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GAUGE-FIXING IN COVARIANT STRING FIELD THEORY

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ABSTRACT

We discuss the gauge-fixing procedure of the covariant formulation of the open bosonic string recently proposed by Neveu, Nicolai and West. We show that this theory has new symmetries. However, it does not seem possible to recover the correct number of physical degrees of freedom.

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1. Introduction

The problem of covariant formulation of string field theory has recently received much attention¹⁾⁻⁹⁾. Some common features of the various formulations proposed are a large set of local gauge invariances and the introduction of supplementary string fields in order to obtain a local field theory. Among those, one formulation with a "minimal" set of supplementary string fields (hereafter referred to as the finite set) has been proposed by Neveu, Nicolai and West⁵⁾. The aim of this letter is to consider the gauge-fixing procedure of the finite set formulation of the open bosonic string at $D=26$ in order to determine if this theory contains the correct number of physical degrees of freedom.

The problems of the finite set are that if we restrict ourselves to the local symmetries presented in Ref.5), a) the number of classical fields minus the number of ghosts does not coincide with the number of physical degrees of freedom obtained in the light-cone gauge¹⁰⁾; b) it is impossible to recover the physical conditions $L_1\Psi = L_2\Psi = 0$. However, as in other formulations^{4),7),8)}, we found that the theory has additional symmetries acting only on the supplementary fields and modifying completely the ghost structure. Namely this leads to the phenomenon of ghosts for ghosts repeating itself. Doing this, we have improved the situation described above but unfortunately the counting of physical degrees of freedom still fails at the 10^{th} level. It is also possible to show that this result is supported by purely classical considerations. One cannot exclude on logical grounds that we could have missed some point in both arguments even if it seems to us they are rather independent. Note also that the authors of

Ref.5 have proposed⁸⁾ an extension of the finite set with more fields and symmetries having manifestly the correct counting.

This paper is organized as follows. In section 2 we recall the main features of the finite set formulation and its ghost counting problem. Some useful technical tools are developed in section 3 and used in section 4 in order to find a necessary condition for the elimination of the supplementary fields. Section 5 is devoted to the new symmetries and their ghosts.

2. The ghost counting problem

The finite set formulation of the open bosonic string at $D=26$ can be summarized by its equations of motion:

$$\frac{\delta S}{\delta \psi} = (L_0 - 1) \psi + L_{-1} \phi^1 + L_{-2} \phi^2 = 0$$

$$\frac{\delta S}{\delta \phi^1} = L_1 \psi + 2 \phi^1 + L_{-1} \zeta^2 + L_{-2} \zeta^3 = 0$$

$$\frac{\delta S}{\delta \phi^2} = L_2 \psi + 4 \phi^2 + 3 \zeta^2 + L_{-1} \zeta'^3 + L_{-2} \zeta^4 = 0$$

$$\frac{\delta S}{\delta \zeta^2} = L_1 \phi^1 + 3 \phi^2 - (L_0 + 1) \zeta^2 = 0 \quad (1)$$

$$\frac{\delta S}{\delta \zeta^3} = L_2 \phi^1 - (L_0 + 2) \zeta'^3 = 0$$

$$\frac{\delta S}{\delta \zeta'^3} = L_1 \phi^2 - (L_0 + 2) \zeta^3 = 0$$

$$\frac{\delta S}{\delta \zeta^4} = L_2 \phi^2 - (L_0 + 3) \zeta^4 = 0$$

The action is invariant under the following gauge transformations:

$$\begin{aligned} \delta \psi &= L_{-1} \Lambda^1 + L_{-2} \Lambda^2; \quad \delta \phi^1 = -L_0 \Lambda^1; \quad \delta \phi^2 = -(L_0 + 1) \Lambda^2 \\ \delta \zeta^2 &= -L_1 \Lambda^1 - 3 \Lambda^2; \quad \delta \zeta^3 = -L_1 \Lambda^2; \quad \delta \zeta'^3 = -L_2 \Lambda^1; \quad \delta \zeta^4 = -L_2 \Lambda^2 \end{aligned} \quad (2)$$

Note that the procedure can be extended to arbitrary D ¹¹⁾ by introducing an additional string field ζ'^2 with the gauge transformations

$$\delta \zeta'^2 = -3 \Lambda^2 \quad (3)$$

The third equation of motion becomes

$$\frac{\delta S}{\delta \phi^2} = L_2 \psi + 4 \phi^2 + 3 \zeta^2 + \left(\frac{D-26}{6} \right) \zeta'^2 + L_{-1} \zeta'^3 + L_{-2} \zeta^4 = 0 \quad (4)$$

We have also an additional equation of motion for ζ'^2

$$\frac{\delta S}{\delta \zeta'^2} = \left(\frac{D-26}{18} \right) (3 \phi^2 - (L_0 + 1) \zeta'^2) = 0 \quad (5)$$

However, in the rest of this paper we will only consider the case D=26. The number of field components of the string field Ψ at level N is then $T^{26}(N)$.

By convention, the lowest level corresponds to N=0 and

$$\sum_{N=0}^{\infty} T^D(N) X^N = \prod_{n=1}^{\infty} (1 - X^n)^{-D} \quad (6)$$

From Eq.(1) we see that the lowest components of $\zeta^2, \zeta^3, \zeta'^3$ and ζ^4 appear respectively at level 2,3,3 and 4 (taking the levels of Ψ as a reference).

On the other hand,

a pair of ghost antighost corresponds to each of the gauge parameters Λ^1 and Λ^2 whose lowest components appear respectively at level 1 and 2. Taking into account the fact that ϕ^1 and ϕ^2 are auxiliary fields we obtain that the number of classical fields minus the number of ghosts at level N is

$$T^{26}(N) - 2T^{26}(N-1) - T^{26}(N-2) + 2T^{26}(N-3) + T^{26}(N-4) \quad (7)$$

From the light-cone formulation¹⁰⁾, we know that the number of physical degrees of freedom should be $T^{24}(N)$. Using (6) and

$$\prod_{n=1}^{\infty} (1 - x^n) = \sum_{k=-\infty}^{+\infty} (-1)^k x^{\frac{(3k+1)k}{2}} \quad (8)$$

$$= 1 - x - x^2 + x^5 + x^7 - x^{12} - x^{15} + \dots$$

we get

$$\begin{aligned} T^{24}(N) = & T^{26}(N) - 2T^{26}(N-1) - T^{26}(N-2) + 2T^{26}(N-3) \\ & + T^{26}(N-4) + 2T^{26}(N-5) - 2T^{26}(N-6) \\ & - 2T^{26}(N-7) + T^{26}(N-10) - \dots \end{aligned} \quad (9)$$

Clearly (7) fails to reproduce (9) at level 5 and beyond. Eq.(9) is represented pictorially in table 1 (remember that (9) is obtained by multiplying (6) by the square of (8)). We see that if (2) exhausts all the gauge symmetries of the action, we can only fulfill the first nine boxes (which form a 3 by 3 square) of table 1.

3. Technical tools

In this section, we develop some useful technical tools, namely the solutions of $AL_1 = BL_2$ and $L_n K^N = X_n^N(L_0 + N)$. The details are tedious, but we believe that they present some interest independently of the particular problem considered here.

In order to solve

$$AL_1 = BL_2 \quad (10)$$

we use the following expansion

$$A_n = \sum_{i=1}^{p(n)} a_n^i \mathcal{L}_i^{(n)} ; B_{n-1} = \sum_{i=1}^{p(n-1)} b_{n-1}^i \mathcal{L}_i^{(n-1)} \quad (11)$$

where

$$\{ \mathcal{L}_i^{(n)} \} = \{ (L_n)^{\lambda_n} (L_{n-1})^{\lambda_{n-1}} \dots (L_1)^{\lambda_1} : \sum_{j=1}^n \lambda_j j = n \} \quad (12)$$

and $p(n)$ is the number of partitions of n . Using the independence theorem of Brower and Thorn¹²⁾ we see that the number of independent solutions of (10) of the form (11) is

$$p(n-1) + p(n) - p(n+1) \quad (13)$$

However, if we subtract from those the solutions of the type $A_{n,i} = \mathcal{L}_i^{(n-m)} A_m$, $B_{n-1,i} = \mathcal{L}_i^{(n-m)} B_{m-1}$ we obtain the following recursion formula for the number of "primary" solutions R_n

$$R_m = p(n-1) + p(n) - p(n+1) - \sum_{2 \leq m < n} p(n-m) R_m \quad (14)$$

Defining

$$f(x) = \sum_{n=2}^{\infty} R_n x^n \quad (15)$$

and using (8) we get

$$x f(x) = \prod_{p=1}^{\infty} (1 - x^p) - 1 + x + x^2 \quad (16)$$

Finally

$$R_m = \delta_{m, \frac{(3k+1)k}{2} - 1} ; |k| = 2, 3, 4, \dots \quad (17)$$

The first solution occurs at $n=4$ and has the explicit form

$$\begin{aligned} A_4 &= L_4 - \frac{1}{2} L_3 L_1 + L_2 L_2 + \frac{1}{6} L_2 L_1 L_1 \\ B_3 &= L_3 + L_2 L_1 + \frac{1}{6} L_1 L_1 L_1 \end{aligned} \quad (18)$$

Those operators satisfy the commutation relations

$$\begin{aligned} [A_4, L_{-1}] &= 3 B_3 - 2 A_4^1 (L_0 + 1) \\ [B_3, L_{-1}] &= -2 A_4^2 (L_0 + 2) \\ [A_4, L_{-2}] &= -3 L_{-1} A_4^1 + (6 A_4^{11} + 4 B_3^1) (L_0 + 2) \\ [B_3, L_{-2}] &= -3 L_{-1} A_4^2 + (6 A_4^{12} + 4 B_3^2) (L_0 + 3) \end{aligned} \quad (19)$$

where

$$A_4 = A_4^1 L_1 - A_4^2 L_2 ; A_4^1 = A_4^{11} L_1 - A_4^{12} L_2 ; \dots \quad (20)$$

In this particular case the decomposition (20) is unique. As expected from (17) we found the next primary solution at $n=6$ and its general form is

$$A_6 = L_6 - \frac{1}{2} L_5 L_1 + \frac{1}{6} L_4 (L_1)^2 - \frac{1}{24} L_3 (L_1)^3 + \frac{1}{120} L_2 (L_1)^4 - \frac{1}{3} (L_2)^3 + (a L_2 + b L_1 L_1) A_4 \quad (21)$$

$$B_5 = \frac{2}{3} L_5 - \frac{1}{3} L_3 L_2 - \frac{1}{3} (L_2)^2 L_1 + \frac{1}{120} (L_1)^5 + (a L_2 + b L_1 L_1) B_3$$

The arbitrary coefficients a and b reflect the fact that primary solutions are defined up to non-primary solutions. However, imposing that A_6 and B_5 should satisfy Eq.(19) fixes unambiguously

$$a = \frac{138}{350} \quad ; \quad b = \frac{-13}{350} \quad (22)$$

The next non-zero R_n which appear at $n=11$ and $n=14$ are negative. From (14), we interpret this result as an "overlap" of the solutions generated by the two first primary solutions. Namely, there should be operators C_7, C_5, C_{10} and C_8 such that

$$\begin{aligned} C_7 A_4 &= C_5 A_6 \\ C_{10} A_4 &= C_8 A_6 \end{aligned} \quad (23)$$

but we have not checked this statement explicitly. Finally we mention that R_n is just the number of even minus odd partitions of $n+1$ with unequal numbers. This could provide us with a simpler construction.

We will now construct the solutions of

$$L_n K^N = X_n^N (L_0 + N) \quad ; \quad n = 1, 2 \quad (24)$$

with

$$K^N = 1 + \sum_{n=1}^{\infty} L_{-i}^{(n)} K_{ij}^{N(n)} L_j^{(n)} \quad (25)$$

It is easy to show that (24) and (25) are in fact equivalent to systems of inhomogeneous linear equations with as many equations as parameters. Explicitly, we found up to order 2

$$\begin{aligned}
 K^4 &= 1 + \frac{1}{10} L_{-1} L_1 + \frac{1}{350} \left[22 L_2 L_2 + 3(L_{-1} L_1 L_2 + L_{-2} L_1 L_1) \right. \\
 &\quad \left. + 2(L_{-1} L_1 L_1 L_1) \right] \\
 X_1^4 &= \frac{1}{5} L_1 + \frac{2}{175} L_{-1} (3L_2 + 2L_1 L_1) \\
 X_2^4 &= \frac{1}{175} (58L_2 + 12L_1 L_1)
 \end{aligned} \tag{26}$$

In order to find a general procedure we introduce the matrices¹²⁾

$$\mathcal{M}_{ij}^{(n)}(h) = \langle h | \mathcal{L}_i^{(n)} \mathcal{L}_{-j}^{(n)} | h \rangle \tag{27}$$

with

$$\begin{aligned}
 \langle h | h \rangle &= 1 ; L_0 | h \rangle = h | h \rangle ; L_m | h \rangle = 0 \quad ; \quad m > 0 \\
 \{ \mathcal{L}_{-i}^{(n)} \} &= \{ \mathcal{L}_i^{(n)} + \}
 \end{aligned} \tag{28}$$

and the projector (note the unusual ordering)

$$P(L_0) = \prod_{n=1}^{\infty} \left(1 - \mathcal{L}_{-i}^{(n)} \mathcal{L}_j^{(n)} (\mathcal{M}^{(n)})_{ij}^{-1} (L_0 - n) \right) \tag{29}$$

By construction

$$L_n P(L_0) \psi = 0 \quad ; \quad n > 0 \tag{30}$$

On the other hand, for $n > 0$ $L_n P(-N)$ is non-zero but vanishes on eigenstates of L_0 with eigenvalue $-N$. Consequently

$$K^N = P(-N) \tag{31}$$

Explicit calculation up to order 2 agrees with (26). Using the fact that $P(L_0)$ is a projector it is easy to see that K^N is in fact hermitian. It remains to be shown that $\mathcal{M}^{(n)}(-N-n)$ is invertible for the values of N which we will consider later, namely 4 and 6. The zeroes of $\det \mathcal{M}^{(n)}(h)$ at $D=26$ are given by¹³⁾

$$h_{rs} = \frac{1}{24} (25 - (3s+2r)^2) \quad (32)$$

with r and s strictly positive integers such that $rs \leq n$. A detailed analysis shows that the set of negative integer h_{rs} is

$$h_{rs} = -\frac{(3k+1)k}{2} + 1 \quad ; \quad |k| = 1, 2, 3, \dots \quad (33)$$

corresponding to

$$s = 1 \bmod 2 \quad ; \quad r = 1 \bmod 3 \text{ or } 2 \bmod 3 \quad (34)$$

By a systematic inspection we see that $K^4(K^6)$ could be¹ ill-defined at order 7 and 10 (5 and 8) due to the non-invertibility of $\mathcal{M}^{(7)}(-11)$ and $\mathcal{M}^{(10)}(-14)$ ($\mathcal{M}^{(5)}(-11)$ and $\mathcal{M}^{(8)}(-14)$). This should be related to (23). We will come back to this point later.

¹ In fact some poles of $P(L_0)$ disappear when we reorder as in (25) .

4. Elimination of the supplementary fields

In the gauge $\phi^1 = \phi^2 = 0$, the equations of motion reduce to

$$L_1 \psi + L_{-1} \zeta^2 + L_{-2} \zeta^3 = 0 \quad (35.a)$$

$$L_2 \psi + 3 \zeta^2 + L_{-1} \zeta'^3 + L_{-2} \zeta^4 = 0 \quad (35.b)$$

$$(L_0 - 1) \psi = (L_0 + 1) \zeta^2 = (L_0 + 2) \zeta^3 = (L_0 + 2) \zeta'^3 = (L_0 + 3) \zeta^4 = 0 \quad (35.c)$$

If we consider the theory on-shell, we can still use a residual gauge invariance associated with gauge parameters satisfying

$$L_0 \Lambda^1 = (L_0 + 1) \Lambda^2 = 0 \quad (36)$$

However, we will show that this is not enough to eliminate all the supplementary fields ζ and recover

$$(L_0 - 1) \psi = L_1 \psi = L_2 \psi = 0 \quad (37)$$

If we want to eliminate the ζ on-shell we have to consider fields and gauge parameters satisfying (35) and (36) and solve

$$\begin{aligned} \zeta^2 &= L_1 \Lambda^1 + 3 \Lambda^2 ; & \zeta'^3 &= L_2 \Lambda^1 \\ \zeta^3 &= L_1 \Lambda^2 ; & \zeta^4 &= L_2 \Lambda^2 \end{aligned} \quad (38)$$

From the previous section, we know that there are constraints among the r.h.s. of (38). A necessary condition to solve (38) is that the l.h.s. satisfy the same set of constraints, for instance with notation (20)

$$\begin{aligned} A_4 \zeta^2 - B_3 \zeta'^3 - 3 A_4^1 \zeta^3 + 3 A_4^2 \zeta^4 &= 0 \\ A_4 \zeta^3 - B_3 \zeta^4 &= 0 \end{aligned} \quad (39)$$

Note that equations (39) are gauge-invariant. This means we have to show that (39) holds by the equations of motion. The only combination of equations of motion independent of Ψ and which does not reduce to Klein-Gordon equations is

$$A(L_{-1} \zeta^2 + L_{-2} \zeta^3) = B(3 \zeta^3 + L_{-1} \zeta'^3 + L_{-2} \zeta^4) \quad (40)$$

Using now (19) and (35.c) we get in the case of A_4

$$L_{-1}[A_4 \zeta^2 - B_3 \zeta'^3 - 3A_4^1 \zeta^3 + 3A_4^2 \zeta^4] + L_{-2}[A_4 \zeta^3 - B_3 \zeta^4] = 0 \quad (41)$$

From the complex conjugate of Eq.(10), we see that the solution of (41) is

$$[A_4 \zeta^2 - B_3 \zeta'^3 - 3A_4^1 \zeta^3 + 3A_4^2 \zeta^4] = A_4^+ \eta^{10} + A_6^+ \eta^{12} + \dots \quad (42)$$

$$[A_4 \zeta^3 - B_3 \zeta^4] = -B_3^+ \eta^{10} - B_5^+ \eta^{10} - \dots$$

with

$$(L_0 + 9) \eta^{10} = (L_0 + 11) \eta^{12} = 0 \quad (43)$$

Eq.(42) is weaker than (39) and the gauge symmetries (2) are not sufficient to eliminate completely the supplementary fields on-shell. It is interesting to remark that (42) differs from (39) at level 10.

5. The new symmetries and their ghosts

Using the technical tools developed in section 3 we have solved the conditions to have new symmetries acting only on the supplementary fields. We obtained that the symmetries of the action up to level 11 are

$$\begin{aligned}
 \delta \psi &= L_{-1} \Lambda^1 + L_{-2} \Lambda^2 \\
 \delta \phi^1 &= -L_0 \Lambda^1 + A_4^+ K^4 \Lambda^5 + A_6^+ K^6 \Lambda^7 \\
 \delta \phi^2 &= -(L_0 + 1) \Lambda^2 - B_3^+ K^4 \Lambda^5 - B_5^+ K^6 \Lambda^7 \\
 \delta \zeta^2 &= -L_1 \Lambda^1 - 3 \Lambda^2 - 2 A_4^{1+} K^4 \Lambda^5 + A_4^+ X_1^4 \Lambda^5 - 2 A_6^{1+} K^6 \Lambda^7 \\
 &\quad + A_6^+ X_1^6 \Lambda^7 \\
 \delta \zeta^3 &= -L_1 \Lambda^2 + 2 A_4^2 K^4 \Lambda^5 - B_3^+ X_1^4 \Lambda^5 + 2 A_6^2 K^6 \Lambda^7 - B_5^+ X_1^6 \Lambda^7 \quad (44) \\
 \delta \zeta^3 &= -L_2 \Lambda^1 + \left[(6 A_4^{11+} + 4 B_3^{1+}) K^4 \Lambda^5 - 3 A_4^{1+} X_1^4 \Lambda^5 + A_4^+ X_2^4 \Lambda^5 \right. \\
 &\quad \left. + (A_4 \leftrightarrow A_6, B_3 \leftrightarrow B_5, \Lambda^5 \leftrightarrow \Lambda^7, \dots) \right] \\
 \delta \zeta^4 &= -L_2 \Lambda^2 - \left[(6 A_4^{12+} + 4 B_3^{2+}) K^4 \Lambda^5 - 3 A_4^2 X_1^4 \Lambda^5 + B_3^+ X_2^4 \Lambda^5 \right. \\
 &\quad \left. + (A_4 \leftrightarrow A_6, B_3 \leftrightarrow B_5, \Lambda^5 \leftrightarrow \Lambda^7, \dots) \right]
 \end{aligned}$$

At level 12 and beyond, the procedure we proposed to construct K^4 and K^6 should presumably be modified. This will become clearer after the following remarks. The new symmetries are in fact a non-local redefinition of the gauge parameters

$$\begin{aligned}
 \Lambda^1 &\rightarrow \Lambda^1 - \frac{1}{L_0} (A_4^+ K^4 \Lambda^5 + A_6^+ K^6 \Lambda^7) \\
 \Lambda^2 &\rightarrow \Lambda^2 + \frac{1}{L_0 + 1} (B_3^+ K^4 \Lambda^5 + B_5^+ K^6 \Lambda^7)
 \end{aligned} \quad (45)$$

A non-local redefinition is a non-trivial change of variables. This means that the new gauge parameters cannot be simply reabsorbed into the old ones. Usually those redefinitions are irrelevant because they lead to non-local gauge transformations of the fields. In the case considered here (45) is the only such redefinition which yields local gauge transformations ((44) is perfectly local). That is why we can fix the gauge with the conditions $\phi^1 = \phi^2 = 0$ (this fixes the redefined parameters) but obviously the gauge transformations of those conditions are themselves invariant under

$$\begin{aligned}\delta \Lambda^1 &= A_4^+ (K^4 \Omega^5 + X_1^{4+} \Omega^6 + X_2^{4+} \Omega^7) + A_6^+ (K^6 \Omega'^7 + X_1^{6+} \Omega^8 + X_2^{6+} \Omega^9) \\ \delta \Lambda^2 &= -B_3^+ (K^4 \Omega^5 + X_1^{4+} \Omega^6 + X_2^{4+} \Omega^7) - B_5^+ (K^6 \Omega'^7 + X_1^{6+} \Omega^8 + X_2^{6+} \Omega^9) \\ \delta \Lambda^5 &= (L_0 + 4) \Omega^5 + L_{-1} \Omega^6 + L_{-2} \Omega^7 \\ \delta \Lambda^7 &= (L_0 + 6) \Omega'^7 + L_{-1} \Omega^8 + L_{-2} \Omega^9\end{aligned}\tag{46}$$

Note that we have used the hermiticity of K^N which implies

$$K^N L_{-n} = (L_0 + N) X_n^{N+}\tag{47}$$

Again we find that $\delta \Lambda^5$ is invariant under

$$\begin{aligned}\delta \Omega^5 &= L_{-1} \Sigma^6 + L_{-2} \Sigma^7 \\ \delta \Omega^6 &= -(L_0 + 5) \Sigma^6 + A_4^+ \Sigma^{10} \\ \delta \Omega^7 &= -(L_0 + 6) \Sigma^7 - B_3^+ \Sigma^{10}\end{aligned}\tag{48}$$

where we have consistently disregarded level 12 terms like $\delta \Omega^6 = A_6^+ \Sigma^{12}$.

Those variations are themselves invariant under

$$\begin{aligned}\delta \Sigma^6 &= A_4^+ \Xi^{10} \\ \delta \Sigma^7 &= -B_3^+ \Xi^{10} \\ \delta \Sigma^{10} &= (L_0 + 5) \Xi^{10}\end{aligned}\tag{49}$$

A similar construction can be done for $\delta\Lambda^7$. At level 12 we see from (23) that $\delta\phi^1$ and $\delta\phi^2$ could have new invariances acting simultaneously on Λ^5 and Λ^7 . This is precisely the level where the procedure of section 3 could become ill-defined for the individual terms involving Λ^5 or Λ^7 . We believe that using (23) it would be at least possible to define properly the sums

$$\begin{aligned} & A_4^+ K^4 \Lambda^5 + A_6^+ K^6 \Lambda^7 \\ & B_3^+ K^4 \Lambda^5 + B_5^+ K^6 \Lambda^7 \end{aligned} \quad (50)$$

However we have not solved explicitly this hard technical problem because the important feature of the ghost structure we want to stress already appears at level 10.

We will now consider the BRS invariance (not to be confused with Siegel's formulation¹⁾) corresponding to the nested invariances (44) to (49), disregarding terms of level 12 and beyond. With that restriction, Λ^5 and Λ^7 can be treated independently and we will only consider the former, the inclusion of the latter being similar. We obtained the following BRS Lagrangian

$$\mathcal{L}^{BRS} = \mathcal{L}_{INV.} + \mathcal{L}_{GF} + \mathcal{L}_G \quad (51.a)$$

with

$$\begin{aligned} \mathcal{L}_{GF} = & -\frac{1}{\alpha_1} b^1 \phi^1 - \frac{1}{2\alpha_1} b^1 b^1 - \frac{1}{\alpha_2} b^2 \phi^2 - \frac{1}{2\alpha_2} b^2 b^2 - \frac{1}{\alpha_5} \bar{f}^5 G^5 \\ & - \frac{1}{\alpha_6} \bar{b}^6 H^6 - \frac{1}{\alpha_7} \bar{b}^7 H^7 - \frac{1}{\alpha_{10}} \bar{f}^{10} G^{10} \end{aligned} \quad (51.b)$$

and

$$\begin{aligned} \mathcal{L}_G = & \bar{G}^1 (-L_0 G^1 + A_4^+ K^4 G^5) + \bar{G}^2 (-(L_0 + 1) G^2 - B_3^+ K^4 G^5) \\ & + \bar{H}^5 ((L_0 + 4) H^5 + L_{-1} H^6 + L_{-2} H^7) + \bar{G}^6 (-(L_0 + 5) G^6 + A_4^+ G^{10}) \\ & + \bar{G}^7 (-(L_0 + 6) G^7 - B_3^+ G^{10}) + \bar{H}^{10} (L_0 + 9) H^{10} \end{aligned} \quad (51.c)$$

One can check that $\Delta \mathcal{L}^{BR5} = 0$ for the following transformations

$$\Delta \psi = L_{-1} G^1 + L_{-2} G^2$$

$$\Delta \phi^1 = -L_0 G^1 + A_4^+ K^4 G^5$$

$$\Delta \phi^2 = -(L_0 + 1) G^2 - B_3^+ K^4 G^5$$

$$\Delta \psi^2 = -L_1 G^1 - 3 G^2 - 2 A_4^{1+} K^4 G^5 + A_4^+ X_1 G^5$$

$$\Delta \psi^3 = -L_1 G^2 + 2 A_4^{2+} K^4 G^5 - B_3^+ X_1^4 G^5$$

$$\Delta \psi^{13} = -L_2 G^1 + (6 A_4^{11+} + 4 B_3^{1+}) K^4 G^5 - 3 A_4^{1+} X_1^4 G^5 + A_4^+ X_2^4 G^5$$

$$\Delta \psi^4 = -L_2 G^2 - (6 A_4^{12+} + 4 B_3^{2+}) K^4 G^5 + 3 A_4^{2+} X_1^4 G^5 - B_3^+ X_2^4 G^5$$

$$\Delta G^1 = A_4^+ (K^4 H^5 + X_1^{4+} H^6 + X_2^{4+} H^7)$$

$$\Delta G^2 = -B_3^+ (K^4 H^5 + X_1^{4+} H^6 + X_2^{4+} H^7)$$

$$\Delta G^5 = (L_0 + 4) H^5 + L_{-1} H^6 + L_{-1} H^7$$

$$\Delta H^5 = L_{-1} G^6 + L_{-2} G^7$$

$$\Delta H^6 = -(L_0 + 5) G^6 + A_4^+ G^{10}$$

$$\Delta H^7 = -(L_0 + 6) G^7 - B_3^+ G^{10}$$

$$\Delta G^6 = A_4^+ H^{10}$$

$$\Delta G^7 = -B_3^+ H^{10}$$

$$\Delta G^{10} = (L_0 + 9) H^{10}$$

$$\Delta N^{10} = 0$$

$$\Delta \bar{G}^1 = \left(\frac{1}{\alpha_1}\right) b^1$$

$$\Delta \bar{G}^2 = \left(\frac{1}{\alpha_2}\right) b^2$$

$$\Delta \bar{H}^5 = \left(\frac{1}{\alpha_5}\right) \bar{b}^5$$

$$\Delta \bar{G}^6 = \left(\frac{1}{\alpha_6}\right) \bar{b}^6$$

$$\Delta \bar{G}^7 = \left(\frac{1}{\alpha_7}\right) \bar{b}^7$$

(52)

$$\begin{aligned}\Delta \bar{N}^{10} &= \frac{1}{2_{10}} \bar{f}^{10} \\ \Delta b^1 &= \Delta b^2 = \Delta \bar{f}^5 = \Delta \bar{b}^6 = \Delta \bar{b}^7 = \Delta \bar{f}^{10} = 0\end{aligned}$$

The property $\Delta^2 = 0$ follows from the identities

$$\begin{aligned}X_1^4 L_{-1} - L_{-1} X_1^{4+} &= X_1^4 L_{-2} - L_{-1} X_2^{4+} = 0 \\ X_2^4 L_{-2} - L_{-2} X_2^{4+} &= X_1^{4+} A_4^+ - X_2^{4+} B_3^+ = 0\end{aligned}\tag{53}$$

which can be obtained from (24) and the hermiticity of K^4 . Note that G^5, H^6, H^7 and G^{10} disappear after integration over the Lagrange multipliers and are irrelevant for the counting of physical degrees of freedom. The remaining ghosts are $(H^5, \bar{H}^5)$ which are ghosts for ghosts (hereafter denoted (ghosts)²), $(G^6, \bar{G}^6)$ and $(G^7, \bar{G}^7)$ which are (ghosts)³ and the (ghosts)⁴ $(H^{10}, \bar{H}^{10})$. Those (ghosts)ⁿ appear in pairs with their antighosts. They are fermionic (bosonic) if n is odd (even). One can check that $\mathcal{L}^{BRS}$ is invariant under a global symmetry which we shall call BRS-charge, which is 0 for the classical fields and their Lagrange multipliers and which is increased by one unit under the action of $\mathcal{J}$. The (ghosts)ⁿ have BRS-charge n while their antighosts have the opposite charge. There is no room for single "hidden" ghosts¹⁴⁾, i.e. ghosts of BRS-charge 0. Taking the case of the antisymmetric tensor as a guide¹⁵⁾ we see that such a ghost could appear if there was a condition of the type

$$V \phi^1 + W \phi^2 = 0\tag{54}$$

This would allow a non-trivial BRS transformation for b^1 and b^2 while the condition $\Delta^2 = 0$ would require the introduction of a new ghost with BRS-charge 0. However, if we replace the auxiliary fields by their equations

of motion in (54) we obtain a set of consistency conditions on V and W which can be related by the Virasoro algebra and lead to $V=W=0$. We can repeat the counting of section 3 (with a positive contribution for the bosonic ghosts). We obtain for $N \leq 11$

$$\begin{aligned} T^{26}(N) - 2T^{26}(N-1) - T^{26}(N-2) + 2T^{26}(N-3) + T^{26}(N-4) \\ + 2T^{26}(N-5) - 2T^{26}(N-6) - 2T^{26}(N-7) + 2T^{26}(N-10) \end{aligned} \quad (55)$$

The last term of (55) differs from (9) by a factor 2. This is related to the apparently unavoidable doubling of ghosts discussed above. Eq.(55) can be represented pictorially as a vertical and a horizontal strip of table 1, both starting at level 5. Those two strips overlap at level 10 which gives the wrong factor 2.

6. Conclusions

We found that the finite set formulation has more symmetries than the ones presented in Ref.5). Taking them into account, we failed to recover the correct counting of physical degrees of freedom at level 10 and beyond. This can be understood in terms of the classical arguments given in section 4. Those arguments are not affected by the new symmetries because it is possible to prove that Eq.(39) are invariant under any symmetry of the action. Consequently the obstruction found at level 10 remains valid and agrees with the result of section 5.

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$+T^D(N)$	$-T^D(N-1)$	$-T^D(N-2)$	$+T^D(N-5)$	$+T^D(N-7)$	$-T^D(N-12)$	---
$-T^D(N-1)$	$+T^D(N-2)$	$+T^D(N-3)$	$-T^D(N-6)$	$-T^D(N-8)$	$+T^D(N-13)$	---
$-T^D(N-2)$	$+T^D(N-3)$	$+T^D(N-4)$	$-T^D(N-7)$	$-T^D(N-9)$	$+T^D(N-14)$	---
$+T^D(N-5)$	$-T^D(N-6)$	$-T^D(N-7)$	$+T^D(N-10)$	$+T^D(N-12)$	$-T^D(N-17)$	---
$+T^D(N-7)$	$-T^D(N-8)$	$-T^D(N-9)$	$+T^D(N-12)$	$+T^D(N-14)$	$-T^D(N-19)$	---
$-T^D(N-12)$	$+T^D(N-13)$	$+T^D(N-14)$	$-T^D(N-17)$	$-T^D(N-19)$	$+T^D(N-24)$	---
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	

Table 1

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