ELECTRON STORAGET RINGS FOR THE PRODUCTION OF SYNCHROTRON RADIATION

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1. INTRODUCTION

An introductory overview is given on storage ring sources of synchrotron radiation (SR) -- in particular, sources of rather hard X-rays (hv > 1 keV). Requirements set by the users, properties of the devices used to produce the radiation, and finally design criteria to be followed to optimize a storage ring for the particular purpose, are discussed.

2. THE PHYSICS AND THE EXPERIMENTAL REQUIREMENTS

The study of the interaction of X-rays with matter goes back to the beginning of the century and has resulted in an amazingly large number of applications and findings of primary importance. Interesting reviews can be found\(^1,\)\(^2\), from which I will widely quote.

The basic processes characterizing the interaction of X-ray photons with matter are absorption and scattering: on the first is based, for example, radiography; on the second the study of crystals and molecules. The importance that radiographic techniques have acquired in everyday life need not be stressed. As an example from the past in the field of crystallography, it suffices to quote the unravelling of the structure of the DNA molecule, which 'will stand out as one of the greatest scientific achievements of this century'.

Up to a few years ago, when the new sources of X-rays became available from electron accelerators and storage rings, X-rays were only produced by means of X-ray tubes. In the conventional X-ray tube, the radiation is produced by bremsstrahlung of electrons as they strike the anode and by the de-excitation of electronic core levels in the atoms of the anode excited by incident electrons. Both processes are very inefficient, and more than 99% of the electron energy is converted into heat. The dissipation of this heat sets practical limitations on the X-ray intensity that can be obtained from X-ray tubes. The spectrum consists of a continuous background with lines superimposed at fixed energies, and the angular distribution of emitted photons is approximately isotropic. Polarization is only partial and is a function of energy for the continuous part of the spectrum\(^3\).

Synchrotron radiation suffers from none of these problems: in principle any intensity can be produced either in a continuous spectrum or in sharp tunable lines; it has excellent directional properties, well-defined polarization and, in addition, it can be made to have a very fast time structure.

The availability of many-orders-of-magnitude more intense, tunable sources opens up entirely new scenarios of possibilities of interest to a multitude of disciplines and applications. This is well illustrated in Fig. 1, where fields of activity and experimental methods are listed together with their relative frequency of occurrence at existing SR sources. Here it is sufficient to mention that an extremely wide spectrum of problems, all of importance, ranging from energy technology to semiconductor device production, from the
basics of catalysis and corrosion to the properties of metals and alloys, from the detailed structure of protein molecules to preventive diagnostics of heart diseases (Fig. 2), can be studied.

The basic requirements of an experimenter can be summarized, with some degree of over simplification, in just a few points:

a) The largest possible number of the desired energy photons should reach the sample, so that samples containing fewer and fewer atoms (more dilute solutions, monoatomic surface layers, very small biological samples) can be studied.

b) The photons reaching the sample should have the smallest possible divergence.

For unfocused beams this simply translates into the requirement of a high brightness, $\Phi_\Omega$ (see Fig. 3),

$$\Phi_\Omega = \frac{d^2n}{dt \, d\Omega \, (\Delta\lambda/\lambda)},$$

where $(\Delta\lambda/\lambda)$ is the desired energy bandwidth.
Fig. 2 Synchrotron X-ray of the arteries in a pig's heart with a 20 mg/l concentration of iodine (from Ref. 2)

A. UNFOCUSED BEAM

![Diagram of unfocused beam]

Requirements:
1. A large number of photons should reach the sample;
2. The beam should have a small divergence (diffraction experiments).

BRIGHTNESS

\[ \Phi / \Omega = \frac{d^4 n}{dt \, d\Omega \, (d\lambda / \lambda)} \]

- \( n \) - number of photons;
- \( t \) - time;
- \( \lambda \) - wavelength;
- \( \Omega \) - solid angle

\[ [\Phi / \Omega] = \text{photons per second} \text{ (mrad)}^2 \text{ 0.1\% bandwidth} \]

B. FOCUSED BEAM

![Diagram of focused beam]

Requirements:
1. A large number of photons should reach the sample (detector);
2. The image of the source should be small;
3. The divergence of the beam reaching the sample (detector) should be small.

BRILLIANCE

\[ B = \frac{d^4 n}{dt \, d\Omega \, ds \, (d\lambda / \lambda)} \]

- \( s \) - source size;

\[ B = \text{photons per second} \text{ (mrad)}^2 \text{ (mm)}^2 \text{ 0.1\% bandwidth} \]

Fig. 3 The requirements of brightness and brilliance for unfocused and focused beams.
If the beam is focused onto the sample, then the smaller the source dimensions and divergence, the smaller the spot size and divergence of the photon beam at the sample. The figure of merit is now the brilliance $B$ (see Fig. 3):

$$B = \frac{d^n}{dtds} \frac{d^2n}{(\Delta\lambda/\lambda)} ;$$  \hspace{1cm} (2)

$B$ is the most important figure of merit for hard X-ray sources. The increase in brilliance obtained over the last decades from improving their design is dramatic and is shown in Fig. 4.

3. THE PRODUCTION OF SYNCHROTRON RADIATION

An electron beam travelling along a curved path will lose energy by radiation -- the synchrotron radiation. In a storage ring the energy lost in a turn is restored by the RF system, so that the average electron beam energy $E_0$ is a constant.

In storage rings built for HEP, synchrotron radiation generated in the bending magnets is a somewhat unwanted feature: the fast increase of radiated power with energy ($P \propto E^3/\rho$, where $\rho$ is the bending radius) makes it expensive to move to very high energies (think of LEP with its 2 km of RF cavity).

High power and intensity are, on the contrary, welcome to SR users who first set up their beam lines, looking at bending magnets, on practically all HEP rings.

The geometry of a bending magnet source is illustrated in Fig. 5a. The spectrum is continuous, and a 'critical energy',

$$\epsilon_c = \frac{3}{4\pi} \lambda e^2 m_0 c^2 \frac{\gamma^3}{\rho_0}$$  \hspace{1cm} (3)
\( \lambda_{ce} \) is the electron Compton wavelength, is usually defined that divides the power spectrum into two equal halves\(^{a,b}\).

Since the vertical angular aperture of the emerging radiation fan is much smaller than its radial aperture (\( \theta_v \) being of the order of \( 1/\gamma \)), the number of photons per unit time and per unit horizontal aperture angle is often integrated over the vertical distribution, giving the 'flux' \( \phi \):

\[
\phi = \frac{d^2n}{d\theta_R dt}.
\]

The horizontal aperture is defined by slits, because the fan radiated by any given magnet is usually much larger than is needed by any beam line.

Because of the natural vertical collimation and the small effective source size, these sources are intrinsically 'brilliant' compared with X-ray tubes.

Note that on a given machine, \( \phi \) is a fixed quantity so that the critical energy of the radiation can only be changed by adjusting the operating energy. Also, increasing the experiment acceptance \( \theta_R \) only increases the flux to the sample, without affecting the brilliance.

A way to improve this is to use an 'insertion device', i.e. a special magnet to be placed on a straight section of the main lattice. The main lattice should, of course, ideally 'see' the device as a straight section.

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(a) bending magnet; (b) wiggler; (c) modulator

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Fig. 5 Different sources of synchrotron radiation:

\( 0 < 10 - 20 \text{ mrad} \)

\( \frac{1}{\theta} \)
A sequence of alternating magnetic fields, a 'wiggler' magnet, producing an orbit (usually lying in the horizontal plane) that starts and ends tangentially to the main orbit, is an obvious solution. It has the advantage that the local radius of curvature $\rho_w$ is now, to within reasonable limits, an independent variable, so that the critical energy can be changed by changing the field (i.e. $\rho_w$) even when the storage-ring energy is kept constant.

The horizontal radiation aperture angle from one 'pole' is now

$$\theta_M = \frac{K}{\gamma}, \quad K = \frac{eB_w \lambda_w}{2\pi mc^2}, \quad (5)$$

and depends on the wiggler magnetic field $B_w$ and spatial period $\lambda_w$. Depending on the value of $K$, $\theta_w$ can be much larger or of the order of (or indeed smaller than) the 'natural' aperture of the radiation.

The magnet is called a 'wiggler' if $K >> 1$; otherwise it is an 'undulator'. Besides freeing the value of $\rho_w$ (usually fields higher than those in the bends are used to produce higher-energy X-ray beams, and a single-period, very high field wiggler is often called a 'wavelength shifter'), wiggler magnets give higher flux than bending magnets for the same overall power. This is because of the compressed fan and because several 'poles' can radiate into the same fan angle (see Fig. 5b). For the same source size, the brilliance is now also increased by (approximately) the number of poles $N$ (see Ref. 6).

When $K \leq 1$, the aperture of the radiation due to the wiggler in the trajectory becomes comparable to the natural aperture of the radiation (Fig. 5c); the magnet becomes an 'undulator'. Interference effects between the radiation beams from different poles set in. The spectrum is modified and discrete lines appear at wavelengths (in the forward direction):

$$\lambda_i = \frac{\lambda_u}{\gamma_i^2} \left( 1 + \frac{K^2}{2} \right), \quad (i = 1, 3, 5, \ldots), \quad (6)$$

where $\lambda_u$ is the spatial period of the magnetic structure and $i$ is the harmonic number (only odd harmonics appear in the forward direction).

The angular aperture of the radiation (from a single electron) is narrowed down, because of interference, to

$$\theta_u \approx 1/(\gamma_i \lambda_u), \quad (7)$$

so that, for large $N$, the brilliance can become extremely high.

4. Effective Source Size

The radiation is produced by an electron beam that has a finite size in the four-dimensional (transverse) phase space $(x, x', z, z')$. For a Gaussian beam the envelope is an ellipsoid with standard deviations $\sigma_x, \sigma_{x'}, \sigma_z, \sigma_{z'}$.

In a storage ring the product $\varepsilon_x = \sigma_x \cdot \sigma_{x'}$, the betatron radial emittance, is a constant of the motion determined by the lattice; $\varepsilon_z$ is proportional to $\varepsilon_x$ through the coupling constant $K$ of horizontal to vertical oscillations. The value of $K$ can be controlled by...
means of the lattice tunes (or of coupling elements such as tilted quadrupoles), and its
ultimate value depends on machine errors and imperfections; it is usually much less than 1
(0.1-0.01).

One has, at a point where \( \sigma_x, \sigma_z \), and the dispersion vanish,
\[
\varepsilon_x = \sigma_x/\sigma'_x = \frac{\sigma_X}{\sigma'_X}, \quad \varepsilon_z = \kappa \sigma_z/\sigma'_z = \frac{\sigma_Z}{\sigma'_Z}.
\]
(8)

The values of \( \sigma_x, \sigma'_x, \sigma_z, \sigma'_z \), are of course functions of the position along the machine.
From Eq. (8) it is, however, clear that small beam sizes and divergences need small emittances.
It is also seen that by properly choosing the \( \beta \) functions at the radiation source
point, one can balance (within limits) the beam dimensions against divergence or vice versa.
In the vertical plane, by keeping the coupling small, beam sizes smaller than those in the
horizontal plane can be achieved. \( K \) values as small as 0.01 have been obtained on existing
machines.

Because of diffraction effects, a radiation beam of wavelength \( \lambda_R \) also has a finite
intrinsic emittance -- the same in both planes -- given by
\[
\varepsilon_R = \lambda_R = \sigma_R/\sigma'_R.
\]
(9)

Since the actual source dimensions are, in the end, the r.m.s. combination of the electron
beam dimension \( \sigma_B \) and the diffraction values \( \sigma_R \) (for Gaussian beams),
\[
\sigma_{\text{effective}} = (\sigma_B^2 + \sigma_R^2)^{1/2}
\]
(10)
in all four dimensions. The beam emittance has to be compared with the radiation one, and
should ideally be the smallest in order to achieve the highest brilliance in the beam.

When dealing with X-rays of the order of 10 keV, the corresponding wavelength is
\( \lambda_R \approx 10^{-10} \) m: this sets the scale for the ideally desirable beam emittance. It is found
that it is very difficult (although ideally not impossible) to achieve \( \varepsilon_x = 10^{-10} \) m even
for a perfect lattice and in the zero current limit.

Note that when discussing undulators where interference effects became important,
we have implicitly assumed that the angular beam spread would be smaller than \( \sigma'_R = 1/(\gamma \max) \).
Otherwise, interference will tend to be cancelled.

Since \( \varepsilon_B > \varepsilon_R \), whilst ideally one would like to have
\[
\sigma'_B = (\varepsilon_B/\beta)^{1/2} \leq \sigma'_R,
\]
(11)
a rather large \( \beta \) should be provided at places where undulators are to be located.

Conversely, for a wiggler magnet or a bending-magnet source the angular beam spread
plays hardly any role because it is usually much smaller than the aperture of the fan of
radiation produced by the curvature in the trajectory. The objective is then to have
\( \sigma_B \leq \sigma_R \), and this calls for low values of \( \beta \).
5. LOW-EMITTANCE LATTICES

The horizontal emittance in a storage ring is given by the expression

$$\varepsilon_x = CE^2 \frac{\langle H/d \rangle}{\langle 1/d^2 \rangle},$$  \hspace{1cm} (12)

where $\langle \rangle$ means the average around the storage ring (in practice in the magnets because $\rho = \rho_0 = $ elsewhere); $H$ is called the lattice invariant and is given by

$$H = \gamma D^2 + 2\alpha D' D' + \beta D'^2,$$  \hspace{1cm} (13)

where $D_x$ is the horizontal dispersion function. Note that for an isomagnetic lattice ($\rho = \rho_0 = $ constant) Eq. (12) becomes

$$\varepsilon_x = C_q \frac{\langle H \rangle}{\langle J_x \rangle}, \quad C_q = 3.84 \times 10^{-13} \text{ m}$$  \hspace{1cm} (14)

($J_x$ is the damping partition number, usually $\sim 1$, and $E$ has been replaced by the dimensionless $\gamma$).

From Eq. (14) it is clear that in order to make $\varepsilon_x$ small, one should make $\langle H \rangle$ small. [Since $\langle H \rangle$ is proportional to $\rho_0$, see for example Eq. (18), the bending radius drops out of Eq. (14)].

From Eq. (13), one sees that in order to make $\langle H \rangle$ small, $D_x$ (but also $D'_x$, $\alpha_x$, $\beta'_x$) should be small in the magnets. The physics is that dispersion should be small at places where photons are radiated, because the energy change associated with the emission of a photon causes the onset of betatron oscillations (proportional to $D_x$) that contribute to the equilibrium of $\varepsilon_x$.

In the following we will see that minima for $H$ can be found for various types of lattices, involving proper choices for $D_x$, $\beta_x$, and their derivatives in the bending magnet.

A very simple lattice that has low dispersion in the bending magnet and therefore can be made to have a low emittance, is the so-called 'double-focusing achromat' (a Chasman-Green lattice from the names of the physicists who first proposed it).

Its basic 'cell' is shown below:

![Diagram](image)

Here $B_1$ and $B_2$ are bending magnets, and $Q_F$ is a focusing quadrupole. Some additional focusing is, of course, needed on either side of the basic cell. The dispersion $D_x$ is zero in straight sections $S_1$ and $S_2$, so that the basic cell is an achromat: note that the cell is symmetric around $S_3$, so that in fact the basic lattice building-block is the first half-cell. Zero dispersion in $S_1$ and $S_2$ is also useful because the RF, injection, and insertion devices are best located in dispersion-free straight sections.
The dispersion $D_X$ obeys the equation

$$D_X''(s) + g(s)D_X(s) = \frac{1}{\rho(s)},$$

(15)

so that when one assumes $D_X(s_0) = D_X'(s_0) = 0$ and considers a uniform field magnet ($n = 0$, $\rho = \rho_0 = \text{const}$), it is easy to find that

$$D_X(s_1) = \rho_0 \left(1 - \cos \phi_B \right),$$

$$D_X'(s_1) = \sin \phi_B,$$

(16)

where $\phi_B = \ell_B/\rho_0$.

The symmetry condition $D_X'(s_1) = 0$ determines (for a given $L/2$) the strength of quadrupole $Q_B$ that makes the cell achromatic.

As mentioned above, quadrupole doublets (or triplets) can be inserted in $S_1$ and $S_2$ to shape the $\beta$ functions. Since $D_X$ is only excited by the bending field, these quadrupoles do not affect the dispersion.

The emittance of a lattice built of such cells can easily be computed$^{9-11}$. It is found that when $\beta_X$ is made to have a minimum value,

$$\beta_{x \min} (s^*) = \frac{\sqrt{3}}{B\sqrt{5}} \ell_B$$

(17)

at a point $s^* = s_0 + 0.375 \ell_B$. The invariant $(H)$ has a minimum

$$H_{\min} = \rho_0 \frac{\phi_B^3}{4\sqrt{15}},$$

(18)

and the emittance has a minimum

$$e_{x \min} = C q \gamma^2 \phi_B^4 / (4\sqrt{15})$$

(19)

in practical units,

$$e_{x \min} = 2.48 \times 10^{-13} \gamma^2 \phi_B^3 \text{ (m-rad)}.$$  

(20)

Note that by decreasing $\phi_B$, i.e. by increasing the number of cells, the emittance can be made to decrease very quickly at any given energy. However, the required value of $\beta_{x \min}$ also decreases rapidly and soon becomes difficult to achieve (or gives unacceptably high values of $\beta_X$ at the magnet ends).

For instance, to achieve an emittance $e_{x \min} = 10^{-9}$ m·rad at 5 GeV with a radius $\rho_0 = 20$ m, one would need approximately 42 cells (84 bending magnets, each ∼ 1.5 m long) and a value $\beta_{x \min} = 0.15$ m. Strong focusing is required to obtain such low $\beta$'s at very many (84) places around the machine, and large $\beta$ oscillations, giving large chromaticities (see Section 6), are obtained.
It can be shown\(^{19}\) that to obtain a reasonably well behaved lattice the emittance has to be increased by a factor of 2 to 3 over the ideal value given by Eq. (20).

Also note that lowering the emittance tends to increase the machine circumference. Eventually, very low values of \(\phi_B\) make it even difficult to extract the radiation from the machine because of the interference of beam ports with the lattice elements.

As an example, a variant of such a lattice designed for the European Synchrotron Radiation Facility (ESRF)\(^{11}\) is shown in Fig. 6. It has 32 cells and gives an emittance of \(\sim 7 \times 10^{-9} \, \text{m}\cdot\text{rad}\), about three times as large as predicted from Eq. (20), but with \(\beta_{x_{\text{min}}}\)

of the order of 1 m.

The horizontal beam size in the bending magnet has a minimum of less than 100 \(\mu\text{m}\). Notice that many (\(\sim 30\)) long straight sections are provided for insertion devices, and that in those marked U (undulator) the \(\beta\) functions tend to be rather high (especially \(\beta_{x}\)), whilst in those marked W (wiggler), \(\beta\)'s tend to be low, for the reasons explained in Section 4.

The dependence of emittance on the product \(\gamma^2 \phi_B^3\) exhibited by Eq. (19) is not peculiar to the Chasman-Green lattice, but is a rather general property.

For instance, the emittance of the regular FODO lattices used in all large accelerators is\(^{12}\)

\[
\varepsilon_x = 4 \int_{\varepsilon_C}^{\beta_2} F(\varepsilon_C) \gamma^2 \phi_B^3 = 7.68 \times 10^{-13} F(\varepsilon_C) \gamma^2 \theta^3, \quad (21)
\]
where $F(\mu_c)$ is a function of the betatron phase advance per cell $\mu_c$, and is given by

$$F(\mu_c) = \left( 1 - \frac{3}{4} \sin^2 \frac{\mu_c}{2} + \frac{1}{60} \sin^4 \frac{\mu_c}{2} \right) \left( \sin^2 \frac{\mu_c}{2} \sin \mu_c \right).$$

(22)

It has a minimum at around $\mu_c = 3\pi/4$, where its value is $F(3\pi/4) = 0.62$. The minimum is, however, rather flat in the range $100^\circ \leq \mu_c \leq 160^\circ$. In this range of phase advances

$$\varepsilon_{\text{min}}^x \approx 5 \times 10^{-13} \gamma^2 \phi_B^3 \quad \text{(m-rad)}.$$  

(23)

Notice that the coefficient is about 20 times larger than that in Eq. (20), meaning that for the same minimum emittance one needs about three times as many bending magnets.

In practice, however, it is easier to come close to the minimum emittance with a FODO lattice than with a Chasman-Green type, and the factor on the number of magnets is generally less than or close to 2.

When designing a machine with a very large number of straight sections whilst trying to keep the circumference reasonable, not many FODO cells are allowed in between the straight sections. It is rather hard, especially if you are accustomed to HE colliding-beam storage ring lattices, to even recognize the FODO structure in the lattice of Fig. 7 proposed as an alternative for the ESRF. However, its basic properties, and notably the regular behaviour and the low values of $\beta_x, \beta_z$ in the 'arcs', are recognizable.

Note that $\mu_c = 3\pi/4$ implies very strong focusing, and that chromaticity problems may be expected here too. Very strong focusing is, of course, inherent in the concept of all low-emittance lattices.

As a side remark (but more can be found in Ref. 11 and its bibliography) note that low-emittance lattices, because they usually have low $D_x$, also have a very low momentum compaction factor $\alpha_c$. Recall that the linear part of $\alpha_c$ is given by

$$\alpha_c^L = \frac{1}{\epsilon} \int \frac{B_x(s)}{\beta(s)} \, ds.$$  

(24)

![Fig. 7 A FODO type lattice for the ESRF](image-url)
For the ESRF, $\alpha_C^{(s)} = 3 \times 10^{-5}$. Non-linear terms may therefore become important and have to be watched.

The low value of $\alpha_C^{(s)}$ also affects bunch length (very short bunches can be obtained, at least for vanishing current).

6. SOME OF THE PROBLEMS TO BE SOLVED IN CONNECTION WITH LOW-EMITTANCE LATTICES

6.1 Chromaticity correction

One of the foremost problems is the correction of the lattice chromaticity\(^{13}\). From the linearized equations of motion, the natural (linear) chromaticities $\xi_{TX,z}$ are found to be

$$\frac{\Delta \xi_{TX,z}}{(\Delta p/p)} = \xi_{TX,z} = \frac{1}{4} \int \beta_{x,z}(s)K_{x,z}(s) \, ds,$$  \hspace{1cm} (25)

where $\Delta \xi_{TX,z}$ is the tune shift suffered by an off-momentum particle, and $K(s) = G(s)/(B_0 p_0)$ is the normalized quadrupole gradient.

The physics is very simple: given the quadrupole gradients $K(s)$, which give the right $Q$ for an on-momentum particle, a particle with higher energy will be less strongly focused (and vice versa). A negative value of $\xi_T$ will in general be obtained.

The strong focusing required to obtain a low emittance implies strong gradients and, often, also rather large values of $\beta(s)$, so that $\xi_T$ tends to become large. In the lattices we have been considering, a value of $\xi_T \sim 50-100$ is not unusual; meaning that, unless it is corrected, a particle only a few standard deviations away from the average energy will travel through strong resonances and be lost.

Bearing in mind that Eq. (25) is only the linear part of the chromaticity, the second- and higher-order effects may become important when $\xi_T$ is large.

The way to correct the linear chromaticity is to add extra momentum-dependent focusing. A sextupolar field placed where the dispersion is non-zero will do just that. More precisely, its correction term is

$$\xi_S = \frac{1}{2} \int \beta(s)D_X(s)K_S(s) \, ds,$$  \hspace{1cm} (26)

where $K_S(s)$ is the normalized sextupolar gradient. This has to be chosen in such a way that $\xi_S$ has the opposite sign with respect to $\xi_T$, in both planes:

$$K_S(s) = \frac{G_S}{B_0 p_0}, \quad G_S = \frac{1}{2} \frac{B''}{a^2}.$$  \hspace{1cm} (27)

Note that $\xi_S$ has to be large to compensate for the large $\xi_T$, and that $D_X$ is usually small everywhere. This means that strong sextupoles are needed.

The sextupolar fields introduce strong non-linearities in the equations of motion, and the linear approximation is no longer sufficient. To the second order in $(\Delta p/p)$, changes of the $\beta$-functions and of the dispersion with momentum have to be taken into account. Non-
chromatic and higher-order effects, such as a Q-shift depending on the betatron oscillation amplitude, are also found. These can be corrected by sextupoles in dispersion-free regions of the machine. Finally, resonances are excited by the non-linear terms (e.g. the third-order excited by sextupolar fields).

Unless all these effects are properly corrected, the net result is that the phase-space volume inside which particles are stable, the dynamic aperture, tends to become vanishingly small. Injection then becomes very difficult or impossible, and lifetimes become insufficient.

In order to obtain reasonably large dynamic apertures, complicated arrangements of separately powered sextupoles have to be found for each lattice. For instance, the project (ESRF) lattice shown in Fig. 6 requires five separate sextupole families (rather than the two that are necessary to correct the linear chromaticity only), and the lattice of Fig. 7 about seven families. The arrangement of sextupoles and the limits of the dynamic aperture are studied by tracking many particles around the ring (of course in a computer simulation) for many hundreds of turns.

The dynamic aperture of the ESRF lattice (ESRF-27), and that of a 'detuned' version having twice the emittance (ESRF-50), are shown in Fig. 8. Note that it shrinks for off-momentum particles (\(\Delta p/p \neq 0\)). Figure 9 shows two examples of phase-space plots, obtained by tracking particles with large initial amplitudes of oscillation (near the dynamic aperture limits). The position of the particle in phase space is recorded turn by turn, at a

![Dynamic apertures: ESRF lattices 27/3 and 30/3](image-url)
given azimuth along the ring (in the example: the centre of a long straight section). Note how high-order resonances distort the shape of the phase-space trajectory, which would be a perfect ellipse if the lattice were linear.

The problem of optimizing the chromaticity correction is central to the design of low-emittance rings, and is still attacked largely by trial and error since a complete theory is lacking.

6.2 Lifetimes

The lifetime of the beam in a storage ring is determined by several effects: radial and energy oscillations\(^7\), bremsstrahlung\(^\text{11}\), and scattering on the residual gas atoms, beam-beam bremsstrahlung, intrabeam scattering, etc.
In high-energy colliding-beam facilities one is accustomed to consider bremsstrahlung as the main mechanism leading to the loss of particles. However, for low-emittance rings, when the dynamic aperture and the vacuum chamber aperture are much smaller than usual, scattering on the residual gas and intrabeam scattering have to be considered and may become dominant.

Let us consider scattering: a particle is lost when it is scattered through an angle greater than that limiting the dynamic aperture at the position where the scattering event takes place, or in such a way that the ensuring oscillation amplitude will take it across the boundary determined by the physical chamber aperture. Let \((\mathrm{d}x/\mathrm{d}\Omega)_i\) be the corresponding differential cross-section for an atom of species \(i\), and let \(n_i\) be the density of scattering atoms of species \(i\) in the residual gas.

The scattering lifetime \(\tau_s\) is then given by

\[
\frac{1}{\tau_s} = C \sum_i n_i \int_\Omega \frac{\mathrm{d}z}{(\mathrm{d}x/\mathrm{d}\Omega)_i} \mathrm{d}\Omega ,
\]  

(28)

where \(\mathrm{d}\Omega\) is the solid angle over which the particle is lost.

If the machine acceptance is limited by the dynamic aperture, represented in real space by an ellipse having half axes \(N_x \sigma_x\) and \(N_z \sigma_z\) (with \(N_x, N_z\) integers), it can be shown\(^{14}\) that

\[
\frac{1}{\tau_s} = \frac{4cr^2K}{\gamma^8} \frac{\pi}{\gamma} \left( \frac{\bar{b}_x}{N_x^2 \sigma_x^2} + \frac{\bar{b}_z}{N_z^2 \sigma_z^2} \right) n_i p_i z_i^2 ,
\]  

(29)

where \(\bar{b}_x\) and \(\bar{b}_z\) are averages over the machine, and \(z_i\) is the atomic number of the scattering atom. Note that when \(\sigma_x, \sigma_z, N_x, N_z\) become small and, possibly, \(\bar{b}_x\) and \(\bar{b}_z\) become large, \(\tau_s\) decreases rapidly. Since \(n_i\) is proportional to the residual gas pressure, great care has to be taken in the design of low-emittance SR in order to have low residual gas pressures (in the range of \(10^{-9}\) Torr), large dynamic apertures, and the lowest possible values of \(\bar{b}_x, \bar{b}_z\) and \(z_i\).

Intrabeam scattering causes both a loss of particles (in longitudinal phase space) and a diffusion process in the six-dimensional phase space (see A. Piwinski, these Proceedings). Diffusion in competition with damping will lead to new equilibrium beam dimensions, and the emittance may blow up compared with the single-particle value. In its simplest form this was discovered quite some time ago in AD, the first \(e^+e^-\) storage ring built at Frascati\(^{15,16}\).

The loss rate is determined by the equation

\[
\frac{\mathrm{d}n_b}{\mathrm{d}t} = -\alpha n_b^2 ,
\]  

(30)

where

\[
\alpha = \frac{4\pi^2 e^2}{\gamma^2 \gamma_p} \left( \frac{J(e_p n_p)}{8\pi^{3/2} \sigma_x \sigma_y \sigma_z} \right) ,
\]  

(31)
where \( n_b \) is the number of particles in a bunch, \( \epsilon_p \) is the momentum acceptance, \( p_T = \nu \sigma_x' \), and \( J(\epsilon_p, p_T) \) is a complicated function of \( \epsilon_p \) and \( p_T \). The angular brackets indicate the average over the ring. Given the lattice, \( \alpha \) has to be computed numerically. Again note that when \( \sigma_x', \sigma_y', \) and \( \sigma_z' \) are small and the momentum spread \( \sigma_p' \) (determined by the dynamic acceptance) is also small, \( \tau_T = 1/\alpha \) decreases very rapidly.

At low energies the radiation damping time becomes long and diffusion processes start to blow up the beam. This tends to increase the lifetime but at the expense of emittance. As an example: lifetimes, emittance, and energy spread computed for the ESRF are shown in Fig. 10 as functions of the operating energy at a current of 4 mA per bunch.

Intrabeam scattering sets a limit on the emittances and beam dimensions obtainable at low energies for a given current. At high energy it limits the current that can be stored in a bunch with acceptable lifetime.

Fig. 10 ESRF: Touschek (loss by single scattering) lifetime, emittance, and momentum spread versus operating energy

6.3 Effects of wigglers and undulators

Wigglers, and to a lesser extent undulators, besides providing the desired radiation beams also produce side effects that have to be considered.

It can be shown\(^\text{17}\) that the end fields of a wiggler magnet produce a net focusing in the vertical plane. The fact that wigglers are usually located at places where \( \beta_z \) is low tends to diminish the effect on the lattice, but proper corrections have still to be provided. More important are usually the effects on the momentum spread, the damping time, and the emittance.
The additional radiation produced in a wiggler magnet will tend to increase the quantum fluctuations and lower the damping time. Depending on the value of the dispersion function at the wiggler location (but be careful: the self-generated dispersion has to be taken into account!), one or the other effect will tend to dominate and the beam emittance may either increase or decrease.

If the dispersion is zero (or small), a wiggler will actually decrease the beam emittance. As an example, the effect of a 72 m overall wiggler length on the emittance of the ESRF is given in Fig. 11a as a function of the wiggler peak magnetic field. The lattice dispersion is zero and the emittance decreases up to the point where the dispersion generated by the wiggler itself takes over.

![Graph](image)

**Fig. 11** Effect in ESRF of wigglers on  
a) emittance ($\epsilon_0$ is the unperturbed value),  
b) momentum spread.

On the other hand, the momentum spread is always increased. Figure 11b shows the behaviour of $\sigma_p$ for the ESRF, at various wiggler peak fields, as a function of the overall magnet length. This effect can become a problem when the momentum acceptance is limited by non-linearities.

Undulators, having lower field and shorter periods, produce lesser effects. However, lifetime problems may arise when very small gaps are required\(^{11}\) in order to obtain the desired field values.
Lastly, non-linearities in the wiggler/undulator fields may also be a problem; and, for machines designed to have very many of these devices, their effect has to be assessed in detail.

6.4 Machine alignment

The problem of keeping many lengths of beam lines precisely aligned over long periods of time and long distances is one of the main worries of the designer. Alignment tolerances and closed-orbit detection and correction require state-of-the-art techniques. Also, the stability of the ground in the presence of microseismic activity due to man-produced noise (such as traffic) and natural causes (wind, earthquakes, ocean waves) has to be carefully assessed.

7. THE EUROPEAN SYNCHROTRON RADIATION FACILITY

A European facility dedicated to the production of hard X-rays has been the subject of much study during the past five or six years. Feasibility studies were started as far back as 1979 under the auspices of the European Science Foundation\(^\text{18}\)). The latest work, funded by a number of European governments and hosted by CERN, was carried out by the ESRF\(^\text{11}\)), and is assumed to provide enough information for construction to be started in the near future. Its main parameters are listed in Table 1, and a general view of the lattice is shown in Fig. 12.

![Fig. 12 A general view of the ESRF](image-url)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron energy</td>
<td>5 GeV</td>
</tr>
<tr>
<td>Electron current</td>
<td>100 mA (multibunch). Possibility to increase to 200-300 mA.</td>
</tr>
<tr>
<td>Number of straight sections for insertion devices</td>
<td>30</td>
</tr>
<tr>
<td>Circumference</td>
<td>770 m</td>
</tr>
<tr>
<td>Critical wavelength:</td>
<td></td>
</tr>
<tr>
<td>Bending magnets</td>
<td>$0.9 \times 10^{-10}$ m</td>
</tr>
<tr>
<td>Multipole wigglers</td>
<td>From 0.5 to several $10^{-10}$ m</td>
</tr>
<tr>
<td>Wavelength shifters</td>
<td>From 0.5 to $0.2 \times 10^{-10}$ m</td>
</tr>
<tr>
<td>Minimum wavelength for the fundamental from undulators</td>
<td>$0.86 \times 10^{-10}$ m</td>
</tr>
<tr>
<td>Mode of operation</td>
<td>Multibunch with a possibility of single-bunch operation</td>
</tr>
<tr>
<td>Emittance</td>
<td>$\varepsilon_X = \pi \times 7 \times 10^{-9}$ rad·m</td>
</tr>
<tr>
<td>Straight section length</td>
<td>6 m</td>
</tr>
<tr>
<td>Full length for undulators</td>
<td>6 m</td>
</tr>
<tr>
<td>$\beta$</td>
<td>25.8 m</td>
</tr>
<tr>
<td>$\sigma_X$</td>
<td>0.43 mm</td>
</tr>
<tr>
<td>$\sigma'_{X}$</td>
<td>0.017 mrad</td>
</tr>
<tr>
<td>Full length for multipole wigglers</td>
<td>3 m</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.57 m</td>
</tr>
<tr>
<td>$\sigma_X$</td>
<td>0.064 mm</td>
</tr>
<tr>
<td>$\sigma'_{X}$</td>
<td>0.113 mrad</td>
</tr>
<tr>
<td>Great flexibility for replacement of an undulator by a multipole wiggler and vice versa.</td>
<td></td>
</tr>
<tr>
<td>Maximum number of wigglers/undulators</td>
<td>28-30</td>
</tr>
<tr>
<td>Injector</td>
<td>5 GeV, 10 Hz synchrotron</td>
</tr>
<tr>
<td>Pre-injector</td>
<td>Microtron ($e^-$) or linac ($e^+$)</td>
</tr>
</tbody>
</table>
REFERENCES


2) J.A. Nielsen, ed., The case for a European synchrotron radiation facility (ESRF, Strasbourg, 1982).


13) H. Wiedemann, SLAC PEP Note 220 (1976).


