CERN-TH.4325/85

EVOLUTION WITH TEMPERATURE AND THE POSSIBILITY OF INFLATION
FROM THE SUPERSTRING IN FOUR DIMENSIONS

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A B S T R A C T

We analyze the temperature dependence of the effective potential after superstring compactification and find that it evolves non-trivially around the critical temperature where the gaugino condensate forms. The gauge coupling can be correctly determined by the dynamics only if the initial conditions satisfy certain significant but not unnatural constraints. To avoid difficulties with monopoles and density perturbations, it is desirable to realize inflation in the four-dimensional theory obtained from the superstring. We find that subsequent inflation is possible, but seems to require some fine-tuning of the parameters of the effective potential generated by gaugino condensation.

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CERN-TH.4325/85

November 1985

1. - INTRODUCTION

The heterotic superstring theory^{1),2)} with the gauge group $E_8 \times E_8'$ appears to have good prospects for providing dynamically³⁾⁻⁹⁾ all mass scales less than M_P and all coupling constants^{3),10)} of the low energy world. Although the detailed dynamics of the compactification from ten dimensions down to four dimensions is yet not clear, an attractive possibility for preserving one supersymmetry is that the six compact dimensions should form a Calabi-Yau space³⁾. Unfortunately, not much is known about these manifolds. If one assumes the existence of an as yet unfound Calabi-Yau space which can lead to such desirable properties as three light generations, a suitable low energy gauge group and certain topological restrictions on Yukawa couplings, it is possible⁹⁾ to write down an almost unique supergravity model based on the superstring that satisfies all necessary phenomenological requirements at scales $O(M_W)$.

While some aspects of the low energy theory such as the values of the Yukawa couplings depend on the largely unknown details of the compactified extra dimensions, others such as the value of the gauge coupling constant can in principle be determined in the four-dimensional theory. When the gauge coupling constant is fixed, this has the $SU(n,1)/SU(n) \times U(1)$ structure^{11),12)} of the no-scale models¹²⁾⁻¹⁵⁾ that have been studied for other reasons in the past, and we can then take over the previous no-scale programme of dynamical determination of everything. However, the cosmological evolution of the compactified theory towards the correct vacuum where $g_{GUT}^2 \approx O(1)$ poses some problems. In the absence of SUSY breaking the only minimum of the naive potential is situated at $g_{GUT}^2 = 0$.

The most natural mechanism of SUSY breaking is gaugino condensation in the hidden E_8' . Gaugino zero modes appear in topologically non-trivial background gauge field configurations such as instantons and torons. The latter can give rise to the desired SUSY breaking provided E_8' is broken to some $SU(N)$ subgroup by the Wilson loop mechanism. This condensation will induce a non-perturbative contribution to the effective potential and will create a local minimum $g_{GUT}^2 \neq 0$ with vanishing cosmological constant. However, there still exists another minimum at $g_{GUT}^2 = 0$ which also has zero cosmological constant, and the problem of choosing between them then becomes a problem of initial conditions.

A related question is cosmological inflation^{16),17)}. In order to diminish the primordial abundance of unwanted heavy particles such as monopoles and fractionally charged particles, as well as solve the horizon and flatness problems while keeping density perturbation $\delta\rho/\rho$ acceptably small, it is desirable to realize inflation in the $d = 4$ effective theory. Because superstring models effectively fix the $d = 4$ supergravity model leaving little freedom to adjust parameters, it is not obvious that inflation can be achieved in these models at all, and any inflation obtained may have some special features.

In the present paper we will discuss the cosmological evolution after compactification in superstring theories of the scalar fields entering in the determination of the gauge coupling constant. We will show that, contrary to naive belief, the classical field equations will prevent these fields from rolling to the vacuum where $g_{\text{GUT}}^2 = 0$. The reason for this is the curvature of the scalar field manifold, which couples the fields to each other in a non-trivial way through the kinetic term. We will also consider the prospects for inflation during the stage when the fields are setting down to their minima. The good news is that all the conditions for inflation can be fulfilled. However, the bad news is that some fine-tuning of the gaugino condensation temperature is required.

This paper is organized as follows. In Section 2 we discuss and solve the full classical field equations and show that no fine-tuning of initial conditions is needed to achieve the correct vacuum where $g_{\text{GUT}}^2 \neq 0$. Section 3 contains a detailed study of the surprisingly non-trivial behaviour of the effective potential during the stage when the condensate is forming. In Section 4 we discuss why inflation should take place at temperatures much less than the compactification scale, and hence in four dimensions, and consider the special conditions under which inflation can be obtained during the formation of the condensate.

2. - CLASSICAL EVOLUTION OF THE $d = 4$ EFFECTIVE THEORY

The point-like field theory limit of the superstring is the traditional $N = 1$, $d = 10$ supergravity augmented¹⁾ by terms needed for anomaly cancellation which involve an antisymmetric tensor field B_{mn} , and other terms obtained by integration over heavy degrees of freedom.

When gluino condensation in E_8' takes place so that $\langle \bar{\chi}\chi \rangle \neq 0$, the Lagrangian contains a term^{*)}

$$e^{-1} \mathcal{L}^{(10)} \ni -\frac{3}{4} \phi^{-3/2} \left(H_{mnp} - \frac{\sqrt{2}}{24} \phi^{3/4} \text{Tr} \bar{\chi} \Gamma_{mnp} \chi \right)^2 \quad (1)$$

which makes a contribution to the superpotential W in four dimensions. Here H_{mnp} is the field strength of B_{mn} , ϕ is the dilaton and Γ_{mnp} is the antisymmetric product of three gamma matrices in $d = 10$. Compactification on a Calabi-Yau manifold of the ten-dimensional theory leads⁵⁾ to a conventional $d = 4$, $N = 1$ supergravity model with the Kähler potential

$$G = -\log(S+S^\dagger) - 3 \log(Z+Z^\dagger - 2C_i^\dagger C^i) + \log |W(C^i, S)|^2 \quad (2)$$

where S and Z are gauge singlet chiral superfields, and the C^i are gauge non-singlet matter fields. In addition, compactification provides a chiral function $f_{\alpha\beta}$ coupled to the chiral density $V^\alpha V^\beta$ formed from gauge supermultiplets, simply given by

$$f_{\alpha\beta} = \delta_{\alpha\beta} S \quad (3)$$

In leading order, the real parts of the S and Z fields, $S_R \equiv \text{Re } S$ and $Z_R \equiv \text{Re } Z$, are related to the dilaton ϕ by

$$\phi^{-3} = S_R Z_R^{-3} \quad (4)$$

The superpotential W is given by

$$W = c + h e^{-\gamma S} + f(C^i) \quad (5)$$

where $\gamma = 8\pi^2/C_2(G)$, $\langle H_{mnp} \rangle = C \epsilon_{mnp}$ and $h \approx 0(1)$ is a constant. Obviously, the value of S will be determined by the minimization of the potential obtained from $W(S)$, and this will also determine $g_{\text{GUT}}^2 = 1/S_R$.

As has been argued in Ref. 18), the loop expansion of the heterotic string theory is in powers of ϕ , with the limit $\phi \rightarrow 0$ corresponding to the semi-classical limit. Because of this, and because necessarily $g_{\text{GUT}}^2 \approx 0(1)$, the compactification scale M_C must be close to M_P . For phenomenological reasons it is

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^{*)} Hereafter, unless explicitly mentioned, all formulae will be written in units of the Planck mass.

desirable to have the scale of gluino condensation less than M_C . Therefore at $T \approx M_C$ the potential is

$$V \simeq |C|^2 e^{G_0} / 2S_R \quad (6)$$

where we assume that other effects at the Planck scale fix $Z_R \approx 0(1)$ so that

$$G_0 = -3 \log (Z + Z^\dagger - 2C_i^\dagger C^i) \simeq 1 \quad (7)$$

and we take the matter fields to be at their minima $C^i = 0$ (this assumption will be justified later).

Equation (6) suggests insurmountable difficulties: as V has no minimum, will S_R not roll off to infinity where $g_{GUT}^2 = 0$? Even if gluino condensation will eventually provide a minimum at $g_{GUT} \approx 1$, why should the initial conditions not be such that initially S_R is so large that it will never be able to settle to this minimum? Of course, the form of the Kähler potential (2) is valid as long as we are in the semi-classical limit $\phi \lesssim 1$ of the superstring, and one may argue that the superstring really is strongly coupled and the preceding worries are irrelevant.

In fact, as we now show, for many initial conditions S_R will be driven by the field equations towards, but not necessarily into, the strong coupling regime, and the field S_R can find its way to the $g_{GUT} \neq 0$ minimum in a natural way. Consider the kinetic energy of the complex field S :

$$\mathcal{L}_{kin} = \frac{1}{4S_R^2} \partial_\mu S \partial^\mu S^\dagger \quad (8)$$

Because of the small $O(1/M_P)$ couplings of the S field, it is plausible that S is not thermalized after the Planck epoch, but evolves according to the classical equations of motion. Note that $S_I \equiv \text{Im } S$ is coupled directly to S_R , providing an effective potential which greatly modifies the classical solutions for S_R ^{19),20)}. Consider the classical field equations for the homogeneous complex field $S(t)$ in a Friedman-Robertson-Walker background. One obtains

$$\dot{S}_I(t) = 4Q S_R^2 / R^3 \quad (9)$$

where Q is an integration constant and $R(t)$ is the usual scale factor. Thus the kinetic energy of S_I contributes to the energy density a factor

$$\rho_I = \frac{4 Q^2 S_R^2}{R^6} = 4 \bar{Q}^2 S_R^2 T^6 \quad (10)$$

where the last equality follows from entropy conservation. This serves as an effective temperature-dependent potential for S_R , so that the full effective potential

$$V_{\text{eff}} = V(S_R) + \rho_I \quad (11)$$

now has a minimum at

$$S_R^3 = c^2 R^6 e^{G_0} / 4 Q^2 \quad (12)$$

The position of the minimum is of course temperature- and time-dependent. The existence of such a minimum does not of course guarantee that S_R will find itself here. However, when Q is fixed ρ_I will act as a barrier against fluctuations to $S_R \gg 1$. How effective this barrier is depends on the value of the arbitrary integration constant Q , which we must now discuss.

It is convenient to define a normalized field

$$\phi = \frac{1}{\sqrt{2}} \log S_R \quad (13)$$

It is natural to suppose that any initial condition at $T = M_C$ should satisfy

$$\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi) + \rho_I(\phi) \lesssim 1 \quad (14)$$

otherwise all hell would break loose. Presumably it is higher string modes that render such fluctuations impossible by providing a cut-off for Einstein quantum gravity. While Q itself has no natural order of magnitude, the initial fluctuation in the kinetic energy of S_I does: obviously at $T = M_C$, ρ_I must be $\lesssim 0(1)$, so that (14) is satisfied. Although arbitrarily large values of S_R are still allowed by taking $Q \approx 1/S_R$, in all cases the field equations will take S_R towards $S_R = 0$. If initially $S_R \gg 1$, one can neglect V altogether. It is in any case plausible that the large number of light gauge non-singlet fields will provide a radiation-dominated background so that $R(t) = t^{\frac{1}{2}}$, in which case the field equations can be solved to give

$$\phi(t) = \phi_0 - \frac{4}{\sqrt{2}} \int^{\sqrt{2}} (1 - t^{-1/2}) - \frac{1}{\sqrt{2}} \log \frac{1}{2} \left[(1 + \alpha) + (1 - \alpha) e^{-8 \int^{\sqrt{2}} (1 - t^{-1/2})} \right] \quad (15)$$

where α^2 is the fraction of energy density carried initially by the S_I field, so that $4Q^2 \exp(2\sqrt{2} \phi_0) = \rho(1 - \alpha^2)$, where ϕ_0 is the initial value of ϕ . We see from (15) that $\phi(t) \rightarrow \text{constant}$ as $t \rightarrow \infty$. Not all fluctuations will take S_R to a region in field space where the minimum provided by gluino condensation will appear; but many will. For instance, let us take the initial fluctuation in energy density to be large, $\rho = 1$. Let us furthermore assume that half of this fluctuation is carried by the kinetic energy of $\text{Im } S$ so that $\alpha^2 = \frac{1}{2}$. Then one finds from (15) that $S_R(t=\infty) = 0.015 S_R(t=0)$. Thus in this example the classical evolution of the field equation would carry all initial conditions with $S_R(t=0) \lesssim 67$ to the region where the correct $g_{\text{GUT}}^2 \neq 0$ minimum will be formed. It is amusing that the classical evolution of S_R tends towards a constant, so that g_{GUT} could be fixed even in the absence of gluino condensation. Such a possibility does not, however, appear to be appealing for other reasons^{*}).

The classical field equations may not give a realistic picture of the evolution of S_R . In renormalizable field theories quantum corrections to field equations can cause rapid dissipation of the otherwise endless oscillations about a minimum of the potential. Presumably some dissipation will take place also in our case, although we are unable to compute the needed quantum corrections. Let us for the sake of argument assume that this dissipation is sufficiently rapid that S_R relaxes to the minimum^{**}) (12). Furthermore, if we take at $T = M_C$, $\rho_0 \sim 1$,

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^{*}) As for the Z_R -dependence of the potential, it is given by $V \propto Z_R^{-3}$. This naive potential is unstable and its only minimum is at $Z_R = \infty$. However, following the same steps as those used above for the S_R -field, one can show that the effects of the curvature in the field manifold can induce an effective potential with a minimum at $Z_R \propto (3R^6/8Q_Z^2)^{1/5}$ and a large barrier preventing fluctuations to $Z_R \gg 1$. In the same way, $Z_R(t) \rightarrow \text{constant}$ as $t \rightarrow \infty$ and Z_R can dissipate very rapidly its energy and relax to its minimum. As assumed above one can require, by adjusting the parameter Q_Z , that Z_R takes the values needed for the inflationary scenario of Section 4 to hold, as given in Table 3, for temperatures down to the inflation temperature.

^{**}) Bear in mind that since the minimum (12) of the effective potential with $\dot{S}_R = 0$ is not a solution of the field equations, it is not obvious that dissipative terms will behave in the way we assume here. However, our subsequent discussion remains valid if S_R relaxes to a value close to the minimum.

then S_R will be able to go to the correct minimum at $S_R \sim 0(1)$ if at the condensation temperature T_C obeys the relation

$$e^{G_0} C^2 T_C^{-6} S_R^2(M_C) \lesssim 0(1) \quad (16)$$

If we take $C \sim 10^{-3}$, as suggested by some superstring calculations, and $T_C \approx 10^{-1}$ one finds the initial condition

$$S_R(M_C) \lesssim 0(1) \quad (17)$$

which certainly is restrictive but does not appear unnatural even in this extreme case. Thus we conclude that although not automatic, a natural mechanism exists for S_R to relax to its minimum value g_{GUT}^{-2} . In no case will it roll to $g_{GUT} = 0$, and no fine-tuning of the initial conditions is needed.

3. - EFFECTS OF THE CONDENSATE

In this section we will study the temperature effects on the formation of the condensate and the ensuing evolution of the effective potential. To this end, let us consider the full effective potential

$$V = \frac{1}{2S_R} \left\{ e^{G_0} |\Gamma|^2 + \frac{1}{6} e^{\frac{2}{3}G_0} |f_i|^2 \right\} + \frac{1}{2} D^\alpha D^\alpha \quad (18)$$

where $\Gamma = W + f - 2S_R W_S$, and W and f were defined in (5). This potential is positive definite with global minima at

$$\Gamma = f_i = D^\alpha = 0 \quad (19)$$

At high temperatures (18) is subject to thermal corrections, and the matter fields will choose their symmetric minima at $\langle C^1 \rangle = 0$. The assumption of thermal equilibrium for the gauge non-singlet fields is justified because their interaction cross-sections are $O(1/E^2)$, so that their interaction rates are $O(E \text{ or } T)$ in an adiabatic expansion. This can be compared with the expansion rate $H = O((E^2 \text{ or } T^2) \times \sqrt{N_E})$, where N_E is the number of light effective degrees of freedom. Comparing these rates we see that the gauge non-singlet particles are likely to be in equilibrium for $E \text{ or } T \lesssim 1/\sqrt{N_E}$, which includes the temperatures $T = O(T_C) \lesssim O(10^{-3})$ which interest us. Note the contrast with the gauge singlet fields S and Z whose cross-sections are $O(1/M_P^2)$, and are hence unlikely to be in

thermal equilibrium after the Planck epoch, as was already mentioned in Section 2 and will be discussed again later.

We will exhibit explicitly the one-loop thermal corrections to the potential (18) by using a generalization of the methods used in Ref. 21) for a similar calculation in the no-scale models of $N = 1$ supergravity. The result will also be useful in Section 4, where we show how thermal corrections lift the degeneracy between global minima in the S_R direction. The finite temperature corrections arise from loops of properly normalized scalar fields and their fermionic partners, of gauge vector bosons and gauginos, and of gravitinos. The one-loop contribution is given by

$$\Delta V_{1-loop}(T) = -\frac{\pi^2}{24} (N+1) T^4 + \frac{T^2}{24} \left[\text{Tr } M_0^2 + \text{Tr } M_{1/2} M_{1/2}^\dagger + \text{Tr } M_{3/2}^2 + 3 \text{Tr } M_1^2 \right] \quad (20)$$

where N is the total number of chiral superfields and M_j ($j=0, 1/2, 1, 3/2$) are the normalized mass matrices of spin- j fields.

As we have said above the calculation follows similar lines to Ref. 21) where thermal corrections were computed in the usual no-scale models (i.e., without the S -field). The effect of the S -field is to give an extra contribution to $\text{Tr } M_0^2$ and $\text{Tr } M_{1/2} M_{1/2}^\dagger$ of the form:

$$\Delta (\text{Tr } M_0^2 + \text{Tr } M_{1/2} M_{1/2}^\dagger) = \frac{1}{2S_R} \left\{ 2(N+7) e^{G_0} |\Gamma|^2 + 2|W+f|^2 e^{G_0} + \frac{4}{3} e^{2/3 G_0} |f_i|^2 + 48 S_R^4 e^{G_0} |W_{SS}|^2 \right\} \quad (21)$$

Then, using the results of Ref. 21), one can show that

$$\Delta V_{1-loop}(T) = \frac{T^2}{24} \left\{ \frac{1}{S_R} \left[2 e^{G_0} |f+W|^2 + \frac{1}{9} (N+10) e^{2/3 G_0} |f_i|^2 + \frac{1}{24} e^{G_0/3} |f_{ij}|^2 + (N+7) e^{G_0} |\Gamma|^2 + 24 S_R^4 e^{G_0} |W_{SS}|^2 \right] + 12 g_{GUT}^2 e^{G_0/3} C_2(R_i) |C^i|^2 + O(D^2) \right\} \quad (22)$$

It is clear from (22) that the configuration $\langle C^1 \rangle = 0$ (i.e., $f = 0$, $D^\alpha = 0$) will be energetically favoured at high temperatures.

Putting now all matter fields at their vacuum expectation values, we can write the potential (18) as

$$V = \frac{1}{2S_R} e^{G_0} |\Gamma|^2 \quad (23)$$

where $\Gamma = W(s) - 2S_R W_S$, and the one-loop thermal corrections

$$\Delta V_{1-loop} = \frac{T^2}{24 S_R} e^{G_0} \left\{ (N+7) |\Gamma|^2 + 2 |W|^2 + 24 S_R^4 |W_{SS}|^2 \right\} \quad (24)$$

Since N is usually a big number, $N = O(10^2)$, we can take the large N limit of (24) and, to a good approximation,

$$\Delta V_{1-loop}(\tau) \simeq \frac{T^2}{24} N V \quad (25)$$

so that the one-loop thermal corrections will not change the structure of minima of the potential (23). However, in some interesting cases, where V has degenerate global minima, the non-leading terms in (24) can lift this degeneracy.

We now turn to analyze the potential (23) and study the effects of the gaugino condensate on it. The global minima are easily exhibited by writing the potential

$$V = \frac{e^{G_0}}{2S_R} \left\{ \left[c + h(1 + 2\gamma S_R) e^{-\gamma S_R} \cos \gamma S_I \right]^2 + \left[h(1 + 2\gamma S_R) e^{-\gamma S_R} \sin \gamma S_I \right]^2 \right\} \quad (26)$$

where h is considered real for simplicity, and the global minima of (26) satisfy

$$\gamma S_I = (2n+1)\pi, \quad (\text{i.e. } \cos \gamma S_I = -1) \quad (27a)$$

$$\lambda = \frac{c}{h} = (1 + 2\gamma S_R) e^{-\gamma S_R} \quad (27b)$$

The potential (26) is an effective potential due to non-perturbative effects below the scale of gluino condensation. As such, it is temperature-dependent^{*)}. We parametrize the complicated dependence on temperature of the vacuum expectation value of the gluino condensate due to non-perturbative effects by a function $h(T)$ which is equal to zero for $T > T_c$, where

$$T_c \simeq M_{GUT} e^{-\gamma/3g_{GUT}^2} \quad (28)$$

while $h(T) > 0$ for $T < T_c$, and $h(T) = 0(1)$ for $T \ll T_c$, when the condensate is fully formed. The variation in $h(T)$ means that the shape of the potential (26) is changing non-trivially with the temperature, as we will now discuss.

Using (27a), the potential along the real axis S_R is

$$V = \frac{1}{2} e^{G_0} \gamma h^2 \frac{1}{x} \left[\lambda - (1 + 2x) e^{-x} \right]^2 \quad (29)$$

where $x = \gamma S_R$, and the value of the constant C can be inferred from the condition (27b). Using $\langle S_R \rangle = 1/g_{GUT}^2$ as at $T \ll T_c$, we find

$$C \simeq \left(1 + 2\gamma/g_{GUT}^2 \right) e^{-\gamma/g_{GUT}^2} \quad (30)$$

Now we will follow the evolution of the potential (29) as the temperature cools down. For $T > T_c$ (i.e., $\lambda \rightarrow \infty$) the potential has no global minimum

$$V = \frac{1}{2} e^{G_0} \gamma |C|^2 \frac{1}{x} \quad (31)$$

and its shape is shown in Fig. 1a. For $T < T_c$ the shape of the potential will depend on the value of $\lambda(T)$ as indicated in Fig. 1. There are four critical values of λ corresponding to four qualitative changes of regime in the potential (29) as λ decreases with T . For $\lambda = \lambda_1$, a stationary point appears, as shown in Fig. 1b. For $\lambda = \lambda_2$, Fig. 1d, a global minimum with zero vacuum energy is present in the potential for the first time. For $\lambda = 1$, Fig. 1f, the global minimum is shifted to $S_R = 0$. Finally, for $\lambda = \lambda_3$, as shown in Fig. 1h, the global minimum becomes an unstable stationary point with zero slope which may give rise to an inflationary epoch. The forms of the potential for intermediate ranges of λ are

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^{*)} This phenomenon is similar to non-perturbative QCD where strong interactions form quark and gluon condensates below a critical temperature $T_c \sim 200$ MeV. The quark condensates there break chiral symmetry, just as the gluino condensate here breaks supersymmetry.

shown in Fig. 1c, ($\lambda_1 \gg \lambda \gg \lambda_2$), 1e ($\lambda_2 \gg \lambda < 1$), and 1g ($1 \gg \lambda \gg \lambda_3$), respectively. The critical values of the parameter λ_i ($i=1,2,3$) are tabulated in Table 1.

The location of the global minimum increases when the temperature cools down and finally, at $T \ll T_c$, when the condensate is fully formed, $h(T) = 0(1)$, $\lambda(T) = C$ and $x_+ = \gamma/g_{\text{GUT}}^2$, as in Fig. 1i.

Before concluding this section we would like to make a few comments about the effective potential (29). It is clear that we can trust the form of this potential which does not include string loop effects only in the region where $\phi^3 = S_R^{-1} Z_R^3 \lesssim 1$, i.e., in the region where the superstring is semi-classical. This means that we can trust our potential in the region $S_R \gtrsim Z_R^3$, although it is not yet excluded that the loop expansion in the string theory may lead to a similar potential. Let us be conservative and require $\phi^3 \lesssim 1$. In that case, adopting $Z_R \lesssim 0.1$ one sees that the region $S_R \gtrsim 10^{-3}$ is a respectable one. This means that the radius of the compact manifold is $\lesssim 0.3$ Planck lengths and the Kaluza-Klein excitations are almost as heavy as the other string excitations. However, at temperatures a few orders of magnitude below the Planck scale, which are of interest for inflation, both types of excitations would be decoupled and our four-dimensional field theory is expected to be reliable.

4. - THE POSSIBILITY OF SUPERSTRING INFLATION

A period of early cosmological inflation^{16),17)} is a very aesthetic explanation of the observed homogeneity, isotropy and flatness of the present-day universe. In this section we will consider ways of producing inflation in superstring models.

Let us first establish when inflation should take place. For this analysis it is important to remember that the compactification scale M_C , the string excitation scale M_s and the Planck scale M_P must all be roughly equal. Analysis of gravitational waves and density fluctuations $\delta\rho/\rho$ in four-dimensional field theory models of inflation have revealed²²⁾ serious problems if the field energy density during inflation is $O(M_P^4)$. In our case, physics at the Planck scale is not described by a simple field theory in $d = 4$, but we can expect similar conclusions to apply. Below $M_C \lesssim M_P$ Einstein gravity, perhaps improved by the addition of higher derivative terms, is the effective theory. Quantum fluctuations of this theory are cut off at $M_s \sim M_C$. This means that following

compactification the metric $g_{\mu\nu}$ will initially be subject to large fluctuations typically $O(M_C)$. These will lead to large scalar and tensor perturbations of the energy momentum tensor. If one splits the metric into $g_{\mu\nu} = g_{\mu\nu}^{(0)} + M_C h_{\mu\nu}$, where $h_{\mu\nu}$ is a fluctuation, the fluctuation in energy density will also scale²³⁾ as $\delta\rho/\rho \sim M_C^2$. These perturbations will propagate as gravitational waves and other distortions of the classical background, and it is very difficult to understand how the universe would end up being isotropic and homogeneous at later times if an inflationary era would not smooth out the fluctuations²²⁾. These considerations suggest that cosmologically useful inflation can only take place at temperatures $T \ll M_C$, and that Kaluza-Klein type inflation²⁴⁾ during compactification or inflation during the string phase²⁵⁾ would not solve the very problems for which inflation was first proposed. Let us also remind the reader that Kaluza-Klein type inflation does not appear feasible in the context of superstrings, since a very large number of compact dimensions is needed to obtain a Kasner-type separation of the compact and four-dimensional scales. Moreover, as discussed by Wen and Witten²⁶⁾, the compactified heterotic string theory contains monopoles and superheavy fractional charged particles that are stable. The overabundance of such particles would be a cosmological disaster, and therefore one needs inflation to dilute their number density. Finally, E_6 breaking by Wilson lines might result in degenerate vacua separated by domain walls. Also in this case one would need inflation at scales $T \ll M_C$.

In the past there have been several attempts²⁷⁾ to build an inflationary model in $d = 4$, $N = 1$ supergravity models with $SU(n,1)$ symmetry. In these models it was assumed that among the matter fields C_i one gauge singlet field played the part of the inflaton. In superstring models such a field might be found in the coset E_6/G_0 , where G_0 is the low energy gauge group, possibly $G_0 = SU(3) \times SU(2) \times U(1) \times U(1)$. Here we will focus on another possibility. We have at our disposal another gauge singlet scalar field S^* , the potential of which might serve as the source of the necessary vacuum energy.

A quick look at Fig. 1h shows that if the field S_R is driven to the stationary point $x_- = 1/3/2$ by the temperature evolution of the potential (29), then the universe may enter a de Sitter inflationary era. This may happen if the evolution of the S_R field is the one indicated with black dots in the movie of

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*) As discussed elsewhere, we assume that Z_R is fixed by gravitational corrections to be $O(1)$.

Fig. 1. This necessary condition cannot be provided by the one-loop temperature corrections (22), (24), since they do not change the global shape of the potential. However, as discussed in Section 2, corrections to the effective potential for S_R due to the curvature of the Kähler manifold can produce^{19),20)} a suitable value for S_R : $x \lesssim 1+3/2$ when $T = T_c$, even if the naive tree-level potential does not possess any minimum (as is the case here), and thus solve simultaneously the problems of the stability of the theory and of the initial conditions for inflation. We will return to this question later in the light of the inflationary parameter values introduced shortly.

In this case, the field will be trapped in the stable minimum of Figs. 1c,d. However, as we have seen above, for $\lambda < \lambda_2$ the global minimum of Fig. 1d splits into the two degenerate global minima in Fig. 1e. In that case the one-loop thermal corrections can break this degeneracy. Expanding Eq. (24) around $\Gamma = 0$ we get

$$\Delta V_{1-loop}(\tau) = \frac{1}{3} T^2 h^2 \gamma e^{G_0} x (1 + 3x^2) e^{-2x} \quad (32)$$

which favours small values of x , i.e., the left-hand minimum in Fig. 1e. After that, the field will follow the evolution of the black points in Figs. 1e-i.

Hereafter we will study the conditions for the universe to enter an inflationary de Sitter phase space when $\lambda \gtrsim \lambda_3$. If this happens all interaction rates will be negligible compared to the expansion rate of the universe during the de Sitter era and the condensate, i.e., the value of h gets frozen. We assume that the condensate takes essentially the value it would have at a finite temperature equal to the Hawking temperature of the de Sitter phase.

We observe that during inflation, $h(T) \approx C$, which can be very small, depending on the group G into which E_8' is broken down. This value of C provides the scale of inflation and energy density perturbations which is usually introduced by hand in inflationary models. We introduce the parameter m_0 defined by

$$m_0^2 = V \left(1 - \frac{\sqrt{3}}{2} \right) \quad (33)$$

so that

$$m_0 = 0.35 e^{G_0/2} Y^{1/2} C \quad (34)$$

A condition for inflation is $\lambda(T_H) \gtrsim \lambda_3$, where $T_H = H/2\pi$ is the Hawking temperature and H the Hubble constant, given by

$$H = m_0 / \sqrt{3} \quad (35)$$

If $\lambda(T_H) \gtrsim \lambda_3$, there is a barrier between the local minimum, where the field is trapped when the inflation starts, and the global minimum. Inflation can proceed à la Hawking-Moss (HM)^{*} provided that the height of the barrier satisfies the relation $\mu^2 \leq 4H^2$, where μ^2 is the curvature of the potential at the local minimum.

The condition for having the HM phase transition is always satisfied provided that we require enough inflation

$$H\tau = \frac{3H^2}{\mu^2} \geq 60, \quad \text{i.e.} \quad \mu^2 \leq H^2/20 \quad (36)$$

The condition (36) is satisfied provided that the local minimum of the potential is located at $x_m = 1 - 3/2 - \delta x$, where δx is bounded by

$$\frac{\delta x}{x_m} \lesssim 1/76 \quad (37)$$

If we write $\lambda(T_H) = \lambda_3 + \delta\lambda$, then

$$\frac{\delta\lambda}{\lambda} \simeq \frac{4\sqrt{3}}{1 + 4x_m^2} (\delta x)^2 \lesssim 2 \times 10^{-5} \quad (38)$$

which is the amount of fine-tuning that the theory has to provide.

Simple dynamical models suggest that

$$h(\tau) = h_0 \left(1 - \frac{\tau}{\tau_c}\right)^p, \quad p = O(\text{few}) \quad (39)$$

in which case

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^{*}) The phase transition could also be of second order, corresponding to $\lambda(T_H)$ slightly less than λ_3 . In this case the fine-tuning requirements (37) and (38) will be similar.

$$\lambda(\tau) = \lambda_0 \left(1 - \frac{\tau}{T_c}\right)^{-p} \quad (40)$$

and hence

$$\frac{\delta\lambda}{\lambda} \approx \frac{p}{1 - \frac{T_H}{T_c}} \left(\frac{\delta T}{T_H} \right) \left(\frac{T_H}{T_c} \right) \quad (41)$$

where $\delta T = T - T_H$. We see that a small value of $\delta\lambda/\lambda$ is possible if either, (a) $T_H = 0(T_c)$ and

$$\frac{\delta T}{T_H} = \left(h(T_H)/h_0 \right)^{1/p} O(10^{-5})$$

or, (b) $T_H \ll T_c$ in which case $\delta T/T_H \approx (T_c/T_H) \times O(10^{-5}) \gg O(10^{-5})$. In the latter case the present zero-temperature potential (Fig. 1i) would actually have to be very similar to the critical case illustrated in Fig. 1h^{*}). It is of course possible that the naive expectation (39) for the T evolution of h is incorrect, in which case the fine-tuning problem may be alleviated and, furthermore, the possibility $T_H \ll T_c$ would no longer imply that the zero-temperature potential is given by Fig. 1h, which is incompatible with the conditions for inflation and correct energy density perturbations, as given by Eq. (49). For the present, we will just assume that the conditions (37) and (38) are satisfied, and analyze the inflationary predictions of the potential (29). Amazingly enough we will find a clear prediction of the biggest group G into which the second E_8 ' is broken during compactification.

In terms of the normalized field

$$\phi_N = \frac{1}{\sqrt{2}} \log \frac{Y_{SR}}{1 - \frac{\sqrt{3}}{2}} \quad (42)$$

the potential (29) can be expanded as

$$V = V_0 - A \phi_N^3 \quad (43)$$

where $V_0 \equiv V(1-\sqrt{3}/2)$ and $A = 0.18 m_{\text{pl}}^2$, and we have neglected the quadratic term which is justified when $H\tau \gg 60$.

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^{*}) However, this possibility seems to be excluded by the conditions of inflation and correct energy density perturbations, Eq. (49), which give $\langle x \rangle_{T=0} \gtrsim 27$, while $\langle x \rangle \approx 1.4$ in Fig. 1h.

The standard calculation of energy density perturbations leads to

$$\frac{\delta\rho}{\rho} = \frac{1}{\sqrt{2} \pi^{3/2}} \frac{A}{H} \left(\log H k^{-1} \right)^2 \quad (44)$$

where $Hk^{-1} = 10^{21}$ for galactic scales, so that

$$\frac{\delta\rho}{\rho} = 94.22 m_0 \quad (45)$$

The condition $T_H < 0(T_c)$, which is necessary to have a consistent inflationary scenario [see Eqs. (39-41)], provides a relation among $C_2(G)$, where G is the biggest group into which the second E_8' is broken down, α_{GUT} , M_{GUT} and $\delta\rho/\rho$, as

$$\alpha_{GUT} C_2(G) = \frac{2\pi}{3} \frac{1}{6.9 - \log \delta\rho/\rho + \log M_{GUT}/M_P} \quad (46)$$

Fixing the energy density perturbations to the experimental value

$$\frac{\delta\rho}{\rho} \simeq 10^{-4} \quad (47)$$

translates into the following values for m_0 , the Hubble parameter and the Hawking temperature

$$\begin{aligned} m_0 &\simeq 10^{-6} M_P \\ H &\simeq 6 \times 10^{-7} M_P \\ T_H &\simeq 10^{-7} M_P \end{aligned} \quad (48)$$

Using (47) and the absolute bound $M_{GUT} > 10^{-3} M_P$ we get the following upper bound on $C_2(G)$

$$C_2(G) < 0.23 / \alpha_{GUT} \quad (49)$$

Taking the absolute upper bound $\alpha_{GUT}^{-1} < 40$ we obtain $C_2(G) < 9$, which already restricts the breaking of E_8' . A sample of values of α_{GUT} for different values of M_{GUT} and G are given in Table 2.

On the other hand, the value of the C -parameter in the effective superpotential (5) given by (27b) becomes, using Eq. (46), a function of M_{GUT} . Values of C for different values of M_{GUT} are tabulated in Table 3. Once we have $C = C(M_{GUT})$ and using (34) and (48) we can obtain G_0 as a function of M_{GUT} as shown in Table 3. Finally the radius R of the internal manifold is related to Z_R

as ^{6),18)} $R \sim Z_R^{\frac{1}{2}}$ which is, in turn, a function of M_{GUT} . Values of R for the different values of M_{GUT} are shown in Table 3.

We see in Table 3 that for M_{GUT} very close to M_P , the radius of the internal manifold is very small and contributions from higher dimensional operators in the σ -model expansion could alter the inflationary predictions analyzed in this section. The appropriate value of M_{GUT} should be fixed by gravitational radiative corrections after inflation which produce a stable potential for the field Z_R and determine the vacuum expectation value $\langle Z_R \rangle$.

Now the question arises of how close T_H and T_c have to be for the consistency of the whole picture. The answer depends of course on the detailed functional dependence of $h(T)$. If we assume the simple model of Eq. (39), then $(T_c - T_H)/T_c = (h(T_H)/h_0)^{1/p}$ and using the values of Eq. (48) we have $h(T_H)/h_0 \approx 10^{-7}$ so that $(T_c - T_H)/T_c \approx 1\%$ for $p = 3.5$ and $(T_c - T_H)/T_c \approx 10\%$ for $p = 7$. For this range of values, $1/p(T_c - T_H)/T_c \approx 10^{-2}$ so that the fine-tuning needed for inflation is $\delta T/T_H \approx 10^{-7}$, according to Eq. (38). However, as indicated above, all these fine-tunings could be alleviated if the assumed T evolution (39) of h is incorrect. On the other hand, adopting the values in Table 3, the condition $\rho_I \lesssim 0(1)$, Eq. (16), gives $S_R \lesssim 10^6 T_c^3$. As we have discussed above, the parametrization (39), (40) leads to $T_c \approx T_H \approx 10^{-7}$ as the only possibility and we obtain from that the severe constraint $S_R(M_c) \lesssim 10^{-15}$. However, a different parametrization could open up the possibility that $T_c \ll T_H$. In that case the natural condition $S_R(M_c) \lesssim 0(1)$ could perhaps be obtained with $T_c \approx 10^{-2}$.

Let us now summarize the inflationary scenario that may arise from the potential (29). The evolution of $\lambda(T)$ is adjusted such that $\lambda(T_H) \gtrsim \lambda_3$, where the Hawking temperature T_H is obtained as in (48) by the condition $V(\lambda_3, x=1-\sqrt{3}/2) = 12\pi^2 T_H^2$. In this way inflation will start at the temperature T_I given by the condition

$$\frac{\pi^2}{30} g_* T_I^4 = V_{\min}(\lambda(T_I)) \quad (50)$$

where $g_* = 0(10^2)$ is the number of light degrees of freedom, and $\lambda_3 < \lambda(T_I) < 1$, Fig. 1g. Therefore, even though the inflationary era starts when $T \approx T_I > T_H$, it is only when T falls to T_H , which is the minimal temperature in the de Sitter phase, that the condensate will stop its evolution and freeze. There might appear to be a potential danger in this picture, namely that the universe would enter an

inflationary period for some λ : $\lambda_2 < \lambda < \lambda_1$ (Fig. 1c) and that the condensate stops its evolution then. In that case there would be no graceful exit to inflation. However, this does not happen as we will now explain. For T such that $\lambda_2 < \lambda(T) < \lambda_1$ as in Fig. 1c, one can easily see that $V_{\min}(\lambda) < V(\lambda_3, x=1-\sqrt{3}/2)$, where $V_{\min}(\lambda)$ is the height of the potential at the local minimum with positive cosmological constant, whenever $T_H < T < T_c$. This means that even if there is some solution T_I to Eq. (50): $\lambda_2 < \lambda(T_I) < \lambda_1$, leading to a brief inflationary epoch for some range of $T < T_I$, the corresponding Hawking temperature $T_H(\lambda)$ given by $T_H^2(\lambda) = V_{\min}(\lambda)/12\pi^2$ is always smaller than T during this epoch, the condensate does not stop its evolution and a FRW universe is recovered at some $\lambda(T)$ whereafter $\pi^2/30 g_* T^4 > V_{\min}(\lambda(T))$. This clearly will happen for some T_f : $\lambda_2 < \lambda(T_f) < \lambda_1$, since we know that $V_{\min}(\lambda_2) = 0$.

Let us now return to the problem of initial conditions. As is evident from the discussion of Section 2, there will be a large class of initial conditions which take the field S_R to the correct region. To quantify this we have studied numerically the full field equations in a radiation-dominated background adopting the values of Eq. (48). We have also taken $\alpha_{\text{GUT}} = \alpha_3(M_W) = 0.12$ and assumed three complete light 27's of E_6 in the low energy theory, so that $T^2 t = 1.51 g_F^{-\frac{1}{2}} = 0.078$ and a typical time is t_0 : $\log t_0 = 12.46$, corresponding to the typical temperature $T_0 \approx 6 \times 10^{-4}$. Then we should have $S_R(T_0) \lesssim 0.16$. If dissipation is strong enough to take S_R to the minimum (12), then we should have $Q \gtrsim 290$. If S_R does not have time to relax to the minimum, or if quantum corrections do not take it there, then a larger set of initial conditions might result in inflation. The evolution of S_R for some representative choices of initial conditions is presented in Fig. 2, which also shows the time evolution of the minimum of the effective potential for some values of Q .

In all the cases shown the classical field equations keep S_R in the region $S_R < 0.16$ where inflation can take place (this corresponds to $\phi > \phi_1$ in Fig. 2). This can happen for relatively small values of Q , and although initial conditions exist such that inflation cannot occur at all, providing the necessary initial conditions for inflation does not in our case lead to fine tuning. As has been stressed by Linde³⁰⁾, most other inflationary models suffer from the problem of initial conditions, which stems from the fact that the thermal corrections to the inflaton potential are not sufficiently large to guarantee that the inflaton will be at its metastable minimum at the onset of the inflationary era. Here such difficulties can be avoided.

5. - CONCLUSIONS

We have analyzed in this paper the temperature dependence of the effective potential for the gauge singlet field S which is coupled to the gauge fields in the superstring theory after compactification. We have found a surprisingly non-trivial behaviour, as illustrated in Fig. 1. No particular fine-tuning of the initial conditions is required for the real part of the S field to be trapped in the global minimum which gives a non-zero gauge coupling constant in four dimensions.

We have gone on to give reasons why inflation in superstring models should occur in the four-dimensional effective theory. We have shown that the S potential can give such inflation at the price of some fine-tuning of parameters. As is shown in Table 2, inflation with density perturbations $\delta\rho/\rho$ in an astrophysically acceptable range is only possible for a restricted choice of the hidden gauge subgroup left unbroken by the Wilson loop symmetry breaking mechanism. Let us also point out that the inflationary scenario that appears to rise from the superstring is primordial inflation³¹⁾.

Although it does not yet appear very natural, we find it remarkable that there is any hope of inflation using the very constrained form of effective potential for the S field which is provided by gaugino condensation. We are encouraged by our observation that no special choice of initial values for the gauge singlet fields S and Z is necessary for the satisfactory dynamical determination of the gauge coupling, and can hope that more profound understanding will make the tuning of parameters needed for inflation seem more natural. We find it unlikely that inflation could arise in the sector of observable gauge non-singlet chiral fields obtained from the superstring. We can imagine that it might be possible to obtain inflation elsewhere in the gauge singlet sector if this contains more than just the S and Z fields we have studied. However, the present understanding of the possible appearance and couplings of such fields is not enough to investigate such a conjecture. Thus the possible mechanism for inflation presented in this paper is the only one available so far. Perhaps our ugly duckling will eventually turn into a swan.

ACKNOWLEDGEMENTS

One of us (K.E.) is grateful for the financial support of the Academy of Finland.

λ_1	λ_2	λ_3
$8(1+\sqrt{3}/2) e^{-(1+\sqrt{3}/2)}$	$2e^{-1/2}$	$8(1-\sqrt{3}/2) e^{-(1-\sqrt{3}/2)}$

Table 1 - The critical values of the parameter λ in the potential (26).

M_{GUT}/M_P	1	10^{-1}	10^{-2}	10^{-3}
$\alpha_{\text{GUT}}^{-1} (G = \text{SU}(2))$	15.7	13.2	10.9	8.7
$\alpha_{\text{GUT}}^{-1} (G = \text{SU}(3))$	23.1	19.8	16.4	13.0
$\alpha_{\text{GUT}}^{-1} (G = \text{SU}(4))$	30.8	26.5	21.8	17.3

Table 2 - Values of α_{GUT} predicted by Eq. (46) for different values of M_{GUT} and G .

M_{GUT}/M_P	1	10^{-1}	10^{-2}	10^{-3}
C	10^{-19}	10^{-16}	10^{-13}	5×10^{-11}
G_0	58	44	30	18.5
R	5×10^{-5}	5×10^{-4}	5×10^{-3}	3×10^{-2}

Table 3 - Values of the C-parameter in the superpotential (5), the field G_0 and the radius of the internal manifold $R \sim Z_{\frac{1}{2}}^{\frac{1}{2}} R$, in Planck length units, for different values of M_{GUT} .

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FIGURE CAPTIONS

Fig. 1 Evolution with temperature of the effective potential (29).

- (a) For $\lambda > \lambda_1$ the potential has neither global nor local minima;
- (b) At $\lambda = \lambda_1$ a structure begins to develop and a stationary point with zero slope appears at $x = 1+\sqrt{3}/2$;
- (c) As the temperature cools down and $\lambda_2 < \lambda < \lambda_1$, the stationary point splits into a maximum at $x > 1+\sqrt{3}/2$ and a local minimum at $1/2 < x < 1+\sqrt{3}/2$. The local minimum has positive energy which falls with the temperature;
- (d) For $\lambda = \lambda_2$ the local minimum becomes a global one at $x = 1/2$;
- (e) As the temperature decreases further, when $1 < \lambda < \lambda_2$ the global minimum at $x = 1/2$ splits into two global minima at $1/2 < x_+ < 1.256$ and $0 < x_- < 0.5$, with a maximum in between at $0.3 < x_M < 1/2$. The other maximum is located at $3.963 < x'_M < 4.33$;
- (f) For $\lambda \rightarrow 1$, $x_- \rightarrow 0$, $x_+ \rightarrow 1.256$, $x_M \rightarrow 0.3$ and $x'_M \rightarrow 4.33$;
- (g) As the temperature cools down so that $\lambda_3 < \lambda < 1$, the minimum at $x_- = 0$ rises and creates a local minimum (with positive vacuum energy) at $0 < x_- < 1-\sqrt{3}/2$, while the maxima are at $1-\sqrt{3}/2 < x_M < 0.3$ and $x'_M > 4.33$, and the global minimum is at $x_+ > 1.256$;
- (h) When $\lambda = \lambda_3$, x_- and x_M collapse at $x_- = x_M = 1-\sqrt{3}/2$, the global minimum is at $x_+ = 1.400$ and the maximum at $x'_M = 4.45$;
- (i) For $\lambda < \lambda_3$ the stationary point at $1-\sqrt{3}/2$ disappears and the unique global minimum is at $x_+ > 1.400$. The values of λ_i ($i=1,2,3$) are given in Table 1.

Fig. 2 Solutions of the field equations of $\phi = 1/\sqrt{2} \ln S_R$ for different initial conditions:

- (a) $\phi_0 = -3$, $\dot{\phi}_0 = 1$, $Q = 1$ ($\rho = 0.501$);
- (b) $\phi_0 = -3$, $\dot{\phi}_0 = 1$, $Q = 10$ ($\rho = 0.583$);
- (c) $\phi_0 = -3$, $\dot{\phi}_0 = -1$, $Q = 10$ ($\rho = 0.583$);
- (d) $\phi_0 = -1$, $\dot{\phi}_0 = -1$, $Q = 1$ ($\rho = 0.736$);
- (e) $\phi_0 = -1$, $\dot{\phi}_0 = -0.1$, $Q = 1$ ($\rho = 0.241$).

Also shown is the evolution of the minimum of the effective potential for different values of Q and the critical field value ϕ_1 corresponding to $x = 1+\sqrt{3}/2$.

Figure 1
(a) - (c)

Figure 1
(d) - (f)

Figure 1
(g) - (i)

- Figure 2 -