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Gluonic correlations and lattice hadron structure*

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Abstract

An improved correlation technique for measuring gluonic structure in lattice hadrons is proposed. Illustrative results for elementary spatial plaquettes in SU(2) QCD are presented.

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Monte Carlo computer simulations are yielding increasingly detailed information about quantum chromodynamics (QCD). Besides the important topics of mass spectra,¹⁻⁷ magnetic moments,² the static $q\bar{q}$ potential⁸ and the like, hadron internal structure may also be investigated with lattice techniques.

A number of papers investigating the structure of lattice $q\bar{q}$ states have recently appeared. So far, the attention in these investigations has been concentrated on the fermionic structure of these particles, either from the extraction of wave functions,⁹ current-current overlap,¹⁰ or a direct measurement of the rms charge radius.¹¹ We would like to stress here the use of gluonic variables as a complementary source of hadronic structural information; we show that their use need not require an excessive amount of computer time. As an illustration, elementary spatial plaquettes are used to probe the $(\beta a)^2$ content of a Wilson lattice pion.¹²

We are interested in detecting deviations from vacuum expectation values (VEVs) in certain lattice probes, $P(\vec{y}, t_y)$, in the vicinity of a hadronic state, $|G(\vec{x}, t)\rangle$. The spatial correlation length found in these types of measurements is an indicator of the hadron size. To this end, one may construct the quantity

$$\frac{\langle G(\vec{x}, t') | P(\vec{y}, t_y) | G(\vec{x}, t) \rangle}{\langle G(\vec{x}, t') | G(\vec{x}, t) \rangle \langle 0 | P(\vec{y}, t_y) | 0 \rangle} \quad (1)$$

when $\langle 0 | P(\vec{y}, t) | 0 \rangle \neq 0$. One is looking for significant deviations from unity in the measurement of (1) as the probe point, $\vec{y}$, approaches the hadron creation-annihilation space point, $\vec{x}$. If one has the Monte Carlo data $\langle G(\vec{x}, t') | G(\vec{x}, t) \rangle$: $G_1, t=1, \dots, N$, and

corresponding vacuum values (which may need corrections for perturbative contributions) $\langle 0 | P(\vec{y}, t_y) | 0 \rangle$: P_i , $i=1, \dots, N$, then the quantity

$$\langle G \rangle \equiv \frac{1}{N} \sum_i P_i, \quad \langle GP \rangle \equiv \frac{1}{N} \sum_i G_i P_i$$

$$\frac{\langle GP \rangle}{\langle G \rangle \langle P \rangle} \quad (2)$$

is an estimate of (1). In this form, deviations from VEVs come entirely from correlations between the G_i , P_i data. A well known method of identifying a significant correlation is to introduce

$$r = \frac{\langle GP \rangle - \langle G \rangle \langle P \rangle}{\sigma_G \sigma_P}, \quad (3)$$

called the linear correlation coefficient.¹³ The σ_G , σ_P are the standard deviations in the G_i , P_i data. One may show that necessarily $|r| \leq 1$, which means the possible signal is limited in magnitude by

$$\frac{\sigma_G \sigma_P}{\langle G \rangle \langle P \rangle}.$$

Now, each of the amplitudes $\langle 0 | P | 0 \rangle$, $\langle G | G \rangle$, $\langle G | P | G \rangle$ in (1) making up this measurement results from a separate path integral. Therefore, in the estimate of the overall error, one must combine the errors in these amplitudes as for uncorrelated data. Under these circumstances, the Monte Carlo error in measuring (1) is approximately (σ_{GP} is the standard deviation in the $(G_i P_i)$ data)

$$\frac{1}{\sqrt{N}} \sqrt{\frac{\sigma_G^2}{\langle G \rangle^2} + \frac{\sigma_P^2}{\langle P \rangle^2} + \frac{\sigma_{GP}^2}{\langle G \rangle \langle P \rangle^2}}.$$

Since σ_G , σ_P , $\langle P \rangle$, etc. are all fixed in the parent population, it is necessary to reduce the error by increasing the number of

measurements, N , in order to "see" the signal. This is notoriously difficult for hadrons, where costly and time consuming matrix inversions must be done. Another procedure one can follow is to bin the data and to estimate the Monte Carlo errors from the variance of the sample bins. However, it is not necessary to isolate the signal in such a manner if one limits oneself to a measurement of the extent of correlation.

Necessarily, from (2), one must have $r \neq 0$ if a variable is taking on a value different from its VEV, and this r value can often be shown to be statistically significant (i.e., extremely unlikely for uncorrelated data). The spatial dependence of a signal may also be deduced from the relative values of correlation coefficients since the signal, from (3), is proportional to r .

In order to illustrate this technique, we examine the correlation of elementary spatial plaquettes with a pion propagator; the unnormalized signal found will be a relative measure of $(\beta a)^2$ in the presence of a hadron.¹⁴ The Wilson formulation¹⁵ for lattice fermions in the quenched approximation for SU(2) color was used. Quark propagators, each consisting of 4 columns of the inverse fermion matrix, were extracted from 6 different starting space-time points from 10 gauge field configurations. We have chosen to work at a value of the bare hopping parameter $K = 0.157$, very near $K_{critical}$ from the results of Ref. 6. The lattice used was $6^3 \times 15$ at $\beta = 4/g^2 = 2.35$. Using the results of Ref. 7, the lattice spacing at our β value is then ~ 0.18 fm. Antiperiodic boundary conditions are imposed on the quark fields in the space directions. We do not impose this boundary condition in the time direction, but rather set the fermion coupling across the time boundaries equal to zero, i.e., quark propagation across the time boundary is not allowed. This is

similar, but not identical, to the boundary condition considered by Bernard et al.²

The correlation coefficients found, based on the 60 Wilson pseudoscalar propagators, are shown in Table 1. One should be aware that possible correlations among propagators from the same configuration will tend to over-emphasize the significance of the correlation coefficients. In the table, Δt refers to the number of lattice time steps separating the hadron creation and annihilation points. When $\Delta t=1$, we have placed the spatial plaquettes at the annihilation time coordinate. The spatial location of the plaquettes relative to the meson creation-annihilation position is indicated in Fig. 1. Using rotational invariance, we average over 12 equivalent plaquettes located on the $+x$, $+y$ and $+z$ axes, and then look for correlations between the averaged, unsubtracted¹⁶ plaquette data and the hadron propagators.¹⁷ The significance of the resultant r values is then brought out in terms of a standard statistical test¹³ which gives the probability, for totally uncorrelated data, that one would obtain a correlation as large or greater than $|r|$. A value of say, $r=0.3$ in Table 1 would establish a signal present at between the 95 and 99 percent confidence limits.

There is very little doubt that positive correlations, thus increasing the value of $\langle \beta a \rangle^2$ near the pseudoscalar, are being observed in the $\Delta t=0$ data. The $\Delta t=\Delta L=0$ result could come from uncorrelated data with only approximately a 10^{-5} probability, and the $\Delta t=0$, $\Delta L=1$ result can be so explained with less than a 0.002 probability. However, if the value seen at $\Delta L=3$, $r \sim 0.15$, truly represented a signal, it would require approximately three times as many propagators as in this investigation to confirm its presence above the 95 percent confidence

limit. These data are also presented graphically in Fig. 2 where standard statistical errors on the sample r values of Table 1 are shown.¹⁸ These are given as a function of distance from the creation-annihilation point in lattice units; we choose the centre of the plaquette to define this distance. Note that the error bars shown on the $\Delta L=3$ and 4 measurements in Fig. 2 would include $r=0$ if 95 percent confidence levels were given instead.

In summary, we have shown that measurements of unnormalized gluonic properties need not take an excessive amount of computer time. Correlations at a high level have been detected using the linear correlation coefficient indicating an increased value of $\langle \beta^3 \rangle^2$ near a Wilson pion. To obtain this information conventionally would have required at least an order of magnitude increase in the number of propagators to reduce the error bars to a comparable size. Of course, the technique as applied above has the limitation that the data which shows the most significant correlations ($\Delta t=0$) has also the strongest mixing with radial excitations. In this sense our results are for an "average" pion. However, this technique may also be applied in pure glue-gluon correlation studies, such as in the second paper of ref. 12, where these problems do not arise.

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- ¹²Such a study has been carried out for SU(2) glueballs by K. Ishikawa, G. Schierholz, H. Schneider and M. Teper, Nucl. Phys. B227, 221 (1983).
See also M. Fukugita and T. Niuya, Phys. Lett. 132B, 374 (1983), where the gluonic flux surrounding a heavy quark-antiquark pair is investigated.

- ¹³See any book mentioning statistical correlations, for example P.R. Bevington, Data Reduction and Error Analysis for the Physical Sciences (McGraw-Hill, New York, 1969).
- ¹⁴Actually, if the trace of a plaquette is given by $2\cos\theta$, the quantity we use in this correlation study is θ^2 .
- ¹⁵K.G. Wilson, Phys. Rev. D10, 2445 (1974); in New Phenomena in Subnuclear Physics, ed. A. Zichichi (Plenum, New York, 1977).
- ¹⁶Subtracting a perturbative constant from the data will not affect the correlation coefficients found.
- ¹⁷From (3), one has that
$$\frac{\langle GP \rangle}{\langle G \rangle \langle P \rangle} = 1 + r \frac{\sigma_G \sigma_P}{\langle G \rangle \langle P \rangle}.$$
The signal is thus proportional to σ_P . This is why we use averaged data (having a smaller σ_P) whenever possible to look for significant correlation coefficients. The important point to remember is to always use the same number of equivalent plaquettes (or other probe values) in this average if one wants to be able to compare these r values.
- ¹⁸For the theory behind the establishment of confidence limits on linear correlation coefficients, see M.G. Kendall and A. Stuart, The Advanced Theory of Statistics, V.I. (Charles Griffen and Co., London, 1963) Ch.16.

Table 1. Linear correlation coefficients for spatial plaquettes as a function of position (ΔL) and creation-destruction time separation (Δt) for the 0^+1 state. The 95 and 99 percent confidence limits are $|r|=0.254$ and 0.330 , respectively for $N=2=58$ degrees of freedom.

		ΔL			
		0	1	2	3
Δt	0	.556	.408	.212	.154
	1	.253	.007	.066	.041

FIGURE CAPTIONS

1. Illustration of the positions of spatial plaquettes used in the flux correlation study.
2. The $\Delta t=0$ correlation values of Table 1 for spatial plaquettes as a function of lattice distance units.

Fig. 1

Fig. 2