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frame Schrodinger-type equation:

$$H|\Psi\rangle = (H_0 + V_{q\bar{q}}(\vec{p}, \vec{r}) + H_A)|\Psi\rangle = E|\Psi\rangle \quad (1)$$

where

$$H_0 = \sqrt{p^2 + m_q^2} + \sqrt{p^2 + m_{\bar{q}}^2} \quad (2)$$

H_A is the annihilation amplitude which we must consider in self conjugate mesons where $q\bar{q}$ annihilation via gluons can contribute to the masses, and $V_{q\bar{q}}(\vec{p}, \vec{r})$ is the quark-antiquark potential which is, due to relativistic effects, momentum dependent in addition to being coordinate dependent. We find $V_{q\bar{q}}(\vec{p}, \vec{r})$ by equating the scattering amplitude of free quarks with the effects between bound quarks inside a meson. In the non-relativistic limit V goes to the QCD motivated form:

$$V \rightarrow br + (\text{Breit-Fermi}) \quad (3)$$

However, in deriving the relativistic potential $V_{q\bar{q}}(\vec{p}, \vec{r})$ it is shown that retaining the full Dirac matrix element results in two important effects:

- 1) the coordinate $\vec{r}$ becomes smeared out over distances of the order of the inverse quark masses.
- 2) the various potentials become dependent on the momenta of the interacting quarks.

In this approach we have neglected off-mass-shell behavior of the valence quarks and also the more complex components of Fock space such as $(q\bar{q}g)$ and $(q\bar{q}q\bar{q})$. Both of these effects should be present and would introduce additional effects in the quark-antiquark potential so in light of this, rather than take the scattering amplitude too seriously, we take it as a framework on which to build our effective potential. In this spirit we include the smearing by convoluting a smearing function with the potential. The energy-dependence of the potential results in the various factors of m_i^{-1} in the non-relativistic treatment being replaced with, roughly speaking, factors of $(p^2 + m_i^2)^{-1/2}$ which are in turn controlled by the radius of confinement. As an illustration of these effects consider the hyperfine interaction which in the non-relativistic limit goes like $V_{hy} \propto \delta^3(\vec{r}_{q\bar{q}})/(m_q m_{\bar{q}})$. In our relativized quark model this interaction becomes

$$V_{hy} \propto \frac{1}{E_q E_{\bar{q}}} \left(\frac{E_q E_{\bar{q}}}{m_q m_{\bar{q}}} \right)^{\epsilon} \frac{\sigma^3}{\pi^{3/2}} e^{-\sigma^2 r^2} \quad (4)$$

where ϵ is a parameter which reflects our ignorance of the exact relativistic form but which we expect to be close to zero. This procedure is very crude but the relativistic corrections are only significant in the light quark sector. In addition we include the QCD running coupling constant, $\alpha_s(Q^2)$, which we let run according to perturbative QCD down to $\theta(\Lambda)$ where it is frozen down to $Q^2 = 0$.

I do not wish to present a detailed report of our results. Suffice it to say that we have been able to satisfactorily describe in a unified framework

MESON PHYSICS IN A RELATIVIZED QUARK MODEL WITH CHROMODYNAMICS

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ABSTRACT

Meson physics was studied in a relativized quark model which includes the important features of quantum chromodynamics. It was found that all mesons, from the π to the T , can be described successfully in this model. The key ingredient of the model is a universal one-gluon-exchange plus linear confinement potential motivated by QCD. However, it is crucial to the success of the model that relativistic effects be included. The spectroscopic results of the model are supported by an extensive analysis of strong, electromagnetic, and weak couplings. The model will be described briefly followed by a more detailed discussion of high spin mesons.

INTRODUCTION

Models motivated by Quantum Chromodynamics predict, in addition to the standard $q\bar{q}$ mesons and qqq baryons other states such as *glueballs*¹, *hermaphrodite* states (or hybrids)², and *baryonia*³. To distinguish these "new" states (and therefore new physics) from the more prosaic $q\bar{q}$ states it is important to understand the properties of the standard $q\bar{q}$ mesons.

Ideally we would solve QCD exactly to calculate these properties, but at present, despite much effort especially by means of numerical studies of QCD on the lattice, we have yet to extract hard numbers from the theory. In another vein we have a very successful phenomenological model of hadron physics, the quark model, in which hadron properties are described in terms of their valence quarks. Recently we have completed a very extensive analysis of meson physics in the framework of a relativized constituent quark model in which we have included the dominant features expected from quantum chromodynamics⁴. The model has had considerable success in describing the systematics of the mass spectra, electromagnetic transitions, and the strong, weak, and electromagnetic decays. In this report we will sketch out the model and present a few results. We then turn to the study of high spin mesons and what then can teach us about QCD.

THE MODEL

To include relativistic effects we do not carry out the relativization from first principles but rather construct a quark potential motivated by the expected relativistic properties. We approximate mesons by the $q\bar{q}$ sector of Fock space, in effect integrating out the degrees of freedom below some distance scale, μ . This results in an effective potential, $V(\vec{p}, \vec{r})$, whose dynamics are governed by a $\gamma_\mu \otimes \gamma^\mu$ one-gluon-exchange interaction at short distance and a linear $I \otimes I$ confining interaction. Mesons are then described by the rest

the hundreds of known mesons (from the π to the heaviest $b\bar{b}$ state) along with their decays. As a representative example we show the strange meson spectrum in Fig. 1 and some strange meson properties are given in Table I.

HIGH L MESONS

Next, I would like to discuss high spin mesons and what we can learn from them. First we note that the $b\bar{b}$ and $c\bar{c}$ systems probe the region from 0.25 fm to 1 fm so to study larger distance we need to study the radial and orbital excitations of the lighter quark systems. In practice orbital excitations are easier to find experimentally so I will concentrate on them. By studying these mesons we hope to learn about the nature of the quark-antiquark interaction, both its shape and its Lorentz structure. For instance, in lowest order of the v^2/c^2 expansion we find that the spin-orbit interaction is of the form

$$V_{Ls} = \frac{\tilde{L} \cdot \tilde{S}}{2m^2} (3V' - S'). \quad (5)$$

Where V represents a Lorentz vector interaction ($\gamma^\mu \otimes \gamma_\mu$) and S a Lorentz scalar interaction ($I \otimes I$). Therefore, if the long distance interaction is Lorentz scalar the multiplets will undergo an inversion, $M^3(L_{L-1}) > M^3(L_L) > M^3(L_{L+1})$, for some value of L where the long range potential begins to dominate over the short range one-gluon-exchange interaction. If, on the other hand, it is a vector interaction the ordering of the low-mass multiplets will persist to high L .

The masses of the isovector, strange, and $\bar{s}\bar{s}$ mesons have been calculated up to $L=8$.⁵ The leading states are plotted on a Chew Frautschi plot in Fig. 2. It can be seen that these states lie on linear Regge trajectories. This is a consequence of both the relativistic kinetic energy and the linear confinement potential. For instance, to obtain a linear Regge trajectory with a non-relativistic kinetic energy would require a potential of the form $r^{2/3}$.

Table I: The strange mesons of fig. 1: Theory versus experiment

State	Property	Theory	Experiment
$1^1S_0(K)$	Mass (GeV)	0.47	0.49
	$\text{Amp}(K \rightarrow \mu\nu)(f_K)$	0.47	0.32
$1^3S_1(K^*)$	Mass (GeV)	0.90	0.89
	$\text{Amp}(K^* \rightarrow K\pi) (\text{MeV}^{1/2})$	+7.9	7.2 ± 0.1
	$\text{Amp}(K^{*+} \rightarrow K^+\gamma)(\text{MeV}^{1/2})$	0.26	0.23 ± 0.01
	$\text{Amp}(\tau \rightarrow K^*\nu)(f_{K^*})$	0.30	0.28 ± 0.07
$1^3P_2(K^*)$	Mass (GeV)	1.43	1.43
	$\text{Amp}(K^* \rightarrow K^*\pi) (\text{MeV}^{1/2})$	-3.7	5.0 ± 0.5
	$\text{Amp}(K^* \rightarrow \rho\pi) (\text{MeV}^{1/2})$	-2.1	3.0 ± 0.4
	$\text{Amp}(K^* \rightarrow \omega K) (\text{MeV}^{1/2})$	+1.2	2.0 ± 0.4
	$\text{Amp}(K^* \rightarrow K\eta) (\text{MeV}^{1/2})$	-0.8	2.2 ± 1.7
	$\text{Amp}(K^* \rightarrow K^+\gamma) (\text{MeV}^{1/2})$	0.48	0.50 ± 0.05
$1^3D_3(K^*)$	Mass (GeV)	1.79	1.78 ± 0.01
	$\text{Amp}(K^* \rightarrow K\pi) (\text{MeV}^{1/2})$	-4.6	4.9 ± 1.0
	$\text{Amp}(K^* \rightarrow K^*\pi) (\text{MeV}^{1/2})$	+2.8	seen
	$\text{Amp}(K^* \rightarrow \rho K) (\text{MeV}^{1/2})$	-2.3	seen
	$\text{Amp}(K^* \rightarrow \omega K) (\text{MeV}^{1/2})$	+1.3	seen
$1^3F_4(K^*)$	Mass (GeV)	2.11	2.06 ± 0.03
	$\text{Amp}(K^* \rightarrow K\pi) (\text{MeV}^{1/2})$	-2.7	4 ± 1
	$\text{Amp}(K^* \rightarrow K^*\pi) (\text{MeV}^{1/2})$	+1.7	-
	$\text{Amp}(K^* \rightarrow \rho K) (\text{MeV}^{1/2})$	+1.6	-
	$\text{Amp}(K^* \rightarrow \omega K) (\text{MeV}^{1/2})$	-0.9	-

Fig. 2. Chew-Frautschi plot for the ρ -f, K^* , and ϕ trajectories.

CONCLUSIONS

We find that the properties, including masses and decays, of all mesons from the lightest $w\bar{w}$ mesons to the heaviest $b\bar{b}$ mesons are described with reasonable success in this unified model. The physics underlying the mesons persists from the lightest isovector mesons to the heaviest $b\bar{b}$ states. This is illustrated in figure 3. For this analysis to be successful it was necessary to include both relativistic effects and the properties expected from QCD.

Fig. 3: A graphic illustration of the universality of meson dynamics from the π to the T , showing the splittings of 3P_2 and 1S_0 from 3S_1 in the $b\bar{b}$, $c\bar{c}$, $s\bar{s}$, $u\bar{u}$, and $d\bar{d}$ families.

In light of our results we believe that our model provides a reasonable background of $q\bar{q}$ states against which to look for new physics. Finally, we advocate the continued study of high spin mesons as one of the best ways of studying the nature of confinement.

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REFERENCES

- 1] T. Barnes, F.E. Close, and S. Monaghan, Nucl. Phys. **B198**, 380 (1982).
- 2] N. Isgur and J. Paton, Phys. Lett. **124B**(1983)247; T. Barnes and F.E. Close, Phys. Lett. **116B** 365(1982); M. Chanowitz and S. Sharpe, Nucl. Phys. **B222**, 211(1983).
- 3] R.L. Jaffe, Phys. Rev. **D15**, 267(1977).
- 4] S. Godfrey and N. Isgur, Phys. Rev. **D31** (in Press); S. Godfrey, "The Quark Structure of Matter", proceedings of the Yukon Advanced Study, edited by N. Isgur, G. Karl, and P.J. O'Donnell (World Scientific Publishing, Singapore, 1985).
- 5] S. Godfrey, Phys. Rev. **D31** (in Press).