CERN-TH.4287/85

QUARK PERCOLATION IN NUCLEI

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ABSTRACT

The percolation of quarks and gluons in finite size nuclei with realistic density distributions is studied as a function of the interaction-radius of the nucleon. Cluster probabilities and effective mean cluster sizes are calculated in order to fit the EMC data as well as the A-dependence of the SLAC data on deep inelastic lepton-scattering off nuclei.

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1. - INTRODUCTION

The difference of structure functions of bound nucleons compared to free nucleons^{1),2)} has shown the relevance of quark degrees of freedom for an understanding of nuclei. The high resolution electromagnetic probe interacts with pointlike quarks leaving an experimental trace of the dynamics of quarks in the nucleus. A model of the nucleus based on baryons and mesons alone together with their on-shell intrinsic structure functions may be adequate at low resolution, as successful nuclear physics calculations have shown. At high resolution the wave function of the nucleus evolves and it is hardly possible that the Fock space spanned by hadronic colour zero baryons and mesons is complete. At short distances between nucleons, the antisymmetrization of the many quark wave function does no longer allow to identify a quark as belonging to one or the other hadron. It is meaningless to separate quarks in mesons from quarks in nearby nucleons.

In this spirit the quark cluster model³⁾ has been invented. It is based on an order parameter χ which differs inside and outside of the quark clusters. A colour dielectric constant⁴⁾ χ may play such a role. The field χ is obtained as an average over Wilson loops with a real size which approaches the QCD scale $(1/\Lambda_{\text{QCD}})^4$.

$$\chi(x) = \frac{1}{N_c} \text{Tr} \lim_{L^4} \exp i \oint_x dx_\mu A^\mu$$

One has $\chi \approx 1$ inside the hadron allowing quarks and gluons to propagate freely, whereas $\chi = 0$ outside the hadron makes the quark infinitely heavy and forbids gluon fields. An effective colour dielectric Lagrangian may be calculable in vacuum by a blockspinning procedure on the lattice. In the nucleus the many body problem is much more complex. Intuitively we assume the colour dielectric field to be randomly distributed outlining small islands on which quarks and gluons can live. Outside of these islands the nuclear χ -field will be very small, suppressing colour degrees of freedom but not totally eliminating them. In Ref. 4) it was shown how such a model can lead to nuclear binding via a spread of the quark wave functions of the bound nucleon. The self-energy of a nucleon isolated in a Wigner-Seitz cell and surrounded by the nuclear dielectric medium is lower than the free nucleon mass. A crystal of nucleons, of course, does not correspond to our understanding of nuclei as a liquid or gas of nucleons. Therefore, we want to investigate in this paper what kind of structures develop when we allow for a random distribution of the colour dielectric field as it is generated by clustering of nucleons of a given size. Such a problem is analogue

to percolation studies of electric conductivity in amorphous materials. Indeed a long time ago such a model³⁾ has been used to study the interaction radius of the nucleon in He³. It has led many others⁵⁾⁻⁷⁾ to apply it to deep inelastic scattering. Extending our previous work, we want to show the transition from isolated nucleons to a colour conducting nucleus when the effective confinement zone of the nucleon becomes larger. In the colour dielectric model, the confining barrier m_0 separating quarks in different nucleons evolves with Q^2 as the quark mass (cf. Fig. 1)

$$\frac{m_0(Q^2)}{m_0(Q_0^2)} = \left(\frac{\ln Q_0^2/\Lambda^2}{\ln Q^2/\Lambda^2} \right)^{2d}$$

where $d = 4/(11-2/3f)$ is the anomalous dimension of the evolution of the quark mass and $(f = 2)$ is the number of quark flavours. The starting value $m_0(Q_0^2) \approx 1$ GeV characterizes low energy nuclear physics. It gives a rms radius of 0.62 fm for the valence quarks and a potential depth for the nucleon of $V_N = -50$ MeV. With increasing Q^2 the confining barrier becomes smaller and the effective quarks of the size $1/Q^2$ look more delocalized. Recently⁸⁾ another value of $m_0 \approx 8$ GeV has been proposed which would lead to a smaller violation of the charge symmetry in mirror nuclei. This criticism, however, has not taken into account the gluon contribution to the proton-neutron mass difference. Whatever the exact value of m_0 is, the existence of this larger mass scale produces higher twist effects in deep inelastic lepton nucleus scattering which behave as m_0^2/Q^2 . These become unimportant when the partial confinement of quarks in nucleons has disappeared, i.e., when the nucleus becomes fully colour conducting. For the free nucleon there is no scale m_0 and quarks are totally confined. Therefore, we would not expect large higher twist effects there. We note that both cases are in agreement with the operator product expansion, which states that each moment of the structure functions is given by a unique Wilson coefficient (depending on Q^2) times a hadronic matrix element (independent of Q^2) plus higher twist contributions which vanish in the Bjorken limit, $Q^2 \rightarrow \infty$, $\nu \rightarrow \infty$, $x = Q^2/2m\nu$ fixed. The crucial point for lepton-nucleus scattering is that these higher twist contributions need not necessarily introduce a rapid variation of the α_s -scaling corrections near $Q^2 \approx m_0^2$. A similar case is known in the $e^+e^- \rightarrow$ hadrons reaction where the appearance of new flavours does not change α_s abruptly. In Fig. 2, the variation with Q^2 of the rms radius of the valence quarks is calculated for nucleons in different nuclei (²D, ³He, ..., ¹⁹⁷Au) in the colour dielectric model⁴⁾. These nuclei differ by their average densities and colour dielectric constants. The denser the nuclear medium, the more the quarks leak out of their bags and the stronger the average nuclear potential. This is seen in the ordering of the curves for fixed Q^2 . With increasing Q^2 the tunneling of quarks

becomes stronger when the barrier is already low at small Q^2 . Therefore, we see a stronger increase of the rms radius in gold than in the deuteron. In Fe the nucleon radius increases by 10% between $Q^2 = 10 \text{ GeV}^2$ and $Q^2 = 100 \text{ GeV}^2$. In addition, nucleons will overlap more with increasing nucleon radius. Therefore, the confinement region of quarks will also increase. In this paper we are going to investigate this effect of quark percolation in the nucleus.

In Chapter 2, we will give a Monte Carlo method to study the percolation of nucleons in finite size nuclei. Chapter 3 will be devoted to the numerical results of the simulation. We will show the reduced cluster size which can serve as a signal for percolation in finite systems. In chapter 4 we use the effective confinement size to calculate the change of structure functions and compare to experimental data. We concentrate on the A-dependence and Q^2 -dependence of the ratio of structure functions, since the x-behaviour has been discussed before.

2. - MONTÉ CARLO SIMULATION OF QUARK CLUSTERING IN FINITE NUCLEI

The charge density distributions of nuclei are well measured for low momentum transfer. Assuming little difference for the matter distribution, one can deduce the rms radii of nuclei

$$\langle r_A^2 \rangle = \frac{\int \rho_A(r) r^4 dr}{\int \rho_A(r) r^2 dr} \quad (2.1)$$

Depending on the nucleon number A, we use a Gaussian parametrization ($A = 3, 4$) or a Fermi distribution ($A > 12$). For Be^9 we use a harmonic oscillator model

$$\text{Gaussian:} \quad \rho_A(r) = \rho_0 e^{-\alpha r^2} \quad (2.2)$$

$$\text{Harmonic well:} \quad \rho_A(r) = \rho_0 (1 + \alpha \frac{1}{2} r^2) e^{-\frac{1}{2} k^2 r^2} \quad (2.3)$$

$$\text{Fermi-distr.:} \quad \rho_A(r) = \rho_0 \left(1 + \frac{W r^2}{c^2} \right) / \left(1 + \exp\left(\frac{r-c}{Z}\right) \right) \quad (2.4)$$

The parameters are given in Refs. 9) and 10). If one varies the nucleon radius, one has to decide whether the nuclear size should be kept fixed. We think that consistently with the free nucleon, where there is absolute confinement, the nuclear radius should not vary. Therefore, we reduce the rms radii to rms body-radii by subtracting the rms radius of the nucleon

$$\langle r^2 \rangle_{body} = \langle r_A^2 \rangle - \langle r_N^2 \rangle \quad (2.5)$$

The parameters of the density parametrization $(\tilde{\alpha}, \tilde{k}, \tilde{w}, \tilde{c}, z)$ are adjusted in such a way that the rms body radii are reproduced. Especially one can deduce from (2.2) to (2.4)

$$\langle r^2 \rangle_{body} = \frac{3}{2} \tilde{\alpha} \quad \text{Gaussian} \quad (2.6)$$

$$\langle r^2 \rangle_{body} = \frac{3}{2} \frac{1}{\tilde{k}^2} \frac{2 + 5\tilde{\alpha}}{2 + 3\tilde{\alpha}} \quad \text{Harmonic well} \quad (2.7)$$

and $\langle r^2 \rangle_{body}$ for the Fermi distribution is numerically estimated. The Monte Carlo program proceeds in three steps: firstly, it sets up the positions of the nucleons in the following way. For each nucleon i ($i = 1, \dots, A$), a distance x_i from the centre-of-mass is chosen with the conditional probability distribution $p(x_i) = x_i^2 \rho(x_i)$. Then half of the direction cosines $\cos \theta_i$ of the polar angles and half of the azimuthal angles ϕ_i are chosen equally distributed in the intervals:

$$\begin{aligned} -1 \leq \cos \theta_i \leq 1 ; \quad & (i = 1, \dots, \frac{A(+1)}{2}) \\ 0 \leq \phi_i < 2\pi . \end{aligned} \quad (2.8)$$

The other half of the direction angles are adjusted in such a way that the centre-of-mass of the nucleus is at zero, i.e.

$$\sum_{i=1}^A \vec{X}_i = 0 \quad (2.9)$$

These are three equations for A unknowns. Therefore, the space of the solutions has dimension $(A-3)$. Except for ${}^3\text{He}$, there is an infinite number of solutions. Only one solution is searched for by means of the minimization of the function

$$f(\vec{X}_1, \dots, \vec{X}_A) = \left(\sum_{i=1}^A x_i \right)^2 + \left(\sum_{i=1}^A y_i \right)^2 + \left(\sum_{i=1}^A z_i \right)^2 \geq 0 \quad (2.10)$$

with the help of the method of conjugate gradients [Ref.11]. This constraint plays an important role for small A . The set of nuclei we investigated consists of ${}^3\text{He}$, ${}^9\text{Be}$, ${}^{12}\text{C}$, ${}^{56}\text{Fe}$, ${}^{197}\text{Au}$. For each nucleus 10 000 random assignments of nucleon positions are made. In order to count the nucleon clusters, we take into account all connected nucleons with distances $r < 2R_c$. For the definition of all radii it is good to check with Table 3. The parameter R_c is a fit parameter. It

may be compared to the equivalent hard sphere radius of the nucleon $R_u = \sqrt{5/3} \langle r^2 \rangle^{1/2}$. The ratio of the overlap volume Ω of the two nucleons with hard sphere radii $R_u = 1$ fm to the free nucleon volume is given as a function of their distance r

$$\frac{\Omega}{\frac{4\pi}{3} R_u^3} = 1 - \frac{3}{2} \frac{r}{2 R_u} + \frac{1}{2} \frac{r^2}{4 R_u^2} \quad (2.11)$$

At $r = R_u$, this ratio becomes $3/8$, so about half of the time one picks a quark it comes out of the overlap volume. This leads to an estimate for the critical half distance $R_c \approx \frac{1}{2} R_u$ as given by Ref. 12). A strict separation of a ($\chi = 1$) and ($\chi = 0$) region is no longer possible for two extended nucleons which merge. We therefore would expect $f = R_c/R_u = R_c/1\text{fm}$ with $0.5 \leq f \leq 1$. At a distance $r = 2R_u$ ($f = 1$), the bags start fusing. Associated with that distance an attraction between the nucleons develops. At a distance $r = R_u$ ($f = 0.5$), the repulsion sets in with an enhancement of exotic components of the six quark system. It is part of the exploratory nature of our study to get an experimental hint, how big the parameter f is and how the nucleus looks as a function of the confinement size of the nucleon.

In order to analyze the cluster structure of the nucleus we construct a matrix B_{ij}

$$B_{ij} = \begin{pmatrix} 0 & & & \\ & \ddots & & \\ & & \Delta_{ij} & \\ & 0 & & \\ & & & \ddots & \\ & & & & 0 \end{pmatrix} \quad (2.12)$$

where the $\Delta_{ij} = |\vec{x}_i - \vec{x}_j|$ are the distances between the i^{th} and the j^{th} nucleon. From this matrix, a vector $\vec{c}$ is constructed where the i^{th} component of $\vec{c}$ gives the number of the cluster to which the i^{th} nucleon belongs. We define that two nucleons belong to the same cluster when their hard spheres begin to overlap:

$$\Delta_{ij} \leq 2 R_c \quad (2.13)$$

In Figs. 3 an example for this part of the programme is shown. Fig. 3a shows the situation during the cluster analysis. For example, cluster 1 was found consisting of the nucleons 1 and 12, and cluster 5 was found consisting of the nucleons 7, 8 and 11. Later on, nucleon 20 was found to overlap with nucleon 7 and

belonging to cluster 5 (Fig. 3b). On the way scanning all nucleons whether they overlap with the actual nucleon 20 or not, it was found that nucleon 20 overlaps also with nucleon 12 which belongs to the cluster 1. If two clusters are merging, we choose the smallest cluster number for the number of the merged cluster. In this case we are relabelling the nucleons 7, 8, 11 and 20 to belong to cluster numbers 1 (cf. Fig. 3c) and so on. From the vector $\vec{c}$ it is easy to find which nucleon belongs to a certain cluster, what is the greatest distance between nucleons in one cluster and how many nucleons belong to a cluster. This method of labelling is called "single-labelling" technique. The multiple-labelling technique used to study percolation on a two-dimensional lattice of Ref. 13) cannot be used for our problem because here the nucleons are randomly distributed in three dimensions. The volume of each cluster is evaluated using a fine three-dimensional grid and counting the grid points which lie inside a certain cluster. This approximates the clusters by small cubes. The accuracy of the evaluation of the volume is determined by the number of grid points used. The relative error of the cluster volume was about 1%.

3. - NUMERICAL RESULTS OF MONTE CARLO SIMULATION

In order to understand our simulation, it is advantageous to get a naive picture of the nucleus first. In the spirit of percolation theory, we do not include correlations between nucleons or weights for different arrangements according to their interaction energy. A real calculation of the correlated ground state can only be done, when one has an Hamiltonian which is consistent with the ground state properties of nuclear matter: i.e., it has to give binding and nuclear matter saturation. This has not yet been done for the colour dielectric model.

Let us look at simple results of the simulation. In Fig. 4 we display the distance of each nucleon to its nearest neighbour d_{nn} for $R_c = 0.75$ and $A = 56$ (iron). Contrary to the expectation from an ordered arrangement of nucleons on a lattice, the nearest neighbour of each nucleon comes quite close on the average:

$$\langle d_{nn} \rangle = \frac{1}{A} \sum_{i=1}^A \min_{j \neq i} |\vec{x}_i - \vec{x}_j| \approx 0.5 \text{ fm} \quad (3.1)$$

As stated before, this distance may change if the interaction is taken into account. The behaviour of this correlation length d_{nn} with average density is $d \propto (1/\rho)^{3/2}$ different from a lattice model, where one would calculate a weaker

decrease of d_{nn} with the average density ρ , namely $d_{nn} \propto (1/\rho)^{1/3}$. This must be due to the fact that we study finite nuclei. The next question concerns the shape of the formed clusters. There are two possible extremes; the spherical cluster and the stretched linear chain. In the first case, the volume of the cluster $(4\pi/3)R_L^3$ increases linearly with the number of nucleons n in the cluster, or the radius R_L of the cluster is related to the nucleon radius R_c , as

$$R_L \approx n^{1/3} R_c \quad (3.2)$$

For the linear shape the length L of a one-dimensional chain of nucleons increases as

$$L \approx n R_c \quad (3.3)$$

Let us look at the Monte carlo calculation. The simulation gives for the maximal distance d_{max}^n between two nucleons of an average cluster of n nucleons

$$d_{max}^n = \text{av} \max_{i,j \in C_n} |\vec{x}_i - \vec{x}_j| \quad (3.4)$$

a behaviour which is inbetween the two extreme cases $d_{max}^n = R_L$ and $d_{max}^n = L$. In Fig. 5 we show the cluster number n versus the distance d_{max}^n in fm. The drawn curves give the two extreme cases $n = d_{max}^n{}^3 / R_c^3$ and $n = d_{max}^n / R_c$. The curve in between is evaluated with the assumption that the clusters have an ellipsoidal shape with the large principal axis a and the smaller principal axis $b = c$. A fit to the simulated data gives a ratio of axis $a/b = 1.25$ from the behaviour $n = (d_{max}^n / R_c)^3 (b/a)^2$.

There are various plots possible to show the percolation transition. In the three-dimensional Figs. 7 and 8, we give the probabilities p_i to have i -nucleons in a cluster or the number n_i of clusters with i -nucleons as a function of the critical radius R_c for $R_c = 0.25, 0.5, \dots, 1.75$ fm of a nucleon. One sees that in general around $R_c = 1$ fm all cluster sizes are equally represented. This parameter represents the boundary between a nucleonic nucleus and the percolated quark nucleus. Another way to quantify the percolation transition is to calculate the reduced cluster size

$$I'_{av} = \frac{8}{A^2} \left[\sum_{i=1}^{n_{max}} p_i i^2 - p_{n_{max}} n_{max}^2 \right] \quad (3.5)$$

This variable is small if there are only very small clusters ($p_i = 1 = p_{nmax}$ for $i = 1$), then increases and becomes again small when the full percolation has taken place ($p_i = 1$ for $i = nmax$). In between it shows a maximum when the system separates into two almost equal size clusters. In Fig. 9 we show the results for the reduced cluster size l'_{av} for different nuclei. One can see a maximum for the large nuclei at $R_c \approx 1-1.2$ fm. In the light nuclei He^3 , Be^9 , C^{12} the transition is not a very sharp one.

For the rescaling of structure functions in deep inelastic scattering, the effective confinement size is the relevant quantity. The maximal distance d_{max}^n between two nucleons given in Eq. (3.4) can be associated with the inverse momentum cut-off for a gluon from the n -nucleon cluster. The effective confinement size $\langle r_{eff}^2 \rangle$ can then be defined as an average over the d_{max}^n distribution of all clusters weighted with the number of quarks from which the gluon has originated, i.e.,

$$\langle r_{eff}^2 \rangle^{1/2} = (3/5)^{1/2} \cdot \left[\left(\frac{\frac{1}{2} \sum_{d_{nmax}} 3n d_{nmax}^n}{\sum_{d_{nmax}} 3n} \right)^2 + R_c^2 \right]^{1/2} \quad (3.6)$$

The quantity $\langle r_{eff}^2 \rangle^{1/2}$ becomes equal to the r.m.s. radius of the nucleus when quark percolation is complete. In Figs. 9a,b we show the dependence of $\langle r_{eff}^2 \rangle^{1/2}$ on R_c .

Between $0 < R_c < 0.5$ fm, there is almost no difference between $\langle r_{eff}^2 \rangle^{1/2}$ and $\sqrt{3/5} R_c$. Only at $R_c = 0.5$ fm the different densities of the light and heavy nuclei show up. The full nuclear size is reached at very large R_c only. If R_c really depends on Q^2 , as speculated on the basis of the colour dielectric model, this feature should affect the A -dependence of the EMC-effect. How does our approach compare to estimates of the mean confinement size based on two- or three-body correlations? Take $R_c \approx 0.75$ and $A = 56$, then the percentage of nucleon = 3 quark, 2-nucleon = 6 quark, 3-nucleon = 9 quark clusters is $p_3 = 30.3\%$, $p_6 = 15.5\%$, $p_9 = 11.6\%$. They add up to 57% of the total. Therefore, any estimate of the A -dependence of the SLAC data based on a large R_c , taking into account only 3, 6, 9 quark clusters, is questionable. For other R_c 's above $R_c = 0.5$ fm the situation can be seen in Table 1. For He^3 the probabilities for $R_c = 0.5$ fm were previously found to be 89%, 10% and 1%. In the Monte Carlo method we find $p_3 = 86.5\%$, $p_6 = 12\%$ and $p_9 = 1.5\%$ which compares well with the old result. For large A an extrapolation¹⁴⁾ rescaling the three-body distribution of He^3 does not hold, because of sizeable percentage of large clusters (Table 2).

4. - COMPARISON WITH EXPERIMENT

In the previous section, we have presented a simplified theoretical model for quark percolation in nuclei. What are the experimental consequences produced by such a picture? Using the rescaling model^{15),16)} we can fit the effective confinement size R_c which reproduces the A-dependence of the SLAC data.

In our original papers^{14),15)} we estimated the Q^2 of the SLAC data as too low to have full colour conductivity. The percolation model presented here allows to interpolate between isolated nucleons and fully deconfined quarks. With the rescaling formula^{15),17)}, we obtain

$$\frac{F_2(x, Q^2, A)}{F_2(x, Q^2, d)} = 1 + \frac{\partial \ln F_2(x, Q^2, p)}{\partial \ln Q^2} \cdot \frac{\bar{\alpha}_s}{\alpha_s(Q^2)} \cdot \ln \left(\frac{r_{\text{eff}}^2(A)}{r_{\text{eff}}^2(d)} \right) \quad (4.1)$$

where $\partial \ln F_2(x, Q^2, d) / \partial \ln Q^2 = -0.24^{18)} \pm 0.04$ at $Q^2 = 10 \text{ GeV}^2$ and $\bar{\alpha}_s / \alpha_s(Q^2) = 0.8$ is chosen. Note that phenomenology alone does not allow an independent determination of the effective QCD-coupling constant $\bar{\alpha}_s$ and of the ratio of radii in Eq. (4.1). The error on the scaling violation parameter $\partial \ln F_2 / \partial \ln Q^2$ is however so large that small changes in $\bar{\alpha}_s$ do not matter. We can then calculate the ratio of structure functions with the effective confinement scale $\langle r_{\text{eff}}^2 \rangle$ for each nucleus. For the deuteron we use the Reid soft core¹⁹⁾ wave function. For heavier nuclei $A > 2$ the effective confinement size $\langle r_{\text{eff}}^2 \rangle^{1/2}$ is calculated in the simulation as a function of the critical radius R_c of the nucleon. A best fit to the SLAC data is achieved for $R_c = 0.63 \text{ fm}$. The A-dependence of the ratio $F_2(x = 0.62, A) / F_2(x = 0.62, d)$ is plotted in Fig. 10. The colour percolation radii for the different nuclei corresponding to these data are:

	d	Be ⁹	C ¹²	Fe ⁵⁶	Au ¹⁹⁷
$\langle r_{\text{eff}}^2 \rangle^{1/2}$	0.52 fm	0.63 fm	0.67 fm	0.76 fm	0.80 fm

Recall that $\langle r_{\text{eff}}^2 \rangle > \sqrt{3/5} R_c$, so that the effective r.m.s. radii may be smaller than the hard sphere radii. The percolation model gives a stronger variation than the colour dielectric model which determines the effective nucleon radius for different nuclei at $Q^2 = 10 \text{ GeV}^2$ in Fig. 2.

	d	Be ⁹	C ¹²	Fe ⁵⁶	Au ¹⁹⁷
$\langle r_N^2 \rangle^{1/2}$	0.65	0.66	0.70	0.75	0.81

In this model each nucleon is isolated in its own Wigner-Seitz cell and cannot communicate with other nucleons. The possibility of overlapping colour dielectric zones is missing. What can we learn from the value of $R_c = 0.63\text{fm}$. We can compare it with the equivalent hard sphere radius of the nucleon and obtain for the ratio $f = R_c/R_u = 0.63$ with $R_u \sim 1.0\text{fm}$ showing that nucleons have to come rather close before losing their identity. This value of f is near the estimate of Ref. 12) based on the volume argument. R_c of the present analysis is somewhat higher than the large x -data analysis of He^3 at smaller Q^2 . The latter analysis pertains to lower Q^2 and $x > 1$. This region may be more difficult or there may be a Q^2 -dependence of R_c . At $R_c = 0.63$ the probabilities for n -quark clusters in Fe^{56} are $p_3 = 50\%$, $p_6 = 20\%$, $p_9 = 10\%$, $p_{\text{rest}} = 20\%$. These probabilities shed some doubts on an analysis of the SLAC data using only two- or three-body correlations as it has been done in Ref. 20). Of course, there is some arbitrariness how to define a new confinement scale in the presence of clustering. But even with the definition of Ref. 15), our results propose that an extended cluster calculation is necessary. It is interesting to extend the analysis to the EMC-data at the same time. Since the colour dielectric model makes R_c depend on Q^2 , we can investigate the rescaling factor $\xi = (\langle r_{\text{eff}}^2 \rangle(\text{Fe}) / \langle r_{\text{eff}}^2 \rangle(\text{d}))^{\bar{\alpha}_s/\alpha_s}$ as a function of Q^2 . For this purpose we attach a second scale to the $\langle r_{\text{eff}}^2 \rangle^{1/2}$ versus R_c plot (Fig. 9b) by identifying $R_c(Q^2)$ with

$$R_c(Q^2) = (5/3)^{1/2} \cdot \langle r_N^2 \rangle^{1/2} \cdot f \quad (4.2)$$

where $\langle r_N^2 \rangle^{1/2}$ is the r.m.s. radius of the valence quark distribution of Fe given in Fig. 2 and the factor f (cf. Section 3) gives the dynamical reduction obtained empirically. Finally $\sqrt{5/3}$ converts the r.m.s. radius to the equivalent hard sphere radius. In principle, we could have identified $R_c(Q^2)$ with a different Q^2 -scale for each nucleus. But we think the input of the colour dielectric model is too crude to justify such a separation. The resulting Q^2 -dependence of the rescaling factors is given in Table 3.

The rescaling factors vary between 1.71 and 8.7 when we change Q^2 from 10 GeV^2 to 1000 GeV^2 . In Fig. 11 we compare the values of the rescaling parameter extracted in Ref. 21) from iron/deuterium data as a function of Q^2 with our theoretical values. Both scales are logarithmic. In the same plot we give a calculation with fixed $R_c = 0.63 \text{ fm}$. The experimental data are ambiguous. One

parameter extracted in Ref. 21) from iron/deuterium data as a function of Q^2 with our theoretical values. Both scales are logarithmic. In the same plot we give a calculation with fixed $R_c = 0.63$ fm. The experimental data are ambiguous. One has to wait new muon collaboration²²⁾ or FNAL data to determine $\xi(Q^2)$ more accurately. Note the EMC data points can also be reproduced in the colour-conductivity picture with other radii. One may imagine a situation where the transition between the nucleonic regime and colour conductivity is a very fast one. The signature of such a transition would be a variation of $\xi(Q^2)$ which peaks in-between, because the dense nuclei percolate first and the deuteron last. This would make the Q^2 -dependent ratio of confinement radii maximal at intermediate values of Q^2 . At the existing Q^2 (SLAC and EMC) we would argue now that the situation is far from asymptotic still. The deuteron especially does not show any large confinement zone, yet we should add that the mean field calculation used for Fig. 1 is not very reliable for a small nucleus like deuterium. Even at $Q^2 = 1000$ GeV², the effective percolation radius in iron is only $\langle r_{\text{eff}}^2 \rangle^{1/2}(\text{Fe}) = 1.21\text{fm}$ compared to $\langle r_{\text{eff}}^2 \rangle^{1/2}(\text{Fe}) = 0.7$ fm at $Q^2 = 10$ GeV². The transition to colour conductivity in the present classical model is very slow. It may be that a dynamical picture of the nucleus in the infinite momentum frame gives a different result. When one includes in the rescaling formula Eq. (4.1) the Q^2 dependence of $d\ln(F_2(x, Q^2))/d\ln Q^2$ then the product of $(d\ln F_2/d\ln Q^2) \ln \xi$ does not vary (see Table 2); i.e., the Q^2 variation of the EMC-effect is very weak. This seems to be in agreement with existing experiments. Since the colour dielectric model gives a reasonable behaviour of the confinement radius with Q^2 , it would be worthwhile to use the model directly to calculate the nucleon structure function in the nucleus. Note the present theory only predicts the change of the structure function with confinement size. A direct calculation of the structure function of the bound nucleon is currently investigated.

5. - CONCLUSION

Using Monte Carlo simulations of finite nuclei we have calculated the effective colour confinement size in He³, Be⁹, C¹², Fe⁵⁶ and Au¹⁹⁷ as a function of the critical distance between nucleons where clustering starts. Analyzing the SLAC data with the so-determined rescaling factors, we find as critical clustering distance 1.26fm. A Q^2 dependence of quark confinement in the nucleus suggested by the colour dielectric model produces a slow transition from isolated clusters to a colour conducting nucleus at very large Q^2 . We give quantitative predictions for the resulting Q^2 -dependence of the EMC-effect which can be tested at FNAL and CERN.

We would like to thank Prof. O. Nachtmann for a critical reading of the manuscript.

R_c	P_3	P_6	P_9	P_{rest}
0.25	0.947	0.051	0.001	0.001
0.50	0.665	0.196	0.069	0.070
0.75	0.303	0.155	0.116	0.426
1.00	0.118	0.068	0.046	0.768
1.25	0.046	0.021	0.001	0.932

Table 1: Probabilities p_n for n nucleon clusters as a function of R_c

Q^2 [GeV ²]	$\frac{\langle r_{eff}^2 \rangle(Fe)}{\langle r_{eff}^2 \rangle(d)}$	$\xi = \left(\frac{\langle r_{eff}^2 \rangle(Fe)}{\langle r_{eff}^2 \rangle(d)} \right) \bar{\alpha}_s / \alpha_s$	$\frac{\partial \ln F_2^{exp}}{\partial \ln Q^2} \Big _{x=0.55}$
5	1.71	1.46	
10	1.96	1.71	-0.20 ± 0.04
20	2.14	2.04	
50	2.35	2.6	
100	2.58	3.23	-0.10 ± 0.05
500	3.34	6.10	
1000	3.47	8.7	?

Table 2: Rescaling factor $\xi = \left(\langle r_{eff}^2 \rangle(Fe) / \langle r_{eff}^2 \rangle(d) \right) \bar{\alpha}_s / \alpha_s$ as a function of Q^2 . For $R_c = 0.63$ independent of Q^2 the ratio $\langle r_{eff}^2 \rangle(Fe) / \langle r_{eff}^2 \rangle(d) = 2.13$. The scaling violation parameters are taken from Ref. 18).

	Meaning	Q^2 variation
$\langle r_A^2 \rangle^{\frac{1}{2}}$	r.m.s. radius of the nucleus A	No
r	distance between two nucleons	No
R_c	critical hard sphere radius of nucleon ($R_c = fR_u$); $2R_c$ is the critical distance between nucleons for clustering	No
R_u	experimental hard sphere radius of nucleon $R_u = 1.0\text{fm}$	No
$\langle r_{\text{eff}}^2 \rangle^{\frac{1}{2}}$	effective r.m.s. radius of confinement cf. Figs. 9a,b	Yes
$R_c(Q^2)$	critical hard sphere radius of nucleon as a function of Q^2 , $R_c(Q^2) = \sqrt{5/3} f \langle r_N^2 \rangle^{\frac{1}{2}}$ cf. Fig. 9b	Yes
$\langle r_N^2 \rangle^{\frac{1}{2}}$	r.m.s. radius of valence quarks of bound nucleons (cf. Fig. 2)	Yes

Table 3: Definition of radii used

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FIGURE CAPTIONS

- Fig. 1 : Effective confinement of quarks in a nucleus with $A = 2$ nucleons as a function of Q^2 . $(Q_1^2 > Q_0^2)$ χ_N is the nuclear dielectric field.
- Fig. 2 : r.m.s. radius of the valence quark distribution of bound nucleons in different nuclei (^2d , ^3He , ...) as a function of Q^2 .
- Fig. 3 : Cluster analysis done in the program with the help of the vector $\vec{c} = (c_1, \dots, c_n)$. The index i of c_i gives the nucleon number and the value of c_i corresponds to the cluster number.
- Fig. 4 : Distance of each nucleon in Fe^{56} to its nearest neighbour d^{nn} . This curve shows how strongly nucleons are overlapping in Fe^{56} .
- Fig. 5 : Cluster number n versus maximal distance of two nucleons in the n -nucleon cluster.
- Fig. 6 : Three-dimensional plot for C^{12} of the probability p_i to have i -nucleons in a cluster as a function of R_c the critical radius of the nucleon. R_c varies in steps of 0.25fm. i changes from 1, 2, ..., 12.
- Fig. 7 : Three-dimensional plot for Fe^{56} of the number of nucleons n_i in a cluster with i -nucleons for 10 000 random assignments of nucleon positions. R_c varies in steps of 0.25fm. i changes from 1 to 56.
- Fig. 8 : The reduced cluster size I'_{av} [Eq. (3.5)] is shown as a function of the critical radius R_c of the nucleon. Note for sufficiently large nuclei the percolation transition occurs at $R_c \approx 1-1.2\text{fm}$.
- Fig. 9a: Effective r.m.s. confinement size $\langle r_{\text{eff}}^2 \rangle^{\frac{1}{2}}$ as a function of the critical radius R_c of the nucleon for different nuclei (D , ^3He , ^9Be , ^{12}C , ^{56}Fe , ^{197}Au).
- Fig. 9b: Same figure as 9a on a larger scale. Also is shown the variation of $R_c(Q^2)$ as a function of Q^2 taken from Eq. (4.2) for Fe^{56} .
- Fig. 10: A-dependence of the ratio $F_2(x = 0.62, A)/F_2(x = 0.62, \text{d})$ as a function of $\log^{10} A$ for ^9Be , ^{12}C , ^{56}Fe , ^{197}Au . The squares are the experimental results and the dots the result of the calculation.

Fig. 11: Rescaling factor $\xi(Q^2) = (\langle r_{\text{eff}}^2(\text{Fe}, Q^2) \rangle / \langle r_{\text{eff}}^2(\text{d}, Q^2) \rangle)^{\bar{\alpha}_s / \alpha_s(Q^2)}$ as a function of Q^2 for $R_c = 0.63$ fm independent of R_c (dashed curve) and for $R_c(Q^2)$ of Fig. 9b (full curve).

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9a

Fig. 9b

Fig. 10

Fig. 11