CERN-TH.4309/85

TOPOLOGICALLY NON-TRIVIAL GAUGE CONFIGURATIONS
AND THE HETEROTIC STRING

C. Gomez
CERN - Geneva

ABSTRACT

The contribution to the heterotic string of topologically non-trivial gauge configurations is studied. In the σ -model approach, it is proved that this kind of gauge configurations are not contributing to gauge anomalies. The $\beta = 0$ condition is also analyzed.

CERN-TH.4309/85
November 1985

It has been recently argued^{1),2)} that non-perturbative effects associated with topologically non-trivial gauge configurations can modify the potential for the dilaton field in the low-energy limit of superstring theories. It is, on the other hand, well known^{3),4)} that a consistent compactification of the ten-dimensional string theory must satisfy the constraint $\int_{Q_4} dH = 0$ as a consequence of the gauge invariant nature of H :

$$H = dB - \omega_3 + \omega_{3L} \quad (1)$$

If we do not allow string-like singularities for the B -field, the constraint becomes: $\int_{Q_4} F \wedge F - \int_{Q_4} R \wedge R = 0$ for Q_4 any four-dimensional compact submanifold of space-time. If this is the case for the low energy limit of the string theory⁵⁾, then only special combinations of gauge instantons and gravitational instantons can produce real physical effects. Very unlikely, this situation would change dramatically our understanding of the θ -structure of the vacuum for gauge theories. In particular for the E_8 case, the minimum Pontryagin number consistent with the constraint

$$\text{tr } F \wedge F = 30 \text{tr } R \wedge R \quad (2)$$

is three. In fact, from the index theorem we get:

$$V_+ - V_- = \frac{\dim V}{24 \cdot 8\pi^2} \int \text{tr } R \wedge R - \frac{1}{8\pi^2} \int \text{tr } F \wedge F \quad (3)$$

using the constraint (2) we get:

$$V_+ - V_- = \frac{1}{8\pi^2} \int \text{tr } F \wedge F \Big|_{ad} \left(\frac{\dim V}{24 \cdot 30} - 1 \right) \quad (4)$$

which for E_8 ($\dim V = 248$) is integer only for E_8 configurations with $\Pi_3(E_8)$ equal to 3α , $\alpha \in \mathbb{Z}^*$.

In this letter, we will study some aspects of the vacuum structure of the low-energy limit of the string theory and we will prove that pure gauge instantons (or any other topologically non-trivial gauge configuration) are consistent with the world sheet formulation of the string in terms of non-linear σ -models and with the general constraint $\int_{Q_4} dH = 0$. We will start considering the kind of

*) This fact was pointed out to me by G. Veneziano.

ground states, for string theories, founded by Candelas et al.⁶⁾. These kinds of compactifications to $M_4 \times K_6$ with the field strength H equal to zero and the spin connection embedded in a $SU(3)$ subgroup of the gauge group are possible backgrounds to the string σ -model with β -function equal to zero^{7),8)}. If K_6 is not simply connected, $\Pi_1(K_6) \neq 0$, we can define a non-trivial (torsion classes different from zero) flat fibre bundle ξ on K_6 in order to break the gauge group. If this is the case, the residual gauge symmetry contains only the set of gauge transformations $g(X, \xi)$ (X on M^4 , ξ on K_6) which does not change the "vacuum" defined by the flat fibre bundle ξ on K_6 ⁹⁾. At this level and in order to have the theory free of global anomalies, we must impose the following topological constraint on the torsion classes of the flat fibre bundle ξ ¹⁰⁾:

$$P_1(\xi; \mathbb{Z}) = P_1(T; \mathbb{Z}) \quad (5)$$

where P_1 represents the first Pontryagin class in the integer cohomology $H^4(K_6; \mathbb{Z})$ and where T represents the tangent fibre bundle on K_6 .

Very likely, after imposing condition (3), the residual gauge symmetry will be a non-Abelian group $SU(N)$. In these conditions and in the temporal gauge, classical vacuum gauge configurations are given by gauge connections $A_i = g(\underline{x}) \partial_i g(\underline{x})^{-1}$ with g independent on K_6 variables. Notice that all these gauge connections are good ground states for the string theory, in particular, they satisfy the topological constraint (5) and the $\beta = 0$ condition, moreover if we compactify the space to S_3 they can be classified in different topological classes by the $\Pi_3(SU(N)) = \mathbb{Z}$. These configurations just correspond to the usual $|n\rangle$ states of gauge theories¹¹⁾. At this level of discussion, we can try to understand instantons as tunnelling processes between states $|n\rangle$ and $|n+1\rangle$. In the temporal gauge picture this tunnelling must correspond to a situation with the state $|0\rangle$ at time $-\infty$ and the state $|1\rangle$ at time $+\infty$. The state $|0\rangle$ is characterized by $A = B = H = 0$ and the state $|1\rangle$ by $A_i = g_{(1)}^{-1} \partial_i g_{(1)}$, $H = 0$ and B the gauge transformation of $B = 0$ $g_{(1)}$ ¹²⁾:

$$B = \alpha(g_1) \quad (6)$$

with $\alpha(g_1)$ a two-form locally defined by:

$$\alpha(g_1) = -\frac{1}{3} \text{tr} (g_1^{-1} dg_1)^3 \quad (7)$$

This form is well defined only locally since the gauge transformation g_1 is

topologically non-trivial; in other words, the three-form $1/3 \text{tr}(g_1^{-1} dg_1)^3$ is in a non-trivial class of $H_3(S^3) = \mathbb{Z}$. Therefore, we see that the field B, corresponding to a tunnelling process from $|n\rangle$ to $|n+1\rangle$ is similar to the kind of generalized Dirac monopoles studied in Ref. 13). In fact, if we use, as a complementary point of view, a compactification of space-time to S^4 , then the tunnelling configuration for the field B corresponds to two configurations B^N, B^S defined on the hemispheres with a transition function on the S^3 -equator with non-trivial Π_3 . To check that this configuration satisfies $\int_{S^4} dH = 0$ is very easy. In fact, the H-configuration is defined by: $dB = \omega_{YM} = \omega_L = 0$ on one hemisphere and $dB = d\alpha(g)$, $\omega_{YN} = 1/3 \text{tr}(g^{-1} dg)^3$, $\omega_L = 0$ on the other one, with g defining the transition function on the overlapping region (the S^3 -equator). For this configuration we have:

$$\int_{S^4} dH = \int_{S^3} d\alpha(g) - \frac{1}{3} \text{tr}(g^{-1} dg)^3 = 0 \quad (8)$$

Now let us study whether these tunnelling processes are consistent with the string theory. We will work out this problem in the σ -model representation of the heterotic string^{14),15)}. In this framework, we will first prove that topologically non-trivial background gauge configurations are not producing any gauge anomaly. In the fermionic formulation the world sheet σ -model for the heterotic string contains ten right-moving fermions ψ_i and two groups of 16 left-moving fermions λ_i . The left-moving fermions are coupled to the gauge field pulled back on the world sheet $\Sigma_i (\partial x^i / \partial \xi^\alpha) A_i$ (ξ co-ordinates on Σ), and the right moving fermions are coupled to the pull back of the spin connection $(\partial x^i / \partial \xi^\alpha) \omega_i$. The tunnelling configuration will affect only the movement of the string on M_4 . The movement on K_6 will be anomaly free in our case, because on K_6 we always satisfy Eq. (2), as a consequence of the embedding of the spin connection in a subgroup of the gauge group. Therefore, the only part of the functional measure we must study is the determinant of left moving fermions in the presence of an instanton and the part corresponding to the coupling of the B-field to the string:

$$\det D_{16} \left(\frac{\partial X^i}{\partial \xi^\alpha} A_i \right) \cdot \exp \int_{\Sigma} X^* B \quad (9)$$

Now, in order to visualize the effect of an instanton-like configuration on the string, we will consider a two-parameter family of world sheets Σ and the pull back on this family of the non-trivial gauge configuration A_μ defined on the space-time M_4 . This construction corresponds to define an application $\tilde{X}(\xi, \hat{\xi})$

(ξ on Σ , $\tilde{\xi}$ on S^2) from $Z \times S^2$ into M_4 . Now for this two-parameter family of world sheets we can easily study the problem of anomalies. In fact, it is well known^{16),17)} that the σ -model anomalies are given, from a global point of view, by:

$$A_6 = \int_{\Sigma \times S^2} ch \tilde{X}^* \eta \quad (10)$$

with $\tilde{X}^* \eta$ the gauge fibre bundle defined by $(\partial \tilde{X}^1 / \partial \xi) A_1$. Then an instanton-like configuration living on M_4 can produce an anomaly if $\mathcal{A}$ is different from zero. The explicit value of $\mathcal{A}$, in our case, is:

$$A_6 = \deg \tilde{X} \int_{S^4} tr F \wedge F \quad (11)$$

where we compactify the space-time to S^4 and where $\deg \tilde{X}$ will be different from zero if the application $\tilde{X}: S^2 \times \Sigma \rightarrow S^4$ is topologically non-trivial. At this point, we can study two different cases: i) $\deg \tilde{X} = 0$ and ii) $\deg \tilde{X} \neq 0$. The first case is clearly anomaly-free and it is not going to produce any problem. The case ii) is more interesting and requires a more careful analysis. The gauge anomaly (11) is the first Chern class of a determinant fibre bundle defined on the S^2 -family of world sheets we are using¹⁸⁾. This characteristic class can be visualized as the winding number of the phase of the fermionic determinant on the equator S^1 of the S^2 -family of world sheets. On the other hand, the part in the fermionic measure (9) corresponding to the coupling of the B-field to the string is changing in the same way. In fact, and just using the gauge transformation rule (6) for the B-field, we see that the change of $\int_{\Sigma} \tilde{X}^* B$ around the equator S^1 is $\int_{\Sigma \times S^1} \tilde{X}^* (tr(1/3 g^{-1} dg)^3)$ which exactly cancels the phase of the fermionic determinant:

$$\int_{\Sigma \times S^1} \tilde{X}^* (F_1 F) = \int_{\Sigma \times S^1} \tilde{X}^* \left(\frac{1}{3} tr (g^{-1} dg)^3 \right) \quad (12)$$

Hence we see that this tunnelling configurations do not contribute to the string measure with new gauge anomalies. Very likely the interpretation of the string as an axion string¹⁹⁾, together with the approach to anomalies of Ref. 20), can provide a more physical understanding of the previous mechanism.

The next point we must analyze is the problem of conformal anomalies. It is clear that the ground states defined by pure gauge configurations $g^{-1}(\underline{x}) dg(\underline{x})$,

with $g(\underline{x})$ in a topologically non-trivial class, satisfy the $\beta = 0$ condition, however, the instanton configuration, if it is considered as a background state, gives a non-zero contribution to the one-loop gauge β -function⁷⁾:

$$\beta_{\mu\nu}^G = R_{\mu\nu} - \frac{1}{4} H_{\mu\nu}^2 + 2 \nabla_\mu \nabla_\nu \phi + \frac{\alpha'}{2} (R_{\mu\lambda\sigma\tau} R_{\nu}^{\lambda\sigma\tau} - \frac{1}{2} F_{\mu\nu}^2) \quad (13)$$

which, in principle, can only be compensated by the simultaneous existence of gravitational instantons. Thus the $\beta = 0$ condition on the string σ -model apparently forbids the existence of string-like singularities of the B-field, a field which is not gauge invariant. However, the value of F_μ , F_ν which appears in Eq. (13) must be interpreted as a v.e.v. on the real vacuum state. This v.e.v., in the presence of massless fermions, which is just our case, will be zero on the $|\theta\rangle$ -vacuum that we are just getting as an effect of the tunnelling processes.

In conclusion, we see that topologically non-trivial gauge configurations are perfectly consistent with the string theory if i) string-like singularities for the B-field are allowed and ii) the background field values used in the computation of the σ -model β function are the v.e.v. on the real vacuum we get, after taking into account all possible tunnelling processes.

ACKNOWLEDGEMENTS

It is a pleasure to thank E. Cohen, L. Girardello, C. Reina, R. Stora and G. Veneziano for very useful discussions.

REFERENCES

- 1) M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. 156B (1985) 55.
- 2) E. Cohen, J. Ellis, C. Gomez and D.V. Nanopoulos, Phys. Lett. 156B (1985) 55.
- 3) E. Witten, Phys. Lett. 149B (1984) 351.
- 4) M. Green and J. Schwarz, Nucl. Phys. B255 (1985) 93.
- 5) S. Cecotti, S. Ferrara, L. Girardello and M. Porrati, CERN preprint TH.4253/85 (1985).
- 6) P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B258 (1985) 46.
- 7) C. Callan, E. Martinec, M. Perry and D. Friedan, Princeton preprint (1985).
- 8) C.M. Hull, MIT preprint (1985).
- 9) E. Witten, Nucl. Phys. B258 (1985) 75.
- 10) E. Witten, "Global Anomalies in String Theories", Princeton preprint (June 1985).
- 11) C. Callan, R. Dashen and D. Gross, Phys. Rev. D17 (1978) 2717.
- 12) G. Chapline and N. Manton, Phys. Lett. 110B (1985) 105.
- 13) R. Nepomechie and A. Zee, Seattle preprint 40048-OLP5 (1985).
- 14) C. Hull and E. Witten, Princeton preprint (1985).
- 15) D. Gross, J.A. Harvey, E. Martinec and R. Rohm, Phys. Rev. Lett 52 (1985) 502 and Princeton preprints (1985).
- 16) G. Moore and P. Nelson, Phys. Rev. Lett. 53 (1984) 1519.
- 17) E. Cohen and C. Gomez, Nucl. Phys. B254 (1985) 235.
- 18) M. Atiyah and I. Singer, Proc. Nat. Acad. Sci. 81 (1984) 2597.
- 19) E. Witten, Phys. Lett. 153B (1985) 243.
- 20) C. Callan and J. Harvey, Princeton preprint (1984).