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COULD ONE INVENT THE N-STRING ?

I. Bengtsson

CERN - Geneva

A B S T R A C T

We point out that the recently proposed gauge invariant field theories for free strings can be generalized to include an arbitrary number, N , of Virasoro gauge algebras. The corresponding light front gauge expressions are also given. The resulting models have intercept N and critical dimension 26. Scattering amplitudes and interaction terms are briefly discussed and a possibly fatal problem is noted, but no definite conclusions are drawn.

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1. - INTRODUCTION

The theoretical basis of dual models has changed drastically over the years, but the number of models on the market has remained small, unlike the case of field theories for particles of spin two and lower, where an infinite number of models is available. Of course, the attempts to generalize the dual models have varied in flavour, according to the state of the dual art. Among interesting attempts from the days when a dual model was regarded as an S-matrix theory one could mention the Coon model, which replaced the gamma-functions of the Veneziano model with basic hypergeometric functions. With the advent of the string interpretation, attempts at generalization have concentrated on changing the action of the underlying quantum mechanical object, the string.

Now the perspective seems to be shifting again, to infinite component field theory. Of course, the light front field theory formulation has been around for some time¹⁾⁻³⁾, but the interest has been heightened by the recent gauge invariant formulations [see, e.g., Refs. 4)-6) and references therein]. It appears that a description of interactions in this language will be provided shortly, too⁷⁾. Anyway, with a new way of looking at dual models, new ways to generalize them suggest themselves, and here we explore one of them.

2. - THE FREE THEORY

Let us begin by describing the light front gauge approach, since this yields models which are ghost-free by construction. In a suitable notation²⁾, the light front Hamiltonian for a string field is

$$H = \langle p^+ \psi | h | \psi \rangle \quad (1)$$

where

$$h = \frac{1}{2p^+} \sum_{p=1}^N (L_0^p - 1)$$

$$L_n^p = \frac{1}{2} \sum_{m=-\infty}^{\infty} : \alpha_{n-m}^{p,i} \alpha_m^{p,i} : \quad (2)$$

Here N equals one or two, if N = 2 the field obeys the constraint $(L_0^1 - L_0^2) | \psi \rangle = 0$, and the α^i 's are oscillators obeying

$$[\alpha_n^{\mu\nu}, \alpha_m^{\rho\sigma}] = \eta^{\mu\rho} \delta_{n,-m}^{\sigma\nu} \quad (3)$$

To prove that the Hamiltonian (1) is consistent with Lorentz invariance, one simply constructs the Lorentz generators and checks that they obey the Poincaré algebra. The result of this construction, valid in 26 dimensions only, is

$$J^{\mu\nu} = \langle p^+ \psi | j^{\mu\nu} | \psi \rangle \quad (4)$$

where

$$\begin{aligned} j^{ij} &= x^{[i} p^{j]} + i \sum_{p=1}^N \sum_{n=1}^{\infty} \frac{1}{n} \alpha_n^{\rho[i} \alpha_{-n}^{\rho j]} \\ j^{it} &= x^i p^+ - x^+ p^i \\ j^{it} &= x^+ p^i - x^i p^+ \\ j^{in} &= x^i p^+ - x^+ p^i - \frac{i}{p^+} \sum_{p=1}^N \sum_{n=1}^{\infty} \frac{1}{n} (\alpha_{-n}^{\rho i} L_n^{\rho} - L_{-n}^{\rho} \alpha_n^{\rho i}) \end{aligned} \quad (5)$$

The $j^{\mu\nu}$'s are usually derived as Noether generators of a certain two-dimensional field theory, the "string". Depending on the boundary conditions imposed on the string, we find two cases - open strings, yielding $N = 1$ in the above formulae, and closed strings, yielding $N = 2$. Now if we forget about the two-dimensional field theory, and take formulae (1)-(5) as our starting point, a generalization immediately suggests itself, since the above construction works equally well for any N provided we impose $(L_0^P - L_0^Q) |\psi\rangle = 0$ for all $p, q < N$. Examining the Hamiltonian, we find that the intercept of the leading Regge trajectory is N , i.e., these models contain massless states of spin N , where N is allowed to be an arbitrary positive integer.

A light front action is fine, but a covariant action is even better. In the simplest formulation of covariant string field theory available so far⁵⁾, the field theory action for open strings is simply

$$S = -\langle \psi | Q | \psi \rangle \quad (6)$$

where Q is the BRS-charge of the two-dimensional string action (nilpotent if $d = 26$) and ψ is a suitably constrained functional of the co-ordinates and the ghosts of the string. For closed string, Q is a sum of two identical expressions built out of two independent sets of oscillators and the field theory action is again given by (6), with only a very minor modification⁸⁾. This construction also can be generalized to arbitrary N .

In order to see clearly what is going on, let us consider the following action⁵⁾:

$$\begin{aligned}
 S = & \sum_{p=1}^N \left\{ -\frac{1}{N} \psi (L_0^p - N) \psi - \sum_{n=1}^{\infty} \left\{ \varphi^{pn} L_n^p \psi + \frac{n}{2} \varphi^{pn} \varphi^{pn} \right\} + \right. \\
 & + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left\{ \varphi^{pn} (L_0^p + n + m - 1) \varphi^{pm} - \left\{ \varphi^{pn} (L_n^p \varphi^{pm} + (2n+m) \varphi^{p(n+m)} \right\} \right\} + \\
 & + \sum_{q, p=1}^N \left\{ \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left\{ \frac{1}{2} \chi^{pqm} (L_0^p + L_0^q + n + m - 2) \chi^{qpn} - \chi^{pqm} L_m^q \varphi^{pn} \right\} \right\} \\
 & \quad \quad \quad q \neq p
 \end{aligned} \tag{7}$$

This is a truncated version of the action (6). There seems to be some disagreement in the literature as to whether the truncation is consistent, but this does not matter to us since all the actions in Refs. 4)-6) can be generalized in the same way. It is straightforward to check that the action (7) is invariant under

$$\begin{aligned}
 \delta \psi &= \sum_{p=1}^N \sum_{n=1}^{\infty} L_n^p \lambda^{pn} \\
 \delta \varphi^{pn} &= -2 (L_0^p + n - 1) \lambda^{pn} \\
 \delta \varphi^{pm} &= -L_m^p \lambda^{pn} - (2m + n) \lambda^{p(m+n)} \\
 \delta \chi^{pqm} &= -L_m^p \lambda^{qm}
 \end{aligned} \tag{8}$$

provided we impose the second class constraints

$$(L_0^p - L_0^q) \psi = 0 \tag{9}$$

and the concomitant constraints on the other fields.

Let us now examine the massless sector for the case $N = 3$. First we expand those fields which enter at this level in terms of component fields:

$$\begin{aligned}
 \psi &= A - i A^{\mu\nu} \alpha_{-1,\mu}^1 \alpha_{-1,\nu}^2 \alpha_{-1,\sigma}^3 + \dots \\
 \varphi^{11} &= -\varphi^{1\mu\nu} \alpha_{-1,\mu}^2 \alpha_{-1,\nu}^3 + \dots \quad \text{etc.} \\
 \chi^{121} &= i \chi^{12\mu} \alpha_{-1,\mu}^3 + \dots \quad \text{etc.}
 \end{aligned} \tag{10}$$

Then we find the following action for the massless fields (after eliminating the $\phi^{p\mu\nu}$ -fields, which are just auxiliary fields):

$$\begin{aligned} \mathcal{L}_0 = & \frac{1}{2} A_{\mu\nu\rho} \Box A^{\mu\nu\rho} - \chi_{\mu}^{12} \Box \chi^{\mu 12} - \chi_{\mu}^{13} \Box \chi^{\mu 13} - \chi_{\mu}^{23} \Box \chi^{\mu 23} + \\ & + \frac{1}{2} (\partial_{\mu} \chi_{\nu}^{12} + \partial_{\nu} \chi_{\mu}^{13} - \partial^{\sigma} A_{\sigma\mu\nu})^2 + \frac{1}{2} (\partial_{\mu} \chi_{\nu}^{21} + \partial_{\nu} \chi_{\mu}^{13} - \partial^{\sigma} A_{\sigma\mu\nu})^2 + \\ & + \frac{1}{2} (\partial_{\mu} \chi_{\nu}^{31} + \partial_{\nu} \chi_{\mu}^{32} - \partial^{\sigma} A_{\sigma\mu\nu})^2 \end{aligned} \quad (11)$$

If we now isolate the symmetric piece, $s_{\mu\nu\sigma}$, of the $A_{\mu\nu\sigma}$ -field, and perform field redefinitions according to

$$\begin{aligned} A_{\mu\nu\sigma} &= A'_{\mu\nu\sigma} + S_{\mu\nu\sigma} \\ S_{\mu\nu\sigma} &= \varphi_{\mu\nu\sigma} + \sum_{pq} \frac{1}{6} \eta_{\mu\nu} \eta_{\sigma}^{pq} \\ \chi_{\mu}^{pq} &= \eta_{\mu}^{pq} + \frac{1}{2} \varphi_{\mu}^{\alpha} \end{aligned} \quad (12)$$

we find that

$$\begin{aligned} \mathcal{L}_0 = & \frac{1}{2} \varphi_{\mu\nu\rho} \Box \varphi^{\mu\nu\rho} + \frac{3}{2} \partial^{\sigma} \varphi_{\mu\nu} \partial_{\sigma} \varphi^{\mu\nu} - 3 \partial^{\sigma} \varphi_{\sigma\mu\nu} \partial^{\mu} \varphi^{\nu\alpha} - \\ & - \frac{3}{2} \varphi_{\mu\alpha}^{\beta} \Box \varphi^{\mu\sigma}{}_{\beta} + \frac{3}{2} \partial_{\alpha} \varphi^{\alpha\beta} \partial_{\sigma} \varphi^{\sigma\delta}{}_{\beta} \\ & + \text{terms involving } A'_{\mu\nu\sigma} \text{ and } \eta_{\mu}^{pq} \text{ only} \end{aligned} \quad (13)$$

Neglecting surface terms - which, incidentally, are rather puzzling - this is the conventional action for a massless spin 3 field⁹⁾, as expected.

To summarize this section, we have seen that it is possible to construct ghost-free generalizations of string theories which contain massless particles of arbitrary integer spin. The generalization looks very natural from the field theoretic viewpoint, but is quite obscure from the string point of view, since we have lost the surface interpretation. We must now discuss whether this loss will prove a serious one when we come to the interacting theory.

3. - TOWARDS INTERACTIONS

The question of interactions is much harder. For field theories containing massless higher spins only, we know that three-point couplings exist¹⁰⁾, but the existence of higher-order couplings is doubtful. It is known that a sensible gauge algebra involving spin N only does not exist for $N > 2$ ¹¹⁾. However, for the purpose of building a string theory, higher-order couplings are unnecessary.

Essentially only cubic couplings are needed^{1)-3),7)}. So, a priori, the situation does not look so bad.

Let us take a look at the light front version of the cubic coupling for closed strings^{1),2)}:

$$H^{(3)} = \langle \psi_1 \langle \psi_2 \langle \psi_3 | V \rangle \rangle \rangle \quad (14)$$

where, up to a measure factor,

$$\begin{aligned} |V \rangle \rangle \rangle &= \\ &= e^{\frac{1}{2} p_2^2} \left\{ \sum_{r,s=1}^3 \sum_{p=1}^N \sum_{n,m} (\alpha_{-m}^{p(r)} N_{mn}^{rs} \alpha_{-n}^{(s)} + (p_1^r p_2^s - p_2^r p_1^s) N_m^r \alpha_{-m}^{p(s)}) \right\} |0\rangle_1 |0\rangle_2 |0\rangle_3 \end{aligned} \quad (15)$$

The N's are defined in terms of gamma-functions. Thus, the coupling for the N = 2 case is simply a direct product of two N = 1 couplings, and the generalization again suggests itself. However, things may not be that simple. The "cleanest" way of checking that (15) is indeed the correct answer would be to construct the Poincaré generators of the interacting theory and then check their algebra. Unfortunately, this calculation is extremely difficult and has never been done in its entirety [the most exhaustive discussion to date is given in Ref. 2)].

It is known, however, that strange things happen. Computing scattering amplitudes, one finds that (15) must be supplemented with certain quartic terms - unrelated to the Yang-Mills quartic terms - in the N = 1 case, while it appears to be the complete answer for N = 2^{1),2)}. The fact that N = 1 and N = 2 behave differently clearly calls for caution as far as generalization is concerned.

We have not tackled this problem in earnest, but some simpler things can be done. Thus, consider possible generalizations of the operator formalism for computing S-matrix amplitudes. We might try to define the M+2 tachyon amplitude corresponding to the Figure by

$$A_I = \langle 0, k_0 | V(k_1) D \Delta V(k_2) \dots D \Delta V(k_M) | 0, k_{M+1} \rangle \quad (16)$$

where

$$\begin{aligned}
 D &= \left[\sum_{p=1}^N (L_0^p - 1) \right]^{-1} \\
 \Delta &= \prod_{p=1}^{N-1} [2\pi \delta(L_0^p - L_0^{p+1})] \\
 V(k) &= \exp k \cdot \sum_{p=1}^N \sum_{n=1}^{\infty} \frac{a_n^p}{\sqrt{n}} \exp i k \cdot q \exp -k \cdot \sum_{p=1}^N \sum_{n=1}^{\infty} \frac{a_n^p}{\sqrt{n}} \quad (17)
 \end{aligned}$$

The propagator, as well as the constraint Δ , has been "derived" from the field theory, but the rest is of course pure guesswork. By straightforward manipulation¹²⁾ we find

$$\begin{aligned}
 A_I &= \lim_{\substack{r_0 \rightarrow \infty \\ r_1 \rightarrow 1 \\ \theta_p \rightarrow 0 \\ r_{M+1} \rightarrow 0}} \int_0^{2\pi} [r_i^{N-1} dr_i]_{i=2}^M \int_0^{2\pi} [d\theta_i \dots d\theta_M \delta(\theta_i + \dots + \theta_M)]_{i=2}^M r_0^{2N} \times \\
 &\quad \times \prod_{i=0}^{M+1} \prod_{p=1}^N (r_i e^{i\theta_{ip}} - r_j e^{i\theta_{jp}})^{k_i \cdot k_j} \quad j > i \quad (18)
 \end{aligned}$$

This expression has singularities when r_1 approaches 0 or 1. For all even N , we can extend the integration region for the r_i 's to infinity by adding expressions of the type (16) with all possible orderings of the vertex operators $V(k_i)$. In this way, Eq. (18) is seen to reduce to the usual dual model amplitudes for the cases $N = 1$ or 2.

Now, in the latter two cases, the amplitude is a particular form of a Möbius invariant expression¹²⁾. For $N > 2$, this is no longer true, and this is the root of a pathology which is easily seen. The amplitude (18) clearly has poles in both the $k_2 \cdot k_3$ -channel and the $k_2 \cdot k_4$ -channel (say). However, for $N > 2$, also the residue in the $k_2 \cdot k_3$ channel has poles in the $k_3 \cdot k_4$ channel. But no physical S-matrix can behave in this way, and therefore our amplitude is sick. For $N \leq 2$, this kind of behaviour never occurs (apart from simultaneous poles in all three channels $k_2 \cdot k_3$, $k_3 \cdot k_4$ and $k_2 \cdot k_4$).

Apart from Eq. (16), another obvious thing to try is

$$A = \int \prod_{i=a,b,c} [dz_i]_{i=1}^M |z_a - z_b|^n |z_b - z_c|^n |z_c - z_d|^n \prod_{p>q=1}^M |z_p - z_q|^{2k_p \cdot k_q} \quad (19)$$

and to interpret the z 's as quaternions for $n = 4$ and as octonions for $n = 8$. Then we do have Möbius invariance and a sensible singularity structure, and one might think that $n = 4$ corresponds to our $N = 4$. This is not so, however. In fact, by looking at the four-point amplitude and solving the integrals, one finds that Eq. (19) actually has the spectrum of closed strings not only for $n = 2$ but also for $n = 4$ and 8, with a displaced intercept in the latter two cases. The intercept can easily be moved back again through the usual modifications¹²⁾.

So, what are the conclusions? Certainly not definitive, since a conclusive answer can be obtained only by actually doing the very laborious field theory calculations, but it seems unlikely that interacting models can be obtained for $n > 2$, at least if such models are supposed to resemble the conventional ones. We believe that the existence of the free "N-string" has some interest anyway - the classification of string-like models has moved one step closer to the classification of representations of the Poincaré group in the critical dimension.

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