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ON A POSSIBLE STABILIZATION OF THE
TACHYONIC STRINGS

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A B S T R A C T

We present arguments that if the tachyonic strings get dressed to become flux tubes by non-perturbative effects, then the tachyon instability most likely disappears. We compare this scenario with the situation in QCD.

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It is well known that some string models contain tachyons. This may be an incurable illness. However, it is an old idea¹⁾ that the tachyon is an instability which may disappear by the emergence of a new non-perturbative vacuum.

In this note we shall present arguments that if the string gets dressed to become a flux tube by non-perturbative effects, then the tachyon instability most likely disappears.

In order to look for a possible stabilization of the tachyon one should first understand what causes this instability. Recently the author has shown²⁾ that from the point of view of the partition function the tachyon is caused by the exponentially infinite number of infinitely heavy states. These results can be summarized most easily by writing the partition function for a parameter space $R \times \beta$ in the Boltzmann form

$$Z = \sum_n d(n) e^{-\beta E_n(R)}, \quad (1)$$

where the energy levels $E_n(R)$ are derived in Ref. 2) for boundary conditions where the end-points of the open string are fixed in the spatial direction. In the "imaginary time" direction we take periodic boundary conditions with period β . Further, in Eq. (1) $d(n)$ is the level density³⁾ which increases exponentially with n .

String models have a duality property. For the partition function this means that an $R \times \beta$ domain can equally well be considered as a $\beta \times R$ domain. Thus, instead of the Boltzmann form (1) one can also write

$$\bar{Z} = \sum_n d(n) e^{-R \bar{E}_n(\beta)}. \quad (2)$$

Z and $\bar{Z}$ are equal, except for effects coming from the lack of symmetric boundary conditions. This has been discussed in Ref. 2), and it was shown that for large β and/or large R , this lack of symmetry does not affect the leading behaviour of Z and $\bar{Z}$. In Eq. (2) the energy levels $\bar{E}_n(\beta)$ are given as in Ref. 2),

$$E_n(R) = \sigma \sqrt{R^2 - \frac{d-2}{12} \frac{\pi}{\sigma} + \frac{2\pi n}{\sigma}},$$

$$\bar{E}_n(\beta) = \sigma \sqrt{\beta^2 - \frac{d-2}{3} \frac{\pi}{\sigma} + \frac{8\pi n}{\sigma}}, \quad (3)$$

where $n = 0, 1, 2, \dots$, and where σ is the string tension. The difference between E_n and $\bar{E}_n$ is due to the different boundary conditions. Equations (3) are valid for the bosonic string. For the Neveu-Schwarz string very similar expressions are valid.

One can now show²⁾ that the partition function (1) has the asymptotic behaviours

$$\sum_n d(n) e^{-\beta E_n(R)}$$

$$\xrightarrow{\beta \rightarrow \infty} e^{-\sigma \beta \sqrt{R^2 - R_c^2}}, \quad (4a)$$

$$\xrightarrow{R \rightarrow \infty} e^{-\sigma R \sqrt{\beta^2 - \beta_H^2}}, \quad (4b)$$

where^{4),5)}

$$R_c^2 = \begin{cases} \frac{\pi(d-2)}{12\sigma} & \text{Nambu-Goto,} \\ \frac{\pi(d-2)}{8\sigma} & \text{Neveu-Schwarz,} \end{cases} \quad (5)$$

and where^{6),7)} β_H is the inverse Hagedorn temperature given by

$$\beta_H^2 = \begin{cases} \frac{\pi(d-2)}{3\sigma} & \text{Nambu-Goto,} \\ \frac{\pi(d-2)}{2\sigma} & \text{Neveu-Schwarz.} \end{cases} \quad (6)$$

Similarly the partition function (2) has the behaviour

$$\sum_n d(n) e^{-R \bar{E}_n(\beta)}$$

$$\xrightarrow{R \rightarrow \infty} e^{-\sigma R \sqrt{\beta^2 - \beta_H^2}} \quad (7a)$$

$$\xrightarrow{\beta \rightarrow \infty} e^{-\sigma \beta \sqrt{R^2 - R_c^2}} \quad (7b)$$

The results (4a) and (7a) follow trivially from Eq. (3), since only the lowest energy level can contribute in the limits $\beta \rightarrow \infty$ and $R \rightarrow \infty$, respectively. The results (4b) and (7b) follow from the existence of saddle points in the limits $R \rightarrow \infty$ and $\beta \rightarrow \infty$, respectively. The mean value of n which contributes to Eq. (4b) is given by

$$\langle n \rangle_{(4b)} = \frac{\sigma R^2}{2\pi((\beta/\beta_H)^2 - 1)} \quad (R \gg R_c), \quad (8)$$

as can be seen from the saddle point expansion in Ref. 2). The corresponding quantity for Eq. (7b) is given by

$$\langle n \rangle_{(7b)} = \frac{\sigma \beta^2}{8\pi((R/R_c)^2 - 1)} \quad (\beta \gg \beta_H). \quad (9)$$

The average area can also be computed⁴⁾,

$$\langle \text{Area} \rangle / \beta R = (1 - \frac{R_c^2}{2R^2}) / \sqrt{1 - R_c^2/R^2}. \quad (10)$$

From Eqs. (4) and (7) one observes that Eq. (4a) is the same as Eq. (7b) [and Eq. (7a) is the same as Eq. (4b)]. However, the origin of the right-hand sides of (4a) and (7b) is completely different: In Eq. (4a) the result is obtained by keeping only the lowest energy in the Boltzmann sum. In Eq. (7b) the result emerges from the exponentially increasing level density for very large masses, as seen in Eq. (9). Thus, in Eq. (4a) the square root behaviour, which becomes meaningless for $R < R_c$, arises directly from the tachyon. Therefore, by "duality" this tachyonic behaviour can be understood as emerging from the very high energy states, as seen in Eqs. (7b) and (9). This is, in some loose sense, the old finite energy sum rule duality idea, namely, that the high energy states are "equivalent" to the low energy states.

Now, let us return to the question of stabilizing the tachyon. From the remarks above it is clear that if the high energy states could be cut off, then the tachyon would disappear.

However, it is important to notice that there is really no problem for large R (or large β), since the imaginary part does not show up until R is below R_c (or β is below β_H). Therefore, the stabilization problem occurs for $R \rightarrow R_c$ (or $\beta \rightarrow \beta_H$).

From Eq. (10) we see that for $R \rightarrow R_c$ the average area becomes infinite.

Also, it has been shown by Alvarez⁴⁾ that the average transverse fluctuations $\langle x_{\perp}^2 \rangle$ remain bounded relative to the infinite area. Thus, it follows that for $R \rightarrow R_c$ the string oscillates violently with an infinite length. This costs infinite energy, which is only possible because the number of states increases exponentially, thereby causing an infinite "entropy" (infinitely large phase space). In other words, although from (10) the average Nambu-Goto action becomes infinite, this is compensated by an infinite phase space (coming from the exponentially increasing level density), as is clear, e.g., from Eq. (9), from which we see that $\langle n \rangle \rightarrow \infty$ for $R \rightarrow R_c$.

Thus, the tachyon would be stabilized if we can avoid the infinitely long chaotic (∞ phase space) string.

Now let us imagine that non-perturbatively the string becomes dressed to become a flux tube^{*}). For large distances (relative to the transverse width) there is essentially no difference between a string and a flux tube. From Eq. (9) we also see that for $R \gg R_c$ (with β finite but large) the infinitely heavy states cannot contribute. However, as we approach the critical distance R_c a crucial difference between the string and the flux tube emerges. For R close to R_c the flux tube would try to oscillate more and more, just like the string does. However, as we fold the flux tube many times to achieve a long length and a large phase space, we soon run into trouble because of the finite width. Thus, it is impossible to obtain an infinite length by folding the flux tube (with finite transverse oscillations). Thus, the length and the area are expected to be finite, and hence the dominant action is also expected to be finite. Hence, the dominant phase space (the "entropy") is also expected to be finite in order to achieve balance between action and "entropy". Thus, $\langle n \rangle$ should also be finite. Consequently the exponentially rising level density must be cut off, since this is the only way to avoid the result (9) for $\langle n \rangle$.

Of course, the finite temperature case can be treated in a completely analogous way. Replacing the string by a flux tube then means that the passage of the Hagedorn temperature becomes smoothened out.

The string model results exhibited, e.g., in Eqs. (4) and (7) indicate that the "bare" string picture represents a one-phase model. As long as $R > R_c$ or $\beta > \beta_H$ the phase may actually be well described by strings. With the replacement of strings by flux tubes the dynamics may well become a description of two phases⁷⁾. Roughly speaking a flux tube is described by the "interior" and the

^{*}) This can, e.g., be motivated by the possibility that non-perturbatively point particles can become extended.

"exterior". When the flux tube is folded more and more for $R \rightarrow R_c$ or $\beta \rightarrow \beta_H$, we get more and more interior stuff, and the exterior stuff is pushed away from the region where the flux tube has its transverse oscillations. Thus, a sort of bag is formed, where the interior stuff fills the inside of the bag. Therefore, there exists the possibility that the interior is filled with other particles than those belonging to the string excitations.

We do not know any way of showing that non-perturbative effects can turn the string into a flux tube. However, there exists, of course, a well known example, namely QCD, where one believes that what is said above actually happens. At short distances one has asymptotic freedom. As the distance increases strings will start to form. This is rather analogous to what happens in the flux tube model: The transition between flux tubes and free particles (gluons) is expected to occur at a distance of order R_c which from (5) is of order $\frac{1}{2}$ fermi^{4),5)}. Similarly, β_H represents the critical temperature for deconfinement, which also has the right order of magnitude^{6),7)}.

It is amusing that from the point of view of QCD, the short distance phase is easy (perturbative), whereas the large distance phase is complicated and non-perturbative (and perhaps only accessible by computer calculations). For strings it is just the opposite: The large distance phase is perturbative and hence easy, whereas the short distance phase is complicated and non-perturbative. Thus, in a possible deconfined string phase one must expect the gluon field to be a composite operator of some sort (e.g., a small closed string, which is likely to be formed for $R \rightarrow R_c$ or $\beta \rightarrow \beta_H$ due to the relatively chaotic flux tube oscillations, which can burn off small closed strings due to self-touchings or self-crossings).

Finally, let us remark that we do not know, unfortunately, if the tachyon actually stabilizes. Thus, what we have done is to present a possible scenario for a stabilization. However, even if the tachyon does not stabilize, the tachyonic string models may still be quite useful as incomplete one-phase models for a two-phase system like QCD, at least at large distances and low temperatures.

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