$M3$ transition in the two-rotor model

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(Received 2 April 1986)

It is shown that the two-rotor model, like the interacting boson model, predicts an appreciable magnetic octupole strength for the $J^\pi=3^+$ member of the collective $K^\pi=1^+$ band, which was predicted and found experimentally to be mainly excited by the orbital component of the magnetic dipole operator. The importance of the present prediction for a complete understanding of the nature of this magnetic mode is emphasized.

After its experimental discovery,¹ the orbital magnetic dipole collective excitation, known as two-rotor or scissor mode, has been the object of rather intensive experimental as well as theoretical investigations.² The mode, first predicted by the two-rotor model (TRM)^{3,4} and independently in the context of interacting boson model (IBM-2)^{5,6} and random phase approximation (RPA),^{7,8} is characterized by an appreciable $M1$ strength¹ and a quite small $E2$ transition probability.⁹

In the TRM, the $M1$ strength represents the signature of the lowest energy mode of a rotational oscillation of protons against neutrons, considered as two rigid rotors, about a common axis orthogonal to their symmetry axes. A $K^\pi=1^+$ rotational band is thereby generated with the $M1$ and $E2$ strengths concentrated on its first $J^\pi=1^+$ and $J^\pi=2^+$ members, respectively. The characterization of the $M1$ mode as two-rotor mode is further and decisively supported by the close similarity between the TR and the experimental $M1$ form factors.^{1,10}

The TRM estimates for both strengths are, however, much larger than the experimental values. The discrepancy is to be ascribed to the semiclassical nature of the model, where too many degrees of freedom are frozen because of the assumed rigidity of the two rotors and can be removed to a large extent by relating the TRM to the schematic RPA or, better, to the IBM-2.¹¹

In RPA it is easy to take into account two major effects, which reduce drastically the strengths. The coupling of the low frequency TR mode to the high frequency isovector quadrupole excitation depresses dramatically the $E2$ strength down to a value close to the experimental one⁹ and contributes to the reduction of the $M1$ strength by lowering the frequency of the mode.⁸ Another important reduction of the $M1$ strength is brought about by the renormalization of the rigid moment of inertia induced by the pairing correlations.¹² The important role of pairing in renormalizing the $M1$ strength has also been shown in a recent shell model analysis.¹³

Once these major corrections have been made the RPA $M1$ strength results to be about three times the TR value but still about 2.5 times the experimental integrated strength, ascertained so far to be between 2 and $2.5 \mu_N^2$ in ¹⁵⁶Gd.¹⁴ The reason for this remaining discrepancy is not clear as yet. It has been suggested in the shell model analysis¹³ that the method adopted for projecting out

states of good angular momenta might be relevant for the reduction of the strength value.

A more satisfactory agreement with experiments can be obtained in the IBM-2 framework. In IBM-2 the mode emerges as F -spin mixed symmetry state. The calculations, performed by adjusting the parameters of the so called Majorana force to reproduce the experimental energy of the mode and by exploiting the shell-model structure of the S and D bosons, give results close to the experiments for the $M1$ and $E2$ strengths as well as for the $M1$ form factor.^{1,9}

The TRM is also closely related to IBM-2.¹¹ The TRM Hamiltonian, apart from a Coriolis term, has exactly the same form as the classical IBM-2 Hamiltonian.¹⁵ Such a close connection is confirmed by the similar behavior of the TR and IBM-2 $M1$ form factors.¹⁰ In IBM-2, however, the mode is generated by correlations of valence neutron pairs versus valence proton pairs and is therefore much less collective than in the TRM, consistent with experiments. There are other limitations of the TRM versus IBM which might be worth mentioning for the present purposes. Apart from a coupling term which turned out to be negligible, the TRM Hamiltonian is strictly harmonic⁴ and as such should be adequate for the description of the lowest $K^\pi=1^+$ mode but not for higher energy bands, in general quite sensitive to anharmonic effects. Although its Hamiltonian contains the total proton and neutron angular momenta, which include the spin, the TRM in its present form allows only for contributions to the magnetic strengths coming from the convection current.

The experimental analysis carried out so far have indeed confirmed that the $M1$ strength is little fragmented and gets its main contribution from the orbital motion. What about the high spin members of the $K^\pi=1^+$ band?

In a recent IBM-2 calculation¹⁶ it was shown that the low-lying $J^\pi=3^+$ states with mixed symmetry in neutron and proton pairs can be strongly coupled to the ground state by the F -spin vector part of the $M3$ operator with a strength mostly concentrated on the state belonging to the $K^\pi=1^+$ band. It was found, however, that unlike for the $M1$ transition, the spin contribution is comparable to the orbital one.

These results cannot be considered conclusive for several reasons. Microscopic calculations¹⁷ seem to indicate that the fermion-boson mapping leads to a boson $M3$ operator

with a more involved structure than the one used in Ref. 16. Reliable shell model estimates of the gyromagnetic ratios entering into the $M3$ operator are not available. Those based on generalized seniority indicate that in some regions, like the rare earth nuclei, spin and orbital contributions are comparable, while in Pd and Pt isotopes the orbital contribution is dominant.¹⁸ The fragmentation of the strength among states belonging to different bands depends strongly on the choice of the Majorana parameters.¹⁹ What seems to be established is the increasing importance of the spin contribution with increasing multipolarity.^{20,21}

In the present paper it is shown that the TRM itself predicts an appreciable $M3$ strength for the $J^\pi=3^+$ member of the lowest $K^\pi=1^+$ band. Such a transition is naturally promoted by the coupling of the rotational oscillation to the quadrupole deformation, as one can infer by inspecting the expressions here obtained for the TR operators.

Because of the harmonic character of the model, the strength is concentrated only into the mentioned $J^\pi=3^+$ state. The IBM calculations^{16,18} seem to indicate that this is indeed the most collective state.

Although the spin is not included, it is not excluded that its contribution could be incorporated into the $M3$ operator by an appropriate effective gyromagnetic ratio as in IBM, where the spin contributes to the gyromagnetic ratio without altering the form of the boson operator or the way bosons are correlated. The explicit inclusion of the spin, in principle already incorporated into the proton and neutron total angular momenta, would not change the correlation characterizing the mode in any case.

The TRM, though with limitations, should therefore represent a simple and transparent tool for a preliminary and systematic analysis of the magnetic properties of the $K^\pi=1^+$ collective mode. It provides, in fact, simple expressions for $M3$ strength and form factor, both derived by a straightforward procedure. The most meaningful information is provided by the form factor. This, if nothing else, should give an unambiguous indication of how the proton-neutron relative orbital motion affects these magnetic properties. It is at any rate interesting to test the validity of such a simple model by exploring if the TR correlation characterizing the lowest spin state of the $K^\pi=1^+$ band is still dominant at higher spins.

The procedure adopted is close to the one followed in Ref. 4 for the $M1$ form factor.

The most appropriate expression of the $M\lambda$ operator for our purposes is

$$m(M\lambda, \mu) = \frac{1}{(\lambda+1)mc} \int d\mathbf{r} \rho_p(\mathbf{r}) \mathbf{l}_p \cdot \nabla r^\lambda y_{\lambda\mu}(\hat{\mathbf{r}}), \quad (1)$$

where m is the proton mass, $\mathbf{l}_p$ the proton angular momentum, and ρ_p the charge density normalized to the total number of protons to account for the difference in the proton and neutron numbers.

The TR operator can be easily obtained by making use of the classical relations

$$\begin{aligned} \mathbf{l}_p &= m\mathbf{r}(\mathbf{r} \cdot \boldsymbol{\omega}_p) - mr^2\boldsymbol{\omega}_p, \\ \mathbf{J} &= \mathcal{J}_p\boldsymbol{\omega}_p, \end{aligned} \quad (2)$$

where $\mathcal{J}_p$, $\boldsymbol{\omega}_p$, and $\mathbf{J}_p$ are, respectively, the moment of inertia, the angular velocity, and the angular momentum of the proton rigid rotor. After some straightforward algebra, one obtains

$$m(M\lambda, \mu) = -\frac{1}{2(\lambda+1)c} \frac{S_x}{\mathcal{J}_p} \times \int d\mathbf{r} \rho_p(\mathbf{r}) (r^2 \partial_x - x\mathbf{r} \cdot \nabla) r^\lambda Y_{\lambda\mu}(\hat{\mathbf{r}}), \quad (3)$$

where $S_x = (\mathcal{J}_p - \mathcal{J}_n)_x$ is the generator of the rotation of protons against neutrons about the x axis chosen to be their common rotational axis.

For $\lambda=1$ The known $M1$ operator⁴ is obtained;

$$m(M1, \mu) = \delta_{\mu, \pm 1} \sqrt{3/32\pi} S_x \mu_N.$$

The corresponding strength is easily obtained;

$$B(M1) = \frac{3}{16\pi} \mathcal{J} \omega \mu_N^2, \quad (4)$$

where $\mathcal{J} = \mathcal{J}_p + \mathcal{J}_n$, and ω is the energy excitation of the mode. It is worth noticing that this expression coincides with the one derived in schematic RPA.⁸

For $\lambda > 1$ the integral in Eq. (3) vanishes unless the density deformation is taken explicitly into account. A deformed density appropriate for an axially symmetric shape was then considered

$$\rho_p(\mathbf{r}) = \rho_p \{r [1 - \beta Y_{20}(\hat{\mathbf{r}})]\},$$

where β is related to the commonly used deformation parameter δ by $\beta = \sqrt{4\pi/5} \frac{2}{3} \delta$.

Expanding up to the second power in β , one finds that the only nonvanishing component of the $M3$ operator is the one with $\mu=1$. This couples the ground state to the lowest $J^\pi=3^+$ state. The others, belonging to higher energy bands, are decoupled as a consequence of the harmonic character of the model. The expression of the reduced matrix element of the nonvanishing component is

$$\begin{aligned} \langle n=0, K^\pi=1^+, J^\pi=3^+ | m(M3) | 0 \rangle \\ = (1 + \frac{4}{3}\delta)^{\frac{1}{5}} \sqrt{14/3\pi} \delta m Z \frac{\langle r^4 \rangle}{\mathcal{J}_p} \sqrt{\mathcal{J}_r \omega}, \end{aligned} \quad (5)$$

where $\mathcal{J}_r = (4\mathcal{J}_p\mathcal{J}_n)/\mathcal{J}$. The Z dependence can be singled out easily. It is obtained for the strength

$$\begin{aligned} B(M3, 0 \rightarrow K^\pi=1^+, J^\pi=3^+) \\ = (1 + \frac{4}{3}\delta)^2 \frac{14}{75\pi} \delta^2 m^2 A \langle r^4 \rangle^2 \sqrt{4NZ} \frac{\omega}{\mathcal{J}}. \end{aligned} \quad (6)$$

Clearly, like for the dipole case, the $M3$ strength is practically insensitive to the neutron excess. The hexadecapole term in ρ_p was also considered, but its contribution turned out to be negligible.

Numerical estimates were made by using the rigid body value for $\mathcal{J}$ and the standard expression for $\langle r^n \rangle$.²²

If the TRM expression⁴ is used for the excitation energy, $\omega \approx 42\delta A^{-1/6}$, one obtains

$$B(M3, 0^+ \rightarrow 3^+) \approx 0.086 (1 + \frac{4}{3}\delta)^2 \delta^3 A^{17/6} \mu_N^2 \text{ fm}^4. \quad (7)$$

If the empirical energy¹⁰ is used instead, $\omega \approx 66\delta A^{-1/3}$,

the strength results to be

$$B(M3, 0^+ \rightarrow 3^+) \approx 0.134(1 + \frac{4}{3}\delta)^2 \delta^3 A^{8/3} \mu_N^2 \text{ fm}^4. \quad (8)$$

For $A = 156$ and $\delta \approx 0.26$, the two expressions give, respectively, $B(M3) \approx 0.4 \mu_N^2 b^2$ and $B(M3) \approx 0.3 \mu_N^2 b^2$.

It is not obvious how much these estimates would be affected by removing the rigid body assumption. Equation (4) shows that the $M1$ strength can be easily corrected by renormalizing $\mathcal{J}$ and ω , which can be done by taking into account the pairing correlations and the coupling to the high frequency mode as discussed previously. For the $M3$ case it has just been shown that it is easy to account for the corrections due to the frequency. It is not clear, instead, how the pairing renormalization of $\mathcal{J}$, which would roughly double the estimated value of $B(M3)$, is counterbalanced by the renormalization of the term appearing in the numerator of Eq. (6). It should be noted, however, that while the $M1$ operator contains the whole proton density, the contribution to the $M3$ strength comes only from the first and second order terms of the expansion in the deformation parameter, the leading term vanishing. It is therefore reasonable to expect that the $M3$ strength is not necessarily overestimated, at least not as much as the $M1$ strength. The present estimates should otherwise be reduced at least by a factor ~ 2 because of pairing. A much stronger renormalization would lead to a strength value which probably would be difficult to observe.

For a more reliable and complete characterization of the state the $M3$ form factor was computed, though only in plane-wave Born approximation (PWBA). The operator considered is quite similar to the one defined by Eq. (1):²³

$$T(M\lambda, \mu) = \frac{i}{[\lambda(\lambda+1)]^{1/2} mc} \int d\mathbf{r} \rho_p(\mathbf{r}) \mathbf{l}_p \cdot \nabla \times j_\lambda(q\mathbf{r}) Y_{\lambda\mu}(\hat{\mathbf{r}}), \quad (9)$$

where $j_\lambda(x)$ is the spherical Bessel function and ρ_p is again normalized to the proton number. This operator can be written in a form quite similar to Eq. (3), with $j_\lambda(qr)$ instead of r^λ , by making use of Eq. (2). The following result can then be used:

$$\begin{aligned} \partial_i [j_\lambda(qr) Y_{\lambda\mu}(\hat{\mathbf{r}})] &= \partial_i \left[\left(\frac{j_\lambda(qr)}{r^\lambda} \right) r^\lambda Y_{\lambda\mu}(\hat{\mathbf{r}}) \right] \\ &= \frac{x_i}{r} \frac{d}{dr} \left(\frac{j_\lambda(qr)}{r^\lambda} \right) r^\lambda Y_{\lambda\mu}(\hat{\mathbf{r}}) \\ &\quad + \frac{j_i(qr)}{r^\lambda} \partial_i [r^\lambda Y_{\lambda\mu}(\hat{\mathbf{r}})]. \end{aligned}$$

Since the first term, inserted in Eq. (3), clearly gives a vanishing contribution, the procedure adopted for deriving the $M3$ strength can be repeated. The following expression is thereby obtained:

$$\begin{aligned} |F(q)|^2 &= |\langle n=0, K^\pi=1^+, J^\pi=3^+ | T(M3) | 0 \rangle|^2 \\ &= \frac{32\pi}{7.15^2} \delta^2 \sqrt{4NZ/A} \frac{\omega}{j} I(q)^2, \end{aligned} \quad (10)$$

where

$$I(q) = q \int dr r^4 \rho(r) \left[(1 + \frac{1}{2}\delta) j_2(qr) + \frac{1}{6}\delta q r j_1(qr) \right],$$

and ρ is the total spherical nuclear density normalized to A . The integration can be carried out explicitly for a hard sphere density distribution, or numerically, but straightforwardly, for the more realistic Fermi distribution.

The form factors corresponding to the two distributions are shown in Fig. 1. They are rather close for small momentum transfer but differ considerably at high q 's.

A detailed comparison with the IBM form factor report-

FIG. 1. TRM $M3$ form factor vs q in fm^{-1} for ^{156}Gd using a Fermi density distribution (full line) and a hard sphere distribution (dashed line).

ed in Ref. 16 cannot be made since the present form factor is plotted vs q instead of q_{eff} . One can notice, however, that the two form factors can be compatible only at low q 's. Around $q \sim 1 \text{ fm}^{-1}$ they differ quite strongly. The TRM form factor presents, in fact, a minimum which is

partially filled in the IBM case.

An experimental analysis capable of testing the importance of the TRM contribution to the $M3$ strength needs a distorted-wave Born approximation (DWBA) calculation of the form factor. Such an analysis is under way.²⁴

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