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New Heterotic String Theories in Uncompactified Dimensions < 10

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New heterotic string theories in uncompactified dimensions < 10

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Abstract

It is shown that infinitely many heterotic string theories exist in uncompactified dimensions less than 10, that are one-loop finite (for massless external legs). Tachyons are removed by compactifying into tori $(10-d)$ and $(26-d)$ dimensions of the right moving superstring and left moving bosonic string sectors respectively. The condition for modular invariance is shown to be equivalent to self duality condition on even Lorentzian lattices with $(10-d)$ and $(26-d)$ timelike and spacelike directions respectively. The construction results in a $(10-d)(26-d)$ parameter family of one-loop finite string theories. Zero mass sector of these theories for $d = 4$ and 6 correspond to $N = 4$ and 2 supergravity coupled to super Yang-Mills with many possible groups, some of which cannot be obtained by compactifying $d = 10$ heterotic string theory.

Superstring theories⁽¹⁾ and in particular the heterotic string theory⁽²⁾ has attracted a lot of attention recently. The latter has two important features - namely it is finite at 1-loop level (and possibly to all orders) and the zero mass sector describes anomaly free chiral field theory⁽³⁾. Due to these reasons they have become serious candidates for describing nature, and many attempts have been made to obtain $d = 4$ theories by compactifying the extra dimensions into Calabi-Yau manifolds⁽⁴⁾, starting either from the $d = 10$ field theory (ie massless sector) or by directly defining the string theory on these compact manifolds. The latter is possible due to some remarkable features of the Calabi-Yau manifolds, such as vanishing of the β function in the corresponding 2-dim field theory and of the one loop correction to the central charge in the Virasoro algebra⁽⁵⁾.

More recently attempt has been made to compactify via orbifolds obtained from 6-torus by the action of some appropriate discrete group, and it appears that most of the features of Calabi-Yau manifolds could be obtained this way⁽⁶⁾. This is, of course, very attractive, as orbifolds are simpler to handle than Calabi-Yau manifolds.

All the approaches so far, however, have involved two steps - first 26 dimensional left moving sector of the heterotic string is compactified to 10 dimensions and then the 10 dimensions of left and right moving sector are compactified to lower dimensions. As the scales of these two steps are of the same order, one can envisage an alternative approach of directly compactifying 26-d and 10-d dimensions of left and right moving sectors into tori.

In this letter, I shall show that such construction yields $(10-d)(26-d)$ parameter family of tachyon free, one loop finite string theories. This, of course, does not preclude the existence of other consistent theories on more complicated manifolds. The massless sector of these theories in $d = 6$ and 4 correspond respectively to $N = 2$ and 4 supergravity coupled to super Yang-Mills with many possible groups (of rank $26-d$). Many of these groups cannot be obtained by a simple torus compactification of 10-dimensional heterotic string theory, and thus may provide new possibilities for phenomenology. For this, however, one will have to break $N = 4$ (in $d = 4$) down to $N = 1$ by invoking some discrete symmetries, such as in the orbifold approach⁽⁶⁾, but I shall not dwell upon this here.

Crucial ingredient in the construction is the generalisation of the

condition for modular invariance. Let p and q be the number of compactified dimensions in the left and right moving sectors respectively

$$\begin{aligned} X^A &= q^A + 2L^A u + \sum_{n \neq 0} \frac{i}{2n} \alpha_n^A e^{-i2nu} \quad A = 26-p+1, \dots, 26 \\ \tilde{X}^I &= \tilde{q}^I - 2\tilde{L}^I v + \sum_{n \neq 0} \frac{i}{2n} \tilde{\alpha}_n^I e^{-i2nv} \quad I = 10-q+1, \dots, 10 \end{aligned} \quad (1)$$

where $u = \tau + \sigma$, $v = \tau - \sigma$, and $(L^A, \tilde{L}^I) \in \Gamma$ - a lattice in $p + q$ dimensional vector space. Periodicity condition of the closed string at $\sigma = 0$ and π , implies that $(X^A, \tilde{X}^I)$ describe a $(p+q)$ - torus defined by the lattice $2\pi\Gamma$. Note that we take here a general lattice that mixes the left and right moving sectors, but this does not pose any problem. Indeed in the orbifold construction⁽⁶⁾, action of discrete symmetries on 6-torus and that on the left moving 16-torus defining the gauge group are coupled, similarly in ref (7), the group constraint necessary for modular invariance mixes the two sectors.

The space of maps: $S^1 \rightarrow T^{p+q}$ can be thought of as the loop space $\mathcal{M}^{p+q}$ and has an abelian group structure under pointwise multiplication of maps. Let us symbolically denote a group element as $e^{i(X, \tilde{X})}$ with the property

$$e^{i(X, \tilde{X})} e^{i(X', \tilde{X}')} = e^{i(X+X', \tilde{X}+\tilde{X}')}$$

and

$$e^{2\pi i(L, \tilde{L})} = e^{i(0,0)} = 1 \quad (2)$$

At the first quantized level one is actually interested only in the unitary, positive energy ray representation of eq (2) on a Hilbert space. This is achieved by making use of the 2-cocycle (see Segal⁽⁸⁾ for notation)

$$C((X, \tilde{X}), (X', \tilde{X}')) = e^{-i\{S(X, X') + S(\tilde{X}, \tilde{X}')\}} (-1)^{\sigma((L, \tilde{L}), (L', \tilde{L}'))} \quad (3)$$

where

$$S(X, X') = \frac{1}{4\pi} \int_0^\pi \left(\frac{d}{du} X^A \right) X^{A'} du + \frac{1}{4\pi} (X^A(\pi) - X^A(0)) X^{A'}(0)$$

It is clear that C defines a 2-cocycle on Γ provided

$$L^A_L L^{A'} - \tilde{L}^I_L \tilde{L}^{I'} = \text{integers}, \quad (L, \tilde{L}), (L', \tilde{L}') \in \Gamma$$

In other words, Γ , regarded as a "Lorentzian" lattice with signature $(\underbrace{++ \dots +}_p \underbrace{- - \dots -}_q)$ is integral. In fact I will restrict Γ to be also even, justification for this will be given following eq (6) below. Note that S just gives the usual classical Poisson Bracket structure for oscillators. σ is a bilinear form, needed to get the correct commutation relations for charged generators^(2,9)

$$\sigma: \Gamma \times \Gamma \rightarrow \mathbb{Z}/2$$

$$\sigma((L, \tilde{L}), (L', \tilde{L}')) = \frac{1}{2} (L^2 - \tilde{L}^2) \bmod 2 \quad (3')$$

The first quantized operators then satisfy the relation

$$U(e^{i(X, \tilde{X})}) \cdot U(e^{i(X', \tilde{X}')}) = c((X, \tilde{X}), (X', \tilde{X}')) U(e^{i(X+X', \tilde{X}+\tilde{X}')}) \quad (4)$$

Now one constructs a Fock space $H_{(0,0)}$ on which the oscillators α_n and $\tilde{\alpha}_n$ ($n \neq 0$) act in the usual way. Next one defines infinite copies of this Fock space by specifying the action of the constant operators $U(e^{i(q, \tilde{q})})$ as follows

$$U(e^{i(q, \tilde{q})})|_{H_{(k, \tilde{k})}} = e^{i(q \cdot k - \tilde{q} \cdot \tilde{k})}$$

Clearly this makes sense only if $(k, \tilde{k}) \in \Gamma^*$ where Γ^* is the dual of Γ . By making use of eqns (3-4) we see that the winding operators

$$U(e^{i(2Lu, -2\tilde{L}v)}) H_{(k, \tilde{k})} \rightarrow H_{(k+L, \tilde{k}+\tilde{L})} \quad (5)$$

Thus it is clear that if Z is the order of Γ^* / Γ , then there are Z irreducible representations of the algebra of string operators.

Generator of rigid translation of σ parameter must vanish, if we require that there be no preferred point $\sigma=0$ on the closed string. Since a rigid translation in $\sigma(\text{or } \tau)$ does not mix annihilation and creation operators α_n

and α_{-n} ($n > 0$), they can be determined uniquely[#], modulo constants. These constants are obtained as usual by requiring the closure of Lorentz algebra in the physical space-time. The resulting constraint is

$$\frac{1}{2} (k^2 - \tilde{k}^2) = \tilde{N} - N + 1 = \text{integers} \quad (6)$$

Here the constant 1 appears for the 26 dimensional left moving bosonic string, whereas for 10-dimensional right moving superstring this constant is zero. N and $\tilde{N}$ are the oscillator numbers, and hence non-negative integers, in the compactified as well as uncompactified and fermionic degrees of freedom. Eq (6) implies that only the even subset of Γ^* is physical. In fact from eq (5) it is clear that only winding operators corresponding to the even subset of Γ interpolate between physical states, thus we do not lose any generality by restricting Γ to be even.

The generator for τ translation gives the mass formula

$$\frac{1}{4} m^2 = \frac{1}{2} k^2 + \frac{1}{2} \tilde{k}^2 + N + \tilde{N} - 1 \quad (7)$$

Eqn (6) and (7) imply that $m^2 \geq 0$, signifying the absence of Tachyons.

Finally we come to the question of interactions. As mentioned above there are Z inequivalent representations of the algebra of string operators and at the level of free strings it is consistent to pick any one of them, that is even, as the state space. Now I will show that modular invariance and hence one loop finiteness requires that $Z = 1$ (ie $\Gamma^* = \Gamma$). Interaction vertices describing the emission of massless particles are defined as usual. From eq (6) and (7), it is clear that massless states require

$$(\tilde{k}^2 = \tilde{N} = 0) \text{ and either } (k^2 = 2, N = 0) \text{ or } (k^2 = 0, N = 1) \quad (8)$$

Consider, for example, a one-loop diagram with 4 external charged massless states (ie with $k^2 = 2, \tilde{k} = 0, N = \tilde{N} = 0$) and $(k, \tilde{k}) \in \Gamma$). One can go through the analysis of ref (2), where this diagram has been calculated for $d = 10$. In the present case, as $d < 10$, the difference lies in the fact that $(10-d)$ of the additional left moving (besides the 16 in ref (2)) as well as $(10-d)$ of the right moving bosonic momenta are discrete, instead of

[#] Even though C is not invariant under diffeomorphisms group, the latter acts on the associated central extension of maps $S^1 \rightarrow T^{p+q}$ (see ref (8)).

being continuous. This means that we can obtain our results by dividing by a factor $(\frac{-2}{\text{Im}\tau})^{\frac{10-d}{2}}$ and summing over L and $\tilde{L}$. Since the external momenta $\tilde{K}_1^A = 0$, this is particularly simple. The result is (see ref (2) for notation)

$$A(1,2,3,4) = \bar{\epsilon} K \int \prod_{i=1}^3 d^2 v_i d^2 \tau \left(\frac{2}{\text{Im}\tau} \right)^5 \prod_{1 \leq i < j \leq 4} \chi(e^{2\pi i v_{ji}}, e^{2\pi i \tau})^{\frac{1}{2} p_i \cdot p_j} \\ e^{2\pi i \bar{\tau}} f(e^{-i2\pi \bar{\tau}})^{-24} \prod_{1 \leq i < j \leq 4} \left[2\pi i e^{-i\pi \bar{v}_{ji}/\bar{\tau}} \frac{\theta_1(\bar{v}_{ji}|\bar{\tau})}{\theta_1'(0|\bar{\tau})} \right]^{K_i \cdot K_j} \\ \left\{ \left(-\frac{\text{Im}\tau}{2} \right)^{\frac{10-d}{2}} \sum_{(L, \tilde{L}) \in \Gamma} \exp \left[-\pi i \bar{\tau} (L - \sum_{i=1}^4 K_i \bar{v}_i / \bar{\tau})^2 + i\pi \tau \tilde{L}^2 \right] \right\} \quad (9)$$

where $v_{ji} = v_j - v_i$, $v_4 = \tau$, f and χ are as defined in ref (2) and p_i and K_i are the momenta in the space-time and compactified directions of the i^{th} external particle. The curly bracket in the above expression contains the new terms - the factor $(\frac{-\text{Im}\tau}{2})^{\frac{10-d}{2}}$ and $\exp i\pi \tau \tilde{L}^2$. Note that the dependence on v_1, v_2 and v_3 is unchanged, hence the periodicity condition

$$v_i \rightarrow v_i + 1, \quad v_i \rightarrow v_i + \tau \quad i = 1, 2, 3$$

remains unchanged. Under the transformation $\tau \rightarrow \tau + 1$, using the fact that

$$\sum_{j>i} \bar{v}_{ji}^2 K_i \cdot K_j = - \left(\sum_i K_i \bar{v}_i \right)^2$$

which follows from the conservation law $\sum K_i = 0$, and the fact that the Jacobi functions $\theta_1(\bar{v}|\bar{\tau})$ are invariant we see that eq (9) gets multiplied by a factor $e^{-i\pi(L^2 - \tilde{L}^2)}$. As Γ is even this factor = 1. Finally to check the invariance of (9) under $\tau \rightarrow -1/\tau$, we use the following properties (see eq

(B.7) and (B.4) of ref (2))

$$e^{2\pi i \bar{\tau}} f(e^{-i2\pi \bar{\tau}})^{-24} \rightarrow e^{-2\pi i / \bar{\tau}} f(e^{i2\pi / \bar{\tau}})^{-24} (-\bar{\tau})^{+12}$$

$$\frac{\theta_1(\bar{v}_{ji} | \bar{\tau})}{\theta_1'(0 | \bar{\tau})} = \frac{\theta_1(-\bar{v}_{ji}/\bar{\tau} | -1/\bar{\tau})}{\theta_1'(0 | -1/\bar{\tau})} (-\bar{\tau}) e^{i\pi \bar{v}_{ji}^2 / \bar{\tau}}$$

and the Poisson summation formula

$$\begin{aligned} & \sum_{(L, \tilde{L}) \in \Gamma} \exp(-i\pi \bar{\tau} (L - \sum_i K_i \bar{v}_i / \bar{\tau})^2 + i\pi \tilde{L}^2) \\ &= \frac{1}{\sqrt{Z}} (-i\bar{\tau})^{-\frac{26-d}{2}} (i\tau)^{-\frac{10-d}{2}} \sum_{(L, \tilde{L}) \in \Gamma^*} \exp\left(\frac{i\pi}{\bar{\tau}} (L - \sum_i K_i \bar{v}_i)^2 - \frac{i\pi}{\bar{\tau}} \tilde{L}^2\right) \\ & \quad \exp - i\pi (\sum_i K_i \bar{v}_i)^2 / \bar{\tau} \end{aligned}$$

Substituting these identities in eq(9) and using the fact that the first row of (9) is invariant by itself, as it represents (d=10) superstring II amplitude, we conclude that (9) is invariant under the transformation

$$\tau \rightarrow -\frac{1}{\tau}, \quad v_i \rightarrow -\frac{v_i}{\tau}, \quad i = 1, 2, 3 \text{ if and only if}$$

$$\Gamma = \Gamma^*$$

This of course means that $Z = 1$. To sum up, we see that the one-loop amplitudes are modular invariant provided the lattice $\Gamma = \{(L, \tilde{L})\}$ is an even, self dual "Lorentzian" lattice with signature $\frac{++\dots+}{26-d} \frac{---\dots-}{10-d}$.

Modular invariance allows one to restrict the domain of τ integration to $-\frac{1}{2} \leq \text{Re } \tau \leq \frac{1}{2}$, $\text{Im } \tau > 0$ and $|\tau| \geq 1$. The only divergence that can appear now is of the form $(v_{ji})^{-2}$ and $(v_{ji})^{-1}$ representing tachyon and gauge boson intermediate states, but they vanish as argued in ref (2), due to the integration of the phases of v_i which ensure the constraint eq (6). Thus the theory is 1-loop finite.

What is known about the existence of such even, self dual lattices $\Gamma_{(p,q)}$

(here p is the number of positive signature and q that of negative signature). There is a general theorem⁽¹⁰⁾ which states that they exist if and only if $p - q = 8n$ for some integer n . This is as well, as in our case $p - q = 16$. Moreover all such lattices for a given q ($q > 0$) and n ($n \geq 0$) are isomorphic to the lattice $n\Gamma_8 \oplus qP_2$, where Γ_8 is the root lattice of E_8 and P_2 is a two dimensional lattice with signature $(+-)$ and defined by the metric $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ on the simple basis of P_2 . In other words, any even, self dual lattice $\Gamma_{(q+8n,q)}$ can be obtained from $n\Gamma_8 \oplus qP_2$ by means of an $SO(8n+q,q)$ transformation. Since the mass formula (7) is only invariant under the subgroups $SO(8n+q)$ and $SO(q)$, the distinct string theories are characterised by the points of the coset manifold

$$M_d = SO(26-d, 10-d)/SO(26-d) \times SO(10-d)$$

(recall that for us $q = 10-d$ and $n = 2$) which has $(26-d)(10-d)$ parameters.

Before proceeding further, I shall give some examples of such lattices.

i) Setting $d = 10$, we get back the Euclidean self duality condition of 10-dim. heterotic string theory which gives rise to $E_8 \times E_8$ or $\text{spin}(32)/Z_2$. These are the unique choices.

ii) Setting $p = q$ (ie $n = 0$, this would not correspond to heterotic strings), $k, \tilde{k} \in$ weight lattice of a simply laced group G with the constraint $k - \tilde{k} \in$ root lattice of G , we again get a self dual lattice $\Gamma_{q,q}$. This was the construction given in ref (7).

iii) Lattice generated by the following vectors

$(k,0)$ where $k \in$ root lattice of $SO(2q+32)$

$(0,\tilde{k})$ $\tilde{k} \in$ root lattice of $SO(2q)$

$(W_s, \tilde{W}_s)$ W_s and $\tilde{W}_s$ are weights of spinor representations of the two groups.

$(W_v, \tilde{W}_v)$ W_v and $\tilde{W}_v$ are the weights of the vector representations.

iv) $((k_1, k_2), 0)$ where $k_1 \in$ root lattice of E_6

$k_2 \in$ root lattice of E_7

$$((0,0),\tilde{k}) \quad \tilde{k} \in \text{root lattice of } SU(6)$$

$$(w_{27},0), \tilde{w}_{15}) \quad \text{and} \quad ((0,w_{56}), \tilde{w}_{20})$$

$$v) \quad ((k_1, k_2), (\tilde{k}_1, \tilde{k}_2)) \quad k_1, k_2 \in \text{root lattice of } E_6$$

$$\tilde{k}_1, \tilde{k}_2 \in \text{root lattice of } SU(3)$$

$$((w_{27},0), (\tilde{w}_3, \tilde{w}_3)) , \quad \text{and} \quad ((0, w_{27}), (\tilde{w}_3, \tilde{w}_3)).$$

To the above one can add any number of E_8 as well as P_2 lattices. Note that P_2 lattice corresponds to the example (ii) above with $G = SU(2)$.

All of the above examples (for $q > 0$)[#] are isomorphic to $n\Gamma_8 \oplus qP_2$ as a "Lorentzian" lattice, but they give different mass formula (7), resulting in different string theories. In fact as stated above there are $(8n+q)q$, ($n=2$ for our case) parameter family of such theories. If, however, one were to concentrate on the zero mass sector of the theory (ie field theory limit in $(10-q)$ space-time dimensions), then from the condition (8) we get the following massless states

$$\alpha_{-1,L}^i |0_L, j_R\rangle, \quad \alpha_{-1,L}^i |0_L, a_R\rangle, \quad \alpha_{-1,L}^i |0_L, I_R\rangle, \quad i, j = 3, 4 \dots d$$

$$\alpha_{-1,L}^A |0_L, j_R\rangle, \quad \alpha_{-1,L}^A |0_L, a_R\rangle, \quad \alpha_{-1,L}^A |0_L, I_R\rangle,$$

$$|0_L, j_R\rangle(k,0), \quad |0_L, a_R\rangle(k,0), \quad |0_L, I_R\rangle(k,0)$$

where $(k,0) \in \Gamma$ and $k^2 = 2$.

It is clear that for $d = 4$ and 6 , for example, zero mass sector corresponds to respectively $N = 4$ and 2 supergravity coupled to super Yang-Mills theory. The Yang-Mills group G is a product of simply laced groups of total rank

[#]for $q = 0$, there are other self dual lattices Γ_{8n} besides $n\Gamma_8$.

26-d (as there are precisely (26-d) U(1) gauge fields, $\alpha_{-1,L}^A |0_L, i_R\rangle$), but the precise non-abelian structure depends on the set of points $(k, \tilde{k}) \in \Gamma$ such that $k^2 = 2$ and $\tilde{k} = 0$. Unfortunately I do not know of a classification of all possible Yang Mills groups that can be obtained this way, but clearly there are a large number of such possibilities. The generic point in the coset manifold M_d would correspond to no points with $k^2 = 2$ and $\tilde{k} = 0$, this would result in a $U(1)^{26-d}$ gauge field theory. On the other hand combining the examples given above, we can also have the following possibilities.

i) $E_8 \times E_8 \times G_{10-d}$

where G_{10-d} is a simply laced group of rank (10-d).

ii) for $d = 6$, $E_8 \times E_6 \times E_6$

iii) for $d = 5$ $E_8 \times E_7 \times E_6$ or $E_8 \times E_6^2 \times SU(2)$

iv) for $d = 4$ $E_8 \times E_6 \times E_7 \times SU(2)$, $E_8 \times E_6^2 \times G$

where $G = SO(4)$ or $SU(3)$.

v) $SO(52-2d)$

or by rotating some of the axes in the lattice, one may break many of these groups down to some simply laced subgroup and product of several U(1)'s, but always maintaining the total rank = 26-d. Note that these groups (for example $SO(52-2d)$) cannot be obtained by compactifying 10-dim heterotic string theory or the corresponding $N = 1$, $E_8 \times E_8$ or spin (32)/ Z_2 field theories. The examples given above are actually isolated points on M_d , as any infinitesimal transformation by an element of M_d would destroy the zero mass condition (8). In general, if the resulting group is a product of non-abelian groups and $U(1)^n$, then the resulting massless sector is unchanged by infinitesimal transformations of $SO(n, 10-d)/SO(n) \times SO(10-d)$ and therefore defines locally a $(10-d)n$ dimensional surface in M_d . From the low energy field theory point of view all the points in this $(10-d)n$ dimensional surface would describe the same physics, but the masses of "regulator fields" ie. the massive sectors needed to render the theory finite (assuming that the results reported here continue to hold for higher loops) form a $(10-d)n$ dimensional space.

To sum up, we see that there exist infinite number of tachyon free and

one-loop finite string theories in space-time dimensions $d < 10$ and they lie on a $(26-d)(10-d)$ dimensional coset manifold M_d . If one is interested in only the zero mass sector, this manifold M_d splits into finite number (unknown at present) of submanifolds of $(10-d)n$ dimensions where n is the number of $U(1)$ groups in the zero mass-sector. Classification of all allowed Yang-Mills groups is an important open question. If it turns out that higher loops are also finite (or this criterion may select a subset of M_d) then one would conclude that each point of M_d (or of the selected subset of M_d) provides a stable vacuum. The question would then be - which point (if any of the above) does nature select and why.

Direct torus compactification to 4 dimensions, as given here, leads unfortunately to $N = 4$ theories, which are, perhaps, not very appealing phenomenologically. To break supersymmetry completely or to $N = 1$, one can perhaps follow the methods of ref (6), by invoking some discrete symmetry on the right sector and couple it to the one in the left sector. However, this may interfere with the condition of modular invariance and needs a separate investigation. Besides one has to also make sure that the cocycle C in eq (3) is well defined on the quotient of Γ by this discrete symmetry. Another possible scenario would be to start from one of the above $N = 2$ theories in $d = 6$, and compactify down to $M_4 \times S^2$ via monopoles⁽¹¹⁾. If the latter compactification scale can be pushed much below the string scale, zero mass sector could be a good approximation for this.

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