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AXIONS, DILATONS AND WESS-ZUMINO TERMS IN SUPERSTRING THEORIES

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A B S T R A C T

We clarify the nature of the gauge singlet pseudoscalar and scalar particles in the four-dimensional low energy effective theory derived from the superstring. Including the Wess-Zumino terms of the intermediate ten-dimensional field theory does not affect the result of naive dimensional reduction that one pseudoscalar does not couple to $F\tilde{F}$ and is a true Goldstone boson, while the other pseudoscalar couples identically to both the hidden and observable gauge sectors, and is a true axion. The absence of any new physics in the Wess-Zumino terms of the ten-dimensional anomaly-free supergravity theory is emphasized.

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The structure of the low energy four-dimensional effective field theory provided by the superstring¹⁾ is not yet clear. Most authors first take the zero-slope limit of the superstring, giving a traditional ten-dimensional supergravity theory²⁾ with an additional Lorentz-Chern-Simons term³⁾. Then they compactify the six extra dimensions of the field theory, usually requiring that the low energy effective theory be supersymmetric as a first approximation⁴⁾. As a result, compactification on some Calabi-Yau manifold K_6 with zero torsion has been favoured⁴⁾. The field content of the truncated four-dimensional theory includes two scalar fields⁵⁾ corresponding to the dilaton ϕ of the ten-dimensional supergravity theory and to the "compacton" σ which scales the compactification manifold: $g_{IJ} = e^{\sigma} g_{IJ}^0$, and two pseudoscalar fields (D, a) corresponding to components of the antisymmetric tensor field $B_{\mu\nu}$. The vacuum expectation values of the dilaton ϕ and of the compacton σ determine the relative magnitudes of the string tension α' , the Planck mass m_P and the scales of compactification M_c and grand unification M_x , while one combination of the pseudoscalars (D, a) is an axion which solves the problem of the non-perturbative vacuum angle θ , and the other may help solve the problem of the cosmological constant⁶⁾.

In a first approximation, the truncated four-dimensional supergravity theory has a remarkable double $SU(1,1)/U(1)$ ⁷⁾ structure. The Kähler potential for the gauge singlet superfields is⁵⁾:

$$G = -\ln(S + S^+) - 3\ln(T + T^+) + \ln|W|^2 \quad (1)$$

where

$$S = e^{3\sigma} \phi^{-3/4} + 3i\sqrt{2}D \quad (2a)$$

$$T = e^{\sigma} \phi^{3/4} - i\sqrt{2}a \quad (2b)$$

Moreover, the kinetic term for the gauge superfields W_α has⁵⁾ a very specific non-trivial kinetic function:

$$f_{\alpha\beta} W^\alpha W^\beta; \quad f_{\alpha\beta} = \delta_{\alpha\beta} S \quad (3)$$

The "no scale" structure⁷⁾ of the Kähler metric (1) and the independence of $f_{\alpha\beta}$ (3) from T are necessary conditions for the gaugino mass to be much less than the

gravitino mass⁸⁾ and for the "plation" mechanism for a vanishing cosmological constant⁶⁾.

Fear has been expressed^{9),10)} that a more realistic Calabi-Yau compactification might not reproduce the striking structure (1)-(3). In particular, attention has been drawn¹⁰⁾ to the possibility that the Wess-Zumino anomaly cancellation term³⁾ would modify the kinetic term (3) and the Goldstone boson nature of $T_I \equiv \text{Im } T$; the "plation" field.

In this letter we prove that such fears are unfounded and that the Wess-Zumino terms do not modify (1) or (3).

Let us start by recalling some well-known facts about anomalies and Wess-Zumino terms¹¹⁾. The anomaly $G(A)$ is defined by the gauge variation of the one-loop effective Lagrangian $\mathcal{L}_{\text{eff}}(A)$ obtained by the Gaussian integration of the fermions:

$$\delta \mathcal{L}_{\text{eff}}(A) = G(A) \quad (4)$$

Consistent anomalies, i.e., solutions to the Wess-Zumino consistency conditions¹²⁾, can be obtained by solving the equation:

$$\delta G(A) = 0 \quad (5)$$

with δ the exterior differential on the group. The operator δ satisfies the cohomology condition $\delta^2 = 0$ and (5) can be interpreted as a problem in the local cohomology defined by δ on the manifold of gauge field configurations¹²⁾. That the theory is anomalous means that there is no local counterterm W depending on the bosonic fields of our Lagrangian such that:

$$G(A) = \delta W(A) \quad (6)$$

Using a more mathematical language, this means that the anomaly is not an exact form in the local cohomology defined by δ . If this is the case, an additional field, such as the pion in QCD¹¹⁾, must be included in order to get a local functional W satisfying (6): this functional is the traditional Wess-Zumino term. The previous discussion can be extended to gravitational anomalies¹³⁾ just replacing δ by the exterior derivative on the group $SO(n)$ (n = dimension of the space-time).

For the anomaly-free ten-dimensional supergravity theory^{2),3)} we must consider as the manifold of configurations on which the cohomology operator is defined the manifold of A_μ and B-field configurations. Let us call $\hat{\delta}$ the new cohomology operator defined by: $\hat{\delta} A_\mu = \delta A_\mu$; $\hat{\delta} B = 1/30 \text{tr}(Ad\Lambda) - \text{tr}(\omega d\theta)$ with Λ and θ anticommuting parameters. A consistent anomaly in this theory is defined by:

$$\hat{\delta} G(A, B) = 0 \quad (7)$$

in analogy to Eq. (5).

It is well known that solutions to Eqs. (5) and (7) can in principle be obtained, using the transgression formula¹²⁾, from the index density in two more dimensions, which after integration gives us the first Chern class of the determinant fibre bundle $\det \not{D}(A)$. In the case of ordinary gauge theories, this density is given in terms of symmetric polynomials of the curvature. By this procedure we get solutions to the Wess-Zumino consistency conditions up to exact forms. In fact, $G(A)$ and $G(A) + \delta \Gamma(A)$ for Γ any local functional of A are equally good solutions to (5). It is important to notice here that the forms ω_α^1 in terms of which $G(A)$ is defined by the transgression formula are never exact forms in the cohomology of A gauge fields. Thus the anomaly for gauge theories is only an exact form when the fermion representation is anomaly-free.

In the superstring case, the situation is completely different because the forms which appear in the anomaly, obtained by the transgression formula, are exact in the $\hat{\delta}$ -cohomology. Therefore, for the anomaly-free ten-dimensional supergravity theory we must have:

$$\hat{\delta} G(A, B) = 0 \quad (8)$$

$$G(A, B) = \hat{\delta} W(A, B) \quad (9)$$

with W a local functional. Now $G(A, B)$ must be obtained, by the transgression formula, from a new index density, namely the density which gives us the characteristic classes of the determinant fibre bundle $\det \not{D}(A, B)$. Since the fermions are coupled to the B field, which is gauge variant, we must include it when calculating the complete anomaly, e.g., when evaluating the fermion loops which give $\det \not{D}$. Now, using the ambiguity in the definition of consistent anomalies, we see that $G(A, B)$ can always be defined to be zero. So far we have only been computing solutions to the Wess-Zumino consistency conditions and not

integrating explicitly the fermions of the theory. However, solutions to the Wess-Zumino equation must satisfy

$$\hat{\delta} \tilde{\mathcal{L}}(A,B) = G(A,B) \quad (10)$$

with $\mathcal{L}(A,B)$ the effective action resulting from the Gaussian integration of fermions. Thus, if the theory is anomaly-free, i.e., $G(A,B) = \delta W(A,B)$, we can always regularize the theory in such a way that $G(A,B) = 0$ and $\mathcal{L}(A,B) \sim \ln \det \not{D}(A,B)$ is gauge invariant, and the ambiguity in the solution to the Wess-Zumino consistency conditions is then absorbed in a redefinition of $\ln \det \not{D}(A,B)$ ¹²⁾. Thus we see that if the ten-dimensional supergravity theory with the addition of a Lorentz-Chern-Simons term is anomaly-free (8), (9), as in the case of an $O(32)$ or an $E_8 \times E_8$ Yang-Mills group, then the Wess-Zumino term of Ref. 3), denoted by $W_{GS}(A,B)$, can be absorbed in a redefinition of $\ln \det \not{D}(A,B)$. If this is not the case, then the theory will be anomalous in the sense that $G(A,B)$ will not be an exact form in $\hat{\delta}$, and new fields would be necessary in order to get a Wess-Zumino counterterm. The Wess-Zumino term of Ref. 3), namely $W_{GS}(A,B)$ ^{*}, just corresponds to the ambiguity in the solution to (8) and can always be cancelled by choosing a solution to (8) equal to $\hat{\delta} W_{GS}(A,B)$. The moral of this argument is that you do not need to add any counterterm to the theory of Ref. 3), the theory by itself will induce, by one-loop effects, the necessary terms to have $\delta \mathcal{L}(A,B) = 0$. Indeed, this is just what we would expect if we went directly from the ten-dimensional anomaly-free string theory to the anomaly-free four-dimensional theory.

With all this in mind let us briefly consider the axion-dilaton dynamics in the truncated four-dimensional theory. The truncated theory can be obtained by compactification of the ten-dimensional supergravity Lagrangian on a ground state of the string theory. This ground state can in principle be obtained by solving the $\beta = 0$ equation of the σ -model associated with the string¹⁴⁾. A good¹⁵⁾ approximation to this ground state can be obtained, in the approach of Candelas et al.⁴⁾, by solving the Chapline-Manton SUSY transformations with the field strength tensor modified by the addition of the Lorentz-Chern-Simons form³⁾. When the ten-dimensional ground state satisfies the condition:

*) In view of this essential difference between the traditional Wess-Zumino term and its counterpart in ten-dimensional supergravity theory, it might be wiser to avoid calling $W_{GS}(A,B)$ a Wess-Zumino term, and to reserve this terminology for terms obtained by integrating an anomaly, i.e., one-cocycles.

$$P_2(T; \mathbb{R}) = P_2(V; \mathbb{R}) \quad (11)$$

[where $P_2(T(V), \mathbb{R})$ is the Pontryagin number of the tangent T (gauge V) fibre bundle on the internal manifold], then the fermionic field content of the four-dimensional theory is anomaly-free as a consequence of the index theorem which gives the number of four-dimensional chiral fermions in terms of the characteristic classes of T and V ¹⁶⁾.

If a gluino-gluino condensate $\langle \text{tr } \bar{\chi}(1-\gamma_5)\chi \rangle$ is produced by some non-perturbative effects¹⁷⁾, then the set of candidates for Goldstone fermions associated by SUSY with this v.e.v. can depend on the topology of our ground state. In particular, for Calabi-Yau manifolds with second Betti number $b_2 = 1$ the $\langle \bar{\chi}\chi \rangle$ condensate is related by SUSY with two candidates for Goldstone fermions, one of these living in the superfield of the breathing mode which is given by the Kähler harmonic form, and the other one by the massless spin- $\frac{1}{2}$ fermion of the supergravity multiplet. The true Goldstone fermion is thus a linear combination of these. The existence of this new Goldstone fermion which results from our particular ground state with $b_2 = 1$ forces us to consider the possibility of an axion-like coupling of its bosonic partner a defined by $B_{IJ} = \epsilon_{IJ}^a$ ^{18),9),10)}.

Now it is important to remember our previous discussion about Wess-Zumino terms. The point has already been made that the ambiguity in the choice of regularization procedure enables us to avoid introducing $W_{GS}(A,B)$ in the first place, in which case there is nothing to discuss. However, if we were to add to the Lagrangian a local counterterm $W_{GS}(A,B)$ which contains a piece $\propto B \wedge F \wedge F \wedge F$, then dimensional reduction would appear to give a new axion-like coupling in four dimensions. However, the fact that the theory is anomaly-free means that the Green-Schwarz counterterm $W_{GS}(A,B)$ is dynamically induced by fermionic loop effects. Therefore the coupling of the two-harmonic form a to $\tilde{F}\tilde{F}$ can only appear as a result of gaugino loop effects. The only coupling of the B field to the gaugino is contained in the term $H^{\mu\nu\rho} \bar{\chi} \Gamma^{\mu\nu\rho} \chi$. In four dimensions and taking into account the gaugino zero modes on K_6 this vertex can only be different from zero if by dimensional reduction $\partial_\mu B_{IJ} \bar{\chi} \Gamma^{\mu IJ} \chi$, $\mu = 1, \dots, 4$, $I, J = 5, \dots, 10$ gives some four-dimensional coupling $(\partial_\mu a j^\mu) C$ with j^μ a four-dimensional gaugino current and C a constant coming from dimensional reduction. This situation is in principle not possible because the only $SU(3)$ singlet on K_6 is the three-form ϵ_{IJK} .

Finally, it is worthwhile to point out that our conclusion is in agreement with some general results for the heterotic string¹⁹⁾. The real axion coupling is determined by R-invariance: in fact this coupling must change under R-symmetry in such a way as to cancel the change of the gauge kinetic part of the Lagrangian. Thus the real axion field must be contained in the imaginary part of the minimal kinetic function $f_{\alpha\beta}$ (3), which can be interpreted as the gauge coupling constant. In order to be consistent with the string theory, this coupling constant must depend on $\langle\phi\rangle$ (the v.e.v. of the dilaton field) in the same way as the string tension α' :

$$\kappa^2 \sim g_{10}^2 \alpha'(\langle\phi\rangle) \quad (12)$$

If we work on a stable ground state of the heterotic string, the tadpole of the dilaton is zero at one loop¹⁹⁾. This means that the gauge coupling constant defined by (12) is not renormalized by one-loop string effects. On the other hand, for the heterotic string whose string loop expansion is in powers of ϕ^{-2} , the anomaly counterterms are a one-loop effect. Hence, if our ground state is stable, this counterterm cannot change $f_{\alpha\beta}$, even when the stable ground state breaks $E_8 \times E'_8$ to $E_6 \times E'_8$.

In summary: we find that there is no new physics in the anomaly counterterms of $D = 10$ supergravity, that it affects neither the Kähler potential nor the gauge kinetic function obtained from a tree-level analysis, and in particular there is a unique axion D for both the observable and hidden gauge sectors.

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NOTE ADDED IN PROOF

In this paper we have argued that Wess-Zumino terms do not affect the axion in superstring theories, and do not give a mass to the field T_I . After our paper was written, we received a new paper²⁰⁾ in which it is proposed that another effect, namely non-perturbative world sheet instantons, may generate a mass for the T_I field. While this suggestion goes beyond the scope of our paper, we take the opportunity to comment on it by summarizing arguments contained in Ref. 21),

which suggest that the induced mass for the T_I field may be small or zero.

No contribution to $m(T_I)$ can be obtained at the tree level in the bosonic string, because the functional integral representations for tree level amplitudes must²²⁾ be divided by the volume of the conformal Killing vectors of the sphere, namely $\text{Vol}[SL(2,C)]$ which is infinite. This division sets to zero the cosmological constant²²⁾ and also the effective potential for the T field induced by world sheet instantons at the tree level. At the one-loop level in the bosonic string, such a potential may be generated since the volume of the conformal Killing vectors is proportional to the area of the world sheet, but $m^2(T_I)$ is still proportional to the cosmological constant. Note that there would also be a non-trivial potential induced for T_R at the same time.

In the superstring, world sheet instantons generate high order effective interactions involving fermionic zero modes which are D terms rather than F terms²⁰⁾. They cannot generate $T \times \text{Tr} W^2$ terms in four dimensions, because these are F terms rather than D terms. Such a term would in any case appear to violate a recent no-renormalization theorem²³⁾ when gauginos condense. Note also that a non-flat potential for the T field corresponds to a non-zero logarithm of the superstring σ -model generating functional, which appears to break SUSY on the world sheet²⁴⁾.

Finally, we note that the effective interaction proposed in Ref. 20) involves $\overline{27}$ fields, which may well be absent from the low energy spectrum. Purely phenomenological arguments for their absence have recently been given in Ref. 25).

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