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COSMOLOGICAL DIFFICULTIES FOR INTERMEDIATE SCALES IN SUPERSTRING MODELS

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A B S T R A C T

We show that primordial nucleosynthesis puts a severe limit $M_I \lesssim 10^9$ GeV on the intermediate mass scale in superstring theories. More stringent but also more uncertain limits can be obtained from the baryon-to-entropy ratio. All superstring models with more than three generations are ruled out.

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Currently popular superstring theories^{1),2)}, when compactified on a non-simply connected Calabi-Yau manifold to obtain an effective $d = 4$, $N = 1$ supergravity theory³⁾, appear to lead to at least rank-5 gauge group G . This must be further broken down by the conventional Higgs mechanism to obtain the standard low energy group $G_0 = SU(3)_C \times SU(2)_L \times U(1)_Y$. In principle the symmetries in the coset G/G_0 could be broken at the electroweak scale $O(m_W)$, and dynamical calculations exhibiting the feasibility of this case in the minimal model of $G_{\min} = G_0 \times U(1)$ exist⁴⁾. Indeed, from the point of view of $SU(n,1)/SU(n) \times U(1)$ no-scale models⁵⁾⁻⁷⁾, which have been proposed⁸⁾ as the effective $d = 4$ theory of the heterotic⁹⁾ string, such "no hierarchy" is the only possibility. This is because in no-scale models all scales are determined essentially by the same dynamical mechanism and therefore cannot be separated by very many orders of magnitude.

However, as no rigorous treatment of compactification in superstring models exists, one may allow for more general possibilities. In particular, when four generations of ordinary matter are present, one is led to¹⁰⁾ consider models which have a rank-6 gauge group and an intermediate mass scale M_I . In the present paper we will show, however, that all such models lead to an entropy crisis unless $M_I < 10^9$ GeV, and that such a low intermediate scale is in glaring contradiction with renormalization group analyses for the case of four light generations. Therefore superstring models with an intermediate scale do not appear to be a feasible alternative in general and, furthermore, four generation models are ruled out.

Let us consider a softly broken supergravity model with a rank-6 gauge group G : $G_{\min} \subset G \subset E_6$. We wish to break down the symmetries in $G/G_{\min}$ at $\mu = M_I$. The potential V , renormalized at this scale, can then be given a general expansion in terms of an order parameter ϕ which lies on a ray in group space:

$$V = -m \phi^2 + \lambda m \phi^3 + \lambda^2 \phi^4 + \dots \quad (1)$$

where m is the supersymmetry breaking parameter, $m^2 > 0$, and we have dropped higher order terms, which may also be non-renormalizable. Note that the origin of the quartic term in (1) need not be a cubic term in the superpotential. We are not concerned here whether the effective symmetry breaking potential (1) was obtained through radiative corrections or not. Note that the amount of supersymmetry breaking in (4) is always m^2 or greater as possible supersymmetric mass terms give strictly positive contributions.

Let us now assume that the minimum of (1) is such that $\langle \phi \rangle = M_I \gg m_W$. Then, if $C \ll \lambda$, we have

$$M_I \simeq m / \sqrt{2} \lambda \quad (2)$$

If instead $C \gg \lambda$, we get

$$M_I \simeq - 3 m C / 2 \lambda^2 \quad (3)$$

Suppose now that either ϕ has cubic superpotential couplings, i.e., $\phi^3 \subset 27^3$ of E_6 , or $D_\phi^a = \phi^\dagger T^a \phi \neq 0$ for some a . In both cases the natural order of magnitude of the quartic coupling $\lambda^2 \sim O(1)$ [or $O(g^2)$] and the cubic coupling is $C \sim O(1)$, and both (2) and (3) imply that $m \sim O(M_I)$. This would be disastrous, because by construction ϕ carries some charge and therefore couples to some gauge bosons in E_6 . As ordinary matter fields transform non-trivially in $G/G_{\min}$, at least some of them will couple to the same gauge bosons. Thus diagrams such as depicted in Fig. 1 would give rise to SUSY breaking masses $O(M_I^2)$ also in the observable sector^{*}, and we conclude that necessarily $m \lesssim O(m_W)$.

Therefore, ϕ must lie in the direction where D-terms vanish, and there should be no primordial cubic superpotential couplings. Small effective quartic coupling can in this case be obtained from non-renormalizable superpotential terms after heavy degrees of freedom have been integrated out. An example is the effective superpotential

$$W = \frac{h}{n} (N \bar{N})^n \quad (4)$$

where $N(\bar{N})$ is an $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet contained in the $27(\bar{27})$ representation of E_6 ^{**}, and the parameter h is $O(M_c^{3-2n})$, $M_c \simeq M_p$ being the compactification scale. Along the D-flat direction $N = \bar{N}^* \equiv \phi$ the potential corresponding to (4) is

^{*}) Furthermore, if ϕ would not be D-flat, some scalar fields non-singlet under $SU(3)_C \times SU(2)_L \times U(1)_Y$ would get, from D-terms, a vacuum expectation value of $O(M_I)$.

^{**}) In models with Betti-Hodge number $b_{1,1} > 0$ and effective superpotential $W = h/n(27 \bar{27})^n$.

$$V = -2m^2 |\phi|^2 + \frac{4}{n} A h m |\phi|^{2n} + 2h^2 |\phi|^{4n-2} \quad (5)$$

where m and A are supersymmetry breaking parameters. It has a minimum at

$$M_I \equiv \langle \phi \rangle = \left(a m / \lambda \right)^{1/(2n-2)} \quad (6)$$

where a is a constant given by

$$a = -\frac{A}{2n-1} + \left[\frac{A^2}{(2n-1)^2} + \frac{1}{2n-1} \right]^{1/2} \quad (7)$$

These kinds of models have been considered in Ref. 10) for $n = 2$ (i.e., $M_I \approx 10^{10}$ GeV) and in Ref. 11) for $n = 3$ (i.e., $M_I \approx 10^{14}$ GeV).

The mass matrix has a zero corresponding to $\text{Im } \phi$, which is the Goldstone boson eaten by the $G/G_{\min}$ gauge field, while the mass of the physical Higgs $\text{Re } \phi$ is given by

$$m_\phi^2 = V_{\phi\phi^*} + V_{\phi\phi} = 8(n-1)(1 - Aa)m^2 \quad (8)$$

Therefore, for any value of n in (4), the mass of the Higgs ϕ is $m_\phi \sim 0(m)$. As pointed out by Yamamoto¹²⁾, this fact has the consequence that the critical temperature of the symmetry breaking phase transition is very low, and one has an associated entropy release which is far too excessive unless $M_I < 10^9$ GeV as we will now discuss.

The order parameter ϕ may be coupled to other fields such as coloured SU(5) partners G and G^c of the Higgses H and $\bar{H}$ to give them large masses. In any case, it will be coupled to gauge fields. Thus the potential (1) will be subject to thermal corrections

$$\delta V_T = \frac{\tilde{\lambda}^2}{24} T^2 \phi^2 \quad (9)$$

where $\tilde{\lambda}$ is some combination of coupling constants and $\tilde{\lambda}^2 \gtrsim 3g^2$. From (1) one then sees that naively the critical temperature will be given by

$$T_c \approx \frac{m \sqrt{24}}{\tilde{\lambda}} \approx O(M_W) \quad (10)$$

Of course, in reality the phase transition would be of first order, and one needs to evaluate the tunnelling probability $P = D e^{-S/T}$, where $D \sim 0(m)$ is the

determinant factor and S is the bounce action. However, it is easy to verify¹²⁾ that $S/T \sim (T/m)^3 \gg 1$ so that tunnelling probability at $T > T_c$ is very small and the phase transition does not take place until $T \lesssim T_c$.

The source of entropy release is the ratio between the reheating temperature after the phase transition T_R and the decay temperature of ϕ 's, T_D . Actually, when T falls below m the Universe becomes matter dominated and its energy density is given by

$$\rho(T) \sim \frac{M_I^2}{m} T^3 \quad (11)$$

The decay of ϕ into light matter, with a decay rate $\Gamma \sim \alpha^2 m^3/M_I^2$, $\alpha \sim \lambda^2/16\pi^2$, $\lambda = O(1)$, will happen at a temperature such that $\rho^{1/2} \sim \Gamma$, i.e.,

$$T_D \sim \alpha^{4/3} m^{7/3} M_I^{-2} \quad (12)$$

producing an entropy release

$$\Delta \sim \alpha^{-1} m^{-5/2} M_I^3 \quad (13)$$

Of course, when $T_D > T_c$, the true decay temperature coincides with the critical temperature of the phase transition and the Universe does not enter a matter-dominated era. In that case

$$M_I \leq (\alpha m)^{2/3} \lesssim 10^7 \text{ GeV} \quad (14)$$

and the entropy release is given by

$$\Delta \sim \left(M_I / m \right)^{3/2} \quad (15)$$

If we parametrize the entropy release as

$$\Delta = 10^{3x} \quad (16)$$

we get from (13) and (15) the simple formula

$$M_I = m \begin{cases} 10^{x+2} & , \quad x \geq 2 \\ 10^{2x} & , \quad x \leq 2 \end{cases} \quad (17)$$

This entropy generation will dilute the baryon-to-entropy ratio so that, unless there is an appreciable low energy baryogenesis, Δ should be bounded from above¹³⁾. Taking the conservative bound $x \leq 1$ in (16) and $m = O(10^2)$ GeV, we get from (17) $M_I \lesssim 10$ TeV. Even taking the less conservative bound $x \leq 2$ and $m = O(1)$ TeV, which means that the supersymmetric spectrum is very heavy, we get $M_I \lesssim 10^7$ GeV.

If cosmological baryogenesis is to be produced by the usual mechanism of out-of-equilibrium decays of superheavy scalars¹³⁾, the maximum allowed value of entropy release is $\Delta \lesssim 10^4$, which corresponds to an intermediate scale $M_I \lesssim 10^5$ GeV for $m \approx 250$ GeV. We believe that this is a reasonable upper bound, although in more complicated models^{13),12),14)} one might be able to find ways to allow for more entropy release. This is far below any respectable intermediate scale capable of suppressing Higgs mediated proton decay even by dimension-six operators in superstring theories.

However, the dilution of the baryon-to-entropy ratio is not the worst danger coming from the entropy release. If the ϕ lifetime is longer than the age of the Universe during cosmological nucleosynthesis, then Δ should be $\lesssim O(10)$ because otherwise ϕ decays would alter the primordial abundances of light nuclei. We have found that ϕ decays before nucleosynthesis provided that M_I is bounded by

$$M_I < \alpha^{2/3} 10^{10} m \left(m/M_P \right)^{1/6} \quad (18)$$

Taking $m = O(10^2)$ GeV the bound (18) gives $M_I \lesssim 10^8$ GeV while if $m = O(1)$ TeV, $M_I \lesssim 10^9$ GeV.

The bound (18) has to be considered as an absolute upper bound for the intermediate scale of any superstring theory. Otherwise the entropy production, $\Delta > 10^{12}$, would completely destroy the cosmological abundance of light elements as predicted by the standard big-bang model.

The above results will serve us to completely rule out models with four generations of ordinary matter in superstring theories. It is well known that these models require¹⁰⁾ the presence of an intermediate scale at which some species of particles decouple, otherwise the strong coupling constant has a Landau singularity at an energy scale $\mu \approx 10^8$ GeV. However, as a result of the above analysis, the intermediate scale M_I is bounded from above, Eqs. (17) and (18).

To show that there is no window of opportunity for four-generation models, we now study the renormalization group equation predictions of the minimal model with four generations. The gauge group $G = G_{\min} \times U(1)$ is broken down to $G_{\min}$ at $\mu = M_I$ when two singlets $N \in 27$ and $\bar{N} \in \bar{27}$ (in a mirror generation) take a v.e.v., $N = \bar{N}^*$. We assume that only $\bar{N}$ out of $\delta(27+\bar{27})$ survive below the compactification scale and that all heavy quarks ($G+G^c$) and some doublets $H+\bar{H}$ take a mass at M_I , leaving N_h light $H+\bar{H}$ to break $SU(2)_L \times U(1)_Y$. Our results are given in the Table for $N_h = 1$ and 2. We see that the validity of the perturbative approximation breaks down for $M_I \lesssim 10^8-10^9$ GeV and a Landau singularity for α_{GUT} appears at $\mu \approx M_{\text{GUT}}$. This fact, along with the bound (18) can rule out all four-generation models in superstring theories.

Let us notice that the predictions of the Table are found with $\alpha_3(M_W) = 0.12$ and $m \approx 10^2$ GeV in such a way that all supersymmetric partners of ordinary matter are light at scales larger than the electroweak scale. Another boundary value of α_3 and/or another value of m , e.g., $m \approx 0(1)$ TeV, in such a way that some or all supersymmetric partners are heavy for $M_W < \mu < 1$ TeV, would change slightly the predictions of the Table¹⁰⁾ but the main conclusion will remain.

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M_I	10^{12}	10^{11}	10^{10}	10^9	$10^{8.5}$
α_{GUT}	0.15	0.19	0.28	0.50	0.82
$M_{\text{GUT}}/10^{16}$	5.8	7.5	9.7	12.6	14.3
$\sin^2 \theta_W(M_W)$	0.222	0.220	0.218	0.217	0.216

$N_h = 1$

M_I	10^{12}	10^{11}	10^{10}	10^9	10^8
α_{GUT}	0.13	0.16	0.22	0.35	0.85
$M_{\text{GUT}}/10^{15}$	4.4	7.4	12.3	20.5	34.3
$\sin^2 \theta_W(M_W)$	0.242	0.238	0.234	0.230	0.226

$N_h = 2$

Table - Predictions for the minimal superstring model described in the text with four generations and an intermediate scale. N_h is the number of light $(H+\bar{H})$ which can be used to break $SU(2)_L \times U(1)_Y$.

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FIGURE CAPTION

Tadpole contribution to the mass of light scalars. X are the gauge bosons of the group $G/G_{\min}$.