ASPECTS OF THE SUPER-UNIFICATION OF STRONG, ELECTROWEAK AND
GRAVITATIONAL INTERACTIONS

John Ellis, K. Enqvist^{*)}, D.V. Nanopoulos and F. Zwirner⁺⁾

CERN - Geneva

A B S T R A C T

We develop a strategy for extracting low energy phenomenological four-dimensional physics from the superstring. We discuss supersymmetry and gauge symmetry breaking, emphasizing key ingredients in the construction of a realistic model based on Calabi-Yau compactification. The incorporation of a no-scale mechanism for the dynamical generation of the electroweak gauge hierarchy imposes a unique choice of the gauge group $SU(3) \times SU(2) \times U(1)^2$, an almost unique set of matter fields and of Yukawa couplings. Our phenomenological analysis of this model includes the derivation of bounds on the mass of the new neutral gauge boson from the Z^0 boson mass, low energy neutral currents, and cosmology. We calculate the ratios of sparticle masses and give estimates of their magnitudes. These are based on detailed dynamical calculations demonstrating the feasibility of weak gauge symmetry breaking, made possible by radiative corrections to supersymmetry breaking initiated by a gaugino mass.

^{*)} Address after 1 January 1986: Research Institute for Theoretical Physics, University of Helsinki, Finland.

⁺⁾ On leave from International School for Advanced Studies, Trieste, and Istituto Nazionale di Fisica Nucleare, Padova, Italy.

1. INTRODUCTION

Old problems are cast in a new light by the success of the Standard Model. Empirical approaches focus on three problems beyond the Standard Model: those of unification, flavour, and the hierarchy of different mass scales in physics. The hierarchy problem is the only one whose formulation implies that some new physics must appear before 1 TeV. This new physics is surely supersymmetry [1]. The observation of parity violation at low energies suggests that physics is chiral, and argues strongly against extended $N > 1$ supersymmetry and in favour of simple $N = 1$ supersymmetry. The appropriate theoretical framework embracing simple supersymmetry is $N = 1$ supergravity in four dimensions $d = 4$ [2], which is therefore the context for most modern model-building [3, 4]. While supersymmetry and supergravity protected the hierarchy, they did not, in general, explain it. The next step was no-scale supergravity models [5] in which the small mass scales of physics were undetermined at the tree level [6] and could be fixed by radiative corrections which generated a natural hierarchy [5, 7].

Clearly four-dimensional $N = 1$ supergravity is not the end of the story. One could imagine going beyond it to extended $N > 1$ supersymmetry or to more dimensions $D > 4$. Extended supersymmetry runs into the conflict with chirality mentioned earlier, so we are left with $D > 4$ dimensions. In fact, most higher dimensional theories also conserve parity, but one can incorporate chirality by introducing elementary gauge fields [8] and going to an even number of dimensions. The largest even number of dimensions in which $N = 1$ supersymmetry can be incorporated is $D = 10$, so interest naturally focuses on $N = 1$, $D = 10$ Yang–Mills supergravity [9]. Anomalies are a danger for chiral theories, but they can be compensated by a Wess–Zumino term if the Yang–Mills gauge group is $O(32)$ or $E_8 \times E_8$ [10]. Of these possible choices, only $E_8 \times E_8$ gives an acceptable chiral fermion spectrum [11]. These last two results were derived from work on the superstring [12].

Thus we make contact with a less empirical, more Cartesian approach to physics. Ultimately one seeks a calculable quantum theory of gravity. Point field theories of gravity all seem to be unrenormalizable, so one goes to theories of extended objects. Certain string theories contain a massless spin 2 mode which could be identified with the graviton [13], and are believed to be finite. This property only holds in a critical number of dimensions: $D = 26$ for the bosonic string, $D = 10$ for the superstring [14]. The $D = 10$ superstring is actually finite to one loop (and possibly to all orders) and free of anomalies only if the internal gauge symmetry is chosen to be $O(32)$ or $E_8 \times E_8$ [10, 15]. The low-energy limit of the superstring is just the $D = 10$, $N = 1$ supergravity previously arrived at empirically. Much of the phenomenology [11, 16–22] now perpetrated in the name of the superstring actually only exploits its massless modes and their couplings, and therefore follows from the supergravity structure alone. However, the superstring has revealed to us [10] various terms which were not included in the original $D = 10$, $N = 1$ supergravity Lagrangian, but play an essential role in selecting the ground state on which the six surplus dimensions are compactified [11], and, therefore, in the structure of the resulting effective $N = 1$, $d = 4$ supergravity theory. These include the addition to the field strength $H_{\mu\nu\rho}$ for the antisymmetric tensor field $B_{\mu\nu}$ of a Lorentz Chern–Simons term [10]. When supersymmetrized, this gives R^2 terms [23] which are of just the right form to avoid ghosts [24] and are vital in the compactification from $D = 10$ to $d = 4$ [11]. Quantum corrections in the superstring also provide a Wess–Zumino term [10] which cancels the anomalies of the $D = 10$ Yang–Mills supergravity Lagrangian.

We follow ref. 11 in assuming that the compactification from $D = 10$ to $d = 4$ must respect supersymmetry, at least as a first approximation, and hence assume that it is on a Calabi–Yau manifold [25]. In this case the compactification manifold K has $SU(3)$ holonomy and the gauge field strength also has non-vanishing components: $F \wedge F \propto R \wedge R$ and the maximal remaining gauge groups in $d = 4$ are $O(32) \rightarrow O(26) \times U(1)$ or $E_8 \times E_8 \rightarrow E_6 \times E_8$. The former possibility is discarded because it cannot yield chiral fermions [11], while in the latter case the light spectrum contains $N_g \times 27$ chiral supermultiplets with the quantum numbers of the fundamental representation of E_6 , with $N_g = \frac{1}{2} |\chi|$, where χ is the Euler characteristic of K , plus possible additional light gauge singlet supermultiplets. If the compactification manifold is not simply connected, the Euler characteristic may be reduced [11] to give an acceptable number N_g of ‘generations’. Moreover, in this case Wilson loops can break E_6 down to a subgroup of rank ≥ 5 [11, 16], and possibly break the hidden E_8 as well [18]. In this case E_6 is always broken in four dimensions, and although all the unbroken gauge subgroup couplings are equal [16] at a grand unification scale close to the compactification scale [26, 20], the Yukawa couplings of the $27 \times N_g$ gauge non-singlet fields do not respect E_6 Clebsch–Gordan relations [27]. This may enable one to avoid such disasters as rapid proton decay and large neutrino masses.

There is considerable ambiguity in the choice of the Calabi–Yau manifold, and later in this paper we will give phenomenological arguments reducing this in several ways. In particular, we will argue that the low-energy subgroup of E_6 cannot have rank 6, and must be $SU(3)_C \times SU(2)_L \times U(1) \times U(1)$. Also, we argue that in this case there are no additional light gauge non-singlet supermultiplets beyond the N_g complete 27 ‘generations’. We also show that there is an almost unique pattern of Yukawa couplings allowed to be non-zero, which may be imposed by topological considerations or by discrete symmetries. Within the context of this almost unique low-energy theory abstracted from the superstring, we give a general phenomenological analysis of bounds on the new neutral current [21, 28–31] and gauge boson mass. We also calculate the scalar and spin- $1/2$ fermion mass matrices including a general analysis of soft SUSY breaking terms and of D-terms [21].

The most substantial aspect of our calculations is a detailed investigation of scenarios [5, 20, 21] for generating the weak gauge hierarchy $m_W/M_P \ll 1$ using the idea of dimensional transmutation:

$$m_W = M_P \exp \left[- \frac{O(1)}{\alpha} \right] \quad (1.1)$$

Two such scenarios are proposed: one [21] is a variant of the no-scale models proposed previously [5], in which a gauge singlet chiral superfield is left undetermined by dynamics at the Planck scale, and is determined along with gauge non-singlet Higgs vacuum expectation values by radiative corrections at the scale of m_W . The other scenario [21, 32] is a hybrid dimensional transmutation model in which all gauge singlet superfields are regarded as fixed at the Planck scale, while gauge non-singlet Higgs vacuum expectation values and the observable sparticle masses are dynamically determined by radiative corrections to have the form (1.1). We show that models of this type are viable for $m_t \gtrsim 30$ GeV [31], and, although the absolute scale of observable sparticle masses depends on unknown Yukawa coupling parameters, we exhibit correlations between the scale of SUSY breaking, the mass of the new neutral gauge boson, and other observable particle masses.

The structure of this paper is as follows. In Section 2 we introduce our general theoretical framework, starting with notation in Section 2.1, the origin of supersymmetry breaking in Section 2.2, and its communication to the observable sector in Section 2.3. We stress the leading role played by the gaugino masses [20, 21]. Section 2.4 describes our no-scale and hybrid dimensional transmutation strategies in more detail. Section 3 derives phenomenological constraints on models, starting with the motivation of a unique gauge group in Section 3.1. Section 3.2 argues for a unique set of low-mass matter fields, while Section 3.3 derives a two-fold ambiguous set of Yukawa couplings [33]. Section 3.4 derives phenomenological bounds on neutral current parameters and the mass of the second neutral gauge boson coming from its non-observation at the CERN $\bar{p}p$ Collider [21, 29], the Z^0 mass [31], low-energy neutral current measurements [21, 28], and primordial nucleosynthesis [30]. Section 4 contains the bulk of our detailed calculations. Section 4.1 discusses general features of the effective low-energy potential and mass matrices. Section 4.2 discusses the evolution of couplings and minimization of the effective scalar potential. Section 4.3 discusses and discards the possibility that the gaugino masses seeding SUSY breaking in the observable sector are non-universal at the grand unification scale. Section 4.4 presents the results of no-scale model calculations, concentrating on cases with a large extra gauge boson mass. Section 4.5 presents hybrid dimensional transmutation models consistent with all known phenomenological constraints. Finally, Section 5 contains our conclusions and discusses open problems.

2. GENERAL THEORETICAL FRAMEWORK

2.1 Notation

In this section we set up the effective four-dimensional supergravity formalism motivated by compactification from the superstring or $D = 10$ supergravity. We presume that this compactification is supersymmetric, and takes place on some as yet unspecified Calabi-Yau manifold K [11]. As our analysis develops, we will find a number of important constraints on the properties of this manifold of compactification. The effective $d = 4$ supergravity so obtained contains at least two gauge singlet chiral superfields S, T [19], as well as an unspecified number N_g of sets of 27 gauge non-singlet superfields ϕ^i with the quantum numbers^{*)} of the fundamental $\underline{27}$ representation of E_6 , and at least one set of 54 gauge non-singlet fields $\theta^j, \bar{\theta}_j$ with the quantum numbers of a $\underline{27} \oplus \overline{27}$ of E_6 [11]. These are all zero modes on the Calabi-Yau manifold K and $N_g = \frac{1}{2} |\chi(K)|$. As we will see in Section 3.2, it is only in exceptional circumstances that any of the non-singlet $(\theta^j, \bar{\theta}_j)$ appear as low-mass fields, so we will neglect them in what follows. In general the $d = 4$ supergravity theory might contain additional gauge singlet fields as well, but they probably have masses $O(M_P)$ [34], so we restrict our attention here to the gauge singlets S and T , and to the gauge non-singlets ϕ^i .

The real parts S_R and T_R of S and T are different combinations of the dilaton φ already present in the $D = 10$ supergravity theory, and of the ‘compacton’ which scales the six dimensions of K ^{**)} :

$$g_{IJ} = e^{\sigma} g_{IJ}^{(0)}, \quad (2.1)$$

where $g_{IJ}^{(0)}$ is a reference metric normalized so that

^{*)} Lower case latin indices $i, j, \dots$ denote internal gauged degrees of freedom.

^{**)} Upper case latin indices $I, J, \dots$ denote the 6 compactified coordinates y_I .

$$\int_K d^6 y [-\det g^{(0)}]^{1/2} = M_P^{-6} . \quad (2.2)$$

We have

$$S_R = e^{3\sigma} \varphi^{-3/4} , \quad (2.3a)$$

$$T_R = e^{\sigma} \varphi^{3/4} + \phi^i \phi_i^* \quad (2.3b)$$

and

$$S_I = -\frac{2b_0}{3} \alpha , \quad (2.4a)$$

with b_0 the coefficient of the one-loop β -function for the unbroken subgroup of E_8 ,

$$T_I = a , \quad (2.4b)$$

where a and α are pseudoscalar fields parametrizing relevant components^{*)} of the $D = 10$ antisymmetric tensor field $B_{\alpha\beta}$:

$$\varphi^{-3/2} e^{6\sigma} H_{\mu\nu\rho} = -\frac{\sqrt{2} b_0}{9} \epsilon_{\mu\nu\rho\sigma} \partial^\sigma \alpha \Big|_{\mu,\nu,\rho,\sigma=1,2,3,4} \quad (2.5a)$$

$$B_{IJ} = -\frac{1}{\sqrt{2}} \epsilon_{IJ} a \Big|_{I,J=5,6,\dots,10} \quad (2.5b)$$

Thus our notation is essentially that of ref. 19: the fields defined in ref. 26 are related to ours by

$$X = T, \quad Y = S . \quad (2.6)$$

The Kähler potential $\mathcal{G}$ for the chiral fields (S, T, ϕ^i) has a remarkable $[SU(1,1)/U(1)] \otimes [SU(N,1)/SU(N) \times U(1)]$ structure [6, 5, 7]:

$$\mathcal{G} = -\ln(S+S^*) - 3\ln(T+T^* - 2\phi^i \phi_i^*) + \ln |P(\phi^i)|^2 , \quad (2.7)$$

where $P(\phi^i)$ is a superpotential obtained from the projections of $D = 10$ gauge couplings on to zero modes in four dimensions [27]. Note the separation of the S and (T, ϕ^i) sectors. The $SU(N,1)/SU(N) \times U(1)$ structure of the latter has a simple origin [20]: Calabi-Yau compactification to $d = 4$ maintaining supersymmetry enforces a vanishing cosmological constant $\Lambda = 0$ at the tree level [11]. The property $\Lambda = 0$ holds for a continuum of values of the field variables ϕ^i which parametrize zero modes

^{*)} Greek indices $\alpha, \beta, \dots$ denote 10-dimensional coordinates, while $\mu, \nu, \dots$ denote the 4 uncompactified coordinates x_μ .

corresponding to distortions of the Calabi-Yau manifold K . The unique [35] $d = 4$ supergravity manifolds which guarantee $\Lambda = 0$ for a range of field values are the $SU(N,1)/SU(N) \times U(1)$ models [7].

In addition to the Kähler potential (2.7), the characterization of the $d = 4$ supergravity theory requires [2] specification of a symmetric chiral function f_{ab} which determines the kinetic terms of the four-dimensional gauge superfields W^a . Calabi-Yau compactification yields [19]

$$f_{ab} = \delta_{ab} S \quad (2.8)$$

at the tree level. The universality expressed in (2.8) is a consequence of string vertex properties [16]. Since the $d = 4$ gauge coupling is given by

$$g^2 = \frac{1}{\text{Re } f} \quad : \quad g^2 = \frac{1}{S_R} \text{ in leading order} \quad (2.9)$$

and loop diagrams renormalize g , the simple expression (2.8) will be renormalized by the matter fields in the observable sector [20]. It should be emphasized that the forms (2.7) and (2.8) remain unchanged in the presence of the Wess-Zumino terms needed for anomaly cancellation in the $N = 1, D = 10$ supergravity point field theory [36]. Thus, S_I remains the ‘axion’, that solves the strong CP problem, while T_I remains the ‘plation’ that may solve the cosmological constant problem [37].

So far we have not specified the gauge group. It is $E_8 \times E_8$ in the underlying superstring or $D = 10$ supergravity theory, broken [16] to some subgroup of $E_6 \times E_8$ in $d = 4$. E_6 is the maximal subgroup commuting with the $SU(3)$ subgroup of E_8 corresponding to the holonomy group of Calabi-Yau manifolds [11]. If the manifold K is not simply connected, and is obtained from a covering $\tilde{K}$ with discrete symmetry group G : $K = \tilde{K}/G$, then $E_6 \times E_8$ can be broken by Wilson loops $U = \langle 0 | P \exp \oint A_I^\alpha dx^I | 0 \rangle$ around the holes in K , which represent G as a subgroup U_G in the space of $E_6 \times E_8$ matrices [16]. The surviving low-energy group V is that commuting with U_G : $[V, U_G] = 0$. Note that the universality (2.8) of tree level gauge couplings is maintained [16] for all generators in V . The possible patterns of Wilson loop breakings of E_6 have been classified [16], and in Section 3.1 we will select uniquely the maximal breaking, arguing on the basis of the no-scale or dimensional transmutation scenarios [5, 20, 21] introduced in Section 2.4, which exploit the flatness of the Kähler potential $\mathcal{G}$ (2.7). A potential role [18] for E_8 breaking is discussed in Section 2.2.

2.2 The origin of supersymmetry breaking

Calabi-Yau compactification was motivated by the requirement of SUSY in $d = 4$ [11]. A natural mechanism for spontaneous SUSY breaking is offered by the condensation of gauginos in the hidden sector [17]. In this section we review how the desired condensation can occur [18], and discuss the form of local SUSY breaking gravitino mass $m_{3/2}$ that it gives.

Inspection of the supersymmetry transformation laws for the spin- $1/2$ Majorana fermion λ and for the additional components ψ_I of the gravitino in the $D = 10$ supergravity theory reveals [17] terms

$$\delta\lambda, \delta\psi_I \propto \epsilon (\bar{\chi} \Gamma_{IJK} \chi) \epsilon, \quad (2.10)$$

where the χ are $E_8 \times E_8$ gauginos, and Γ_{IJK} is a totally antisymmetric product of three $D = 10$ Dirac matrices. If $\langle 0 | \text{Tr}(\bar{\chi} \Gamma_{IJK} \chi) | 0 \rangle \neq 0$, then the transformation laws (2.10) contain pieces linear in ϵ , a signature that λ and ψ_I are pieces of a Goldstone fermion whose absorption by the gravitino gives $m_{3/2} \neq 0$. Condensation of gauginos in the observable E_6 sector, if not forbidden by massless chiral multiplets, cannot give a phenomenologically interesting value of $m_{3/2}$, but condensation in the E_8 sector can.

If there are only two [18] four-dimensional gaugino zero modes χ_a , they give a condensate of the form

$$\langle 0 | \bar{\chi}_a \Gamma_{IJK} \chi_a | 0 \rangle = A \epsilon_{IJK}, \quad (2.11)$$

which has the desired [17] effect. Gaugino zero modes appear in topologically non-trivial background gauge field configurations, of which the best-known is the instanton [38] which was first found in an S^4 compactification of R^4 . Instantons have topological number 1 leading to 60 gaugino zero modes if E_8 is unbroken. This would generate $\langle 0 | \text{Tr}(\chi^{60}) | 0 \rangle \neq 0$, which is not exactly what we want [18] (2.10, 2.11). If one imposes quasi-periodic T^4 toroidal boundary conditions on R^4 , one finds additional non-trivial background gauge configurations, the torons [39] with topological number k/N in $SU(N)$, where k is an integer $< N$. Torons do not exist in E_8 because $\Pi_1(E_8) = 0$. If E_8 is broken to some $SU(N)$ subgroup, instantons will have $2N$ gaugino zero modes, again not what we want, while the lowest $k = 1$ toron gives 2 gaugino zero modes as desired [18]. These are the supersymmetric transforms of the toron fields themselves, namely the torinos. There are no light matter fields when E_8 is broken to $SU(N) \times \dots$ by the Wilson loop mechanism, since all components of the adjoint $\underline{248}$ of E_8 which are not adjoints of $SU(N)$ are non-singlets of the direct sum $G \oplus U_G$. Hence there can be no matter zero modes, only the two torinos, and the desired condensate (2.11) can [18] form if E_8 is broken to some $SU(N)$ subgroup^{*)}. This is certainly a sufficient condition for gaugino condensation leading to SUSY breaking, but it may not be necessary. We cannot exclude the possibility that condensation (2.11) may still occur through some other mechanism even if E_8 is unbroken. Since the Wilson loop breaking of E_8 to some $SU(N)$ subgroup is possible with almost all realistic Calabi-Yau manifolds, gaugino condensation is in any case not an important constraint on the choice among them.

Gaugino condensation provides [17] a non-trivial superpotential for the gauge-singlet superfield S coupled to $W^a W^b$ (2.8), whose form is dictated by R invariance:

$$P(S) \ni h \exp \left(- \frac{3S}{2b_c} \right) \quad (2.12)$$

where h is an unknown constant $O(1)$, and b_c is the first non-trivial term in the beta-function for the subgroup C of E_8 which condenses:

$$\beta_c = -b_c g_c^3 \quad : \quad b_c = \begin{cases} 3N/16\pi^2 & \text{for } SU(N) \\ 45/8\pi^2 & \text{for } E_8 \text{ itself} \end{cases} \quad (2.13)$$

^{*)} It is easy to check that $\langle 0 | \text{Tr}(\bar{\chi}^0 \chi^0) | 0 \rangle \neq 0$ in the limit as the dimension L of the torus T^4 is taken to infinity.

Since the effective scalar potential for the S field given by $\mathcal{G}$ (2.7) is [2]

$$V = e^{\mathcal{G}} [\mathcal{G}_i \mathcal{G}_{\bar{j}} g^{i\bar{j}} - 3] = \left(\frac{1}{16 S_R T_R^3} \right) |P|^2 \mathcal{G}_s \mathcal{G}_{\bar{s}} g^{s\bar{s}} \quad (2.14)$$

where $g_{i\bar{j}} \equiv \partial_i \partial_{\bar{j}} \mathcal{G}$ and $g^{i\bar{j}}$ is its inverse, and

$$\mathcal{G}_s = \frac{1}{2 S_R P} (-P + 2 S_R P'), \quad \mathcal{G}_{s\bar{s}} = \left(\frac{1}{2 S_R} \right)^2, \quad (2.15)$$

the minimal superpotential (2.12) gives a minimum $V = 0$ at $S_R = \infty$. Thus the dynamical determination of S and hence of g^2 would require an additional contribution $\delta P(S)$ to $P(S)$. The simplest possibility is a constant c which could be provided by condensation $\langle 0 | H_{IJK} | 0 \rangle \neq 0$ of the field strength $H_{\alpha\beta\gamma}$ of the antisymmetric tensor $B_{\alpha\beta}$ [17]. In this particular case, global consistency conditions on the superstring viewed as a 2-dimensional σ -model suggest that $c = O(1/18\pi)$ as in the case of compactification on a group manifold [40]. Any other feature of string dynamics which generated a contribution $\delta P(S)$ to $P(S)$ would be equally satisfactory and presumably also give $\delta P(S)$ not much smaller than unity. Assuming this to be the case, generically there will be a value of $S \neq 0$ or ∞ for which $\mathcal{G}_s = 0$ (2.15) and hence $V = 0$ (2.14). This would give a vanishing cosmological constant $\Lambda = 0$ at the tree level in four dimensions, while local SUSY would be broken:

$$m_{3/2} = e^{\mathcal{G}} = \left(\frac{1}{16 S_R T_R^3} \right)^{1/2} |P|, \quad (2.16)$$

with $|P|$ also fixed to be not much smaller than unity. Since $1/S_R = g_4^2 = O(1)$ is also fixed^{*)}, the only uncertainty in (2.16) is the value of T_R , and $m_{3/2} = O(T_R^{-3/2})$. Scenarios for the dynamical determination of T_R will be presented in Section 2.4, and calculations based on them in Section 4. It is important to notice that (2.14) also implies $m_{S_I} \sim O(M_P)$, i.e. the axion becomes invisible, not because it is superlight but because it is superheavy. Since in this case it decays very rapidly, no bound on $\langle S_I \rangle$ can be derived from the cosmological relaxation constraint [42].

2.3 Supersymmetry breaking in the observable sector

Now we discuss SUSY breaking among the observable sparticles. The masses m_i of the scalar components of the superfields ϕ^i vanish at the tree level because of the $SU(N,1)/SU(N) \times U(1)$ structure [7] (2.7). Gaugino masses, which are given [2] by the formula

$$m_{1/2} = \frac{1}{4 \text{Re} f} e^{\mathcal{G}/2} \mathcal{G}_i g^{i\bar{j}} \partial_{\bar{j}} f^* \quad (2.17)$$

also vanish at the tree level because f is a function of S only, $g_{i\bar{j}}$ does not mix the S and (T, ϕ^i) sectors, and $\mathcal{G}_s = 0$ when $V = 0$ (2.14). At the one-loop level, the leading quadratic divergences to the effective

^{*)} It has been shown recently that, contrary to opposite claims [26], $g_4 = 0$ cannot be reached by the classical field equations, but for a considerable range in initial field values these drive [41] S_R towards the region $S_R = O(1)$ where the minimum is located.

potential (2.14) maintain [21, 32, 43] the $SU(N,1)/SU(N) \times U(1)$ structure of (2.7) since they are proportional to e^g :

$$\delta V = \frac{\Lambda^2}{32\pi^2} \text{Str } \mathcal{M}^2 = -\frac{\Lambda^2}{8\pi^2} e^g. \quad (2.18)$$

They therefore have the same effect on V (2.14) as changing the coefficient (-3) of $\ln(T + T^+ - 2\phi^i\phi_i^+)$ in equation (2.7) to $(-3 + \Lambda^2/8\pi^2)$. Conversely, an initial value $(-3 - \Lambda^2/8\pi^2)$ of this coefficient would be renormalized to give the ‘flat’ coefficient (-3) . It is possible to assume [21] that the form (2.7) can be regarded as correct after these quadratic divergences have been absorbed in this way, but this is not necessary [44], as including (2.18) does not [32] give a mass to the gauge non-singlet fields ϕ^i . In the low-energy effective field theory, the logarithmically divergent contributions to V do not [21, 32, 43] maintain the $SU(N,1)/SU(N) \times U(1)$ structure (2.7). However, it is a general feature of $SU(N,1)/SU(N) \times U(1)$ models that these field-theoretic logarithmic corrections do not give a mass to gauge non-singlet fields such as the ϕ^i at the one-loop level [32, 44]. Moreover, it is not clear that the field-theoretical formulae accurately reflect the structure of loop corrections in the superstring. It is encouraging to note that the non-compact symmetries of supergravities which were the prototypes for this kind of $SU(N,1)/SU(N) \times U(1)$ structure are apparently maintained by loop corrections [45]. Even if this result does not extend to the superstring, the known field-theoretical results lead to $m_i = 0$ at the tree and one-loop levels [44], and no-one has exhibited a non-zero scalar mass in any calculation based on the superstring.

As for the gaugino mass, it has already been observed [20] that the tree level formula (2.8) for the gauge kinetic function f must be renormalized by observable sector loops. The correct renormalization of $\text{Re } f = 1/g^2$ is obtained by

$$\text{Re } f = S_R + b_0 \ln |\Phi^i|^2 + \text{higher order terms}, \quad (2.19)$$

where b_0 is the observable sector beta function defined analogously to (2.13). For the observable sector gauge group E_6 with N_g 27 representations,

$$b_0 = \frac{27}{16\pi^2}. \quad (2.20)$$

The logarithmic corrections (2.19) can [20] be represented by

$$f = S + b_0 \ln |U|^2 + \text{higher order terms}, \quad (2.21)$$

where U is a gauge singlet product of chiral superfields which can be taken to have the same form as prospective terms in the superpotential $P(\phi^i)$. In the presence of such a non-zero superpotential,

$$g_i g^{i\bar{j}} = -\frac{1}{P^*} \partial_{\bar{j}} P^*, \quad \partial_{\bar{i}} f^* = b_0 \frac{U_{\bar{i}}^*}{U^*}, \quad (2.22)$$

so that formula (2.17) for the gaugino mass gives [20]

$$m_{1/2} = -\frac{g^2}{4} \left(\frac{1}{16 S_R T_R^3} \right)^{1/2} \partial^{\bar{i}} P^* \frac{U_{\bar{i}}^*}{U^*} \quad (2.23)$$

in one-loop order. This is the first (and as yet the only) explicit example of a non-vanishing SUSY breaking mass parameter in the observable sector [46].

To have $m_{1/2} \neq 0$, we need $(\partial P / \partial \phi^i) \neq 0$, i.e. a non-vanishing F-term in the language of global SUSY. The simplest example of such a term in the minimal extension of the Standard Model offered by the superstring is [20]

$$P \ni \lambda H \bar{H} N, \quad U \ni H \bar{H} N, \quad (2.24)$$

where H and $\bar{H}$ are conventional $Y = \pm 1/2$ Higgs doublets, and N is an $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet field contained in the $\underline{27}$ representation of E_6 . The Standard Model requires $\langle 0|H|0 \rangle \equiv v$, $\langle 0|\bar{H}|0 \rangle \equiv \bar{v} \neq 0$, and the trilinear coupling (2.24) then guarantees $\langle 0|N|0 \rangle \equiv x \neq 0$. In this case, formula (2.23) becomes

$$m_{1/2} = -\frac{b_0 g^2}{4} \left(\frac{1}{16 S_R T_R^3} \right)^{1/2} \frac{\lambda (|H \bar{H}|^2 + |H N|^2 + |\bar{H} N|^2)}{H \bar{H} N} \quad (2.25)$$

which is non-zero. The field-dependence of this formula should not worry anyone: all masses are field-dependent in supergravity—how could $m_{1/2}$ (2.17) be any different? When we expand the gaugino mass term about any fixed field value we are going to obtain terms that break SUSY softly with $m_{1/2} = \text{constant}$. Then the remaining task is to find the correct vacuum about which to expand, as will be discussed in Section 2.4.

The qualitative form of $m_{1/2}$ (2.25) leads to some interesting consistency conditions. We may represent it generically as

$$m_{1/2} = \left(\frac{\alpha}{4\pi} \right) \left(\frac{1}{16 S_R T_R^3} \right)^{1/2} O(\lambda v) b_0. \quad (2.26)$$

After renormalization one expects $m_H = O(m_{1/2})$, with $\langle 0|H|0 \rangle = O(m_H/\lambda \text{ or } g)$ and $S_R^{-1/2} = g$. Thus equation (2.26) can be used to determine the order of magnitude of T_R :

$$T_R = O(g^2) \quad (2.27)$$

where we have not made explicit simple numerical factors. Consistency in fact suggests that these factors are not negligible. Taking $m_{1/2} = O(v \times \lambda \text{ or } g)$ on the left-hand side of equation (2.26), we see that $T_R^{-3/2} = O(100)$. A similar value is found in the phenomenological analysis of Section 4, and is an advantage in

superstring cosmology [41]. The combination of this newer result with the older result (2.9) tells us [20] that the values of the dilaton φ and compacton e^σ must be^{*)}:

$$e^\sigma = O(1), \quad \varphi = O(g^{8/3}). \quad (2.28)$$

Since $g = O(1)$, we see that the scales of string excitation ($e^{-\sigma}$), compactification ($e^{-\sigma/2}$) and E_6 grand unification ($e^{-2\sigma}$) must all be $O(M_P)$. Moreover, the gravitino mass $m_{3/2}$ (2.16) must be $O(M_P)$ [20, 21].

At this point, the consistency of the philosophy of reducing from the superstring to $d = 4$ supergravity via an intermediate stage of $D = 10$ supergravity might appear dubious. However, Calabi-Yau compactification can be directly motivated by purely superstring considerations [11], and much if not all of the $d = 4$ supergravity structure (2.7, 2.8) can be deduced directly [35] from the superstring without recourse to intermediate $D = 10$ supergravity. One could worry that with $m_{3/2} = O(M_P)$, the philosophy of retaining SUSY during compactification has been contradicted, but SUSY is still a good approximation in the observable sector (in particular, there are no soft scalar masses at tree level nor at one-loop level), and $m_{3/2}$ may still be ‘small’ in some sense if P is small—e.g. if $c = O(1/18\pi)$ as was mentioned earlier. We see no reason for despair [26] that so many mass scales are close to each other.

2.4 Strategies for generating scales

Having seen that consistency demands $T_R = O(g^2) = O(1)$ (2.27), we now propose scenarios for the dynamical determination of this and other scales, including the hierarchy

$$m_i, m_{1/2}, m_W \ll M_P. \quad (2.29)$$

The key idea here is that of dimensional transmutation [47], whereby a small coupling α at $O(M_P)$, through the multiplicative but logarithmically slow renormalization of parameters, can generate a hierarchically small scale

$$Q_0 = M_P \exp \left[-O(1)/\alpha \right]. \quad (2.30)$$

This idea was first applied [47] to supergravity models with a fixed but small supersymmetry breaking scale $\tilde{m}$. A SUSY breaking scalar mass squared such as m_H^2 starts from a positive value at a large renormalization scale $Q = O(M_P)$, and is then driven negative [48] at some much smaller renormalization scale Q_0 given by (2.30). Then gauge symmetry breaking becomes possible with $m_W = O(Q_0)$, and the last of the hierarchies (2.29) is obtained [47]. In realistic models, driving some scalar $m_i^2 < 0$ requires a large Yukawa coupling λ coupling ϕ^i to other chiral superfields, an observation which is used to constrain models in Section 3.

To obtain the hierarchies $m_i, m_{1/2} \ll M_P$, we need a model in which the scale of SUSY breaking is undetermined at the tree level, as occurs in the $SU(N,1)/SU(N) \times U(1)$ models [7]. In no-scale scenarios based on this structure, the SUSY breaking parameters m_i and $m_{1/2}$ are determined dynamically [5, 7], by

^{*)} This argument was originally derived independently of the consistency arguments of ref. 26 which also imply the propinquity of large mass scales.

minimization of the effective potential, to be $m_i, m_{1/2} = O(Q_0)$ (2.30). In most but not all cases, this has involved the dynamical determination of a field equivalent to T_R , utilizing radiative corrections at scales $Q = O(m_W)$. It has been difficult to see how in the context of conventional cosmology the T_R field in these no-scale models can relax to the minimum and not leave behind an excess of vacuum energy in the form of coherent waves [49]. This problem is a generic feature of all models where the SUSY breaking scale is determined through a vev in the hidden sector. Oscillations of this light ‘Polonyi’ field about its minimum can then be dissipated only by gravitational interactions, which with $\Gamma \sim m_{T_R}^3/M_P^2$ are far too slow to transform the coherent field energy into radiation.

We have previously presented [21] calculations based on the extension of this no-scale scenario to superstring models, with $v, \bar{v}, x$ and T_R being determined dynamically much as before. Earlier models of this type had small values of x/v which gave a second neutral gauge boson lighter than the phenomenological constraints [21, 28–31] discussed in Section 3.4. As an improvement, Section 4.4 of this paper provides existence proofs of no-scale models with $x/v > 1$ which have a heavier neutral gauge boson, and are acceptable from the point of view of particle physics. However, because $m_{T_R}^2 \sim m_{1/2}^4/M_P^2 \sim O(m_W^4)/M_P^2$, the cosmological relaxation problem [49] still persists.

An alternative strategy [21] is to assume that T_R is fixed by dynamics at scales $O(M_P)$ —perhaps loops in the full superstring theory which seem to violate $SU(1,1)/U(1)$ [21, 32, 43, 35, 44]—while $v, \bar{v}$ and x are determined dynamically by dimensional transmutation. In this hybrid scenario, one calculates the renormalization of the effective SUSY breaking parameters from $Q = O(M_P)$ downwards, assuming $m_{1/2} \ll Q$ as suggested by the formula (2.25). Then when some m_i^2 goes negative at a scale Q_0 given by (2.30), we find generically that soon afterwards all the parameters $v, \bar{v}$ and $x \neq 0$. Then the gaugino mass (2.25) is also non-zero, and the evolution of the soft SUSY breaking parameters stops when $Q \approx m_{1/2}$, at which point the physical vacuum parameters are dynamically determined. Calculations based on this hybrid scenario are presented in Section 4.5. The cosmological advantage of this model is that because now $m_{T_R}^2 \approx O(1)M_P^2$, T_R no longer has difficulty relaxing to its minimum energy configuration. In this case the hidden sector is completely decoupled from the global SUSY breaking in the observable sector, and consequently $m_W \ll M_P$ is consistent with $m_{3/2} \sim M_P$. Indeed, this possibility provides us the only natural solution we know for the notorious Polonyi [49] and axion [42] problems: both the axion and the Polonyi field are superheavy and decay very quickly, so there is no need to dilute their number densities.

We find that in both these scenarios, an intermediate scale between m_W and M_P is disfavoured. We have already mentioned that all three vacuum expectation values $v, \bar{v}$ and x tend to have similar magnitudes, and to avoid this requires an unnatural adjustment of parameters. It can be seen from formula (2.25) for the gaugino mass that if any one of $v, \bar{v}$ and x is much larger than the others—presumably x —then $m_{1/2} = O(x) \gg O(m_W)$, and the usual argument for SUSY to protect the gauge hierarchy naturally disappears [31]. Therefore, in Section 3 we do not allow models with intermediate mass scales, which rules out many otherwise acceptable low-energy gauge groups.

3. PHENOMENOLOGICAL CONSTRAINTS ON MODELS

3.1 A unique low-energy gauge group

The Wilson loop mechanism [50] can be used to break E_6 down to any one of a number of possible low-energy gauge groups [16]. These include a unique phenomenological viable possibility of rank 5,

namely $SU(3)_C \times SU(2)_L \times U(1)_L \times U(1)_R$, which can only be realized if the discrete symmetry group G mentioned in Section 2.1 is non-Abelian, and its representation U_G in E_6 is non-diagonalizable. Decomposing E_6 via its maximal subgroup $SU(3)_C \times SU(3)_L \times SU(3)_R$, U_G must take the form

$$U_G = \begin{pmatrix} & & \\ & \mathbb{1} & \\ & & \end{pmatrix} \otimes \begin{pmatrix} \alpha_G & 0 & 0 \\ 0 & \alpha_G & 0 \\ 0 & 0 & \alpha_G^{-2} \end{pmatrix} \otimes \begin{pmatrix} \beta_G^{-2} & 0 & 0 \\ 0 & \vdots & \vdots \\ 0 & \beta_G \times V_G & \end{pmatrix}, \quad (3.1)$$

where V_G is a set of $SU(2)$ matrices which are not simultaneously diagonalizable [16]. There are many possible groups of rank 6 including $SU(3)_C \times SU(2)_L \times U(1)^3$, $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)^2$, $SU(3)_C \times SU(3)_R \times SU(2)_L \times U(1)$, and many examples including $SU(4)$ and $SU(5)$ subgroups^{*}. We will now show that our dynamical assumptions set out in Section 2 select uniquely [33] the rank 5 group.

The analysis of gauge coupling renormalization in ref. 16 found that only the groups $SU(3)_C \times SU(2)_L \times U(1)_L \times U(1)_R$, $SU(3)_C \times SU(2)_L \times U(1)^3$ and $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)^2$ could give satisfactory values of $\sin^2 \theta_W$ and of M_X without intermediate mass scales. As explained in Section 2.4, our gaugino mass formula (2.25) and the no-scale and hybrid scenarios for hierarchy generation exclude an intermediate mass scale. Thus we exclude the other gauge groups, and retain only $SU(3)_C \times SU(2)_L \times [U(1)^2 \text{ or } U(1)^3 \text{ or } SU(2)_R \times U(1)^2]$.

For either of the rank 6 groups to be acceptable, one must be able to break them down to $SU(3)_C \times SU(2)_L \times U(1)_Y$, i.e. reduce their rank by 2. Each $\underline{27}$ of E_6 makes available to us two $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlets, namely the N field already introduced, and a field with the quantum numbers of the ν^c_L —the ‘right-handed neutrino’ ν_R . To get from rank 6 to rank 4, at least one representative of each of these types of field must acquire a vacuum expectation value, and they must be larger than $\langle 0|H|0 \rangle = v$ and $\langle 0|\bar{H}|0 \rangle = \bar{v}$, since the extra gauge bosons must be heavier than the Z^0 . As mentioned already in Section 2.4, and discussed in more detail in Section 4, in our scenario for the dynamical generation of the gauge hierarchy, fields can only acquire vacuum expectation values if they have Yukawa couplings λ to other fields [5, 7, 47, 48]. Then the combination $\lambda \langle \text{fields} \rangle \neq 0$, and we have a large new contribution to the fermion mass matrix. As will be discussed in Section 3.3, the only possible Yukawa couplings of the ν^c are [33]

$$\lambda_L H L \nu^c, \quad \lambda_D D d^c \nu^c, \quad (3.2)$$

where L is a conventional lepton doublet, d^c is a conventional conjugate charge $1/3$ quark singlet, and D is one of the new charge $-1/3$ quark singlets in the $\underline{27}$ of E_6 . The first of these, λ_L , would give an unacceptably large neutrino mass $\lambda_L v$, as well as large mixing $\propto \lambda_L \langle \nu^c \rangle$ between leptons and shiggses which violate lepton number conservation [33]. The second of these, λ_D , would give large mixing between the d and D quarks which would violate weak universality. Thus we cannot obtain $\langle 0|\nu^c|0 \rangle \neq 0$ in our dynamical scheme, and $\langle 0|N|0 \rangle \neq 0$ is the only possibility [33]. Thus we can only reduce the rank of our gauge group by 1, the low-energy groups of rank 6 can be excluded and the rank 5 group $SU(3)_C \times SU(2)_L \times U(1)_L \times$

^{*}) The latter have insuperable problems with proton decay and/or further symmetry breaking, and may be discarded out of hand [16].

$U(1)_R$ is the unique choice. In this case the embedding of the extra $U(1)_R$ into E_6 is completely fixed. Therefore the coupling strengths of particles to the corresponding extra neutral gauge boson are also completely fixed [21], and the low-energy neutral current phenomenology is well-defined, as discussed in Section 3.4.

3.2 A unique set of matter fields

We now seek to specify the matter representation of our group. The possible ambiguities are in the number of generations N_g , in the possible addition of some split multiplets from additional $\overline{27} + 27$ representations of E_6 , and in the presence of additional gauge singlet chiral fields.

Reference 16 has already removed the first of these ambiguities, since acceptable values of $\sin^2 \theta_W$ and of M_X are only obtained for $N_g = 3$. We assume that one can find a suitable Calabi-Yau manifold with $|\chi| = 6$, so that $N_g = |\chi|/2 = 3$, and with a non-Abelian discrete group G , so that the low-energy group has rank 5.

The number of additional $\overline{27} + 27$ representations of E_6 is equal to a certain topological number of the manifold, namely the Hodge number $b_{1,1}$, which is ≥ 1 for Calabi-Yau manifolds [11]. In general, the Wilson loop gauge symmetry breaking mechanism [50] will leave no members of the $\overline{27} + 27$ invariant, and hence light, but careful choice of the parameters in U_G can leave some of them invariant and hence light [16]. In the case of our rank 5 group with matrices U_G of the form (3.1), the only such choices of parameters are $\alpha_G = \beta_G$ for all G , and $\alpha_G = \beta_G^{-2}$ for all G . In the first case, the invariant members of the $\overline{27} + 27$ have the quantum numbers of the e^c and its complex conjugate. Clearly these fields cannot have vacuum expectation values, and we see no useful role for them in the low-energy theory. In the case $\alpha_G = \beta_G^{-2}$, the extra low-mass fields have the quantum numbers of the doublet Higgs H with $Y = +1/2$ and its complex conjugate H^c . The first of these could couple to the conventional generations of particles and acquire a vacuum expectation value, but the H^c can do neither, since the $\overline{27}$ does not couple to pairs of 27 representations. Thus H^c cannot contribute to the low-energy mass matrix. Since the job of mass generation can be performed by the H and $\bar{H}$ in conventional E_6 generations, we see no useful role for these possible additional H and H^c . Since making them light in the first place requires a special and otherwise unmotivated choice of parameters, we assume the minimal low-energy model in which they are absent.

Any additional gauge singlet chiral superfields would only have very weak couplings to matter $O(1/M_P)$, and are likely [34] to have masses $O(M_P)$ and therefore decay rapidly—like the S field in Section 2.2, and like the T field in the hybrid scenario of Section 2.4. Therefore they are irrelevant to low-energy phenomenology and can be neglected.

Thus we are left with the unique minimal choice of 3 generations of gauge non-singlet matter fields. In Table 1 we show the field content of one generation which corresponds to a full $\overline{27}$ of E_6 , together with their quantum numbers with respect to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$, where $U(1)_E$ is the combination of Y_L and Y_R which is orthogonal to $U(1)_Y$.

Note that there is a pair of superfields $(H_a, \bar{H}_a)$ with the conventional Higgs quantum numbers in each of the three E_6 generations. Of these, presumably only one has couplings to the third generation of quarks which are large enough to induce a vacuum expectation value by radiative corrections. Thus the other two pairs $(H_a, \bar{H}_a)$ $a = 1, 2$ have no vacuum expectation value, and we call them unhiggses.

3.3 An almost unique set of Yukawa couplings

In addition to the couplings of the Standard Model, the low-energy theory obtained from the superstring may contain the following further Yukawa couplings [33]:

$$\lambda H \bar{H} N + \lambda_D D D^c N + (\lambda' D^c L Q + \lambda'' D e^c u^c) + (\lambda''' D Q Q + \lambda'''' D^c u^c d^c), \quad (3.3)$$

as well as the possibilities (3.2). Note that at least one of the sets of couplings in parenthesis is needed to make D unstable. The couplings (3.3), together with the couplings (3.2) and the conventional Yukawa terms $h^{(U)} Q H u^c + h^{(D)} Q \bar{H} d^c + h^{(L)} L \bar{H} e^c$ are the only couplings allowed by E_6 invariance. These remain the only possible couplings when E_6 is broken by the Wilson loop mechanism, though the coupling constants need not be linked by the usual E_6 Clebsch–Gordan relations [27]. Considerations [46] of the more general Yukawa couplings allowed by the symmetries of the low-energy gauge group would be misleading, because many of these couplings would not be allowed by the underlying E_6 structure. The coupling $\lambda_L H L \nu^c$ would give an unacceptable neutrino mass even if $\langle 0 | \nu^c | 0 \rangle = 0$, so we set it to zero. There is no comparable disaster with $\lambda_D D d^c \nu^c$ as long as $\langle 0 | \nu^c | 0 \rangle = 0$, so we retain it as a possibility.

The remaining couplings do not give a mass to either the left- or right-handed neutrino. It is possible to postulate pentilinear couplings $(\underline{27})^3 (\underline{27} \times \overline{\underline{27}})$ which would appear able to give neutrino masses, but these could only be very small in models with no intermediate scale, such as ours. Furthermore, as mentioned in Section 3.2, the only light component of the $\overline{\underline{27}}$ we could have which might have a vacuum expectation value is the H^c , and in our dynamical framework $\langle 0 | H^c | 0 \rangle = 0$ because the H^c has no Yukawa couplings. Hence we could have only neutrino masses which would be unobservably small at best, generated by some as yet unspecified higher order coupling. We assume here and in Section 3.4 that the ν and ν^c are of negligible mass.

The two sets of couplings in the first and second parentheses of (3.3) do not violate baryon number B separately, but only in combination. The first set (λ', λ'') conserves B if we assign $B(D) = +1/3$, $B(D^c) = -1/3$, while the second set (λ''', λ''') conserves B if we assign $B(D) = -2/3$, $B(D^c) = +2/3$. Thus we can admit as non-zero couplings either (λ', λ'') or (λ''', λ''') . In the first case we can also admit $\lambda_D D d^c \nu^c$, but in the latter case we cannot, since by the argument of the previous paragraph the ν^c is light and we must forbid $B \rightarrow \nu^c$ decay. We are left with the following twofold ambiguity [33] in our choice of Yukawa couplings needed for D decay:

$$(\lambda' D^c L Q + \lambda'' D e^c u^c + \lambda_D D d^c \nu^c) \text{ or } (\lambda''' D Q Q + \lambda'''' D^c u^c d^c). \quad (3.4)$$

However, the choice between these has no impact on the dynamical calculations in Section 4, since neither of these sets of couplings can be large enough to drive field vacuum expectation values.

We have seen already in Section 2.2 that the coupling $\lambda H \bar{H} N$ plays an essential role in generating $m_{1/2} \neq 0$. Moreover, as discussed in Section 4, the coupling λ can by itself drive $m_N^2 < 0$ as required for gauge symmetry breaking. Such a coupling is also needed to avoid the appearance of a massless Higgs field in the particle spectrum. When $m_N^2 < 0$ one gets also $\langle N \rangle \neq 0$, and in a sense this serves as an effective

$\mu H\bar{H}$ -coupling with the additional bonus that now the phenomenologically desirable scale of $\mu = \lambda\langle N \rangle \ll O(1)$ TeV has a natural explanation. However, a coupling of the form kDD^cN is essential for giving masses to the unconventional D quarks. Therefore we must retain

$$\lambda H\bar{H}N + kDD^cN \quad (3.5)$$

The coupling k also has in general an important dynamical role in helping to drive $m_N^2 < 0$, as we discuss in Section 4.

The combinations (3.4, 3.5) are our almost unique choices of Yukawa couplings [33]. How plausible are these choices?

The superstring offers two mechanisms for setting Yukawa couplings naturally to zero: topologically because of the non-intersection of complex hypersurfaces in the manifold of compactification [27] or by discrete symmetries [16]. As already mentioned, some of the couplings which would be $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$ invariant are not there because of the primordial E_6 invariance.

The topological mechanism seems very powerful: ref. 51 gives an example where only 111 out of 8436 possible Yukawa couplings are in fact non-zero. Though no-one has an example of a Calabi-Yau manifold which gives the combination (3.4, 3.5), discrete symmetries may be unnecessary eventually. In the mean time, it is encouraging that precisely these combinations can be obtained by postulating simple discrete symmetries. To get $(\lambda', \lambda'', \lambda_D)$ we can postulate [33] invariance under $Z_2 \otimes Z'_2$:

$$\begin{cases} Z_2 : & [SU(3) \quad \underline{3}, \bar{\underline{3}}] \longrightarrow (-1) [SU(3) \quad \underline{3}, \bar{\underline{3}}] \\ Z'_2 : & \nu^c \longrightarrow (-1) \nu^c \end{cases} \quad (3.6)$$

and to get $(\lambda''', \lambda^{iv})$ we can postulate $Z_2 \otimes Z'_2$:

$$\begin{cases} Z_2 : & (L, e^c, \nu^c) \longrightarrow (-1) (L, e^c, \nu^c) \\ Z'_2 : & \nu^c \longrightarrow (-1) \nu^c \end{cases} \quad (3.7)$$

Thus we have no qualms about either of our choices (3.4, 3.5) of Yukawa couplings. It should be stressed that our 'almost' unique choice of Yukawa couplings automatically evades all kinds of nasty problems with flavour-changing neutral currents (FCNC) and weak universality [33]. With respect to FCNC, box diagrams could be of some importance but as we will see later all our spectrum is 'heavy', so their contribution is of no great concern. Similar arguments apply to CP violating effects, $(g-2)_{e,\mu}$, the electric dipole moment of the neutron, etc. (for details, see ref. 33).

3.4 Neutral current bounds on vacuum expectation values

Three vacuum expectation values $\langle 0|H|0 \rangle = v$, $\langle 0|\bar{H}|0 \rangle = \bar{v}$ and $\langle 0|N|0 \rangle = x$ characterize our low-energy gauge boson masses [21]. The mass of the $W^\pm$ is unaffected by our unique modification of the low-energy gauge group:

$$m_{W^\pm} = \frac{38.65}{\sin \theta_W} \quad (3.8)$$

after including Standard Model radiative corrections. The neutral gauge bosons mix via the matrix [21]:

$$\mathcal{M}^2 = m_Z^2 \begin{pmatrix} 1 & \frac{1}{3} \sin \theta_W \frac{4v^2 - \bar{v}^2}{v^2 + \bar{v}^2} \\ \frac{1}{3} \sin \theta_W \frac{4v^2 - \bar{v}^2}{v^2 + \bar{v}^2} & \frac{1}{9} \sin^2 \theta_W \frac{25x^2 + 16v^2 + \bar{v}^2}{v^2 + \bar{v}^2} \end{pmatrix} \equiv m_Z^2 \begin{pmatrix} 1 & a \\ a & b \end{pmatrix} \quad (3.9)$$

where $\sin^2 \theta_W \equiv g'^2/(g_2^2 + g'^2)$ and $m_Z^2 \equiv m_W^2/\cos^2 \theta_W$ as usual. The eigenstates of the mass squared matrix (3.9) are given by

$$m_{Z_{1,2}}^2 = \frac{1}{2} \left[(1+b) \mp \sqrt{(1-b)^2 + 4a^2} \right] m_Z^2. \quad (3.10)$$

The mass eigenstates are then

$$\begin{aligned} Z_1 &= \cos \theta Z_0 + \sin \theta Z_E, \\ Z_2 &= -\sin \theta Z_0 + \cos \theta Z_E, \end{aligned} \quad (3.11)$$

where

$$\tan 2\theta = \frac{2a}{1-b}. \quad (3.12)$$

Working in the left-handed basis of Table 1, the couplings of fermions to these mass eigenstates are given by:

$$\begin{aligned} Z_1^\mu \bar{f}_L \gamma_\mu f_L &: g_Z Y_Z^f \cos \theta + g_E' Y_E^f \sin \theta, \\ Z_2^\mu \bar{f}_L \gamma_\mu f_L &: -g_Z Y_Z^f \sin \theta + g_E' Y_E^f \cos \theta, \end{aligned} \quad (3.13)$$

where $g_Z \equiv \sqrt{g_2^2 + g'^2}$ and $Y_Z^f \equiv \cos^2 \theta_W T_{3L}^f - \sin^2 \theta_W Y^f$. The magnitude of couplings is fixed by the renormalization group result $g' = g_E$. The couplings which are equal to g_3 and g_2 at the grand unification scale are $g_1 \equiv \sqrt{5/3} g'$, $g_E \equiv \sqrt{5/3} g_E$.

For large enough x/v , $m_{Z_2} \gg m_{Z_1}$ and m_{Z_1} approaches the unmixed value m_Z from below. In the limit of small mixing, one has:

$$m_{Z_1}^2 \simeq m_Z^2 \left(1 + \frac{a^2}{1-b} \right); \quad m_{Z_2}^2 \simeq m_Z^2 \left(b - \frac{a^2}{1-b} \right). \quad (3.14)$$

Figure 1 displays the contours $m_{Z_2} = 200, 400$ and 600 GeV for a relevant range of x/v and $\bar{v}/v$.

The most direct way to look for a second neutral gauge boson is as a peak in $\bar{p}p \rightarrow (\ell^+\ell^-) + X$ [21, 29]. As discussed in a previous paper [21]:

$$\frac{\sigma(\bar{p}p \rightarrow Z_2 + X)}{\sigma(\bar{p}p \rightarrow Z_0 + X)} \approx \left(\frac{1}{4} \text{ to } \frac{1}{5} \right) \quad (3.15)$$

for a Z_2 of equal mass to the conventional Z^0 , and this ratio decreases in a calculable manner for $m_{Z_2} > m_{Z^0}$. If the Z_2 has all possible chiral superfield decay modes kinematically open to it, i.e. $N_g \times 27$ bosons and fermions, then in the absence of mixing for $N_g = 3$

$$B(Z_2 \rightarrow e^+e^-) = 1.08 \% \quad (3.16)$$

while if only the conventional ($N_g = 3$) $\times 15$ fermions are available:

$$B(Z_2 \rightarrow e^+e^-) = 3 \% \quad (3.17)$$

Using (3.16), one can establish [21, 29] the conservative lower bound

$$m_{Z_2} \geq 110 \text{ GeV} \quad (3.18)$$

from the non-observation of Z_2 in $\bar{p}p \rightarrow (\ell^+\ell^-)$ spectra. This limit is improved [29] to $m_{Z_2} > 140$ GeV in the case where only conventional fermionic decay modes are available.

However, the off-diagonal terms in (3.9) produce an effect which may be observable at existing accelerators [31]. Because of mixing, the experimental value of $m_W^2/m_{Z_1}^2$ is no longer $\cos^2 \theta_W$. Introducing the parametrization

$$\sin^2 \bar{\theta}_W \equiv 1 - \frac{m_W^2}{m_{Z_1}^2} = 1 - \left(\frac{m_Z^2}{m_{Z_1}^2} \right) \cos^2 \theta_W, \quad (3.19)$$

it is obvious that $\sin^2 \bar{\theta}_W < \sin^2 \theta_W$. Contours of $\Delta \equiv \sin^2 \theta_W - \sin^2 \bar{\theta}_W$ for a relevant range of x/v and $\bar{v}/v$ are also plotted in fig. 1^{*)}. Recent UA1 and UA2 [52] data on the $W^\pm$ mass, combined with equation (3.8), yield

$$\sin^2 \theta_W = \begin{cases} 0.214 \pm 0.005 \pm 0.014 & : \text{UA1} \\ 0.227 \pm 0.005 \pm 0.008 & : \text{UA2} \end{cases}, \quad (3.20)$$

which we average to get^{**)}

^{*)} Note that these beyond-the-Standard-Model corrections to m_W^2/m_Z^2 are potentially much larger than the Standard Model radiative corrections.

^{**)} Note that we do not use low-energy neutral current data to estimate $\sin^2 \theta_W$, since they may be 'polluted' by additional neutral current contributions of the type discussed shortly.

$$\sin^2 \theta_W = 0.224 \pm 0.004 \pm 0.007 \quad . \quad (3.21)$$

Recent UA1 and UA2 data [52] on the gauge boson mass ratio, combined with equation (3.19), yield

$$\sin^2 \bar{\theta}_W = \begin{cases} 0.194 \pm 0.031 & : \text{UA1} \\ 0.229 \pm 0.030 & : \text{UA2} \end{cases} \quad , \quad (3.22)$$

which we average to get

$$\sin^2 \bar{\theta}_W = 0.212 \pm 0.022 \quad . \quad (3.23)$$

Comparing (3.21) and (3.23) we find

$$\Delta \equiv \sin^2 \theta_W - \sin^2 \bar{\theta}_W = 0.012 \pm 0.023 \quad . \quad (3.24)$$

Thus present experimental precision is at the level of $\Delta = 0.01$ to 0.05 , which is the range of contours shown in fig. 1. Taking the 1σ limit $\Delta < 0.035$ from (3.24), we find

$$m_{Z_2} \geq 170 \text{ GeV} \quad \text{for} \quad \frac{\bar{v}}{v} \leq 1 \quad . \quad (3.25)$$

General theoretical prejudice favours this range of $\bar{v}/v$, and the specific dynamical calculations of Section 4 favour $\bar{v}/v \leq 0.6$, in which case the bound (3.25) can be strengthened to $m_{Z_2} \geq 230 \text{ GeV}$.

There are bounds on the parameters of this minimal superstring model which are provided by the agreement of low-energy neutral current data with the Standard Model [21, 28]. Before mixing, the couplings of the new neutral gauge boson are unique and completely specified as discussed earlier. Armed with this information, we have studied low-energy $\nu N \rightarrow \nu X$ scattering [21], $e^+e^- \rightarrow \ell^+\ell^-$ annihilation [21] and $\nu_\mu e$ elastic scattering [30]. We found that the most stringent constraint was provided by νN scattering, which yielded

$$m_{Z_2} \geq 150 \text{ GeV} \quad (3.26)$$

in the case of small mixing, while present-day e^+e^- annihilation gives a lower bound much weaker than those already listed. The effects in e^+e^- annihilation at the highest LEP energies are currently under investigation [53]. In the case of $\nu_\mu e$ scattering, we find [30]

$$R_{\nu_\mu} \equiv \frac{\sigma(\nu_\mu e)}{\sigma(\nu_\mu e)_{\text{STANDARD MODEL}}} = \left(\frac{m_Z}{m_{Z_1}} \right)^2 \cdot \frac{(1 - \alpha)^2}{(\gamma_Z^{e_L})^2 + \frac{1}{3} (\gamma_Z^{e_R})^2} \cdot \left\{ (\gamma_Z^{e_L})^2 \left[\frac{1}{2} + N E_L \frac{1}{2} + \sin 2\theta (E_L + N)/2 \right]^2 + \frac{1}{3} (\gamma_Z^{e_R})^2 \left[\frac{1}{2} + N E_R \frac{1}{2} + \sin 2\theta (E_R + N)/2 \right]^2 \right\} \quad (3.27)$$

where

$$\begin{aligned}
 z &\equiv \frac{m_{Z_1}^2}{m_{Z_2}^2} ; \\
 f(z) &\equiv \cos^2 \theta + \frac{z}{1-z} ; \quad \bar{f}(z) \equiv \sin^2 \theta + \frac{z}{1-z} ; \\
 E_{L,R} &\equiv \frac{g_E' Y_E^{e_L, e_R}}{g_Z Y_Z^{e_L, e_R}} ; \quad N \equiv \frac{g_E' Y_E^{\nu_L}}{g_Z Y_Z^{\nu_L}} .
 \end{aligned} \tag{3.28}$$

Using these formulae, we find that for $\Delta \leq 0.03$ in fig. 1:

$$0.97 \leq R_{\nu_\mu} \leq 1.04 \tag{3.29}$$

for $0 \leq \bar{\nu}/\nu \leq 1$. Neither present nor forthcoming experiments [54] will be able to measure $\sigma(\bar{\nu}_\mu e)$ with the precision needed to observe such a small deviation from the Standard Model. Thus (3.18, 3.25 and 3.26) are our best limits on the Z_2 mass. They can be translated into the bound

$$\frac{x}{\bar{\nu}} \geq 3 \tag{3.30}$$

for plausible values of $\bar{\nu}/\nu$. As we will see in Section 4, this bound is naturally satisfied for a range of parameters in our dynamical models, and does not require unnatural fine-tuning, in contradiction to a claim [55] in the recent literature.

Cosmology has the potential to provide a more stringent constraint on the model parameters [30]. As discussed in Section 3.3, the right-handed neutrino ν^c is essentially massless, and could therefore affect primordial nucleosynthesis. The presence of a total of 6 left- and right-handed neutrinos seems to conflict with the conventionally quoted [56] upper bound $N_\nu < 4$ on the total number of equivalent left-handed neutrinos. However, even if one accepts this bound, it can be reconciled with the presence of 3 right-handed neutrinos because their cosmological density will be suppressed if $m_{Z_2} \gg m_{Z^0}$ and the ν^c decouple early [57]. Requiring that the ν^c decouple at a temperature $T_d > 300$ MeV before hadron annihilation, we find [30]

$$m_{Z_2} \geq 780 \text{ GeV} \quad \text{for} \quad \frac{\bar{\nu}}{\nu} \sim 0.4 \tag{3.31}$$

However, as discussed elsewhere [30], we believe the nucleosynthesis constraint can conservatively be relaxed to $N_\nu < 5.5$, in which case $T_d \sim 1/2 m_\mu$ is sufficiently early, and then we find [30]

$$m_{Z_2} \geq 330 \text{ GeV} \quad \text{for} \quad \frac{\bar{\nu}}{\nu} \sim 0.4 \tag{3.32}$$

If one is prepared to relax the cosmological constraint slightly further to $N_s < 6$, a possibility we cannot exclude [30], then there is no cosmological lower bound on m_{Z_2} . We regard the cosmological lower bound (3.32) as rather permeable, and will not immediately discard in Section 4 models which do not respect it. But we are even able to provide some models which obey the much more severe constraint (3.31), though in this case the problem of fine-tuning does start to appear.

4. LOW-ENERGY GAUGE AND SUPERSYMMETRY BREAKING

4.1 General features

The considerations developed in Section 3 have led us to a unique choice of the gauge group and of the ‘light’ particle content in the effective $N = 1$, $d = 4$ supergravity model below the compactification scale M_c (which we identify operationally with the grand unification scale M_X). Some residual freedom is left in the choice of the non-zero Yukawa couplings in the superpotential. For the subsequent discussion of symmetry breaking, however, we need to consider only those Yukawa couplings which play a role in determining the VEVs of the scalar fields; therefore we can (uniquely) restrict ourselves to the following simplified form of the superpotential:

$$\mathcal{f} = h Q_3 u_3^c H + \lambda H \bar{H} N + \kappa D D^c N, \quad (4.1)$$

with obvious contractions of group indices.

Some comments on the superpotential (4.1) are in order. First of all, we neglect everywhere intergenerational mixing. Secondly, to avoid dangerous tree-level flavour-changing neutral currents [33], we assume that only two Higgs doublets H and $\bar{H}$ have large Yukawa couplings and therefore can develop vacuum expectation values (VEVs). Moreover, in order to simplify the discussion, for the fields N_a , D_a and D_a^c we consider the couplings of only one generation which we denote by N , D , D^c : the extension to the more general case does not possess qualitatively different features. Finally, we assume that among the Yukawa couplings of ordinary quark and leptons the only non-negligible one is h , associated to the top-quark mass. As we shall see, this assumption can be justified a posteriori.

Given the superpotential (4.1) and the mechanism of supersymmetry breaking described in 2.2, the low-energy Lagrangian at scales $Q < M_X$ will include a soft supersymmetry-breaking part of the general form

$$-\mathcal{L}_{soft} = \sum_i m_i^2 |z^i|^2 + \left[(h A_h Q_3 u_3^c H + \lambda A_\lambda \bar{H} H N + \kappa A_\kappa D D^c N) + h.c. \right] + \frac{1}{2} M_a (\lambda_a \lambda_a + h.c.) \quad (4.2)$$

where the z^i are all the scalar fields, the λ_a are the gaugino fields associated to the different factors of the gauge group, and the A -parameters have the dimension of a mass.

The part of the scalar potential relevant to our analysis will be that containing the Higgs fields, H , $\bar{H}$ and N only [21]:

$$\begin{aligned}
V_{\text{Higgs}} = & m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 + m_N^2 |N|^2 + (\lambda A_\lambda H \bar{H} N + \text{h.c.}) + \\
& \lambda^2 (|H|^2 |N|^2 + |\bar{H}|^2 |N|^2 + |\bar{H} H|^2) + \frac{g_2^2}{2} \left(H^\dagger \frac{\vec{\tau}}{2} H + \bar{H}^\dagger \frac{\vec{\tau}}{2} \bar{H} \right)^2 + \\
& \frac{3}{5} \frac{g_1^2}{2} \left(\frac{1}{2} |H|^2 - \frac{1}{2} |\bar{H}|^2 \right)^2 + \frac{3}{5} \frac{g_E^2}{2} \left(\frac{5}{6} |N|^2 - \frac{2}{3} |H|^2 - \frac{1}{6} |\bar{H}|^2 \right)^2.
\end{aligned} \tag{4.3}$$

We postpone to the following paragraph a detailed discussion of the scalar potential minimization and assume, for the time being, the following pattern of real VEVs:

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \bar{H} \rangle = \begin{pmatrix} \bar{v} \\ 0 \end{pmatrix}, \quad \langle N \rangle = x. \tag{4.4}$$

The parameters appearing in the low-energy Lagrangian depend on the energy scale Q according to the renormalization group equations (RGE): we have derived these, in the one-loop approximation, according to the prescriptions of Ref. [58] and they are listed in Appendix A. To solve the RGEs one must also specify the boundary conditions at the unification scale $Q = M_X$. According to our no-scale hypothesis, we shall assume:

$$g_3(M_X) = g_2(M_X) = g_1(M_X) = g_E(M_X) \equiv g_U; \tag{4.5}$$

$$M_3(M_X) \equiv M_{3U}, \quad M_2(M_X) \equiv M_{2U}, \quad M_1(M_X) \equiv M_{1U}, \quad M_E(M_X) \equiv M_{EU}; \tag{4.6}$$

$$h(M_X) \equiv h_U, \quad \lambda(M_X) \equiv \lambda_U, \quad k(M_X) \equiv k_U; \tag{4.7}$$

$$m_i^2(M_X) = 0; \quad A_h(M_X) = A_\lambda(M_X) = A_k(M_X) = 0. \tag{4.8}$$

In words, we have, as required [16] by the superstring, unification of the properly normalized gauge coupling constants at $Q = M_X$. The values of the Yukawa couplings h , λ and k at the unification scale M_X are the free parameters of our model. All the soft supersymmetry-breaking parameters in the scalar sector are assumed to be vanishing at M_X : non-vanishing masses for scalars will however be generated as an effect of non-gravitational radiative corrections. At M_X the only seed of supersymmetry breaking in the 'observable' sector is therefore given by the primordial gaugino masses (4.6) [20]. If the gaugino condensation in E_6' takes place before E_6 is broken, the seed of global SUSY breaking in the observable sector will be given by *universal* gaugino masses:

$$M_{3U} = M_{2U} = M_{1U} = M_{EU} \equiv m_{1/2} , \quad (4.9)$$

with $m_{1/2}$ given by (2.25). However, a priori one cannot exclude the more general case (4.6), which could arise if E_6 were broken before gaugino condensation: such a possibility will be considered in 4.3, where we show that it is highly unlikely to give realistic phenomenology, while in the rest of our analysis we shall take (4.9) for granted.

We have now all the tools to analyse low-energy gauge and supersymmetry breaking, whose interplay has been extensively discussed in 2.2. It is therefore clear how to proceed: given the Yukawa couplings (4.7) at the scale M_X , one should run the relevant parameters, solving the corresponding RGEs, down to the symmetry breaking scale $\bar{Q}$. Here one should minimize the effective potential (keeping $m_{1/2}$ fixed), determining v , $\bar{v}$ and x . This determines, through (2.25), the gaugino mass $m_{1/2}$.

What fixes the scale $\bar{Q}$ at which we should stop the RGE clock? Two possible answers to this question, the ‘no-scale’ and the ‘hybrid’ dimensional transmutation scenarios [21], have been described in 2.4. In both cases, independently of the mechanism which determines $\bar{Q}$ and $m_{1/2}$, given the VEVs v , $\bar{v}$ and x one can work out the resulting mass spectrum.

The W-boson mass is given by the usual formula:

$$m_W = g_2(m_W) \sqrt{\frac{v^2 + \bar{v}^2}{2}} , \quad (4.10)$$

while the neutral gauge boson mass matrix has already been given in (3.9). The masses of the top-quark and of the D-quarks (at the scale m_W) are given by:

$$m_{top}(m_W) = h(m_W) \cdot v , \quad (4.11)$$

$$m_D(m_W) = k(m_W) \cdot x . \quad (4.12)$$

Note that the masses of the D-quarks are completely independent of those of ordinary quark and leptons, being proportional not only to different Yukawa couplings, but also to different VEVs! Moreover, since we need $x > v, \bar{v}$ to agree with experimental limits on neutral currents, the present non-observation of D-quarks is naturally explained in the present model.

A bit less trivial is the computation of the scalar masses. When there is non-negligible mixing among different fields (e.g. in the case of the stop squarks or of the D-squarks with the coupling k), one ends up with mass matrices of the general form:

$$\begin{pmatrix} \Phi_L^* & \Phi_R^* \\ \Phi_L & \Phi_R \end{pmatrix} \begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{LR}^2 & m_{RR}^2 \end{pmatrix} . \quad (4.13)$$

When the mixing is negligible the full expression for the scalar masses is:

$$m^2(z^i) = m_i^2 + m^2(f^i) - \frac{g_2^2}{2} T_{3L}^i (v^2 - \bar{v}^2) + \frac{\frac{3}{5}g_1^2}{2} Y^i (v^2 - \bar{v}^2) + \frac{\frac{3}{5}g_E^2}{2} Y_E^i \left(\frac{5}{3}x^2 - \frac{4}{3}v^2 - \bar{v}^2 \right), \quad (4.14)$$

where i is an index denoting the generic chiral superfield $\phi^i \equiv (z^i, f^i)$, m_i^2 is the soft supersymmetry-breaking contribution, and T_{3L}^i , Y^i and Y_E^i are the $SU(2)_L$, $U(1)_Y$ and $U(1)_E$ quantum numbers of the z^i respectively, as given in Table 1. The explicit expressions of the mass terms (4.13) and (4.14) are collected in Appendix B. Separate consideration is deserved by the mass terms associated to the Higgs multiplets, whose neutral components acquire non-vanishing VEVs. Making use of the minimization condition for the scalar potential, the mass matrices for the charged and neutral fields contained in H , $\bar{H}$ and N are given by:

$$\begin{array}{cc} H^+ & \bar{H}^{-*} \\ H^{+*} & \begin{pmatrix} -\lambda A_1 \frac{x\bar{v}}{v} + \left(\frac{g_2^2}{2} - \lambda^2\right) \bar{v}^2 & -\lambda A_1 x + \left(\frac{g_2^2}{2} - \lambda^2\right) v\bar{v} \\ -\lambda A_1 \frac{xv}{\bar{v}} + \left(\frac{g_2^2}{2} - \lambda^2\right) v^2 \end{pmatrix} \\ \bar{H}^- & \end{array} \quad (4.15)$$

and:

$$\begin{array}{cccccc} & H_R^0 & H_I^0 & \bar{H}_R^0 & \bar{H}_I^0 & N_R & N_I \\ \begin{array}{c} H_R^0 \\ H_I^0 \\ \bar{H}_R^0 \\ \bar{H}_I^0 \\ N_R \\ N_I \end{array} & \begin{pmatrix} \left(\frac{g_2^2}{2} + \frac{3}{5}g_1^2 + \frac{16}{15}g_E^2\right)v^2 & 0 & \left(4\lambda^2 - g_2^2 - \frac{3}{5}g_1^2 + \frac{4}{15}g_E^2\right)v\bar{v} & 0 & \left(4\lambda^2 - \frac{4}{3}g_E^2\right)v x & 0 \\ -2\lambda A_1 \bar{v}x/v & +2\lambda A_1 x & & +2\lambda A_1 \bar{v} & & \\ -2\lambda A_1 \bar{v}x/v & 0 & -2\lambda A_1 x & 0 & -2\lambda A_1 \bar{v} & \\ \left(\frac{g_2^2}{2} + \frac{3}{5}g_1^2 + \frac{g_E^2}{15}\right)\bar{v}^2 & 0 & \left(4\lambda^2 - \frac{g_E^2}{3}\right)\bar{v}x & 0 & & \\ -2\lambda A_1 \frac{vx}{\bar{v}} & & +2\lambda A_1 v & & & \\ -2\lambda A_1 \frac{vx}{\bar{v}} & & 0 & -2\lambda A_1 v & & \\ & & & & \frac{5}{3}g_E^2 x^2 & 0 \\ & & & & -2\lambda A_1 \frac{v\bar{v}}{x} & \\ & & & & & -2\lambda A_1 \frac{v\bar{v}}{x} \end{pmatrix} \end{array} \quad (4.16)$$

respectively, where we have exploited the symmetry of the matrices to give only their upper triangular parts, and for convenience we have decomposed H^0 , $\bar{H}^0$ and N into their real and imaginary parts. Discarding the unphysical Goldstone bosons, we are left with a charged scalar field of mass:

$$m_{\pm}^2 = m_W^2 - \lambda A_\lambda x \left(\frac{\bar{v}}{v} + \frac{v}{\bar{v}} \right) - \lambda^2 (v^2 + \bar{v}^2) \quad (4.17)$$

(as will be discussed later, the second term in (4.17) turns out to be positive in our conventions) and four neutral scalar fields whose masses can be calculated by diagonalizing the matrix (4.16). In the physically relevant limit $v/x, \bar{v}/x \rightarrow 0$ with $v/\bar{v}$ fixed, the eigenvalues of (4.16) are given by:

$$\begin{aligned} m_1^2 &= -2\lambda A_\lambda \frac{v}{\bar{v}} x, \\ m_2^2 &= -2\lambda A_\lambda \frac{\bar{v}}{v} x, \\ m_3^2 &= -2\lambda A_\lambda \left(\frac{\bar{v}}{v} + \frac{v}{\bar{v}} \right) x, \\ m_4^2 &= \frac{5}{3} g_E^2 x^2. \end{aligned} \quad (4.18)$$

All squared masses are positive definite at the minimum. As we have already discussed in Section 2.4, in realistic cases one needs $x \gg v, \bar{v}$. As a result, in general all the eigenvalues (4.18) are heavier than m_W as also is $m_{H^\pm}$, although equations (4.16) and (4.17) could *a priori* have allowed them to be lighter. Note, however, that the charged unhiggses can well be lighter than the $W^\pm$.

Passing finally to supersymmetric fermions, the chargino mass matrix is simply given by:

$$\begin{array}{cc} & \begin{array}{cc} \tilde{W}^+ & \tilde{H}^+ \end{array} \\ \begin{array}{c} \tilde{W}^- \\ \tilde{H}^- \end{array} & \begin{pmatrix} M_2 & g_2 v \\ g_2 \bar{v} & -\lambda x \end{pmatrix} \end{array}, \quad (4.19)$$

while the neutralino mass matrix has the more complicated structure:

$$\begin{array}{c} \begin{array}{c} \tilde{W}^3 \\ \tilde{B} \\ \tilde{B}_E \\ \tilde{H}^0 \\ \tilde{H}^0 \\ \tilde{N} \end{array} \end{array} \begin{pmatrix} \tilde{W}^3 & \tilde{B} & \tilde{B}_E & \tilde{H}^0 & \tilde{H}^0 & \tilde{N} \\ M_2 & 0 & 0 & -\frac{g_2 v}{\sqrt{2}} & \frac{g_2 \bar{v}}{\sqrt{2}} & 0 \\ 0 & M_1 & 0 & \sqrt{\frac{3}{10}} g_1 v & -\sqrt{\frac{3}{10}} g_1 \bar{v} & 0 \\ 0 & 0 & M_E & -\sqrt{\frac{8}{15}} g_E v & -\sqrt{\frac{1}{30}} g_E \bar{v} & \sqrt{\frac{5}{6}} g_E x \\ -\frac{g_2 v}{\sqrt{2}} & \sqrt{\frac{3}{10}} g_1 v & -\sqrt{\frac{8}{15}} g_E v & 0 & \lambda x & \lambda \bar{v} \\ \frac{g_2 \bar{v}}{\sqrt{2}} & -\sqrt{\frac{3}{10}} g_1 \bar{v} & -\sqrt{\frac{1}{30}} g_E \bar{v} & \lambda x & 0 & \lambda v \\ 0 & 0 & \sqrt{\frac{5}{6}} g_E x & \lambda \bar{v} & \lambda v & 0 \end{pmatrix} \quad (4.20)$$

This concludes our survey of the relevant formulas for the discussion of the low-energy spectrum: further comments will be found in the following subsections.

4.2 Evolution of couplings and scalar potential minimization

Given the gauge group and the particle content of the effective low-energy theory, a first simple exercise is the computation of the grand unification scale M_X and of the renormalized value of the weak angle $\sin^2 \theta_W(m_W)$. Assuming the validity of the supersymmetric one-loop RGEs for the gauge couplings in the whole energy range $m_W \leq Q \leq M_X$, and taking as physical inputs $\alpha_3(m_W) = 0.11$, $\alpha(m_W) = 1/128$, one finds:

$$\begin{aligned} M_X &\simeq 4.4 \times 10^{17} \text{ GeV}, \\ \sin^2 \theta_W(m_W) &\simeq 0.21. \end{aligned} \quad (4.21)$$

Note also that, within the above approximations, the $SU(3)$ β -function vanishes, so that $\alpha_U = \alpha_3(m_W)$. It is worth observing that in contrast to conventional GUTs which have two *a priori* unknown input parameters M_X , α_U , here we know that M_X cannot be much smaller than M_P , and α_U can in principle be dynamically determined. Thus superstring models have no free parameters, and α_3 , α_2 and α' must obey non-trivial consistency conditions.

Since the above results are relevant for the rest of our phenomenological analysis, it is important to check the validity of the adopted approximations. Two obvious improvements that come immediately to mind are the following:

- I) since in general the effective supersymmetry breaking scale, $m_{1/2}$, does not coincide exactly with m_W , but it is slightly larger, one should consider non-supersymmetric RGEs for $m_W < Q < m_{1/2}$;
- II) Since M_X and therefore the scale of SUSY and $SU(2) \times U(1) \times U(1)$ symmetry breaking is sensitive to two-loop effects, one should consider how these change the one-loop results. A priori, one might think that they are relatively important because to one loop $d\alpha_3/dt = 0$ and therefore α_U is large.

We have investigated explicitly both these effects. Assuming that between m_W and $m_{1/2}$ the particle content is that of the standard model, the non-supersymmetric one-loop coefficients of the β -functions are given by $b_3 = -7$, $b_2 = -10/3$, $b_1 = 4$ and $b_E = 3/2$, respectively. On the other hand, between $m_{1/2}$ and M_X the supersymmetric two-loop RGEs for the gauge couplings [59] are:

$$\frac{dg_a}{dt} = - \frac{b_a}{16\pi^2} g_a^3 - \sum_{ab} \frac{b_{ab}}{(16\pi^2)^2} g_a^3 g_b^2, \quad (4.22a)$$

where $t \equiv \ln Q$, $b_3 = 0$, $b_2 = 3$, $b_1 = b_E = 9$ and:

$$b_{ab} = \begin{pmatrix} 48 & 9 & 3 & 3 \\ 34 & 39 & 3 & 3 \\ 24 & 9 & 9 & 216/25 \\ 24 & 9 & 216/25 & 9 \end{pmatrix} \quad (4.22b)$$

if $a, b = 3, 2, 1, E$ and we assume 3 full 27 's of E_6 . We neglect the two-loop effects due to Yukawa couplings. We have checked that, whenever these two effects are not negligible, they tend to cancel each other. Take for example the evolution of $\alpha_3(Q)$: in the supersymmetric one-loop approximation it is a constant between m_W and M_X ; with the improvements outlined above, it is rapidly decreasing going from $Q = m_W$ to $Q = m_{1/2}$, while it is slowly increasing going from $Q = m_{1/2}$ to $Q = M_X$. Since $m_{1/2}$ must be rather close to m_W (in view of our results we can take as a representative value $m_{1/2} \sim 500$ GeV), the net result is that, even if $\alpha_3(Q)$ is not strictly constant, nevertheless $\alpha_U \approx \alpha_3(m_W)$ to a good approximation.

For example, if we take $\alpha_{e-m}^{-1}(m_W) = 128$ and $\alpha_3(m_W) = 0.111$ we find, taking also the full two loop non-SUSY evolution between m_W and $m_{1/2} \approx 500$ GeV, that $\alpha_U = 0.112$ with $M_X = 4 \times 10^{17}$ GeV. This is clearly in accordance with the one-loop result of eq. (4.21). If one were to stick to one-loop level and take into account the threshold, one would find that $\alpha_3(m_W)$ and α_U differ by 20%. Thus the threshold effects have been counterbalanced by two-loop effects in the gauge coupling sector. Although no calculations of two-loop corrections for soft SUSY breaking parameters exist, we conjecture that the same is probably happening there also. Because of this, in the following we will consider only one-loop supersymmetric RGEs and pay no attention to thresholds even when determining M_X and $\sin^2 \theta_W(m_W)$. We feel that this is a consistent one-loop approach that is likely to resemble most the results corrected for two-loop and threshold effects.

Let us now comment briefly on the evolution of the Yukawa couplings and of the soft supersymmetry breaking parameters of the scalar sector, governed by the RGEs displayed in their full glory in Appendix A. In general, one has systems of non-linear coupled differential equations, which cannot be solved analytically: all our results have been obtained solving the RGEs numerically on a VAX 8600 computer. With the simplified superpotential (4.1), however, there are cases where neglecting some irrelevant Yukawa couplings one can derive good analytical solutions.

A typical example is given by the soft supersymmetry-breaking scalar masses. Taking into account the results of Appendix A, it is easy to see that at the generic scale Q they can be written as:

$$m_{\tilde{\phi}}^2(Q) = C_3(i, Q) M_{3U}^2 + C_2(i, Q) M_{2U}^2 + C_4(i, Q) M_{4U}^2 + C_E(i, Q) M_{EU}^2 + m_Y^2(i, Q), \quad (4.23)$$

where the C_a -coefficients can be calculated analytically, while the Yukawa coupling contribution $m_Y^2(i, Q)$, when it is not negligible, must be calculated via numerical integration. The C_a -coefficients of the different fields, calculated at the representative scale $Q = 300$ GeV, are given in Table 2. Even if the full expressions of the scalar masses, collected in Appendix B, also involve the model-dependent D-term contributions, one can already note some interesting relationships. For example, in the case (4.9) of universal gaugino masses one has [21]:

$$m_{\tilde{q}} : m_{\tilde{e}_L} : m_{\tilde{e}_R} : m_{\tilde{g}} \approx 2 : 0.7 : 0.4 : 1, \quad (4.24)$$

with $m_{\tilde{q}}$ referring generically to the squarks of the first two generations. This means for example that, given the present experimental limit $m_{\tilde{e}_R} \gtrsim 20$ GeV, we can infer from (4.24) that $m_{\tilde{q}} \gtrsim 50$ GeV if D-terms are negligible. In fact they cannot be neglected, as discussed in more detail in ref. 31.

Another important issue to be discussed in some detail is the minimization of the scalar potential. In restricting ourselves to the simplified potential (4.3), we have implicitly assumed that all the scalar fields

different from H , $\bar{H}$ and N have vanishing VEVs. To prove such an assumption would require the minimization of the full scalar potential, which is clearly a fools' errand. Nevertheless, it has been claimed [58, 60] that no other VEVs arise if all the soft masses m_i^2 (apart from m_H^2 , $m_{\bar{H}}^2$ and m_N^2) are positive and the following conditions are satisfied:

$$A_h^2 \leq 3(\tilde{m}_{Q_3}^2 + \tilde{m}_{u_3^c}^2 + m_H^2), \quad (4.25)$$

$$A_\kappa^2 \leq 3(\tilde{m}_D^2 + \tilde{m}_{D^c}^2 + m_N^2). \quad (4.26)$$

Given now the potential (4.3), to discuss its minimization it is convenient to set, by definition:

$$\langle H \rangle \equiv \begin{pmatrix} v^+ \\ v \end{pmatrix}, \quad \langle \bar{H} \rangle \equiv \begin{pmatrix} \bar{v} \\ \bar{v}^- \end{pmatrix}, \quad \langle N \rangle = x. \quad (4.27)$$

In general, one expects all the entries in (4.27) to be non-zero and complex. Making use of gauge invariance, however, one can obtain $v^+ = 0$ and v, x real and non-negative. Taking this into account, one can write instead of (4.3):

$$\begin{aligned} V_{\text{Higgs}} = & m_H^2 v^2 + m_{\bar{H}}^2 \bar{v}^2 + m_N^2 x^2 + \lambda A_\lambda x v (\bar{v} + \bar{v}^*) \\ & + \lambda (v^2 x^2 + |\bar{v}|^2 x^2 + |\bar{v}|^2 v^2) + \frac{g_2^2 + \frac{3}{5}g_1^2}{8} (v^2 - |\bar{v}|^2)^2 \\ & + \frac{g_E^2}{120} (5x^2 - 4v^2 - |\bar{v}|^2)^2 + \left(\frac{g_2^2 + \frac{3}{5}g_1^2}{8} + \frac{g_E^2}{120} \right) |\bar{v}^-|^4 \\ & + \left[m_{\bar{H}}^2 + \lambda^2 x^2 + \frac{g_2^2}{4} (v^2 + |\bar{v}|^2) - \frac{3/5 g_1^2}{4} (v^2 - |\bar{v}|^2) - \frac{g_E^2}{60} (5x^2 - 4v^2 - |\bar{v}|^2) \right] |\bar{v}^-|^2, \end{aligned} \quad (4.28)$$

One can immediately note that in (4.28) the only term where the phase of $\bar{v}$ is relevant is $\lambda A_\lambda x v (\bar{v} + \bar{v}^*)$, which is clearly minimized for $\bar{v} \in \mathbb{R}$: in particular, $\bar{v}$ will have the opposite sign to λA_λ . One can therefore write $V_{\text{Higgs}} = V_{\text{neutral}} + V_{\text{charged}}$, with:

$$\begin{aligned} V_{\text{neutral}} = & m_H^2 v^2 + m_{\bar{H}}^2 \bar{v}^2 + m_N^2 x^2 + 2\lambda A_\lambda x v \bar{v} + \lambda^2 (v^2 x^2 + \bar{v}^2 x^2 + v^2 \bar{v}^2) \\ & + \frac{g_2^2 + \frac{3}{5}g_1^2}{8} (v^2 - \bar{v}^2)^2 + \frac{g_E^2}{120} (5x^2 - 4v^2 - \bar{v}^2)^2, \end{aligned} \quad (4.29)$$

$$\begin{aligned} V_{\text{charged}} = & \left(\frac{g_2^2 + \frac{3}{5}g_1^2}{8} + \frac{g_E^2}{120} \right) |\bar{v}^-|^4 + \\ & \left[\frac{m_{\bar{H}}^2}{4} + \left(\lambda^2 - \frac{g_E^2}{12} \right) x^2 + \left(\frac{g_2^2}{4} - \frac{3g_1^2}{20} + \frac{g_E^2}{15} \right) v^2 + \left(\frac{g_2^2}{4} + \frac{3g_1^2}{20} + \frac{g_E^2}{60} \right) \bar{v}^2 \right] |\bar{v}^-|^2. \end{aligned} \quad (4.30)$$

Let $(x, v, \bar{v})$ be the global minimum of V_{neutral} . Then $(x, v, \bar{v}, \bar{v}^- = 0)$ will be a **local** minimum of V_{Higgs} if and only if

$$m_{\tilde{H}}^2 + \left(\lambda^2 - \frac{g_E^2}{12} \right) x^2 + \left(\frac{g_2^2}{4} - \frac{3g_1^2}{20} + \frac{g_E^2}{15} \right) v^2 + \left(\frac{g_2^2}{4} + \frac{3g_1^2}{20} + \frac{g_E^2}{60} \right) \bar{v}^2 > 0. \quad (4.31)$$

In particular, if all the addenda in (4.31) are separately positive, one can be sure that it is also the **global** minimum: in all cases of interest $g_1^2 = g_E^2 < g_2^2$ and $m_{\tilde{H}}^2 > 0$, so that the mentioned sufficient condition simply amounts to:

$$\lambda^2 - \frac{g_E^2}{12} > 0. \quad (4.32)$$

In the following, therefore, we shall restrict ourselves to the minimization of V_{neutral} , checking a posteriori if conditions (4.31) and (4.32) are satisfied.

Let us discuss finally the minimization of V_{neutral} : one can easily see that if $\lambda \neq 0$ it is automatically bounded from below and that the symmetric minimum $x = v = \bar{v} = 0$ becomes unstable when at least one of the masses $m_{\tilde{H}}^2$, $m_{\tilde{H}}^2$, $m_{\tilde{N}}^2$ becomes negative. However, this does not mean that SUSY is broken, since this requires, as is evident from (2.25), all the VEVs to be different from zero. The precise determination of the minimum of the potential and of x , v and $\bar{v}$ is too difficult to be done analytically, so that a dedicated computer program has been devoted to it in the present work.

4.3 Non-universal gaugino masses

In this Section we will discuss the possibility that E_6 is broken by the Wilson loops [16, 50] before the gaugino condensate [17, 18] in E_6' is fully formed. This situation could happen if the evolution of the coupling constant of the subgroup $H \subset E_6'$ where the gaugino condensation is to occur is not fast enough and/or the GUT scale practically coincides with the compactification scale. Then the seed of SUSY breaking would no more reflect E_6 invariance but only $SU(3) \times SU(2) \times U(1) \times U(1)$ invariance. Then, because $\beta_3 = 0$ at one loop, the gluino mass $M_3 = 0$ according to Eq. (2.24) and in leading order SUSY breaking will be mediated to low-energies only by the gaugino masses M_2 and $M_1 (= M_E)$. At M_X the ratio of these gaugino masses to one loop are given by the ratio of their one-loop beta function coefficients, so that one may write:

$$M_{3U} = 0; \quad M_{2U} \equiv m_{1/2}; \quad M_{1U} = M_{EU} = 3 m_{1/2}. \quad (4.33)$$

It is then easy to compute the masses of the first two generation sparticles. According to the general results of Table 3 (and therefore neglecting D-term contributions, which is a good approximation when $m_{1/2} \gg \text{VEVs}$), we find:

$$m_{\tilde{q}} : m_{\tilde{e}_L} : m_{\tilde{e}_R} : M_2 = 0.25 : 0.68 : 0.72 : 0.34. \quad (4.34)$$

Note that the lightest scalar sparticle is now the squark doublet. The number of (un)seen monojets at the $p\bar{p}$ Collider [61] implies $m_{\tilde{q}} \gtrsim 50$ GeV which thus translates into the bound

$$m_{1/2} \gtrsim 200 \text{ GeV}. \quad (4.35)$$

The spectrum (4.34) is not realistic because at the one-loop level the gluino is massless. It will however acquire a mass through the loop corrections depicted in Fig. 2. One finds that

$$\begin{aligned} \delta m_{\tilde{g}} = & -\frac{g^2}{16\pi^2} A m_q^2 m_{1/2} \frac{1}{m_{RR}^2 - m_{LL}^2} \left\{ \frac{m_{LL}^2}{m_{LL}^2 - m_q^2} \ln m_{LL}^2 \right. \\ & \left. - \frac{m_{RR}^2}{m_{RR}^2 - m_q^2} \ln m_{RR}^2 + m_q^2 \ln m_q^2 \left[\frac{1}{m_{RR}^2 - m_q^2} - \frac{1}{m_{LL}^2 - m_q^2} \right] \right\}, \end{aligned} \quad (4.36)$$

where the notation is that of Eq. (4.13), and we have assumed that

$$m_{LR}^2 \simeq A m_q m_{1/2} \quad (4.37)$$

where m_q is the relevant quark mass. This assumption is valid if the coupling constant of the possible superpotential term coupling ϕ_L and $\phi_L^c \equiv \phi_R^*$ is small.

It is obvious that the contributions of the ordinary quarks to the radiative gluino mass are negligible since the corresponding squarks must be rather heavier because of (4.35). However, now the particle spectrum contains also an extra quark D. This could be quite heavy as its mass m_D is proportional to $x \gg m_w$. The actual value of m_D can be obtained only after the minimization of the Higgs potential, which we do not attempt here. Moreover, we have not made a detailed study of the allowable parameter space in this case, but have considered only the case $k = 0$. Nevertheless, as we will discuss below, we believe that these considerations are sufficient to rule out the possibility of non-universal gaugino masses.

To get an estimate of the magnitude of the induced radiative gluino mass, we will neglect the D-term contributions, a procedure which is valid if $m_{1/2} \gtrsim g_1 x$. As we will see in later sections in realistic cases with equal gaugino masses, x cannot be much larger than $m_{1/2}$. Presumably this will also be true in the case of unequal gauginos. Then (4.36) can be written for one generation of heavy D-quarks as

$$\begin{aligned} \frac{\delta m_{\tilde{g}}}{m_{1/2}} \simeq & 0.013 \rho \left[(1 + 3.85\rho) \ln(0.26 + \rho) \right. \\ & \left. - (1 + 15.38\rho) \ln(0.065 + \rho) + 11.54 \rho \ln \rho \right], \end{aligned} \quad (4.38)$$

where we have taken $A_\lambda \simeq 0.3$ as a representative value, $\alpha_3 = \alpha_U = 0.11$ as usual, and we have defined $\rho \equiv m_D/m_{1/2}$. Let us furthermore assume that all three generations of D-quarks contribute equally. In the realistic case $k \neq 0$ this is of course not quite true, but the general features will not change much. Figure 3 displays the one-loop gluino mass as a function of m_D for different values of $m_{1/2}$. There one sees that if $m_D \simeq m_{1/2}$, the UA1 limit [61] $m_{\tilde{q}} \gtrsim 50$ GeV (which is applicable if the gluino is unstable) would yield

$m_{1/2} \gtrsim 11$ TeV, clearly an unacceptable value. Only if $m_D \gtrsim 3m_{1/2}$ would the SUSY breaking scale be low enough to be realistic. Such a value cannot be excluded without detailed examination of the VEVs allowed by minimization of the potential, although it does not seem very plausible.

However, the radiative gluino mass is so small that gluino is very likely to be the lightest, and thus stable, sparticle, which would be unacceptable [62]. To verify this, let us go back to the neutralino mass matrix (4.19). In the limit $v/x, \bar{v}/x \rightarrow 0$ and $m_{1/2}/x$ fixed the photino state is given as in the Standard Model by $\tilde{\gamma} = (g' \tilde{A}_3 + g_2 \tilde{B}) / (g_2^2 + g'^2)^{1/2}$. Since in realistic cases we find $x \sim O(m_{1/2}) \gg v, \bar{v}$, this limit makes sense physically. With $M_1(M_X)/M_2(M_X) = \xi$ one then finds

$$m_{\tilde{\gamma}} = \left(1 + \frac{5}{3} \xi\right) \sin^2 \theta_w \frac{\alpha_2}{\alpha_U} m_{1/2} . \quad (4.39)$$

Adopting again $\alpha_U = \alpha_3 = 0.11$ and $\sin^2 \theta_w = 0.21$, one finds from (4.33) and (4.39) that

$$m_{\tilde{\gamma}} = 0.43 m_{1/2} . \quad (4.40)$$

Therefore invariably the photino will weigh more than the gluino, which raises a severe difficulty for the case of unequal gaugino masses. This scenario could only be rescued if by some peculiar choice of parameters one of the other neutralinos had a tree level mass smaller than the one-loop gluino mass, but this is unlikely [63] to be the case for any of the neutralinos in the matrix (4.20). This phenomenological analysis thus suggests that the gaugino condensation in E_6 should take place practically at the same scale as E_6 breaking. This can be viewed as another independent argument for having $m_{3/2} \simeq M_P$.

4.4 No-scale models

In this section we consider the case where the value of T_R is not fixed by gravitational radiative corrections. Then there is an extra dynamical degree of freedom represented by the gaugino mass $m_{1/2}$. The existence proof for this class of models, together with some characteristics, was given already in Ref. 21. There however only a small λ region of the parameter space was considered with the consequence that x/v turned out to be less than one, and thus the mass of Z_2 was unrealistic. We have now made a more extensive search in the parameter space with the result that one can easily obtain values of $x/v \gg 1$.

Let us first briefly recapitulate the strategy [5] of no-scale determination of $m_{1/2}$. One starts by choosing a renormalization point Q . This choice does not involve any physics, but for practical purposes it is useful to take $Q = O(m_{1/2})$. The effective potential near $Q = Q_0$ defined by $V_{\text{eff}}(Q_0) = 0$ has then the form

$$V_{\text{eff}} = e^{-4t} P(t) \quad : \quad t = \ln Q \quad (4.41)$$

where $P(t)$ is a function whose form depends on radiative corrections. Unfortunately these radiative corrections are not too well known as was discussed in Section 2.3. One may surmise that these will yield a polynomial expansion of the type

$$P = (t - t_0)^n + \dots$$

Here $t_0 = \ln Q_0$ is the scale where symmetry breaking first starts to appear. The minimum of (4.41) is given by $P = 1/4 P'$ where $P' \sim O(\alpha/\pi)$, and therefore one needs only to retain the first term in the expansion to obtain the position of the minimum

$$t = t_0 + \frac{1}{4} \pi. \quad (4.42)$$

This is then the point where one should stop the running of the RGEs.

Once t in (4.42) is given, one can determine the absolute value of $m_{1/2}$ from the value of m_w . Here we will just give one representative example with $h_u = 0.033$, $\lambda_u = 0.31$, $k_u = 0$ and $n = 1$. In this case one finds

$$\frac{\bar{v}}{v} = 0.41, \quad \frac{x}{v} = 2.7, \quad m_{1/2} = 490 \text{ GeV}, \quad (4.43)$$

with a top mass of 52 GeV. The second Z has in this case a mass of 185 GeV. The sparticle spectrum is quite heavy and has been tabulated for completeness in Table 3. Thus the general features of the spectrum expanded in Ref. 21 do not change even though now $x/v > 1$.

This showcase is not quite realistic because we assumed $k = 0$. As we will show in the next section, $k \neq 0$ tends to improve the situation in the sense that one can obtain larger values of x/v . We expect the same holds true also in no-scale models.

Let us end this section by remarking that the change in the ratios $\bar{v}/v$ and x/v is very rapid in the threshold region $Q \approx Q_0$. Therefore the precise values of these ratios necessitate more detailed knowledge of $P(t)$ or n in Eq. (4.41). In this respect the models with T_R fixed are more stable, as we will now discuss.

4.5 Realizations of the 'hybrid' dimensional transmutation scenario

In this subsection we discuss explicit realizations of the 'hybrid' dimensional transmutation scenario illustrated in 2.4.

Both the solution of the renormalization group equations and the minimization of the Higgs potential can be performed factorizing out the gaugino mass $m_{1/2}$, whose only role is that of fixing the scale of gauge and SUSY breaking. For this reason, it is convenient to express all dimensional quantities in units of $m_{1/2}$, denoting them by a hat ($\hat{}$): e.g., the VEVs of the neutral scalar fields, in units of $m_{1/2}$, will be denoted by $\hat{v}$, $\hat{\bar{v}}$ and $\hat{x}$.

The free parameters of the model are h_u , λ_u and k_u : given a point (h_u, λ_u, k_u) in the parameter space, solving numerically the RGEs one is able to calculate the parameters of the scalar potential $\hat{V}$ at any scale $Q < M_X$; then one can minimize $\hat{V}$ and determine the VEVs $\hat{v}$, $\hat{\bar{v}}$ and $\hat{x}$. Since the evolution of the soft supersymmetry breaking parameters is frozen at scales $Q \ll m_{1/2}$, we are going to identify $m_{1/2}$ with the $\bar{Q}$ at which we stop the renormalization group clock: in principle, both of them can be dynamically determined through Eq. (2.25). Due to the uncertainties in the evaluation of some quantities appearing in (2.25), however, the latter equation can be used in practice only as a consistency condition, the precise value of $m_{1/2} \equiv \bar{Q}$ being determined by the insertion of the experimental input $m_w \approx 82$ GeV in formula (4.10). Of course, if supersymmetry is going to solve the hierarchy problem, $m_{1/2}$ must be within the TeV range [31]: in our numerical searches we have allowed for values of $m_{1/2}$ between 50 GeV and 3 TeV.

We have seen in the previous sections that in order to have an acceptable breaking of the gauge symmetry and of global supersymmetry all the three VEVs v , $\bar{v}$ and x must be non-zero. The crucial parameters in the study of the scalar potential are the soft masses m_H^2 , $m_{\bar{H}}^2$ and m_N^2 , for the symmetric minimum $v = \bar{v} = x = 0$ becomes unstable whenever at least one of them acquires a negative value. The typical Q -dependence of these masses is illustrated in Fig. 4. As can easily be understood by looking at the corresponding RGEs, the behaviour of m_H^2 is mainly determined by the top Yukawa coupling h , $m_{\bar{H}}^2$ has only a very slight dependence on λ , and the evolution of m_N^2 depends heavily on k and to a lesser extent on λ . In particular, for reasonably small values of h_u , λ_u and k_u the only masses that can be driven to negative values in the TeV region are m_H^2 and m_N^2 .

The appearance of symmetry breaking in the TeV region as the renormalization scale Q goes down is illustrated in Fig. 5. The requirement that v , $\bar{v}$ and x must be all non-zero at some scale already selects a limited region of parameter space: for h_u or k_u too small, or λ_u too large, v and $\bar{v}$ cannot become non-zero around the electroweak scale. In the opposite case x and $\bar{v}$ vanish. Restricting ourselves to values of (h_u, λ_u, k_u) which give $v, \bar{v}, x \neq 0$, two qualitatively different scenarios emerge, as can be seen from Fig. 5, corresponding to which soft mass becomes negative first.

For larger values of h_u and/or very small values of k_u and λ_u , one has the situation represented in Fig. 5a: at scales $Q \gg m_W$ both m_H^2 and m_N^2 are positive, so that $v = \bar{v} = x = 0$. At a certain scale Q' m_H^2 becomes negative and at the minimum it is now $v \neq 0$. At a lower scale Q'' , also $\bar{v}$ and x become non-zero simultaneously, due to the presence of the trilinear term $\lambda A_\lambda v \bar{v} x$ in the potential. The above possibility presents the general feature that $v > x$, and is therefore incompatible with the neutral current phenomenology, as discussed in 3.4. Much more intriguing is the opposite situation, which occurs for non-negligible k_u —which is also necessary to give phenomenologically acceptable masses to the D-quarks and/or relatively big values of λ_u and/or relatively small values of h_u , corresponding to values of the top mass around the phenomenologically preferred value of 40 GeV. In this case, illustrated in Fig. 5b, it is m_N^2 that becomes negative first, and this allows solutions with $x > v$, in accordance with phenomenological requirements. In particular, when we calculate renormalization effects for values of h_u corresponding to $m_{\text{top}} \sim 40$ GeV and even relatively small values of k_u (e.g. $k_u \sim 0.05$), we find that one can naturally obtain at low-energy situations in which $m_N^2 < 0$ and $m_H^2, m_{\bar{H}}^2 > 0$, with all these (mass)² being of order $m_{1/2}^2$. Conspiring with the other terms in the potential, this pattern is able to produce, without fine-tuning, ratios x/v of the order of several units, which are enough to satisfy all the phenomenological constraints of Section 3.4.

Note also that, due to the presence of the top Yukawa coupling h , in general one has $m_H^2 < m_{\bar{H}}^2$: a consequence of this fact is that one always ends up with $v > \bar{v}$: typical values of the parameter $\bar{v}/v$ range from 0.4 to 0.6. Our neglect of the b-quark Yukawa coupling with respect to the t-quark one is therefore justified a posteriori.

To proceed further in the analysis, it is convenient to reduce the number of free parameters, by fixing the top Yukawa coupling h_u : for the time being we take $h_u \sim 0.025$, which roughly corresponds, as we shall see, to the phenomenologically preferred value $m_{\text{top}} \sim 40$ GeV. At the end of this subsection, however, we shall also consider the possibility of higher top masses. For fixed h_u , the residual parameter space can be graphically represented in the (k_u, λ_u) plane, as done in Fig. 6: we shall see in the following how different phenomenological constraints lead us to discard the most of the points, leaving however a finite region corresponding to solutions fully consistent with phenomenology.

A very stringent constraint is given by the correct fitting of the electroweak breaking scale. Requiring formula (4.10) to reproduce the experimental value of the W-mass, together with the consistency condition (2.25) and a gaugino mass $m_{1/2}$ in the range 50 GeV–3 TeV, is enough to forbid a considerable region of the (k_u, λ_u) plane, as shown in Fig. 6. Too high values of λ_u and/or too small values of k_u give $v = \bar{v} = 0$ or $m_W < m_W^{\text{exp}} \approx 82$ GeV. Values of λ_u and/or k_u going in the opposite direction give $m_W > m_W^{\text{exp}}$. The role played by the m_W -fit in determining the exact values of $\bar{Q} \equiv m_{1/2}$ [the rough value being dynamically determined by Eq. (2.25)] can be understood by looking at Fig. 7. It is notable that, for the consistency of our identification $m_{1/2} \equiv \bar{Q}$ with formula (2.25), the factor of $T_R^{-3/2}$ in the latter must be of the order of 10^2 , which is an advantage in superstring cosmology [41].

Another important constraint is given by the requirement of an acceptable mass spectrum for the many exotic particles which are present in the model, combined with the consistency of our minimization procedure for the scalar potential:

- From e^+e^- collider data we know that all charged particles which have not been seen up to now must weigh more than $O(20 \text{ GeV})$ [64]. Moreover, from the $p\bar{p}$ -Collider limits on monojets we can infer [61] that $m_{\tilde{g}}, m_{\tilde{q}} \gtrsim O(50 \text{ GeV})$.
- All the $(\text{mass})^2$ of the physical neutral scalar particles must be positive.
- Conditions (4.31) and (4.32) must be satisfied, for the consistency of our minimization procedure.
- Conditions (4.25) and (4.26) must be also satisfied, to avoid charge and colour breaking minima.
- The lightest supersymmetric particle must be neutral and weakly interacting, to satisfy stringent astrophysical bounds [62].

The region of the (k_u, λ_u) plane which satisfies all the above constraints corresponds to the shaded area in Fig. 6. Note that a big region (on the right of the dotted line) could give, as an effect of negative D-term contributions, negative or too small squared-masses for the unhiggses $H_a^\pm$ ($a = 1, 2$). It is not clear that this region is necessarily excluded, since there must be contributions to their masses from Yukawa couplings, which must give an acceptably large mass to the corresponding fermions $\tilde{H}_a^\pm$. The region of Fig. 6 under the wavy line may also be excluded, because there the sufficient condition (4.32) for the consistency of our minimization procedure is not satisfied. To decide if these points are acceptable or not, a more detailed study of the scalar potential is needed, which goes beyond the scope of the present work.

Let us now comment on the general features of the solutions corresponding to points of the surviving region. The two parameters of greatest phenomenological interest are the ratio x/v and the gaugino mass $m_{1/2}$: the first (combined with $\bar{v}/v$) determines all the neutral current phenomenology, beginning with the masses of the two neutral gauge bosons. The second gives the scale of the spectrum of all the supersymmetric particles. Depending on the values of λ_u and k_u , one can obtain values of x/v from ~ 2 to ~ 20 and values of $m_{1/2}$ from ~ 100 GeV to our upper limit of 3 TeV. The values of x/v and of $m_{1/2}$ are not unrelated, for the following reason. Because the allowed range of variation of k_u is rather restricted, the magnitude of $\hat{x}$ is essentially fixed. On the other hand, the magnitude of $\hat{v}$ and the value of $m_{1/2}$ must combine in such a way to give the right value of the W-mass. Therefore high values of x/v correspond in general to high gaugino masses, and vice versa. The points of the $(x/v, m_{1/2})$ plane corresponding to some allowed points in the (k_u, λ_u) plane are presented in Fig. 8. The corresponding points in the $(x/v, \bar{v}/v)$ plane are shown in Fig. 9. The region of very high x/v and $m_{1/2}$ poses some problem of naturalness. In this case the dependence of m_W on $\bar{Q} \equiv m_{1/2}$ is very strong, as in the right-hand side of Fig. 7b, requiring a sort

of fine-tuning of parameters to obtain the correct m_W . This is no longer true, however, for smaller values of x/v and $m_{1/2}$, as shown in Fig. 7a and in the left-hand side of Fig. 7b: one can obtain values of x/v up to ~ 10 (corresponding to gaugino masses $m_{1/2} \leq 1$ TeV). In the remainder of this paragraph we shall consider only this more natural class of solutions.

The general features of the resulting particle spectrum are summarized in Fig. 10, where the most significant masses are given as a function of x/v for some points of the allowed (λ_u, k_u) region. In particular, the mass spectrum obtained for a representative choice of λ_u and k_u , which satisfies all the phenomenological constraints, is explicitly spelled out in Table 4.

The general features of the particle spectrum are the following. In order to satisfy the particle physics bound $m_{Z_2} \geq 230$ GeV (for the typical values $0.4 \leq \bar{v}/v \leq 0.6$ of Fig. 9), $x/v \geq 3$. This requires in turn, according to Fig. 8, $m_{1/2} \equiv m_{\tilde{g}} \geq 200$ GeV and therefore, according to our approximate formula (4.24), $m_{\tilde{q}} \geq 400$ GeV: nothing particularly interesting for present-day $p\bar{p}$ Collider physics. The lightest charged partners of ordinary quarks and leptons are in general the right-handed leptons, with masses $m_{\tilde{e}_R} \geq 80$ GeV which could be interesting for LEP. Due to the presence of non-negligible off-diagonal terms in the mass matrix (3.9), the lightest charged supersymmetric particle is a chargino: for $x/v \geq 3$ it can be as light as 65 GeV. The lightest supersymmetric particle is however a neutralino, in accordance with astrophysical and cosmological constraints. We can have neutralinos as light as 40 GeV. For what concerns the non-supersymmetric sector of our model, for $x/v \geq 3$ and typical values of λ around 0.1 one gets $m_D \geq 140$ GeV. On the other hand, due to negative D-term contributions, there is the possibility of having light charged Higgses, but then masses strongly depend on the choice of the parameters, so that no definite prediction can be made.

Let us comment finally, for the sake of completeness, on the possibility of a top quark mass significantly heavier than 40 GeV. A different value of h_u will correspond, in general, to a different allowed region in the (λ_u, k_u) plane: the case $h_u = 0.035$ (corresponding to $m_{\text{top}} \sim 55$ GeV) is shown in Fig. 11. A general result is that, as long as the top Yukawa coupling increases, m_H^2 tends to acquire lower and lower values around the electroweak scale, making less and less natural the desired hierarchy $x > v$: the phenomenological success of the models considered in this paper tends therefore to ask for a relatively light top mass [31].

5. CONCLUSIONS AND OPEN PROBLEMS

Taking the empirical view that a theory is scientific if and only if it is subject to experimental test and falsifiable, in this paper we have developed the phenomenological aspects of $N = 1$, $D = 10$ supergravity—the best field-theoretical candidate for a unified theory, which may also be the low-energy limit of the superstring. We have studied supersymmetry breaking in this model and shown how gaugino condensation in the hidden sector can occur and trigger supersymmetry breaking. We have gone on to show how this can be fed through to the observable sector via a non-zero gaugino mass, while supersymmetry breaking scalar masses are initially negligible. The phenomenological low energy $SU(N,1)/SU(N) \times U(1)$ structure is probably not maintained by radiative corrections, but they do not give masses to the gauge-non-singlet scalars, at least at the one-loop level. We have then exhibited two variants of the no-scale models proposed previously which may lead to the dynamical generation of all low mass scales including the gauge hierarchy. Consistency demands that the string excitation scale, the Planck

mass, the compactification scale, the grand unification scale not differ by many orders of magnitude. We see no reason why this should not be possible, nor why it should prevent the construction of a realistic low-energy model.

A priori there is considerable ambiguity in the four-dimensional gauge group, which must be some subgroup of the E_6 grand unification group proposed [65] previously. No-scale models do not tolerate intermediate mass scales, which led us uniquely to the minimal low-energy gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$. We obtained a unique set of low-energy particle fields for this model, and an almost unique set of Yukawa couplings. The couplings of the new $U(1)_E$ gauge boson are fixed, and we have developed phenomenological constraints on its mass coming from low-energy neutral current experiments, the observed mass of the Z^0 , and primordial nucleosynthesis. It could have a mass as low as $O(200)$ GeV.

We have made detailed calculations of the generation of the gauge hierarchy in our minimal model, giving general formulae for the sparticle masses including renormalization corrections and D-terms, and estimating their values for different values of the input Yukawa coupling parameters. We have found acceptable no-scale models with $m_W \sim 80$ GeV, $m_{1/2} \sim 150$ GeV to 1 TeV, and a similar range for the new $U(1)$ gauge boson mass. We have found choices of the parameters which are consistent with all phenomenological constraints. Supersymmetric particles may be too heavy to be found at the present $\bar{p}p$ colliders or at LEP, but should be accessible to a higher energy collider such as the SSC or LHC, along with the new neutral gauge boson.

Open problems include that of finding the correct compactification scheme which gives the 3 generation, rank 5 low-energy theory with certain vanishing Yukawa couplings that we need. Also, better understanding of the gauge-singlet fields in the four-dimensional field theory is required. We need to know whether and how they are coupled to each other and to the gauge non-singlet fields, in order to choose between our two variants of the no-scale scenario. We also need to know the values of certain Yukawa couplings before we can make precise predictions for the masses of the sparticles and of the new neutral gauge boson. These are problems to be solved in the (presumably) underlying string theory, and at least the latter also depends on details of the compactification.

Even though there are many open problems, we are very excited that now we have for the first time a fundamental theory which is apparently rich enough to provide the complexity of the low energy physics observed. Moreover, there is hope that in the near future one might be able to extract reliable phenomenological predictions from it, and confront them with experiment. We may soon know whether this is the stuff that dreams are made of, or if we are fishing for tigers in a stream.

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APPENDIX A

In this Appendix we write down explicitly the supersymmetric one-loop renormalization group equations for the masses and couplings of the model considered in the text: an unbroken $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$ gauge symmetry is assumed, with a 'light' matter content given by 3 complete 27 's of E_6 . For what concerns the normalization of the $U(1)$ gauge coupling constants, we shall take g_1 and g_E such that $g_3 = g_2 = g_1 = g_E$ when embedded in E_6 .

Gauge couplings and gaugino masses

$$\frac{dg_a^2}{dt} = \frac{b_a}{8\pi^2} g_a^4, \quad (A.1)$$

$$\frac{dM_a}{dt} = \frac{g_a^2}{8\pi^2} M_a, \quad (A.2)$$

where $t = \ln Q$, $a = 3, 2, 1, E$ and $b_3 = 0$, $b_2 = 3$, $b_1 = b_E = 9$. With the boundary conditions (4.5)–(4.6) the above equations are trivially solved to give:

$$g_a^2(Q) = \frac{g_u^2}{1 + \frac{b_a}{8\pi} g_u^2 \ln\left(\frac{M_X}{Q}\right)}, \quad (A.3)$$

$$M_a(Q) = \frac{g_a^2(Q)}{g_u^2} M_{aU}. \quad (A.4)$$

Note that, assuming $g_1(M_X) = g_E(M_X)$ and $M_{1U} = M_{EU}$, the gauge coupling constants and the gaugino masses associated to the two $U(1)$ -factors evolve in exactly the same way. This leads to considerable simplifications in the following RGEs for the Yukawa couplings and the other soft SUSY breaking parameters, which are however shown in the general case.

Yukawa couplings and trilinear scalar couplings

$$\frac{dh}{dt} = \frac{h}{8\pi^2} \left(-\frac{8}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{13}{30} g_1^2 - \frac{2}{5} g_E^2 + 3h^2 + \frac{1}{2} \lambda^2 \right) \quad (A.5)$$

$$\frac{d\lambda}{dt} = \frac{\lambda}{8\pi^2} \left(-\frac{3}{2} g_2^2 - \frac{3}{10} g_1^2 - \frac{7}{10} g_E^2 + \frac{3}{2} h^2 + 2\lambda^2 + \frac{3}{2} \kappa^2 \right) \quad (A.6)$$

$$\frac{dk}{dt} = \frac{k}{8\pi^2} \left(-\frac{8}{3} g_3^2 - \frac{2}{15} g_1^2 - \frac{7}{10} g_E^2 + \lambda^2 + \frac{5}{2} k^2 \right) \quad (\text{A.7})$$

$$\frac{dA_h}{dt} = \frac{1}{8\pi^2} \left(\frac{16}{3} M_3 g_3^2 + 3 M_2 g_2^2 + \frac{13}{15} M_1 g_1^2 + \frac{4}{5} M_E g_E^2 + 6 A_h h^2 + A_\lambda \lambda^2 \right) \quad (\text{A.8})$$

$$\frac{dA_\lambda}{dt} = \frac{1}{8\pi^2} \left(3 M_2 g_2^2 + \frac{3}{5} M_1 g_1^2 + \frac{7}{5} M_E g_E^2 + 3 A_h h^2 + 4 A_\lambda \lambda^2 + 3 A_k k^2 \right) \quad (\text{A.9})$$

$$\frac{dA_k}{dt} = \frac{1}{8\pi^2} \left(\frac{16}{3} M_3 g_3^2 + \frac{4}{15} M_1 g_1^2 + \frac{7}{5} M_E g_E^2 + 2 A_\lambda \lambda^2 + 5 A_k k^2 \right) \quad (\text{A.10})$$

Soft supersymmetry-breaking scalar masses

Using $a = 1, 2, 3$ as a generation index, and making for notational convenience the identifications $H_3 \equiv H$, $\bar{H}_3 \equiv \bar{H}$, $N_3 \equiv N$, $D_3 \equiv D$, $D_3^c \equiv D^c$, one has:

$$\frac{d\tilde{m}_{a_a}^2}{dt} = \frac{1}{8\pi^2} \left(-\frac{16}{3} M_3^2 g_3^2 - 3 M_2^2 g_2^2 - \frac{1}{15} M_1^2 g_1^2 - \frac{4}{15} M_E^2 g_E^2 + \delta_{a3} h^2 F_h \right) \quad (\text{A.11})$$

$$\frac{d\tilde{m}_{u_a^c}^2}{dt} = \frac{1}{8\pi^2} \left(-\frac{16}{3} M_3^2 g_3^2 - \frac{16}{15} M_1^2 g_1^2 - \frac{4}{15} M_E^2 g_E^2 + \delta_{a3} 2 h^2 F_h \right) \quad (\text{A.12})$$

$$\frac{d\tilde{m}_{d_a^c}^2}{dt} = \frac{1}{8\pi^2} \left(-\frac{16}{3} M_3^2 g_3^2 - \frac{4}{15} M_1^2 g_1^2 - \frac{1}{15} M_E^2 g_E^2 \right) \quad (\text{A.13})$$

$$\frac{d\tilde{m}_{L_a}^2}{dt} = \frac{1}{8\pi^2} \left(-3 M_2^2 g_2^2 - \frac{3}{5} M_1^2 g_1^2 - \frac{1}{15} M_E^2 g_E^2 \right) \quad (\text{A.14})$$

$$\frac{d\tilde{m}_{e_a^c}^2}{dt} = \frac{1}{8\pi^2} \left(-\frac{12}{5} M_1^2 g_1^2 - \frac{4}{15} M_E^2 g_E^2 \right) \quad (\text{A.15})$$

$$\frac{d\tilde{m}_{\chi_a^c}^2}{dt} = \frac{1}{8\pi^2} \left(-\frac{5}{3} M_E^2 g_E^2 \right) \quad (\text{A.16})$$

$$\frac{dm_{H_a}^2}{dt} = \frac{1}{8\pi^2} \left(-3M_2^2 g_2^2 - \frac{3}{5} M_1^2 g_1^2 - \frac{16}{15} M_E^2 g_E^2 + \delta_{a3} 3h^2 F_h + \delta_{a3} \lambda^2 F_\lambda \right) \quad (\text{A.17})$$

$$\frac{dm_{\bar{H}_a}^2}{dt} = \frac{1}{8\pi^2} \left(-3M_2^2 g_2^2 - \frac{3}{5} M_1^2 g_1^2 - \frac{1}{15} M_E^2 g_E^2 + \delta_{a3} \lambda^2 F_\lambda \right) \quad (\text{A.18})$$

$$\frac{dm_{N_a}^2}{dt} = \frac{1}{8\pi^2} \left(-\frac{5}{3} M_E^2 g_E^2 + \delta_{a3} 2\lambda^2 F_\lambda + \delta_{a3} 3k^2 F_k \right) \quad (\text{A.19})$$

$$\frac{d\tilde{m}_{D_a}^2}{dt} = \frac{1}{8\pi^2} \left(-\frac{16}{3} M_3^2 g_3^2 - \frac{4}{15} M_1^2 g_1^2 - \frac{16}{15} M_E^2 g_E^2 + \delta_{a3} k^2 F_k \right) \quad (\text{A.20})$$

$$\frac{d\tilde{m}_{D_a^c}^2}{dt} = \frac{1}{8\pi^2} \left(-\frac{16}{3} M_3^2 g_3^2 - \frac{4}{15} M_1^2 g_1^2 - \frac{1}{15} M_E^2 g_E^2 + \delta_{a3} k^2 F_k \right) \quad (\text{A.21})$$

The functions F_h , F_λ and F_k are defined by:

$$F_h \equiv \tilde{m}_{Q_3}^2 + \tilde{m}_{u_3^c}^2 + m_H^2 + A_h^2 \quad (\text{A.22})$$

$$F_\lambda \equiv m_H^2 + m_{\bar{H}}^2 + m_N^2 + A_\lambda^2 \quad (\text{A.23})$$

$$F_k \equiv \tilde{m}_D^2 + \tilde{m}_{D^c}^2 + m_N^2 + A_k^2 \quad (\text{A.24})$$

Note that, as long as they do not involve the Yukawa couplings h , λ and k , which are assumed to be the only non-negligible ones, the RGEs for the soft scalar masses assume the simple form:

$$\frac{dm_i^2}{dt} = -\frac{1}{8\pi^2} \sum_{a=3,2,1,E} c_a(i) M_a^2 g_a^2, \quad (\text{A.25})$$

where i is an index running over all the scalar fields and the coefficients $c_a(i)$ can be read off from equations (A.11) to (A.21). Taking into account Eqs. (A.3)-(A.4) and the fact that $b_3 = 0$, $b_2 = 3$, $b_1 = b_E = 9$, their solutions can be computed analytically:

$$\begin{aligned}
 m_i^2(Q) = & c_3(i) \cdot \frac{g_U^2}{8\pi^2} \ln \frac{M_X}{Q} \cdot M_{3U}^2 \\
 & + \frac{c_2(i)}{6} \left[1 - \frac{1}{\left(1 + 3 \cdot \frac{g_U^2}{8\pi^2} \ln \frac{M_X}{Q}\right)^2} \right] M_{2U}^2 \\
 & + \frac{c_1(i)}{18} \left[1 - \frac{1}{\left(1 + 9 \cdot \frac{g_U^2}{8\pi^2} \ln \frac{M_X}{Q}\right)^2} \right] M_{1U}^2 \\
 & + \frac{c_E(i)}{18} \left[1 - \frac{1}{\left(1 + 9 \cdot \frac{g_U^2}{8\pi^2} \ln \frac{M_X}{Q}\right)^2} \right] M_{EU}^2 .
 \end{aligned} \tag{A.26}$$

All the other RGEs have been solved numerically.

APPENDIX B

We collect here the explicit expressions used in the computation of the scalar masses. When the mixing is not negligible, as in the case of the top-squarks and of the D-squarks involved in the coupling kDD^cN , the matrix elements appearing in (4.13) are given, leaving to the reader the trivial diagonalization. The masses of the scalars involved in the gauge symmetry breaking are separately computed in the text.

$$\tilde{u}_3, \tilde{u}_3^c: \quad m_{LL}^2 = \tilde{m}_{Q_3}^2 + h^2 v^2 - \frac{g_2^2}{4} (v^2 - \bar{v}^2) + \frac{g_1^2}{20} (v^2 - \bar{v}^2) + \frac{g_E^2}{30} (5x^2 - 4v^2 - \bar{v}^2)$$

$$m_{RR}^2 = \tilde{m}_{u_3^c}^2 + h^2 \bar{v}^2 - \frac{g_1^2}{5} (v^2 - \bar{v}^2) + \frac{g_E^2}{30} (5x^2 - 4v^2 - \bar{v}^2)$$

$$m_{LR}^2 = hA_h v + h\lambda x \bar{v}$$

$$\tilde{u}_a (a=1,2): \quad m^2(\tilde{u}_a) = \tilde{m}_{Q_a}^2 - \frac{g_2^2}{4} (v^2 - \bar{v}^2) + \frac{g_1^2}{20} (v^2 - \bar{v}^2) + \frac{g_E^2}{30} (5x^2 - 4v^2 - \bar{v}^2)$$

$$\tilde{u}_a^c (a=1,2): \quad m^2(\tilde{u}_a^c) = \tilde{m}_{u_a^c}^2 - \frac{g_1^2}{5} (v^2 - \bar{v}^2) + \frac{g_E^2}{30} (5x^2 - 4v^2 - \bar{v}^2)$$

$$\tilde{d}_a (a=1,2,3): \quad m^2(\tilde{d}_a) = \tilde{m}_{Q_a}^2 + \frac{g_2^2}{4} (v^2 - \bar{v}^2) + \frac{g_1^2}{20} (v^2 - \bar{v}^2) + \frac{g_E^2}{30} (5x^2 - 4v^2 - \bar{v}^2)$$

$$\tilde{d}_a^c (a=1,2,3): \quad m^2(\tilde{d}_a^c) = \tilde{m}_{d_a^c}^2 + \frac{g_1^2}{10} (v^2 - \bar{v}^2) - \frac{g_E^2}{60} (5x^2 - 4v^2 - \bar{v}^2)$$

$$\tilde{e}_a (a=1,2,3): \quad m^2(\tilde{e}_a) = \tilde{m}_{L_a}^2 + \frac{g_2^2}{4} (v^2 - \bar{v}^2) - \frac{3g_1^2}{20} (v^2 - \bar{v}^2) - \frac{g_E^2}{60} (5x^2 - 4v^2 - \bar{v}^2)$$

$$\tilde{e}_a^c (a=1,2,3): \quad m^2(\tilde{e}_a^c) = \tilde{m}_{e_a^c}^2 + \frac{3g_1^2}{10} (v^2 - \bar{v}^2) + \frac{g_E^2}{30} (5x^2 - 4v^2 - \bar{v}^2)$$

$$\tilde{\nu}_a (a=1,2,3): \quad m^2(\tilde{\nu}_a) = \tilde{m}_{L_a}^2 - \frac{g_2^2}{4} (v^2 - \bar{v}^2) - \frac{3g_1^2}{20} (v^2 - \bar{v}^2) - \frac{g_E^2}{60} (5x^2 - 4v^2 - \bar{v}^2)$$

$$\tilde{\nu}_a^c (a=1,2,3) : m^2(\tilde{\nu}_a^c) = \tilde{m}_{\nu_a^c}^2 + \frac{g_E^2}{12} (5x^2 - 4v^2 - \bar{v}^2)$$

$$N_a (a=1,2) : m^2(N_a) = m_{N_a}^2 + \frac{g_E^2}{12} (5x^2 - 4v^2 - \bar{v}^2)$$

$$H_a^+ (a=1,2) : m^2(H_a^+) = m_{H_a^+}^2 - \frac{g_1^2}{4} (v^2 - \bar{v}^2) + \frac{3g_1^2}{20} (v^2 - \bar{v}^2) - \frac{g_E^2}{15} (5x^2 - 4v^2 - \bar{v}^2)$$

$$H_a^0 (a=1,2) : m^2(H_a^0) = m_{H_a^0}^2 + \frac{g_2^2}{4} (v^2 - \bar{v}^2) + \frac{3g_1^2}{20} (v^2 - \bar{v}^2) - \frac{g_E^2}{15} (5x^2 - 4v^2 - \bar{v}^2)$$

$$\bar{H}_a^- (a=1,2) : m^2(\bar{H}_a^-) = m_{\bar{H}_a^-}^2 + \frac{g_2^2}{4} (v^2 - \bar{v}^2) - \frac{3g_1^2}{20} (v^2 - \bar{v}^2) - \frac{g_E^2}{60} (5x^2 - 4v^2 - \bar{v}^2)$$

$$\bar{H}_a^0 (a=1,2) : m^2(\bar{H}_a^0) = m_{\bar{H}_a^0}^2 - \frac{g_2^2}{4} (v^2 - \bar{v}^2) - \frac{3g_1^2}{20} (v^2 - \bar{v}^2) - \frac{g_E^2}{60} (5x^2 - 4v^2 - \bar{v}^2)$$

$$\tilde{D}, \tilde{D}^c : m_{LL}^2 = \tilde{m}_D^2 + k^2 x^2 - \frac{g_1^2}{10} (v^2 - \bar{v}^2) - \frac{g_E^2}{15} (5x^2 - 4v^2 - \bar{v}^2)$$

$$m_{RR}^2 = \tilde{m}_{D^c}^2 + k^2 x^2 + \frac{g_1^2}{10} (v^2 - \bar{v}^2) - \frac{g_E^2}{60} (5x^2 - 4v^2 - \bar{v}^2)$$

$$m_{LR}^2 = k A_K x + 1 k v \bar{v}$$

$$\tilde{D}_a (a=1,2) : m^2(\tilde{D}_a) = \tilde{m}_{D_a}^2 - \frac{g_1^2}{10} (v^2 - \bar{v}^2) - \frac{g_E^2}{15} (5x^2 - 4v^2 - \bar{v}^2)$$

$$\tilde{D}_a^c (a=1,2) : m^2(\tilde{D}_a^c) = \tilde{m}_{D_a^c}^2 - \frac{g_1^2}{10} (v^2 - \bar{v}^2) - \frac{g_E^2}{60} (5x^2 - 4v^2 - \bar{v}^2)$$

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Table 1

Transformation properties under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$ of the chiral matter superfields contained in the $\underline{27}$ of E_6 . Group and generation indices are understood. In terms of Y_L and Y_R , associated to the maximal $SU(3)_C \times SU(3)_L \times SU(3)_R$ subgroup of E_6 , the conventional weak hypercharge Y and the new hypercharge Y_E are given by $Y = (Y_L + Y_R)/2$, $Y_E = (Y_L - Y_R)/4$. The properly normalized quantities $\hat{Y}$ and $\hat{Y}_E$, such that $\text{Tr } T_3^2 = \text{Tr } \hat{Y}^2 = \text{Tr } \hat{Y}_E^2$, are given by $\hat{Y} = \sqrt{3/5} Y$ and $\hat{Y}_E = \sqrt{3/5} Y_E$.

According to our conventions, ν^c is the $SU(3)_C \times SU(2)_L \times U(1)_Y$ -singlet contained in the $\underline{16}$ of $SO(10) \subset E_6$, while N is the $SO(10)$ -singlet.

Chiral superfield	$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$ quantum numbers
$Q \equiv \begin{pmatrix} u \\ d \end{pmatrix}$	$(3, 2, \frac{1}{6}, \frac{1}{3})$
u^c	$(\bar{3}, 1, -\frac{2}{3}, \frac{1}{3})$
d^c	$(\bar{3}, 1, \frac{1}{3}, -\frac{1}{6})$
$L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}$	$(1, 2, -\frac{1}{2}, -\frac{1}{6})$
e^c	$(1, 1, 1, \frac{1}{3})$
ν^c	$(1, 1, 0, \frac{5}{6})$
$H \equiv \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	$(1, 2, \frac{1}{2}, -\frac{2}{3})$
$\bar{H} \equiv \begin{pmatrix} \bar{H}^0 \\ \bar{H}^- \end{pmatrix}$	$(1, 2, -\frac{1}{2}, -\frac{1}{6})$
N	$(1, 1, 0, \frac{5}{6})$
D	$(3, 1, -\frac{1}{3}, -\frac{2}{3})$
D^c	$(\bar{3}, 1, \frac{1}{3}, -\frac{1}{6})$

Table 2

Numerical values of the $C_a(i, Q)$ coefficients, appearing in formula (4.23) for the soft supersymmetry breaking scalar masses, calculated for the different fields at the representative scale $Q = 300 \text{ GeV}$.

Scalar Field	C_3	C_2	C_1	C_E
$\tilde{u}, \tilde{d}$	3.261	0.438	0.004	0.014
$\tilde{u}^c$	3.261	0	0.058	0.014
$\tilde{d}^c$	3.261	0	0.014	0.004
$\tilde{e}, \tilde{\nu}$	0	0.438	0.033	0.004
$\tilde{e}^c$	0	0	0.130	0.014
$\tilde{\nu}^c$	0	0	0	0.090
H^+, H^0	0	0.438	0.033	0.058
$\bar{H}^0, \bar{H}^-$	0	0.438	0.033	0.004
N	0	0	0	0.090
$\tilde{D}$	3.261	0	0.014	0.058
$\tilde{D}^c$	3.261	0	0.014	0.004

Table 3

Particle and sparticle spectrum of the 'no-scale' model corresponding to the boundary conditions $h_u = 0.033$, $\lambda_u = 0.31$ and $k_u = 0$, where one finds $m_{1/2} \simeq 490$ GeV. This example is not fully realistic because, due to the choice $k_u = 0$, D-quarks cannot acquire mass, and also because it gives $x/v \simeq 2.7$ and $\bar{v}/v \simeq 0.4$, slightly outside the allowed region, but it demonstrates the possibility of obtaining $m_{Z_2} \gg m_Z$ also in the 'proper no-scale' scenario.

Particle	Mass (GeV)
Top-quark	52
D-quark	—
$Z_1 \begin{cases} (\sin^2 \theta_W = 0.206) \\ (\sin^2 \theta_W = 0.220) \end{cases}$	88.6 89.4
$Z_2 \begin{cases} (\sin^2 \theta_W = 0.206) \\ (\sin^2 \theta_W = 0.220) \end{cases}$	185 192
$H_a^\pm, \bar{H}_a^\pm$	340, 340
$H^\pm$	400
$\tilde{t}_1, \tilde{t}_2$	820, 960
$\tilde{u}_R, \tilde{c}_R; \tilde{u}_L, \tilde{c}_L$	900, 940
$\tilde{b}_R, \tilde{b}_L$	880, 920
$\tilde{d}_R, \tilde{s}_R; \tilde{d}_L, \tilde{s}_L$	880, 960
$\tilde{D}_1, \tilde{D}_2$	880, 880
$\tilde{D}_a, \tilde{D}_a^c$	880, 880
$\tilde{\ell}_R^\pm, \tilde{\ell}_L^\pm$	200, 340
$\tilde{\nu}_R, \tilde{\nu}_L$	190, 320
$\tilde{\chi}^0$ (lightest neutralino)	83
$\tilde{\chi}^\pm$ (lightest chargino)	170
$\tilde{g}$	490

Table 4

Particle and sparticle spectrum of the ‘hybrid’ dimensional-transmutation model for $h_u = 0.025$, (corresponding to $m_{\text{top}} \approx 40$ GeV) and two representative choices of the parameters (k_u, λ_u). Case (a), corresponding to $k_u = 0.055$ and $\lambda_u = 0.070$, gives $m_{1/2} = 840$ GeV, $x/v \approx 9.9$ and $\bar{v}/v \approx 0.4$, and is in agreement both with the particle physics limits and with the cosmological constraints, but is characterized by a rather heavy sparticle spectrum. Case (b), corresponding to $k_u = 0.050$ and $\lambda_u = 0.100$, gives $m_{1/2} = 310$ GeV, $x/v \approx 3.9$ and $\bar{v}/v \approx 0.5$, and is characterized by a lighter sparticle spectrum, but does not satisfy the cosmological limit (3.32)

Particle	Mass (GeV)	
	(a)	(b)
Top-quark	42	41
D-quark	530	180
$Z_1 \begin{cases} (\sin^2 \theta_W = 0.206) \\ (\sin^2 \theta_W = 0.220) \end{cases}$	91.8 92.6	90.5 91.3
$Z_2 \begin{cases} (\sin^2 \theta_W = 0.206) \\ (\sin^2 \theta_W = 0.220) \end{cases}$	650 680	250 260
$H_a^\pm, \bar{H}_a^\pm$	450, 540	160, 210
$H^\pm$	620	260
$\tilde{t}_1, \tilde{t}_2$	1500, 1600	540, 630
$\tilde{u}_R, \tilde{c}_R; \tilde{u}_L, \tilde{c}_L$	1500, 1600	580, 610
$\tilde{b}_R, \tilde{b}_L$	1500, 1600	560, 610
$\tilde{d}_R, \tilde{s}_R; \tilde{d}_L, \tilde{s}_L$	1500, 1600	560, 610
$\tilde{D}_1, \tilde{D}_2$	970, 1900	380, 720
$\tilde{D}_a, \tilde{D}_a^c$	1500, 1500	550, 560
$\tilde{\ell}_R^\pm, \tilde{\ell}_L^\pm$	430, 540	160, 210
$\tilde{\nu}_R, \tilde{\nu}_L$	520, 540	190, 200
$\tilde{\chi}^0$ (lightest neutralino)	140	57
$\tilde{\chi}^\pm$ (lightest chargino)	210	110
$\tilde{g}$	840	310

Figure captions

- Fig. 1** Contours in the $(\bar{v}/v, x/v)$ -plane describing the deviations from the standard neutral current phenomenology. The solid lines correspond to different values (200, 400 and 600 GeV) for the mass m_{Z_2} of the heaviest eigenstate of the Z-boson mass matrix. The dotted lines are associated to different values (0.01, 0.02, 0.03, 0.04 and 0.05) of the parameter $\Delta \equiv \sin^2 \theta_W - \sin^2 \bar{\theta}_W$. As input data we have assumed $\sin^2 \theta_W = 0.22$ and $m_W = 82$ GeV.
- Fig. 2** One-loop contribution to the gluino mass.
- Fig. 3** The one-loop gluino mass $m_{\tilde{g}}$ as a function of the D-quark mass m_D , in the case where the tree-level gluino mass is vanishing. Three possible values of the ratio $m_D/m_{1/2}$ are considered: $m_D/m_{1/2} = 1, 2, 3$.
- Fig. 4** Typical Q-dependence of the soft scalar masses m_H^2 , $m_{\tilde{H}}^2$ and m_N^2 according to the renormalization group equations of Appendix A. Figure 4a shows the evolution of $\hat{m}_H^2 \equiv m_H^2/m_{1/2}^2$ for $h_u = 0.025, 0.035$ and 0.045 (when $\lambda_u = 0.1$ and $k_u = 0.05$). Figure 4b shows the evolution of $\hat{m}_H^2 \equiv m_H^2/m_{1/2}^2$ for $\lambda_u = 0, 0.1$ and 0.3 (when $h_u = 0.035$ and $k_u = 0.05$). Figure 4c shows the evolution of $\hat{m}_N^2 \equiv m_N^2/m_{1/2}^2$ for $k_u = 0, 0.05$ and 0.1 (when $h_u = 0.035$ and $\lambda_u = 0.1$).
- Fig. 5** Dependence of the three vacuum expectation values $v(\circ)$, $\bar{v}(+)$, and $x(\bullet)$ on the renormalization scale Q , as an effect of radiative corrections. Case (a) corresponds to $h_u = 0.035$, $\lambda_u = 0.17$ and $k_u = 0$, case (b) to $h_u = 0.025$, $\lambda_u = 0.1$ and $k_u = 0.05$.
- Fig. 6** Allowed region in the (k_u, λ_u) -plane for $h_u = 0.025$ (corresponding to $m_{\text{top}} \sim 40$ GeV). The region outside the solid lines is excluded if we want to have $m_W \approx 82$ GeV for a gaugino mass $50 \text{ GeV} \leq m_{1/2} \leq 3 \text{ TeV}$. The region on the right of the dotted line may be excluded because it corresponds to negative or too small squared-masses of the charged scalars $H_{\pm}^{\pm}$ if their Yukawa couplings are negligible, which need not be the case. In the region below the wavy line $\lambda^2 - g_E^2/12 < 0$, and thus the minimum obtained with the method described in the text is not necessarily the global one. The shaded area is the allowed one: in the darker region we can have solutions with $m_{1/2} < 1 \text{ TeV}$ and $x/v > 3$, while in the lighter one these two requirements cannot be met simultaneously.
- Fig. 7** Relation between the symmetry breaking scale $\bar{Q}$ (identified with the gaugino mass $m_{1/2}$) and the corresponding value of m_W . Case (a) corresponds to $h_u = 0.025$, $\lambda_u = 0.07$, $k_u = 0.05$, case (b) to $h_u = 0.025$, $\lambda_u = 0.12$ and $k_u = 0.07$.
- Fig. 8** Theoretical ‘scatter-plot’ showing the correlation between the ratio of VEVs x/v and the gaugino mass $m_{1/2}$, for $h_u = 0.025$ (corresponding to $m_{\text{top}} \sim 40$ GeV). The 48 points of this figure correspond to the points of a square lattice in the region of the (k_u, λ_u) -plane which allows for $m_{1/2} < 1 \text{ TeV}$.
- Fig. 9** Theoretical ‘scatter-plot’ showing the region of the $(\bar{v}/v, x/v)$ -plane favoured by our model calculations in the case $h_u = 0.025$ (corresponding to $m_{\text{top}} \sim 40$ GeV). The 48 points of this figure correspond to the points of a square lattice in the region of the (k_u, λ_u) -plane which allows for $m_{1/2} < 1 \text{ TeV}$.

Fig. 10 Theoretical ‘scatter-plot’ showing the dependence of some typical s-particle masses on the ratio x/v , for $h_u = 0.025$ (corresponding to $m_{\text{top}} \sim 40$ GeV). As a representative squark mass $m_{\tilde{q}}$ (denoted by crosses) we have taken that of the right-handed up-squarks of the first two generations. The masses of the left-handed and right-handed charged s-leptons ($m_{\tilde{e}_L}, m_{\tilde{e}_R}$) are denoted by full and empty circles, respectively. The masses of the lightest chargino and neutralino ($\tilde{\chi}^\pm, \tilde{\chi}^0$) are denoted by full and empty squares, respectively. The 5×23 points of the figure correspond to the points of a square lattice in the region of the (k_u, λ_u) -plane which allows for $m_{1/2} < 1$ TeV. Interpolating lines have been drawn for the reader’s convenience.

Fig. 11 Allowed region in the (k_u, λ_u) -plane for $h_u = 0.035$ (corresponding to $m_{\text{top}} \sim 55$ GeV). The region outside the solid lines is excluded if we want to have $x > v$ and $m_W \approx 82$ GeV for a gaugino mass $50 \text{ GeV} \leq m_{1/2} \leq 3 \text{ TeV}$. The region on the right of the dotted line may be excluded because it corresponds to negative or too small $(\text{mass})^2$ of the charged scalars $H_\pm^\pm$ if their Yukawa couplings are negligible, which need not be the case. In the region below the wavy line $\lambda^2 - g_E^2/12 < 0$, and thus the minimum obtained with the method described in the text is not necessarily the global one. The region on the left of the dashed line is ruled out because it corresponds to be unacceptably light D-quarks ($m_D < 20$ GeV). The shaded area is the allowed one: in the darker region we can have solutions with $m_{1/2} < 1$ TeV and $x/v > 3$, while in the lighter one these two requirements cannot be met simultaneously.

Fig. 1

Fig. 2

Fig. 3

Fig. 4a

Fig. 4b

Fig. 4c

Fig. 5a

Fig. 5b

Fig. 6

Fig. 7a

Fig. 7b

Fig. 9

Fig. 10

Fig. 11