scattering cross-sections in the minimal superstring model outlined above [(31) to (35)]. Using the same notation [(27) to (29)] as previously, we find

$$\frac{\sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-)}{\sigma_0(\nu_\mu e^- \rightarrow \nu_\mu e^-)} = \left(\frac{m_{Z^0}}{m_{Z_1}} \right)^4 \frac{(1-r)^2}{(Y_L^e)^2 + \frac{1}{3}(Y_R^e)^2} \times$$

$$\times \left\{ (Y_L^e)^2 \left[f + N E_L \bar{f} + \sin 2\theta (E_L + N)/2 \right]^2 + \frac{1}{3} (Y_R^e)^2 \left[f + N E_R \bar{f} + \sin 2\theta (E_R + N)/2 \right]^2 \right\}$$

(37)

for the ratio to the cross-section in the Standard Model. The corresponding ratio for $\sigma(\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-)$ scattering is obtained from (37) by interchanging the subscripts L and R. Numerically, we find in the most favourable case allowed by the cosmological 'bound' (1), namely $\bar{v}/v = 0.4$, $x/v = 5.1$, that $\sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-)/\sigma_0(\nu_\mu e^- \rightarrow \nu_\mu e^-) = 1.02$. While this deviation is presumably too small to be detected by forthcoming experiments¹⁹⁾, experimentalists should not be discouraged. As we have emphasized above, the cosmological bounds²⁾ on light neutrinos and their couplings are distinctly permeable, and the deviations from the Standard Model could be much larger if m_{Z_E} is as low as other laboratory experiments permit.

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Figure caption

The cosmological bound on the mass m_{Z_2} of the second neutral gauge boson in minimal superstring models. (a) The realistic bound given by eq. (25); (b) the more stringent bound of eq. (14) (for explanations see text). Also shown are the contours of constant x/v (dashed lines).

(a)

(b)