CERN-TH.4265/85

THE WAVE FUNCTIONAL OF A SUPER-CLOCK

S. Elitzur^{*)}

CERN - Geneva

A. Forge

Racah Institute of Physics, Jerusalem

and

E. Rabinovici^{*)}

CERN - Geneva

A B S T R A C T

We discuss a model with local supersymmetrical features in two dimensions. Solving the constraint equations, a single wave functional for a super-clock is found. It exhibits no correlations between geometry and the fermion number.

^{*)} Permanent address: Racah Institute of Physics, Jerusalem.

This work is supported in part by the fund for basic research of the Israeli Academy of Sciences and the American-Israeli Bi-National Science Foundation.

CERN-TH.4265/85

September 1985

1. - INTRODUCTION

It has been conceptually useful to study gauge theories in the Hamiltonian formulation. Abelian and non-Abelian gauge theories have been analyzed in the $A_0 = 0$ gauge. As this is not a complete gauge fixing, the Hamiltonian commutes with the generators of the residual gauge transformations. These in turn are imposed to vanish when acting on the physical states. The eigenstates of the Hamiltonian are thus functionals depending only on singlets under time-independent gauge transformations. Gravitational theories are treated by analogy¹⁾ in the synchronous gauge, that is the gauge in which $g^{01} = 0$; $g^{00} = 1$. In this case the Hamiltonian itself is part of the residual gauge symmetries. These consist, in addition, of the generators of spatial reparametrization transformations. The wave functionals of the system (the Universe) are thus to depend only on spatial reparametrization invariant quantities and must be annihilated by the Hamiltonian. This latter property is more difficult to characterize, it is instead labelled by calling this condition the Wheeler-DeWitt (WD) equation. The WD equation was solved on a subset of possible geometries^{1),2)} and in perturbation theory³⁾ in various dimensions of space-time. Its cosmological implications were studied as well¹⁾⁻³⁾.

In this paper we wish to study the wave functional in the presence of fermions. To this end one needs first to identify the set of spatial reparametrization invariant quantities, involving fermions, on which the wave functional may depend. Next, by solving the WD equation, one would obtain in general information about the correlation between the geometric content of the wave functional and its fermionic structure. In particular, we wish to study a model in which the fermions are coupled in a locally supersymmetric manner. The simplest such system is supergravity in two dimensions.

By a simple counting of degrees of freedom, gravity in two dimensions is an over-constrained system. The metric tensor $g_{\mu\nu}$ consists of three degrees of freedom. Two are removed by working in the synchronous gauge, quantum mechanically two more should be removed by imposing, as mentioned earlier, the WD equation and spatial reparametrization invariance. Banks and Susskind^{4),5)}, appealing to Polyakov's⁶⁾ work in the conformal gauge with zero internal degrees of freedom, have suggested that due to the conformal anomaly in the ghost sector, a modified system with a total of one quantum degree of freedom may emerge. A calculation of the effective action due to the ghost in the synchronous gauge has not been performed.

Adding the two-dimensional gravitino, χ_μ^a , does not enlarge the number of degrees of freedom as the gravitino being a gauge particle is over-constrained as well. In the gauge $\chi_0^a = g_{01} = 0$, $g_{00} = 1$, one gets a model with a bosonic field g_{11} and a fermionic one χ^a . With the same number of fields we construct a Lagrangian which is the supersymmetric extension of the one suggested by Banks and Susskind⁴⁾. The system will have all the residual symmetries including time-independent supersymmetry. On top of being a solution of the WD equation and spatial reparametrization invariant, it must be annihilated by the local supersymmetric generator. We have not been able to show that the supersymmetrical extension of the Lagrangian of Ref. 4) is indeed 2-d supergravity. We thus view it rather as a toy model, containing fermions and some supersymmetrical elements, for the study of the interrelations between fermions and geometry.

It has been suggested by Witten⁷⁾ that the bothersome question of handling several solutions of the WD equation may be resolved in supergravity in which only a first-order equation must be solved. In the model we study, this indeed turns out to be the case, supersymmetry necessitates a negative cosmological constant which leads, already in the WD equation, to only one normalizable solution. This solution is picked also by the first-order equation.

The paper is organized as follows: in Section 2 we describe the supersymmetrical system studied. In Section 3, canonical quantization and the various quantum constraints are discussed. In Section 4, the constraints are solved and the wave functional is obtained.

2. - A LOCALLY SUPERSYMMETRIC MODEL FOR TWO-DIMENSIONAL GRAVITY

Two-dimensional supergravity involves four zweibein components e_μ^a and four Rarita-Schwinger fields χ_μ^m . In the supersynchronous gauge, $\chi_0^a = 0$ thus, off-shell, there is one gravity variable e and two fermionic ones, a single scalar auxiliary field is needed to complete a superfield E .

$$E = e + \bar{\theta}\chi + \bar{\chi}\theta + \theta\bar{\theta}F \quad (2.1)$$

We assign to E the transformation properties of a density in superspace. It is more convenient to define the superfield $\mathfrak{E}$

$$\Phi = 2E^{1/2} \quad (2.2)$$

in terms of which a supersymmetric extension, A_s , of the system in Ref. 4) is expressed by:

$$A_s = \int dx d\theta d\bar{\theta} \frac{1}{2} (d\Phi \bar{d}\Phi - \alpha \Phi^2) \quad (2.3)$$

where by convention $\alpha > 0$. $\vec{\partial}$ are left derivatives and $\bar{d}$, d are

$$\bar{d} = \frac{\vec{\partial}}{\partial \theta} - i\bar{\theta} \frac{\partial}{\partial t} \quad d = \frac{\vec{\partial}}{\partial \bar{\theta}} - i\theta \frac{\partial}{\partial t} \quad (2.4)$$

The action A_s is indeed still invariant under some residual gauge transformations. These can be expressed in superspace $Z^A = (t, x, \bar{\theta}, \theta) = (Z^0, Z^1, Z^{\bar{0}}, Z^{\bar{1}})$ by

$$Z^A \rightarrow Z^A + \zeta^A(Z) \quad (2.5)$$

Time-independent local supersymmetry transformations follow from:

$$\zeta^0 = i(\bar{\varepsilon}\theta + \varepsilon\bar{\theta}) \quad ; \quad \zeta^1 = 0 \quad ; \quad \zeta^{\bar{0}} = \bar{\varepsilon} \quad ; \quad \zeta^{\bar{1}} = \varepsilon \quad (2.6)$$

where ε , $\bar{\varepsilon}$ are time independent. Under these transformations, the superfield Φ is changed by $\delta\Phi$:

$$\delta\Phi = i(\bar{\varepsilon}Q + \bar{Q}\varepsilon)\Phi \quad (2.7)$$

where Q and $\bar{Q}$ are given by:

$$Q = \theta \frac{\partial}{\partial t} - i \frac{\vec{\partial}}{\partial \bar{\theta}} \quad \bar{Q} = -\bar{\theta} \frac{\partial}{\partial t} + i \frac{\vec{\partial}}{\partial \theta} \quad (2.8)$$

The action A_s is also invariant under spatial reparametrizations for which:

$$\zeta^0 = 0 \quad ; \quad \zeta^1 = h(x) \quad ; \quad \zeta^{\bar{0}} = \zeta^{\bar{1}} = 0 \quad (2.9)$$

E transforms under both symmetries as a density which, for (2.9) reduces to

$$\delta E = \frac{\partial}{\partial x}(hE) \quad (2.10)$$

These invariances emulate some aspects of two-dimensional supergravity and in any case by observing the wave functional of this system, one may study the properties of the wave functional in the presence of fermions. Expressing the superfield Φ as:

$$\Phi = q + \bar{\theta}\psi + \bar{\psi}\theta + \theta\bar{\theta}S \quad (2.11)$$

the Lagrangian reads, in component notation, after eliminating the auxiliary field as

$$\mathcal{L} = \frac{1}{2} \dot{q}^2 - \frac{1}{2} \alpha^2 q^2 + i \bar{\psi} \dot{\psi} + \alpha \bar{\psi} \psi \quad (2.12)$$

In the absence of fermions and recalling that by (2.2), $q = 2e^{\frac{1}{2}}$, the bosonic sector reproduces the Lagrangian of Banks and Susskind⁴⁾.

The terms $\frac{1}{2}\alpha^2 q^2$ and $\alpha\bar{\psi}\psi$ are the supersymmetric pair corresponding⁸⁾ in higher dimensions to a gravitino mass term and a cosmological constant which is forced to be negative by supersymmetry. The term $\frac{1}{2}\dot{q}^2$ is the conjectural dynamical content of pure gravity in two dimensions and $\bar{\psi}\dot{\psi}$ that of its supersymmetric Rarita-Schwinger partner. The bosonic and fermionic sectors are coupled only by the quantum constraints of spatial reparametrization invariance and the fermionic Gauss law which will be discussed in the next section.

In component notation, the SUSY transformations (2.7) and (2.8) are expressed as:

$$\delta q = \bar{\epsilon}\psi + \bar{\psi}\epsilon \quad ; \quad \delta\psi = \epsilon(-i\dot{q} + \alpha q) \quad ; \quad \delta\bar{\psi} = \bar{\epsilon}(i\dot{q} + \alpha q) \quad (2.13)$$

$$Q = \psi(\dot{q} - i\alpha q) \quad ; \quad \bar{Q} = \bar{\psi}(\dot{q} + i\alpha q) \quad (2.14)$$

Next we proceed to the quantization of the system.

3. - CANONICAL QUANTIZATION AND THE ALGEBRA OF CONSTRAINTS

Quantizing the Lagrangian in component notation, one ascribes conjugate momenta $p = \dot{q}$ and $\pi_\psi = -i\dot{\bar{\psi}}$ to q, ψ

$$[q(x), p(y)]_- = i\delta(x-y) \quad ; \quad [\bar{\psi}(x), \psi(y)]_+ = \delta(x-y) \quad (3.1)$$

in terms of which

$$Q(x) = \psi(x) (p(x) - i\alpha q(x)) \quad ; \quad \bar{Q}(x) = \bar{\psi}(x) (p(x) + i\alpha q(x)) \quad (3.2)$$

which generates transformations (2.13) by:

$$\delta u(x) = i \int dy [\bar{E}(y) Q(y) + \bar{Q}(y) E(y), u(x)]_- \quad ; \quad u = (q, \psi, \bar{\psi}) \quad (3.3)$$

The measure in q space is dq/q due to spatial reparametrization invariance. The Hermitian representation of (3.1) is thus:

$$p(x) = \frac{1}{i} q^{1/2}(x) \frac{\delta}{\delta q(x)} q^{-1/2}(x) \quad ; \quad \bar{\psi}(x) = \frac{\vec{\delta}}{\delta \psi(x)} \quad (3.4)$$

The wave functional is in general $\Psi(\psi, q)$ and its inner product defined by:

$$\langle \Psi_1 | \Psi_2 \rangle = \int D(\ln q(x)) D\psi(x) \Psi_1(\psi, q) (* \Psi_2(\psi, q)) \quad (3.5)$$

where the duality operation $*$ acts on the form basis for the fermionic degrees of freedom. A zero form being the empty state, a one-form contains one fermion and, in the presence of some cut-off, an N form is the full state. The duality $*$ operation turns n forms into $N-n$ forms. In form space the inner product is $\langle \beta_1 | \beta_2 \rangle = \int \beta_1 \wedge (*\beta_2) = \int \beta_2 \wedge (*\beta_1)$. In terms of (3.4), $Q(x)$ and $\bar{Q}(x)$ are Hermitian conjugates.

With the definition (3.5), $\Psi(q, \psi) d(\ln q)$ expresses the amplitude to find the Universe in a specific m form and $q(x)$ configuration. This wave function is subject to some constraints. The residual gauge symmetries are to annihilate the wave function. The first constraint is invariance under spatial reparametrization invariance. The transformations (3.9) are explicitly given by:

$$\delta u = h \frac{\partial u}{\partial x} + \frac{i}{2} u \frac{\partial h}{\partial x} \quad ; \quad u = (q, p, \psi, \bar{\psi}) \quad (3.6)$$

under which

$$\delta \mathcal{L} = \frac{\partial (\mathcal{L} \delta x)}{\partial x} \quad (3.7)$$

the Hermitian generator of this symmetry, $\pi(x)$, is found to be:

$$\pi(x) = \frac{1}{2} \left(\frac{\partial q(x)}{\partial x} p(x) - q(x) \frac{\partial p(x)}{\partial x} \right) - \frac{i}{2} \left(\frac{\partial \psi(x)}{\partial x} \bar{\psi}(x) - \psi(x) \frac{\partial \bar{\psi}(x)}{\partial x} \right) \quad (3.8)$$

reproducing (3.6) through:

$$\delta u(x) = i \int dy h(y) [\pi(y), u(x)] \quad (3.9)$$

Under a finite reparametrization $x \rightarrow x'$, the superfield Φ transforms as a half density

$$q'(x') = \left| \frac{\partial x}{\partial x'} \right|^{1/2} q(x) \quad ; \quad \psi'(x') = \left| \frac{\partial x}{\partial x'} \right|^{1/2} \psi(x) \quad (3.10)$$

$\psi(x)/q(x)$ is thus a scalar, a fact which will become presently useful. Invariance of $\Psi(q, \psi)$ under $\pi(x)$ implies that:

$$\pi(x) \Psi = 0 \quad (3.11)$$

It will be more convenient to define a functional $\hat{\Psi}(q, \psi)$ by:

$$\Psi = D[q] \hat{\Psi} \quad (3.12)$$

where $D[q]$ is defined by

$$D[q] = \exp \left(\frac{1}{2} \int dx \delta(x) \ln q(x) \right) \quad (3.13)$$

Using this notation one has

$$\pi(x) \Psi = D[q] \hat{\pi}(x) \hat{\Psi} \quad (3.14)$$

where $\hat{\pi}(x)$ is obtained from $\pi(x)$ by inserting the more conventional $1/i \delta/\delta q(x) = \hat{p}(x)$ instead of $p(x)$ as given by (3.4). Solving $\hat{\pi}(x) \hat{\Psi} = 0$ ensures $\pi(x) \Psi = 0$ as well.

The second residual gauge invariance is imposed by the WD equation

$$\mathcal{H}(x) \psi(q, \psi) = 0 \quad (3.15)$$

where

$$\mathcal{H}(x) = \frac{1}{2} \rho^2(x) + \frac{1}{2} \alpha^2 q(x)^2 - \frac{\alpha}{2} [\bar{\psi}(x), \psi(x)] \quad (3.16)$$

and $\mathcal{H}(x)$ is semi-positive definite by:

$$[\bar{Q}(x), Q(y)]_+ = 2\delta(x-y) \mathcal{H}(x) \quad (3.17)$$

The third imposed gauge invariance is the fermionic Gauss law:

$$Q(x) \psi(q, \psi) = 0 \quad ; \quad \bar{Q}(x) \psi(q, \psi) = 0 \quad (3.18)$$

which are first-order equations containing the same information as (3.15). This model bears great similarities to Witten's supersymmetric quantum mechanics. Defining $\hat{Q}$ in analogy to $\hat{\pi}$, one may consider the equations

$$\hat{Q}(x) \hat{\psi}(q, \psi) = 0 \quad ; \quad \hat{\bar{Q}}(x) \hat{\psi}(q, \psi) = 0 \quad (3.19)$$

We conclude this section by writing out the algebra obtained by the constraints:

$$\begin{aligned} [Q(x), Q(y)]_+ &= 0 \quad ; \quad [\bar{Q}(x), \bar{Q}(y)]_+ = 0 \quad ; \quad [Q(x), \bar{Q}(y)]_+ = 2\delta(x-y) \mathcal{H}(x) \\ [Q(x), \pi(y)]_- &= i\delta'_x(x-y) Q(y) \quad ; \quad [\bar{Q}(x), \pi(y)]_- = i\delta'_x(x-y) \bar{Q}(y) \\ [\mathcal{H}(x), \pi(y)]_- &= i\delta'_x(x-y) \mathcal{H}(y) \quad ; \quad [\pi(x), \pi(y)]_- = \frac{i}{2} \delta'_x(x-y) (\pi(x) + \pi(y)) \end{aligned} \quad (3.20)$$

This algebra can be consistently realized over the physical states which are simultaneously annihilated by all these operators. We now solve these constraints.

4. - SOLUTION FOR THE WD SUPERSYMMETRIC EQUATIONS

A) Spatial reparametrization invariants

In the proposed model for two-dimensional gravity, the wave functional depends upon one parameter: the length of the Universe. The question which transcends the particular supergravity model we are studying is: which new global invariant forms appear in the presence of fermions.

The functional space as defined by DeWitt consists of wave functions depending on spatial reparametrization invariant quantities. We denote these by Ω_i . In a general notation,

$$\Omega_i = \int dx W_i(q^{(k)}(x), \psi^{(k)}(x)) \quad ; \quad q^{(k)}(x) = \frac{\partial^{(k)} q(x)}{\partial x^k} \quad ; \quad \psi^{(k)}(x) = \frac{\partial^{(k)} \psi(x)}{\partial x^k} \quad (4.1)$$

Under arbitrary variations $\delta q(x)$ and $\delta \psi(x)$, the corresponding variations of the functions are:

$$\frac{\vec{\delta} \hat{\mathcal{F}}}{\delta u(x)} = \sum_i \frac{\vec{\delta} \Omega_i}{\delta u(x)} \frac{\vec{\partial} \hat{\mathcal{F}}}{\partial \Omega_i} \quad u(x) = (q(x), \psi(x)) \quad (4.2)$$

The variations of Ω_i are given by:

$$\frac{\vec{\delta} \Omega_i}{\delta u(x)} = \sum_{m \geq 0} (-1)^m \frac{\partial^{(m)}}{\partial x^m} \left(\frac{\vec{\partial} W_i}{\partial u^{(m)}(x)} \right) \quad (4.3)$$

The constraint equations (3.11) and (3.19) are written as:

$$\left(\frac{\partial q(x)}{\partial x} \frac{\vec{\delta}}{\delta q(x)} - q(x) \frac{\partial}{\partial x} \frac{\vec{\delta}}{\delta q(x)} + \frac{\partial \psi(x)}{\partial x} \frac{\vec{\delta}}{\delta \psi(x)} - \psi(x) \frac{\partial}{\partial x} \frac{\vec{\delta}}{\delta \psi(x)} \right) \hat{\mathcal{F}} = 0 \quad (4.4)$$

$$\psi(x) \left(\frac{\vec{\delta}}{\delta q(x)} + \alpha q(x) \right) \hat{\mathcal{F}} = 0 \quad (4.5)$$

$$\frac{\vec{\delta}}{\delta \psi(x)} \left(\frac{\vec{\delta}}{\delta q(x)} - \alpha q(x) \right) \hat{\mathcal{F}} = 0 \quad (4.6)$$

We first proceed to solve the condition of spatial reparametrization invariance, (4.4). The spatial volume of the system is still a suitable Ω_1 variable,

$$\Omega_0 = \int dx q^2(x) \quad (4.7)$$

The addition of fermions allows us to construct new such invariants. In the zero form sector, Ω_0 is the only available invariant. In the one-form sector, an additional invariant, Ω_1 , arises by integrating over the position of the single fermion

$$\Omega_1 = \int dx q(x) \psi(x) \quad (4.8)$$

In the two-form sector there exists a whole class of new invariants depending on the invariant distance between the two fermions. This now generalizes to any n-form. It is convenient to present these variables in terms of their Taylor expansion (which coincides with an expansion in covariant derivatives). In a given n-form sector, a general term in this expansion is:

$$\tilde{\Omega}_n^{\{m\}} = \int dx \tilde{W}_{\{m_j\}}^{\{k\}}(q^{(k)}(x)) q^2(x) \prod_{j=1}^n \left(\frac{1}{q^2(x)} \frac{\partial}{\partial x} \right)^{(m_j)} \left(\frac{\psi(x)}{q(x)} \right) \quad (4.9)$$

m_j is the rank of the derivative appearing in the j^{th} term of the product which consists of n terms. $\{m_j\}$ is the set of these integers. Recalling that $\psi(x)/q(x)$ is a scalar, $\tilde{W}_{\{m_j\}}^{\{k\}}(q^{(k)}(x))$ must also be a scalar, that is it may depend only on Ω_0 . We denote:

$$\Omega_n^{\{m\}} = \int dx q^2(x) \prod_{j=1}^n \left(\frac{1}{q^2(x)} \frac{\partial}{\partial x} \right)^{(m_j)} \left(\frac{\psi(x)}{q(x)} \right) \quad (4.10)$$

where $\{m\} = (m_1, m_2, \dots, m_n)$ with $0 \leq m_1 < m_2 < \dots < m_n$ because of Fermi statistics. Ω_0 and Ω_1 coincide with those defined in (4.7) and (4.8). $\Omega_2^{(m)}$ is given for example by:

$$\Omega_2^{(m)} = \int dx q(x) \psi(x) \left(\frac{1}{q^2(x)} \frac{\partial}{\partial x} \right)^{(m)} \left(\frac{\psi(x)}{q(x)} \right) \quad m = 1, 3, 5, \dots \quad (4.11)$$

The calculations simplify by defining

$$\tilde{q}(x) = q(x) \quad ; \quad \tilde{\psi}(x) = \frac{\psi(x)}{q(x)} \quad (4.12)$$

from which it follows:

$$\frac{\delta}{\delta q(x)} = \frac{\delta}{\delta \tilde{q}(x)} - \frac{\tilde{\psi}(x)}{\tilde{q}(x)} \frac{\delta}{\delta \tilde{\psi}(x)} \quad (4.13)$$

$\delta/\delta\tilde{q}(x)$ and $\delta/\delta\tilde{\psi}(x)$ commute, thus (4.5) and (4.6) become:

$$\tilde{\psi}(x) \left(\frac{\delta}{\delta\tilde{q}(x)} + \alpha \tilde{q}(x) \right) \hat{\psi} = 0 ; \quad \frac{\delta}{\delta\tilde{\psi}(x)} \left(\frac{\delta}{\delta\tilde{q}(x)} - \alpha \tilde{q}(x) \right) \frac{1}{\tilde{q}(x)} \hat{\psi} = 0 \quad (4.14)$$

In form notation, the general reparametrization invariant wave function is:

$$\hat{\psi} = \hat{\psi}_0(\Omega_0) + \Omega_1 \hat{\psi}_1(\Omega_0) + \sum_{m \text{ odd}} \Omega_2^{(m)} \hat{\psi}_2^{(m)}(\Omega_0) + (\text{SECTORS} \geq 3) \quad (4.15)$$

It remains to select between all these wave functions the supersymmetric one.

B) The supersymmetric solution

We first study explicitly zero-, one- and two-forms and then present a general solution. $\pi(x)$ and $\chi(x)$ can be diagonalized separately in each form sector.

1) Zero-form sector: $\hat{Q}(x)\hat{\psi}_0 = 0$ reads, using (4.2) and (4.3):

$$\tilde{q}(x) \tilde{\psi}(x) \left(\frac{d\hat{\psi}_0}{d\Omega_0} + \frac{\alpha}{2} \hat{\psi}_0 \right) = 0 \quad (4.16)$$

$\bar{Q}$ automatically annihilates the empty state. On the contrary, $\tilde{\psi}(x)$ will turn the zero-form into a one-form, on top of that $\tilde{q}(x)\tilde{\psi}(x)$ is x -dependent, thus one must demand that the x -independent parentheses vanish. From which follows:

$$\hat{\psi}_0(\Omega_0) = c_0 \exp\left(-\frac{\alpha}{2} \Omega_0\right) \quad (4.17)$$

For the full state $\hat{Q}$ and $\bar{Q}$ would change roles, it is the convention of having $\alpha > 0$ that has chosen the normalizable zero-form solution over the non-normalizable full state.

2) One-form sector: $\hat{Q}(x)\hat{\psi}_1(\Omega_0)\Omega_1 = 0$, reads:

$$\tilde{q}(x) \tilde{\psi}(x) \Omega_1 \left(\frac{d\hat{\psi}_1}{d\Omega_0} + \frac{\alpha}{2} \hat{\psi}_1 \right) = 0 ; \quad \left(\tilde{\psi}(x) \frac{\delta\Omega_1}{\delta\tilde{q}(x)} = \tilde{\psi}(x) \tilde{\psi}(x) = 0 \right) \quad (4.18)$$

However, for $\alpha \neq 0$, $\bar{Q}$ cannot annihilate $\hat{\psi}_1(\Omega_0) = c_1 \exp(-\alpha/2 \Omega_0)$ thus $\hat{\psi}_1(\Omega_0) = 0$.

3) Two-form sector: $\hat{\Psi}_2 = \sum_{m \text{ odd}} \hat{\Psi}_2^{(m)}(\Omega_0)\Omega_2^{(m)}$ with:

$$\Omega_2^{(m)} = \int dx \tilde{q}^L(x) \tilde{\Psi}(x) \left(\frac{1}{\tilde{q}^L(x)} \frac{\partial}{\partial x} \right)^{(m)} (\tilde{\Psi}(x)) \quad m = 1, 3, 5, \dots \quad (4.11)$$

Annihilation by $\hat{Q}(x)$ requires:

$$\tilde{q}(x) \tilde{\Psi}(x) \left[\sum_{m \text{ odd}} \left(\frac{d\hat{\Psi}_2^{(m)}}{d\Omega_0} + \frac{\alpha}{2} \hat{\Psi}_2^{(m)} \right) \Omega_2^{(m)} + \sum_{m \geq 3} \hat{\Psi}_2^{(m)} \frac{\delta \Omega_2^{(m)}}{\delta \tilde{q}(x)} \frac{1}{2\tilde{q}(x)} \right] = 0 \quad (4.19)$$

The last summation starts from $m = 3$ because $(\delta \Omega_2^{(1)} / \delta \tilde{q}(x)) \equiv 0$. Explicit x dependence in (4.19) appears in the factor $\tilde{q}(x) \tilde{\Psi}(x)$ and in $(\delta \Omega_2^{(m)} / \delta \tilde{q}(x)) \times (1/\tilde{q}(x)) (m \geq 3)$. The latter are linearly independent as they contain a different number of derivatives, this requires $\hat{\Psi}_2^{(m)} = 0$ for all $m \geq 3$. The reduced equation (4.19) is $(d\hat{\Psi}_2^{(1)} / d\Omega_0) + (\alpha/2) \hat{\Psi}_2^{(1)} = 0$, the solution of this equation, $\hat{\Psi}_2 = c_2 \exp(-(\alpha/2)\Omega_0) \Omega_2^{(1)}$ cannot be annihilated by $\hat{Q}(x)$, thus also $\hat{\Psi}_2 = 0$.

4) General solutions: This line of reasoning extends to all n -forms. The equation $\hat{Q}(x) \hat{\Psi}$ is in general

$$\tilde{q}(x) \tilde{\Psi}(x) \left[\frac{\partial \hat{\Psi}}{\partial \Omega_0} + \frac{\alpha}{2} \hat{\Psi} + \sum_{\substack{n \geq 2 \\ (m)}} \frac{1}{2\tilde{q}(x)} \frac{\delta \Omega_n^{(m)}}{\delta \tilde{q}(x)} \frac{\partial \hat{\Psi}}{\partial \Omega_n^{(m)}} \right] = 0 \quad (4.20)$$

The $n = 0$ term is pulled out of the sum and the $n = 1$ term, which has already been shown to vanish, is omitted.

Again, all the explicit x dependence in parentheses is in the linearly independent terms $\delta \Omega_n^{(m)} / \tilde{q}(x) \delta \tilde{q}(x)$. These must be made to vanish separately by setting

$$\frac{\partial \hat{\Psi}}{\partial \Omega_n^{(m)}} = 0 \quad [\text{except for } n=2, m=1 \text{ because } (\delta \Omega_2^{(1)} / \delta \tilde{q}(x)) \neq 0] \quad (4.21)$$

The resulting reduced (4.20) was just solved several times, confirming that the solution is:

$$\hat{\psi} = \hat{\psi}_0 = c \exp\left(-\frac{\alpha}{2}\Omega_0\right) \quad (4.22)$$

The wave function is frozen into the empty state. Due to fermion number conservation and the absence of other fermions, it was clear that a solution would reside in some n -form, in the zero-form sector there exists no quantity to correlate with Ω_0 . Only for $\alpha = 0$ there would be identical plane wave normalizable volume independent solutions in every sector.

5. - CONCLUDING REMARKS

By working in the fermionic occupation number basis, we were able to give a general characterization of spatial reparametrization invariant states containing fermions. For $\alpha = 0$, supersymmetry did indeed pick a single solution to the WD equation, the single solution appearing in the case of negative cosmological constant. This Universe does not gravitationally collapse but it is mainly concentrated at a volume of the scale of the cosmological constant. Generically, this implies that no long-range physics survives. Correlations could have formed in principle between bosonic and fermionic occupation numbers conspiring to maintain zero energy, however this was ruled out, the clock has zero-fermion number. The fermionic clock is frozen. It is not clear to us whether this is just a property of the simple model. Indeed the gravitino in general could be an integral part of a super-clock or it could well behave as another piece of matter coupled to the clock. In the case studied, the "gravitino" behaves as the fixed fermionic label of the clock.

ACKNOWLEDGEMENTS

We wish to thank Tom Banks for a useful discussion.

REFERENCES

- 1) B.S. DeWitt - Phys.Rev. 160 (1967) 113;
C.W. Misner, K.S. Thorn and J.A. Wheeler - Gravitation, Freeman, San Francisco (1973).
- 2) J.B. Hartle and S.W. Hawking - Phys.Rev. D28 (1983) 2960;
S.W. Hawking - Nucl.Phys. B239 (1984) 257.
- 3) See, for example:
T. Banks, W. Fischler and L. Susskind - SLAC-PUB-3367 (1984);
C. Hill - Fermilab-Pub 85/37-T (1985).
- 4) T. Banks and L. Susskind - Int.J.Theor.Phys. 23 (1984) 475.
- 5) For 2-d gravity in a covariant formulation, see:
S.C. Teitelboim and R. Jackiw - in Festschrift for B. DeWitt's 60th Birthday, S.C. Teitelboim, Austin Preprint (1983).
- 6) A.M. Polyakov - Phys.Lett. 103B (1981) 207.
- 7) See:
T. Banks - Nucl.Phys. B249 (1985) 332.
- 8) S. Deser and B. Zumino - Phys.Rev.Lett. 38 (1977) 1433.