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Nuclear Properties from Perturbative QCD

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Abstract

Two apparently different descriptions of quark distributions in a nucleus may in fact be connected. A 'duality' between the QCD approach and the conventional model of nucleon binding leads to nuclear properties being simply related to the anomalous dimensions of QCD.

One of the major challenges in nuclear physics is to derive nuclear properties from the fundamental theory of interacting quarks, QCD. In this letter we show that the average nucleon separation energy and the Fermi momentum may be quantitatively related to the anomalous dimensions of QCD.

The details of quark confinement in free or bound nucleons are not yet solved, but we believe that when a free nucleon is placed in nuclear matter the modification to confinement is characterised by a change of a length scale. In earlier work⁽¹⁾ we have shown how the consequences of this increase in confinement size can be analysed within perturbative QCD. It leads to an effective Q^2 -rescaling of the nucleon structure functions which, in turn, successfully describes the EMC effect in the region $0.2 < x < 0.8$.

The EMC effect has generated a lot of activity among theorists and several, at first sight unconnected, explanations have been offered, for example the above mentioned. One of these⁽²⁾ argues that the nuclear binding effectively reduces the energy-momentum of the participating quark by $(m + \bar{\epsilon})/m$ where $\bar{\epsilon}$ is the (negative) average nucleon separation energy and m is the nucleon mass. This is very much in the spirit of the 'enhanced - pion' model proposed earlier by Llewellyn Smith and Ericson and Thomas⁽³⁾ where the Fermi gas model was modified to take account of a reduced fraction of momentum carried by the nucleon.

Indeed, as Jaffe⁽⁴⁾ originally pointed out, the main implication of the EMC effect is a degradation of the valence quarks with a transfer of energy-momentum to some other component, and this is a common feature of all explanations of the effect. In our QCD approach the component which acquires the energy-momentum is gluons + sea quarks, while in the nuclear binding approach it is the binding force itself i.e. meson fields. The essence of our QCD approach is that the change which takes place in going from a free nucleon

to a bound nucleon is characterised by a change of a single scale - the confinement size. If this is true we may use the operator product expansion to relate "long-distance" nuclear properties to short distance properties which may be reliably calculated in perturbation theory. If QCD is the underlying theory for the strong interactions it is necessarily true that any other model which is dominated by a single change of scale can be re-expressed in terms of our QCD analysis and therefore a correspondence can be made between two models which, at first sight, may appear unrelated.

We will argue that the nuclear binding model is an example of a model dominated by a single scale change which is manifested at the phenomenological level by approximate 'x-rescaling'^(2,5). So, over a limited range of x , we can make an equivalence between these two views of a bound nucleon and at the phenomenological level we obtain a connection between Q^2 -rescaling and x-rescaling, for example. If there is a duality between the QCD approach and the conventional nuclear binding model then there are implications at a more fundamental level. It implies that the underlying theory of nuclear binding is QCD, i.e. quark and gluon exchange, which may be related, via sum rules, to averages of non-quark-like properties of the nucleus such as the nucleon binding energy, Fermi momentum and meson potentials. In this context we are encouraged by several attempts currently in progress⁽⁶⁾ to describe features of nuclear structure in terms of QCD.

In this paper we see how the parameters of the nuclear model - $\bar{\epsilon}$ the average separation energy, k_F the Fermi momentum and V the binding potential can be linked to the anomalous dimensions of QCD and quantitatively related to the increase in confinement size.

In our earlier work⁽¹⁾ the modification of a quark distribution in going from a free nucleon to a nucleus was attributed to an increase in the

confinement size, $\lambda_N \rightarrow \lambda_A$ where, for $A = 56$, $\lambda_A/\lambda_N = 1.15$. We had no deep understanding of the origin of this change of length scale but we proposed a simple nucleon overlap model which successfully predicted the A -dependence of this scale. Our starting point is that the quark distribution in a bound nucleon at $Q^2 = \mu_A^2$ is the same as that in a free nucleon but at $Q^2 = \mu_N^2$ where the Q^2 scales are given by

$$\frac{\mu_A^2}{\mu_N^2} = \frac{\lambda_N^2}{\lambda_A^2} \quad (1)$$

In QCD the difference between the quark distributions is due to the radiation of gluons between $Q^2 = \mu_A^2$ and $Q^2 = \mu_N^2$. If we select the valence distribution, the moments of the structure function are related by

$$M_n^A(Q^2) = \left[\frac{\alpha_s(\mu_N^2)}{\alpha_s(\mu_A^2)} \right]^{d_n} M_n^N(Q^2) \quad (2)$$

where

$$M_n^{N,A}(Q^2) = \int_0^1 dx x^{n-1} q_{N,A}(x, Q^2) \quad (3)$$

and $q_{N,A}$ are the quark distributions per nucleon for a free and a bound nucleon. The exponents $d_n = \gamma_0^n / 2\beta_0$ where γ_0^n are the non-singlet anomalous dimensions of perturbative QCD. Eqn. (2) leads to the Q^2 -rescaling equation

$$q_A(x, Q^2) = q_N(x, \xi_A \cdot Q^2) \quad (4)$$

where the shift in Q^2 is given by

$$\xi_A \equiv \xi_A(Q^2) = \left[\frac{\lambda_A^2}{\lambda_N^2} \right]^{\alpha_s(\mu_A^2)/\alpha_s(Q^2)} \quad (5)$$

and we have used

$$\alpha_s^{-1}(\mu_A^2) - \alpha_s^{-1}(\mu_N^2) = \frac{\beta_0}{4\pi} \ln \left[\frac{\mu_A^2}{\mu_N^2} \right] = \frac{\beta_0}{4\pi} \ln \left[\frac{\lambda_N^2}{\lambda_A^2} \right] \quad (6)$$

The rescaling relation (4), which applies to each parton distribution (not just valence quarks) gives a good description of the nuclear quark distributions⁽¹⁾.

Taking $n = 2$, eqn. (2) explicitly quantifies the fraction of energy-momentum lost by valence quarks in going from a free nucleon to one bound in a nucleus. On the other hand the $n=2$ moment for the singlet quark plus gluon distributions has zero anomalous dimensions and eqn. (2) will give zero net difference for the total energy of A free nucleons to a nucleus with A bound nucleons. Thus QCD rescaling will not describe the (small) effects due to mass defect.

So much for the response of the quark distributions to a change of length scale in QCD. Next we look at the nuclear binding model which turns out to be essentially a description in terms of a change of a single scale. We follow the presentation of Jaffe⁽⁷⁾ and the model of Akulinichev et al⁽²⁾. The probability distribution of nucleons in the nucleus is $f_{N/A}(y_A)$ where $y_A = p^+/P^+$, the p^+ and P^+ being the light cone momenta of the struck nucleon and the nucleus. If there was no binding we would have $f_{N/A}(y_A) = A\delta(y_A - \frac{1}{A})$ which would lead to $M_n^A = M_n^N$ for the moments of the quark distributions. When we switch on the binding, the momentum of the struck nucleon is $p = (m + \bar{\epsilon}, \underline{p})$

where $\bar{\epsilon}$ is the average separation energy, typically ~ -30 MeV. Once we integrate over the three-momentum $\underline{p}$ we see that the resultant distribution depends only on the single scale $m^* = m + \bar{\epsilon}$. It is this feature which allows us to make a correspondence with our QCD analysis.

However, the connection between the confinement scale of the QCD analysis and m^* is non-linear. To see this consider the quark distribution of the struck nucleon: m^* will depend logarithmically on μ^2 , the momentum squared of this quark, so a change in m^* in going from a free to a bound nucleon will generate an exponentially large change in μ^2 or, equivalently, the confinement scale.

Of course, the convolution model for nuclear binding involves also the effect of Fermi momentum k_F and this may introduce a further scale. However empirically and in model calculations involving nuclear potentials, we find that k_F is related to m^* and so the convolution model is predominantly a single scale model.

We turn now to a quantitative analysis of this connection between the QCD analysis and the nuclear binding models. Because $\bar{\epsilon}$ is negative the fraction of energy carried by the quarks is less than 1 and, assuming the structure function of the nucleon is unaffected when it is off-shell, this depletion of energy carries through into the $n = 2$ moment, i.e.

$$M_{n=2}^A = z_0 M_{n=2}^N \quad (7)$$

where $z_0 = 1 + \bar{\epsilon}/m$.

In the limit of a narrow distribution, e.g. $f_{N/A}(y_A) \sim A\delta(y_A - \frac{z_0}{A})$ the convolution of this with the nucleon quark distribution would give

$$q_A(x) = q_N\left(\frac{x}{z_0}\right) \quad (8)$$

or 'x-rescaling',^(2,5). So in this limit, the model is characterised by a change of scale $x \rightarrow \frac{x}{z_0}$ and, as we have argued, it is this feature which allows us to make a correspondence with the QCD approach. Since the distribution $f_{N/A}(y_A)$ has a finite width governed by the Fermi momentum k_F , x-rescaling is not exact, but it is a good approximation for $x < 0.5$.

The duality between the two approaches is the identification of eqn. (7) with eqn. (2) - with $n = 2$. Both describe the depletion of energy-momentum from the valence quarks, giving

$$\bar{\epsilon} = m \left[\left(\frac{\alpha_N}{\alpha_A} \right)^2 - 1 \right] \quad (9)$$

So, for example, taking $A = 56$ which corresponds to $\alpha_N/\alpha_A = 0.898^{(1)}$ we get $\bar{\epsilon} = -43$ MeV, which is close to the observed value. We can use the values of λ_A/λ_N for a number of nuclei obtained from our nucleon overlap model⁽¹⁾ and estimate the corresponding average separation energies. These are plotted in fig. 1(a)*.

To the extent that k_F does not introduce a new energy scale we should be able to connect the two approaches for moments higher than $n = 2$. For the Fermi gas distribution we have

$$f_{N/A}(y_A) = \frac{3mA^2}{4k_F^2} \left[1 - \frac{m^2A^2}{k_F^2} \left(y_A - \frac{z_0}{A} \right)^2 \right] \quad (10)$$

* We ignore second order corrections here. These depend quite strongly on the choice of renormalisation scheme and we have, as yet, no systematic way to deal with them.

and with this we compute the ratio of moments

$$M_n^A/M_n^N = A^{n-2} \int dy_A y_A^{n-1} f_{N/A}(y_A) \quad (11)$$

The resulting analogue of eqn. (7) for $n = 3$ is

$$M_{n=3}^A = (z_o^2 + \frac{1}{5} \frac{k_F^2}{m^2}) M_{n=3}^N \quad (12)$$

Identifying with eqn. (2) gives

$$k_F^2 = 5m^2 \left[\left(\frac{\alpha_N}{\alpha_A} \right)^{d_3} - \left(\frac{\alpha_N}{\alpha_A} \right)^{2d_2} \right] \quad (13)$$

which relates the Fermi momentum k_F to the constants of QCD and to the increase in confinement size. Taking $A = 56$ again gives $k_F = 285$ MeV - surprisingly good. The most direct connection with QCD comes from eliminating α_N/α_A in eqn. (9) and (13):-

$$\frac{k_F^2}{5m^2} = \frac{\bar{\epsilon}}{m} \left(\frac{d_3}{d_2} - 2 \right) + O\left(\frac{\bar{\epsilon}^2}{m^2} \right) \quad (14)$$

This relates the parameters $\bar{\epsilon}$, k_F of the nuclear binding model to the anomalous dimensions of QCD without any further model dependent input. Feeding in the values for d_2 , d_3 for $n_f = 4$ gives

$$k_F^2 = - \frac{35}{16} m \bar{\epsilon} \quad (15)$$

which is independent of A . If we take the values of $-k_F^2/m\bar{\epsilon}$ for nuclei analysed by Moniz et al⁽⁸⁾ we get values which vary from 1.7 to 2.4 compared with $35/16 = 2.2$. From eq (15) we see that the magnitude of the Fermi kinetic energy per nucleon, $\langle T \rangle = 3k_F^2/10m = -21\bar{\epsilon}/32$ and of the binding potential per nucleon $V = \bar{\epsilon} - \langle T \rangle = 53\bar{\epsilon}/32$ are both of the order of $\bar{\epsilon}$. Thus for $A = 56$ we have $\langle T \rangle = 26$ MeV and $V = -68$ MeV.

Going to higher moments ($n > 3$) provides consistency tests and allows us to test the stability of our estimates for $\bar{\epsilon}$ and k_F . For example eqn. (11) for $n = 4$ gives

$$M_{n=4}^A = (z_o^3 + \frac{3}{5} z_o \frac{k_F^2}{m^2}) M_{n=4}^N \quad (16)$$

and identifying with eqn. (2) again and then solving the $n = 3, 4$ moments yields new expressions for $\bar{\epsilon}$ and k_F^2 . To demonstrate the stability of the solutions for $\bar{\epsilon}$ and k_F we show, in figs. 1(a), 1(b) the results obtained by solving a pair, at a time, of the equations from the $n = 2, 3, 4$ moments. The A -dependence is determined from the A -dependence of the confinement size λ_A which in turn is governed by the nucleon density via our nucleon overlap model⁽¹⁾.

What happens when we consider very high moments? Our proposed duality would, as $n \rightarrow \infty$, give

$$z_o^{n-1} + O(z_o^{n-4} k_F^2/m^2) = \left(\frac{\alpha_N}{\alpha_A} \right)^{d_n} \quad (17)$$

$$\text{or } \ln z_o \sim d_n/(n-1) \sim \ln n/(n-1)$$

so as $n \rightarrow \infty$, $z_o \rightarrow 1$ or $\bar{\epsilon} \rightarrow 0$, thus exposing the unreliable region of the QCD

approach i.e. $x \rightarrow 1$. Higher order and higher twist terms are expected to become important there and there is no way that Q^2 rescaling can get $q_A(x) > 0$ for $x > 1$.

Let us conclude with a summary. In the QCD description, the difference between quark distributions in a nucleus and in a free nucleon is due to a single change in length scale, $\lambda_N \rightarrow \lambda_A$. Of course the magnitude of the EMC effect resulting from this change depends also on another scale Λ_{QCD} since the relevant parameter in this approach is always α_N/α_A where

$$\frac{\alpha_N}{\alpha_A} \equiv \frac{\alpha_s(\mu_N^2)}{\alpha_s(\mu_A^2)} = 1 - \frac{\beta_0}{4\pi} \alpha_s(\mu_N^2) \ln \left(\frac{\lambda_A^2}{\lambda_N^2} \right) \quad (18)$$

In particular the fraction of energy lost by valence quarks depends simply on this scale change. If one considers some other approach which also depends only on a single scale change, then it is obvious that one can translate one scale change into the other, the matching link being the fractional energy-momentum depletion. From this point of view this 'duality' between the two approaches is inevitable.

The nuclear binding model depends on more than one scale, k_F as well as $\bar{\epsilon}$, but only the latter enters into the $n = 2$ moment. The fact that the values of $\bar{\epsilon}$, k_F obtained from higher moments are reasonable implies that our duality is more than a formal equivalence between two single changes of scale.

The QCD approach has one attractive feature - the sum rules are guaranteed. In particular there is no freedom of choice as to where the energy-momentum lost by the valence quarks ended up - it is the gluon and sea-quark components which see to the 'balancing of the books'. So it follows, in this picture, that these components are relatively enhanced in a

nucleus and indeed there is some indirect evidence that this is true for gluons⁽⁹⁾. In the binding picture, the balancing of energy-momentum is done 'by hand' - i.e. it is carried by the meson field, as in ref. (2). So if the duality idea is correct then we should associate the increase of the gluon/sea-quark distributions with the field responsible for binding the nucleons. This does not rule out the interaction also being effectively described by meson exchange. Presently a lot of theoretical effort is going into the study of confinement in nuclear-like systems⁽⁶⁾. Starting from a many-quark system interacting via quark/gluon exchange the aim is to see if bound states emerge between 'nucleons' and also to investigate the effect of allowing the quark density to increase. What seems to be emerging is a picture where the 'nuclear' states appear accompanied by an effective increase in the confinement size of the quarks. Such efforts are still at an early stage but there are grounds for believing that our proposed duality may have a theoretical justification.

Finally we point out that, at the phenomenological level, the dual description attempts to answer complaints⁽¹⁰⁾ that there are too many explanations for the EMC effect. The attempts to devise tests to discriminate between some of these explanations may therefore be misplaced. Apart from this 'economic' argument however, the real bonus is the discovery that while QCD is believed to be the underlying theory of nuclear forces, the details of confinement mechanisms can actually be sidestepped to get simple expressions for the parameters of nuclear binding in terms of fundamental constants of QCD.

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FIG. 1

1(a) Values of the average separation energy $\bar{\epsilon}$. Experimental points ϕ are from ref (7). The duality estimates are shown by $\updownarrow$, the upper end using the $n=2$ moment, the lower end using the $n=3, 4$ moments.

1(b) Values of the Fermi momentum k_F . Experimental points ϕ from ref (7). The duality estimates are shown by $\updownarrow$, the upper end using the $n=2, 3$ moments, the middle point using the $n=2, 4$ moments and the lower end using the $n=3, 4$ moments.