Meson Electric Form Factor on the Lattice

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Abstract

Theoretical aspects of calculating the electric form factor of lattice hadrons are presented. We use the staggered formulation of lattice fermions and deal specifically with SU(2) color; however, the techniques described are easily adaptable to other situations.

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I. INTRODUCTION

Recently there has been considerable interest in Monte Carlo investigations of hadron size and internal structure.\(^1\)\(^\text{-}^4\) Electromagnetic properties provide clean and experimentally accessible information for this purpose.\(^1\)\(^,\)\(^5\) We have shown, in Ref. 1, that a lattice computation of the electric form factor and the rms charge radius of hadronic states is feasible. Results were presented for the electric form factor of the pseudo-Goldstone meson within the staggered formulation\(^6\)\(^,\)\(^7\) of lattice fermions. It was found that the quarks in a lattice meson are indeed localized in a compact object significantly smaller than the lattice volume. In this paper we present a detailed derivation of the formulae for the three-point function and the electric form factor used in Ref. 1. We consider only the pseudo-Goldstone meson state and use SU(2) color. There are no difficulties, in principle, to extending these considerations to SU(3) color and to other lattice hadron states.

An advantage of the staggered fermion scheme is that a remnant global chiral symmetry is preserved.\(^7\)\(^,\)\(^8\) The pseudo-Goldstone boson associated with the spontaneous breakdown of the global axial symmetry (in the massless limit) can be interpreted as the generic pion. The construction of correlation functions for hadronic states with good quantum numbers in this formalism is not straightforward. It is usually done by using non-local flavored Dirac quark fields made up of staggered fermion fields on hypercubes in the lattice.\(^9\)\(^-\)\(^10\) For some states, in particular, for the pseudo-Goldstone (pion) state, it is known that the mass can be determined from a two-point function of operators which are local combinations of staggered fermion fields. This is advantageous for numerical calculations. The main result of this paper is that pionic
We now use the fundamental identity for zero-temperature field theory:

\[
\phi_b(\epsilon_b, \phi^b, \epsilon^b) \phi^b \epsilon^b = \frac{1}{n} \phi_b(\epsilon_b, \phi^b, \epsilon^b) \phi^b
\]

Treating the \( \phi^b \) as field theoretical variables, we have

\[
\langle \phi(\delta) \rangle \prod_{j=0}^{N} \sum_{\epsilon^b} = \langle \phi \rangle^N
\]

where the zero-momentum lattice propagator is

\[
\tilde{\phi}_0 \equiv \int \frac{d^D \epsilon}{(2\pi)^D} \delta(\epsilon^2 - \epsilon_0^2)
\]

and

\[
\tilde{\phi}_L \equiv \int \frac{d^D \epsilon}{(2\pi)^D} \delta(\epsilon^2 - \epsilon_L^2)
\]

We consider this propagator in terms of the Green function.

\[
(\epsilon^2 - \epsilon_0^2)^{-1} = \int \frac{d^D \epsilon}{(2\pi)^D} \delta(\epsilon^2 - \epsilon_0^2)
\]

The two-point function of a \( b \)-field is given by

\[
\langle \phi(\delta) \phi(\delta') \rangle \prod_{j=0}^{N} \sum_{\epsilon^b} = \langle \phi \rangle^N
\]

and

\[
\langle \phi(\delta) \rangle \prod_{j=0}^{N} \sum_{\epsilon^b} = \langle \phi \rangle^N
\]

where

\[
\langle \phi(\delta) \rangle = \int \frac{d^D \epsilon}{(2\pi)^D} \delta(\epsilon^2 - \epsilon_0^2)
\]

and

\[
\langle \phi(\delta) \rangle = \int \frac{d^D \epsilon}{(2\pi)^D} \delta(\epsilon^2 - \epsilon_L^2)
\]

where

\[
\langle \phi(\delta) \rangle = \int \frac{d^D \epsilon}{(2\pi)^D} \delta(\epsilon^2 - \epsilon_0^2)
\]

II. Definition

Fermion fields.

Can also be constructed using operators which are local in the staggered

matrix elements of the electromagnetic current for low momentum states.
\begin{align*}
\langle 0 \mid \mathcal{T} (\psi_\alpha (-it_A) \bar{\psi}_\beta (-it_B) \ldots ) \mid 0 \rangle = Z^{-1} \int d\bar{\psi}_B d\psi_A e^{-S_{\Phi} - S_{\Phi} \left( \psi_A (t_A) \bar{\psi}_B (t_B) \ldots \right)}
\end{align*}

where \( \bar{\psi} = \psi^* \gamma_\alpha Z \) is the normalization integral and

\begin{align*}
\{ \bar{\psi}_\alpha , \psi_\beta \} &= \delta_{\alpha \beta} , \quad \{ \bar{\psi}_\alpha , \bar{\psi}_\beta \} = 0 .
\end{align*}

In contrast, the independent Grassmann integration variables \( \zeta \) and \( \xi \) anticommute:

\begin{align*}
\{ \zeta_\alpha , \zeta_\beta \} = \{ \zeta_\alpha , \xi_\beta \} = \{ \zeta_\alpha , \bar{\xi}_\beta \} = 0 .
\end{align*}

The latin indices are generic including spacetime, Dirac, flavor and color. Since the transformation from \( q \) to \( x \) is linear, (9) leads to

\begin{align*}
M^+ (t_x) = - \frac{1}{4} Z^{-1} \sum_{\tilde{x}, n_1, n_2} (-1)^{n_1 n_2} \langle \bar{\psi}^A_\alpha (2 \tilde{x} + n_1) \psi^A_\alpha (2 \tilde{x} + n_2) \bar{\psi}^B_\beta (n_1) \psi^B_\beta (n_2) \rangle ,
\end{align*}

where

\begin{align*}
\langle \ldots \rangle = \int d\bar{\psi}_B d\psi_A e^{-S_{\Phi} - S_{\Phi} (\ldots )} .
\end{align*}

The \( \bar{\psi} , \psi \) fields are now being treated as integration variables.

Let us now shift the position on all the starting points of the propagator (12) to the origin. This shift entails the assumption that one may do an even-odd lattice point redefinition on unit hypercubes. This is allowed in the quenched lattice vacuum with periodic or antiperiodic boundary conditions on the quarks. However, coupling the time boundaries together leads to difficulties when the charge operator is introduced since the amount of charge that flows in the forward time direction is no longer a constant in time. Thus, we will prefer to adopt nonperiodic boundary conditions in the time directions for our numerical simulations. For the following derivations we will assume, for simplicity, a lattice of infinite time extent.

Performing the shift to the origin on starting positions as well as the \( n_i , n_i \) sum, (12) becomes

\begin{align*}
M^+ (t_x) &= - 2 Z^{-1} \sum_{\tilde{x}} (-1)^{\tilde{x}} \langle \bar{\psi}^A_\alpha (2 \tilde{x} + n_1) \psi^A_\alpha (2 \tilde{x} + n_2) \bar{\psi}^B_\beta (n_1) \psi^B_\beta (n_2) \rangle \\
&= \sum_{\tilde{x}} \bar{\psi}^A_\alpha (\tilde{x}, 2 \tilde{x} + 1) \psi^A_\alpha (\tilde{x}, 2 \tilde{x} + 1) \\
&\quad \bar{\psi}^B_\beta (\tilde{x}, 2 \tilde{x} - 1) \psi^B_\beta (\tilde{x}, 2 \tilde{x} - 1) \\
&\quad \langle \bar{\psi}^A_\alpha (0) \psi^A_\alpha (0) \rangle ,
\end{align*}

with \((-1)^{\tilde{x}} \equiv (-1)^{\tilde{x}} \), \( \tilde{x} \) denoting spatial positions in the original lattice.

We write the action (1) as

\begin{align*}
S_{\Phi}(U) = \sum_{\alpha_1, \beta_1} \bar{x}_\alpha \frac{1}{2} M_{\alpha \beta} x_\beta ,
\end{align*}

where the fermion matrix \( M_{\alpha \beta} \) is proportional to the unit matrix in \( u , d \) flavor space. Explicitly

\begin{align*}
M_{\alpha \beta} = (2 m_0) \delta_{\alpha \beta} + \sum_\mu a_\mu (x) \left\{ [U_\mu (x)]^{AB} \delta_{\alpha \gamma [x,y-a_\mu } \\
&\quad [U_\mu (x-a_\mu ]^{AB} \delta_{\beta \gamma [x,y+a_\mu } \right\} ,
\end{align*}

in the space time \( (x,y) \) and color \( (A,B) \) indices. From (15) one may show that

\begin{align*}
M^\dagger_{\alpha \beta} = (-1)^{xy} M_{\beta \alpha} x_\alpha .
\end{align*}

This component statement may be written as a matrix relation and inverted to yield

\begin{align*}
[M_{\alpha \beta}^{-1}]^* = (-1)^{xy} M_{\beta \alpha}^{-1} x_\alpha .
\end{align*}
we expect to have 

\[ \sum_{n=0}^{k-1} \tilde{z}_{n} = \tilde{z}_{k} \] 

Following steps similar to the above with the deviations

are distributed over all substraction length scales.

and the partial overlaps over which the different phase factors

In the continuum limit for \( \epsilon / \Lambda \gg \delta / \Lambda \) the two description should

\[ \sum_{n=0}^{k-1} \tilde{z}_{n} = \tilde{z}_{k} \] 

Therefore in (23) we make the replacement:

assigning a different phase factor to each point in the discrete lattice,

which is easier to deal with an expression which

However, upon shifting of positions to the origin this propagator becomes

\[ \sum_{n=0}^{k-1} \tilde{z}_{n} = \tilde{z}_{k} \] 

We also need the two-point function for states with non-zero

etc. and the zero doesn't decrease a zero-momentum state propagator.

where \( N \) is the number of boundary-stable cubes on the doubled

\[ \sum_{n=0}^{k-1} \tilde{z}_{n} = \tilde{z}_{k} \] 

complete set of states in (7) and by using (22) in (69) we establish

\[ \sum_{n=0}^{k-1} \tilde{z}_{n} = \tilde{z}_{k} \] 

(12) \[ \mathcal{N}(0, \pm 1) \rightarrow (1,0) \] 

correlation function there. It can be shown for this

which is essentially just the correlation function for the local

(20) \[ \sum_{n=0}^{k-1} \tilde{z}_{n} = \tilde{z}_{k} \] 

we see from (69) that the propagator can be expressed in terms

where the trace and hermitian conjugation are denoted to color space.

(61) \[ \mathcal{N}(0, \pm 1) \rightarrow (1,0) \] 

(81) \[ \mathcal{N}(0, \pm 1) \rightarrow (1,0) \] 

We also have that: 

...
\[ G(p; t) \longrightarrow Z(p)e^{-\frac{E_p}{t}}, \quad t >> 1 \]  

(26)

assuming the continuum dispersion relation \( E_p = \sqrt{m^2 + p^2} \). This gives

\[ Z(p) = \frac{N_0 \langle 0|\hat{\sigma}(0)|\tilde{\xi}(\tilde{p})\rangle^2}{\delta(1 + e^{-E_p\bar{\beta}})(1 + e^{-E_p\bar{\beta}})}. \]  

(27)

**B. The three-point function**

In this section we discuss matrix elements of the conserved vector (electromagnetic) current. The current operator\(^8\) can be derived using the fact that the action (1) is invariant under the global transformation

\[ \chi(x) \rightarrow e^{i\omega} \chi(x), \]  

(28a)

\[ \tilde{\chi}(x) \rightarrow \tilde{\chi}(x) e^{-i\omega}, \]  

(28b)

for each flavor \( f \). The equivalent transformation for the \( q \)-fields is

\[ q(z) \rightarrow e^{i\Omega(0)\tilde{\Omega}(z)} q(z), \]  

(29a)

\[ \tilde{q}(z) \rightarrow \tilde{q}(z) e^{-i\Omega(0)\tilde{\Omega}(z)}, \]  

(29b)

When the transformation (29) is made local on the doubled lattice by assigning distinct phase factors, \( \Omega(z) \), to each hypercube, we may use

\[ \hat{A} J_q(z) = \frac{\delta S_p(u)}{\delta (\partial_\mu \Omega(z))} \]  

where \( \Delta_0 \Omega(z) = \Omega(z + a_\mu) - \Omega(z) \). We give the explicit expression for the current operator only for the time component (the charge density):

\[ \hat{A} J_q(z) = -\sum_{\tilde{\xi}} \frac{1}{2} a_q(\tilde{\xi}) [\chi(\tilde{\xi} + \tilde{\eta}, 2t_{z+1} + 1)u_{\tilde{\xi}}(\tilde{\xi}, 2t_{z+2} + 1)\chi(\tilde{\xi} + \tilde{\eta}, 2t_{z+2} + 2) \]

\[ + \tilde{\chi}(\tilde{\xi} + \tilde{\eta}, 2t_{z+2} + 2)u_{\tilde{\xi}}^\dagger(\tilde{\xi}, 2t_{z+1} + 1)\chi(\tilde{\xi} + \tilde{\eta}, 2t_{z+1} + 1)]. \]  

(30)

Notice the nonlocality of this charge density as well as the fact that it is positioned between hypercubes in time.

As was mentioned previously in connection with the nonzero-momentum two-point function, it is easier to deal with expressions that assign phase factors to points in the original lattice. We therefore use (24) and make the replacement

\[ \sum_{\tilde{\xi}} e^{i\tilde{q} \cdot \tilde{z}} \hat{A} J_q(z) \rightarrow \sum_{\tilde{\xi}} e^{i\tilde{q} \cdot \tilde{z}} \hat{A} J_q(z), \]  

(31)

\[ \hat{A} J_q(z, t) \equiv \frac{1}{2} a_q(z) \{ \chi(z, t)u_q(z, t)\chi(z, t+1) \]

\[ + \tilde{\chi}(z, t+1)u_q^\dagger(z, t)\tilde{\chi}(z, t) \}, \]  

(32)

Such replacements should not affect the low momentum physics.

The three-point function from which we will extract the electric form factor is

\[ \hat{\Lambda}(\tilde{\xi}, z, t_{z+1}, t_{z+2}) = \langle 0|\hat{T}(|\sum_{\tilde{\xi}} e^{i\tilde{q} \cdot \tilde{z}} \phi(z) \rangle \]

\[ \times \sum_{\tilde{\xi}_1, \tilde{\xi}_2} e^{i\tilde{q} \cdot \tilde{z}} \hat{q}_{\tilde{\xi}_1}(z)\hat{\phi}(0))|0 \rangle. \]  

(33)

Using (31) we get

\[ \hat{\Lambda}(\tilde{\xi}, \tilde{q}, t_{z+1}, t_{z+2}) = -\frac{1}{4} z^{-1} \sum_{\tilde{\xi}, \tilde{\xi}_1} e^{-i\tilde{q} \cdot \tilde{z}} \chi(\tilde{\xi} + \tilde{\eta}, 2t_{z+2} + 1)\chi(\tilde{\xi} + \tilde{\eta}, 2t_{z+1}) \]

\[ \times \langle \tilde{k}(2z_{2+n})\chi^\dagger_0(2z_{2+n})\chi^\dagger(2z_{1+n})\chi(2z_{1+n}) \rangle. \]  

(34)

with \( \rho(z, t) = \sum_{\tilde{\xi}} \hat{q}\tilde{\xi}(\tilde{z}, t) \).

The usual steps of shifting starting positions and summing on \( n, n' \) are
(94)
\[ \phi(x) = \phi \phi(x) \]

Following correspondences:

show that the lattice states and spin 1/2 fields used here have the form

\[ \phi_s \phi \frac{d\phi}{d\phi} \int \frac{d\phi}{d\phi} \]

The states \( \phi_s \) produce Lorentz covariant matrix elements. The continuum

\[ (96) \]
\[ \frac{\phi_s}{\phi_s} (O) = \left( \frac{1}{\phi_s} \right) \left( \frac{1}{\phi_s} \right) \]

expression. We write continuum correspondences as

relate the lattice charge density matrix element to the continuum

\[ (99) \]
\[ = \left( \frac{1}{\phi_s} \right) \left( \frac{1}{\phi_s} \right) \]

In order to extract the form factor from (99), it is necessary to consider the "general form of a typical term in Eq. (33)." We define

\[ (96) \]
\[ \left( \frac{1}{\phi_s} \right) \left( \frac{1}{\phi_s} \right) \]

where that function of the sum function of (33) is rear on the lattice (the

\[ (96) \]
\[ \left( \frac{1}{\phi_s} \right) \left( \frac{1}{\phi_s} \right) \]

Assume the amplitude of \( \phi \frac{d\phi}{d\phi} \int \frac{d\phi}{d\phi} \)

with \( \phi \).

\[ (96) \]
\[ \left( \frac{1}{\phi_s} \right) \left( \frac{1}{\phi_s} \right) \]

\[ (96) \]
\[ \left( \frac{1}{\phi_s} \right) \left( \frac{1}{\phi_s} \right) \]
\[
\langle \pi^+(\tilde{p}) | \rho(0) | \pi^+(\tilde{p}') \rangle \longrightarrow \frac{1}{N_q z E_{\pi^+}} \langle \pi^+(\tilde{p}) | \rho^c(0) | \pi^+(\tilde{p}') \rangle ,
\]

(44)

where \( \rho^c(0) \) is the continuum charge density operator. The electric form factor \(^{16}\) is defined as

\[
\langle \pi^+(\tilde{p}) | \rho^c(0) | \pi^+(\tilde{p}') \rangle = (E_{\pi^+} E_{\pi^-}) F_+(q) .
\]

(45)

Since we now know the relation of \( \hat{A}(\tilde{p}, \tilde{q}; t_2, t_1) \) to \( F_+(q) \), we may solve for the local quantity \( A(\tilde{p}, \tilde{q}; t_2, t_1) \) from (37) when \( t_1, (t_2 - t_1) \gg 1 \)

\[
A(\tilde{p}, \tilde{q}; t_2, t_1) \longrightarrow \left\{ \begin{array}{l}
Z(p) Z(p') \left[ \frac{1 + e^{E_p a}}{1 + e^{-E_p a}} \right]^2 \\
\left[ \frac{1 + e^{E_p a}}{1 + e^{-E_p a}} \right]^2 \end{array} \right\}^{1/2} \epsilon_{\tilde{p}} e^{-E_{\pi^+} t_1} \frac{(E_{\pi^+} E_{\pi^-})}{2 E_{\pi^+} E_{\pi^-}} F_+(q) .
\]

(46)

This is the result we are looking for. It shows that the pion electric form factor can be calculated from a three-point function involving local interpolating fields. It is clear from (46) and (21) that \( A \) satisfies a sum rule (for \( t_2 > t_1 \))

\[
A(\tilde{p}, 0; t_2, t_1) = G(\tilde{p}; t_2) ,
\]

(47)

a relation noted before in Ref. 3. If periodic or antiperiodic boundary conditions were used in time Eq. (47) would not have such a simple form.

To extract the form factor from Monte Carlo data it is convenient to form the combination

\[
\left\{ \frac{A(\tilde{p}, \tilde{q}; t_2, t_1) A(\tilde{p}, \tilde{q}; t_1, t_2)}{G(0; t_2) G(\tilde{q}; t_1)} \right\}^{1/2} \frac{(E_{\pi^+} + m_\pi)}{2 E_{\pi^+} m_\pi} F_+(q) .
\]

(48)

Notice that the \( Z(p) \), \( Z(p') \) factors and the time dependence have all dropped out leaving only the matrix element of interest. Since it is the factors \( Z(p) \) and \( Z(p') \) which contain contributions from states other than the vacuum when we do our calculations with non-periodic time boundary conditions on a finite lattice,\(^{17}\) we expect the ratio to be free of time boundary effects. Finally we note that it is important in calculating the statistical error in \( F_+(q) \) from Monte Carlo data to include the covariances between the various factors in (48).

We conclude with a discussion of how the three-point function can be calculated as the derivative of a two-point function, a technique which has been used in other applications.\(^{18,19}\) Define a new action

\[
S_F(U_0, q, d, q) = S_F(U) - \sum_{x, y} a_x e^{\frac{i q \cdot x + d \cdot x}{2 \beta}} f(x, \tau) .
\]

(49)

This gives a new fermion matrix

\[
M(a_c, q) \chi_{x} y = \chi_{x}, y B + \frac{1}{2} a_c e^{\frac{i q \cdot x + d \cdot x}{2 \beta}} \\
\times \delta_{x', y} [ \delta_{x \cdot x'} e^{d y \cdot \delta_{x \cdot x'}} [U_0(\chi)]^{AB} + \delta_{x \cdot x'} e^{d y \cdot \delta_{x \cdot x'}} [U_0(\chi - a_c)]^{AB} ] 
\]

for each flavor, \( f \). This new matrix has the properties

\[
M^*(a_c, \bar{q}) \chi_{x; y} B = (-1)^{x \cdot y} M(a_c, -\bar{q}) \chi_{x; y} B ,
\]

(50)

and

\[
\sigma_2 M(a_c, \bar{q}) = \sigma_2 M^*(a_c, -\bar{q})
\]

(52)

for \( \sigma_2 \) in color space. One can show as in (17) that (51) gives

\[
[M^{-1}(a_c, \bar{q}) \chi_{x; y} B]^* = (-1)^{x \cdot y} [M^{-1}(a_c, \bar{q}) \chi_{x; y} B]^* .
\]

(53)

In addition, (52) implies
self-contraction. Since one can show that

source on the vacuum. Subsequently, such effects come from current

therefore to be considered, one must expect to include the effect of the
decorrelating characteristics of the channel operator as a source.

of the simulation, where primary spatial sense the method we are

decorrelating factors, which would be implemented in the Monte Carlo part

notice the factor of \( \frac{1}{N} \) above. The appearance of these

Notice the factor of \( \frac{1}{N} \) above.

\[
\int \sum_{\sigma} \sum_{D} \int \sum_{\mathbf{r}_D} \frac{1}{\mathbf{r}_D} \cdot \frac{1}{\mathbf{r}_D} \cdot \| \mathbf{p} \|^2 \cdot \| \mathbf{p} \|^2 \cdot \exp \left( -\frac{\mathbf{p}^2}{2\mu} \right) \cdot \exp \left( -\frac{\mathbf{p}^2}{2\mu} \right)
\]

\[
\int \sum_{\sigma} \sum_{D} \int \sum_{\mathbf{r}_D} \frac{1}{\mathbf{r}_D} \cdot \frac{1}{\mathbf{r}_D} \cdot \| \mathbf{p} \|^2 \cdot \| \mathbf{p} \|^2 \cdot \exp \left( -\frac{\mathbf{p}^2}{2\mu} \right) \cdot \exp \left( -\frac{\mathbf{p}^2}{2\mu} \right)
\]

Notice the factor of \( \frac{1}{N} \) above.

\[
\int \sum_{\sigma} \sum_{D} \int \sum_{\mathbf{r}_D} \frac{1}{\mathbf{r}_D} \cdot \frac{1}{\mathbf{r}_D} \cdot \| \mathbf{p} \|^2 \cdot \| \mathbf{p} \|^2 \cdot \exp \left( -\frac{\mathbf{p}^2}{2\mu} \right) \cdot \exp \left( -\frac{\mathbf{p}^2}{2\mu} \right)
\]

\[
\int \sum_{\sigma} \sum_{D} \int \sum_{\mathbf{r}_D} \frac{1}{\mathbf{r}_D} \cdot \frac{1}{\mathbf{r}_D} \cdot \| \mathbf{p} \|^2 \cdot \| \mathbf{p} \|^2 \cdot \exp \left( -\frac{\mathbf{p}^2}{2\mu} \right) \cdot \exp \left( -\frac{\mathbf{p}^2}{2\mu} \right)
\]
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REFERENCES

11. Due to parity mixing within the staggered formulation, another pion interpolating field is \( q(z)(\gamma_4 \Theta_4^a) q(z) \), assuming the numerical absence of a signal for the \( 0^{-+} \) state. The results (46) and (48) do not depend on which interpolating field is used.
13. Such a relation does not exist for SU(3). This will be one of the main differences in generalizing the present SU(2) color derivation.
precisely.

Reference along the quantum approximation requires some extra
2. Thus, will not be true for SU(3) color. Then use of the source
section, supplemented, V. N. 1984.

1. W. Deppert, E. K. Anosov, and D. Siver (private communication, information

(1983).


1.6. See, for example, C. Zedda in, Quantum Field Theory.

1.5. For the L-dependence case.

\[ \frac{Z}{n} \frac{Z}{n} \ldots \frac{Z}{n} \frac{Z}{n} \ldots \frac{Z}{n} \frac{Z}{n} \]

Orthonormal lattice by \( \gamma \). The lattice positioning on the
allows great structure with the lattice (labeling positions on the
is the description of lattice in terms of \( \gamma \) fields (determining positions).