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SCATTERING OF AN ELECTROMAGNETIC WAVE
BY AN IRREGULARLY SHAPED OBJECT

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ABSTRACT

We consider the scattering of an electromagnetic wave of a centimetric wavelength by an object whose surface is irregular. We study the angular dependence of the intensity scattered in the framework of the eikonal model. Using different types of surface our results show a correlation between the irregularities and the local maxima in the intensity spectrum and a systematic back-scattering enhancement. This approach extends our previous study of light scattering by large particles of fractal shape.

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Measurements ¹⁾ of light scattering by rough particles whose dimensions are greater than the incident wavelength have exhibited large discrepancies from the Mie theory ²⁾. As a first attempt to interpret these results, a modelisation of the diffracted amplitude in the forward direction has been worked out in the framework of the eikonal picture ³⁾ and shown in qualitative agreement ⁴⁾ with measurements of scattering intensity and polarization in microwave and optical radiation. In this approach, which has been extended to the whole angular domain ⁵⁾ for scattering potentials with compact support, the asperities of the surface were modelled by a Fermi type distribution giving therefore only a statistical representation of the roughness. Recently, an expression for the scattering amplitude taking exactly into account the shape of the particle and modelling its surface by a fractal function has been derived ⁶⁾. The angular dependence of the intensity diffracted by a rough surface was characterized by an increase of backscattered intensity and strong oscillations.

A full check of these predictions is difficult for light scattering since present experimental data ¹⁾ have only investigated the angular dependence between 20° and 160° with a resolution of 3° which does not allow the observation of oscillations. However the theoretical predictions for the intensity averaged over the oscillations are in qualitative agreement with experimental data. Therefore, in order to accurately test our model, we will study the scattering of an electromagnetic wave by three different objects exhibited in fig.1, for which precise measurements in the whole angular domain with a resolution of less than 1° can be done. Since our goal is a study of surface effect, we impose the conditions that the objects have the same diameter (16 cm), an index of refraction $n = 1.89 \pm i \ 0.01$ and a cylindrical symmetry along the z axis. We have examined three objects whose surface is made of circles.

Type I (solid line) corresponds to 8 convex quarters of circles of radius 2 cm, type II (dashed line) is made of 16 convex quarters of circles of radius 1 cm and type III (dashed-dotted line) has 8 quarters of circles of radius 2 cm of which 4 are concave. A few comments are in order here. These objects exhibit surface irregularities of size greater than the electromagnetic wavelength chosen to be 0.856 cm. The symmetry along the z axis was taken both for computational convenience and in view of the manufacturing constraints, however the eikonal formalism presented in ref. 6 can be extended to a non symmetric surface.

Let us first describe briefly the eikonal picture. We consider the scattering of an electromagnetic wave of wavelength λ and wavevector k by an object having a cylindrical symmetry, whose mean radius a has to be large compared to λ and whose size of irregularities d has to be greater than λ . According to ref. 6, the scattering amplitude reads :

$$f(\theta) = \frac{ik^2}{2} \int_0^{+\infty} b db J_0(kb \sin \theta) \frac{1 - n^2(b)}{q - \frac{1 - n^2(b)}{2}}$$

$$\left\{ \left[e^{iqz_1} e^{-ik \frac{(1 - n^2(b))}{2} (z_2 - z_1)} - e^{iqz_1} \right] + \sum_{j=2}^{N_b/2} \left[e^{-ik/2 (1 - n^2(b))} \right. \right.$$

$$\cdot \left[\sum_{l=1}^{j-1} (z_{2l} - z_{2l-1}) - z_{2j-1} \right] \left(e^{i(q - k \frac{(1 - n^2(b))}{2}) z_{2j}} \right.$$

$$\left. \left. - e^{i(q - k \frac{(1 - n^2(b))}{2}) z_{2j-1}} \right) \right] \right\} \quad (1)$$

with the following notations : $n(r)$ is the complex index of refraction of the object, $q = 2k \sin^2 \theta / 2$ is the scattered wave vector and

the $z_i(b_0)$ ($i=1, \dots, N_b$) are the solutions of the equation $S(z) = b_0$ where $b = S(z)$, is the equation of the boundary of the scattering object. The scattered intensity is given by :

$$i(\theta) = k^2 |f(\theta)|^2 \quad (2)$$

For the three objects exhibited in fig.1 the equation $S(z) = b_0$ is characterized by two intersecting points z_1 and z_2 .

We compute the scattering of an electromagnetic wave of wavelength $\lambda = 0.856$ cm by the macroscopic objects described in fig.1. This study gives us insights on the sensitivity of $i(\theta)$ on d/λ since for the object of type II we are at the limit of validity of the eikonal picture ($d/\lambda \approx 1$). Figures 2 and 3 show the intensity diffracted by objects of type I and II. In both cases we observe an increase of backscattered intensity and strong oscillations. Moreover one gets a very interesting correlation between the number of quarter of circles in fig.1 and the number of maxima of intensity in the angular domain $85^\circ \leq \theta \leq 180^\circ$. For 16 circles (fig.2) we get 3 maxima at 88° , 129° and 165° , whereas for a surface made of 8 circles we get 2 more additional maxima at 110° and 146° (see fig.3) i.e. at medium angles between two consecutive maxima. Other numerical computations not presented here with respectively 4 and 6 quarters of circles lead also to additional peaks of intensity at medium angles. We have therefore exhibited interference phenomena, as in the Young slits or any diffraction grating experiment ⁷⁾, with a frequency of oscillations proportional to the distance between two consecutive elementary patterns, which corresponds in our case to the radius of the quadrant of the surface.

The intensity scattered by an object of type III is shown in fig.4. It is similar to the previous cases for $\theta \leq 60^\circ$. At larger angles it oscillates more and therefore the magnitude of intensity peaks decreases. An experiment concerning these three objects is in project ⁸⁾ and should

provide us with an interesting check.

Since we have exhibited interference phenomena, it would be natural to study how circles of different radii or disymmetric surfaces with a different number of right N_C^R or left N_C^L quadrants affect these results. Fig.5 gives the intensity diffracted by 16 quadrants of incommensurable radii of roughly $1 \pm .1$ cm. We observe an increase of backscattering intensity and a strong reduction of the maxima. The case of a disymmetric surface with $N_C^R = 6$ and $N_C^L = 4$ circles of incommensurable radii is shown in fig.6. Due to destructive interferences, the oscillations are still present but strongly reduced in magnitude. Although the peaks of intensity have disappeared, the increase of backscattering intensity remains. This increase is analogous to the weak localization theorem⁹⁾ which states a constructive interference effect at 180° .

Further numerical studies, performed at different wavelengths, have led to the same global behaviour. For instance taking $\lambda = 0.02$ cm with an object of type II we observe an increase for the intensity of one order of magnitude in the range $20^\circ \leq \theta \leq 150^\circ$ and a strong increase of forward and backward peaks.

To conclude, we have exhibited important interference phenomena which should be soon experimentally tested. In all cases we observe an increase of the backscattering intensity at 180° which corresponds to a weak localization effect. Moreover the magnitude of the backscattered peak depends on the ratios d/λ and a/λ . Therefore a study of the scattering intensity, especially in the angular domain $150^\circ \leq \theta \leq 180^\circ$, should give us insights on the mean radius a and the degree of roughness of the scattering object (as observed for example in the acoustic domain¹⁰⁾).

On the other hand, the objects we have considered do not lead to a global enhancement of scattered intensity between 150° and 180° , as

obtained in ref. 6 for rough objects whose scattering surfaces were described by a fractal function, simulating asperities extending through several scales. Therefore a global increase of diffracted intensity indicates the degree of roughness of the scatterer.

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Figure captions

Fig. 1 : shapes of the scattering surface $b = S(z)$. Object of type I; full curve. Object type II ; dashed curve. Object of type III ; dotted-dashed curve.

Fig. 2 : scattering intensity for a particle of type I of mean radius 8 cm and index of refraction $n = 1.89 + i \ 0.01$ as a function of the scattering angle Θ . The dashed curve corresponds to the perfect sphere for comparison.

Fig.3 : the same as fig. 2 for an object of type II

Fig.4 : the same as fig. 2 for an object of type III

Fig.5 : the same as fig. 2 for an object of type I made of circles of incommensurable radii.

Fig.6 : scattering intensity for a disymmetric object made of 6 right and 4 left circles as a function of the scattering angle Θ .

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6