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KALUZA-KLEIN APPROACH TO THE HETEROTIC STRING (II)

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ABSTRACT

Further properties of $d = 10$ heterotic superstrings, including the Lorentz Chern-Simons terms, are obtained from spontaneous compactification of the $d = 506$ bosonic string on the group manifolds $G = SO(32)$ or $E_8 \times E_8$. These involve generalizing our previous Kaluza-Klein ansatz for the gauge bosons A_μ of G_L to (A_μ, B_μ) of $G_L \times G_R$, which necessitates the incorporation of Kaluza-Klein scalars to maintain consistency. The heterotic strings are then obtained by identifying B_μ ($SO(8)_{\text{internal}} \subset G$) with ω_μ^+ ($SO(8)_{\text{space-time}}$), where $\omega_\mu^\pm$ is the connection with torsion $\pm \frac{1}{2}H_{\mu\nu\rho}$. In particular, we obtain the exact result $dH = (\alpha'/2)(R_+^{ab} R_+^{ab} - 2F_+^i F_+^i)$, non-linear in H . The Type II strings are obtained in a similar way by setting $A_\mu(SO(8)) = \omega_\mu^-$ and $B_\mu(SO(8)) = \omega_\mu^+$.

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The massless sector of $d = 10$ superstrings describes the interaction of $N = 1$ supergravity coupled to $N = 1$ super-Yang-Mills with gauge groups $SO(32)$ or $E_8 \times E_8$. This runs counter to the traditional Kaluza-Klein philosophy for which the fundamental theory has only space-time symmetries and for which, in particular, the Yang-Mills fields are components of the gravitational field in a higher dimension. In this traditional Kaluza-Klein approach, the Yang-Mills gauge group is merely a subgroup of the general co-ordinate group.

In a previous paper¹⁾, however, it was shown that the gauge bosons of the heterotic string²⁾ in fact have a traditional Kaluza-Klein origin in the $d = 506$ bosonic string¹⁾. Specifically

- 1) The conformal invariance of the closed bosonic string in d space-time dimensions places restrictions on the background fields $\hat{g}_{MN}(x)$, $\hat{B}_{MN}(x)$ and $\hat{\phi}(x)$ ³⁾. These restrictions are equivalent to the Einstein-matter field equations with cosmological constant proportional to $(d-26)/\alpha'$ ⁴⁾.
- 2) The presence of the cosmological constant forbids flat vacuum solutions except when $d = 26$, but for $d > 26$ permits a spontaneous compactification, i.e., a ground-state solution of the form

$$M_d = M_n \times G, \quad (1)$$

where M_n is n -dimensional space-time with zero cosmological constant, and G is a compact group manifold of dimension $k = d-n$.

- 3) This gives rise, in a conventional Kaluza-Klein fashion⁵⁾, to an effective n -dimensional theory with a finite number of massless states and an infinite tower of massive states, all belonging to various representations of $G_L \times G_R$. The massless sector describes the n -dimensional fields $g_{\mu\nu}$, $B_{\mu\nu}$ and ϕ , massless Yang-Mills gauge bosons A_μ^i and B_μ^i corresponding to G_L and G_R respectively where $i = 1, \dots, k$, and massless scalars S^{ij} in the (adjoint, adjoint) representation of $G_L \times G_R$. From a string point of view, this corresponds to decomposing the d -dimensional creation operators $\alpha_M^{(n)}$ and $\tilde{\alpha}_N^{(n)}$ into space-time and extra dimensional components. In particular, acting on the vacuum with

$$\alpha_M^{(-1)} \tilde{\alpha}_N^{(-1)} = \begin{pmatrix} \alpha_\mu^{(-1)} \tilde{\alpha}_\nu^{(-1)} & \alpha_i^{(-1)} \tilde{\alpha}_\nu^{(-1)} \\ \alpha_\mu^{(-1)} \tilde{\alpha}_j^{(-1)} & \alpha_i^{(-1)} \tilde{\alpha}_j^{(-1)} \end{pmatrix} \quad (2)$$

we obtain the above-mentioned massless states⁶⁾.

- 4) In order to focus attention on the massless sector, one makes a Kaluza-Klein "ansatz" for $\hat{g}_{MN}$, $\hat{B}_{MN}$ and $\hat{\phi}$. In Ref. 1), our aim was to obtain the massless bosonic sector of the heterotic string by choosing $n = 10$ and $G = SO(32)$ or $E_8 \times E_8$. Hence our ansatz included only the fields $(g_{\mu\nu}, B_{\mu\nu}, \phi, A_\mu^i)$. This ansatz was consistent in the sense described in Ref. 5), i.e., all solutions of the resulting ten-dimensional theory were solutions of the original 506 dimensional theory. This is to be contrasted with a generic Kaluza-Klein theory where for consistency one must keep only G_R singlets and all the G_R singlets, in particular G_R singlet scalars which, in the present case, would be massive.
- 5) In this way, we reproduced the bosonic sector of the low-energy limit of the heterotic string²⁾, i.e., the $N = 1$ supergravity coupled to Yang-Mills of Chapline and Manton including, of course, the correct Yang-Mills Chern-Simons term^{7),8)}.

Although this represents encouraging evidence that all the properties of the superstring may be derived from the bosonic string⁹⁾, many important questions remain unanswered. In this paper, we address the questions: (i) Where do the Lorentz Chern-Simons¹⁰⁾ terms come from? (ii) Does the derivation of the ten-dimensional β functions from $d = 506$, carried out successfully at one loop in Ref. 1), persist to two loops? Bear in mind that the coefficients of $(\text{Riemann})^2$ in the two-loop β functions are in the ratio 2:1:0 for the bosonic, heterotic and Type II strings respectively⁴⁾, whereas a naive Kaluza-Klein argument would yield the same coefficients. This prompts (iii): Can the equivalence be demonstrated to all orders by deriving the ten-dimensional Fradkin-Tseytlin³⁾ superstring actions from the corresponding bosonic one in $d = 506$?

As we shall see, the key to answering all these questions resides in (a) making an improved Kaluza-Klein ansatz involving not only the gauge bosons A_μ^i of G_L , but also the B_μ^i of G_R ; (b) including Kaluza-Klein scalars S^{ij} in the

(adjoint, adjoint) representation of $G_L \times G_R$ in order to maintain consistency; (c) truncating to an $SO(8)$ subgroup of G_R [in the heterotic case²⁾] or of both G_L and G_R [in the Type II case¹¹⁾] and then identifying the $SO(8)$ gauge potentials with the spin-connection of the transverse space-time $SO(8)$.

Let us begin by deriving the massless sector of the n -dimensional theory which results from the spontaneous compactification of the bosonic string on the group manifold G . Although we are ultimately interested in $n = 10$ and $G = SO(32)$ or $E_8 \times E_8$, at this point $n = d-k$ and G will be arbitrary solutions of the critical dimension formula¹²⁾⁻¹⁵⁾

$$d-26 = \frac{k C_A}{2p + C_A}, \quad (3)$$

where C_A is the second-order Casimir in the adjoint representation and p is an integer. This follows by noting that if we substitute the ground-state values of $\hat{g}_{MN}$, $\hat{B}_{MN}$ and $\hat{\phi}$ quoted in Ref. 1) into the Fradkin-Tseytlin action, the result is nothing but a non-linear σ model on $M_n \times G$ with a Wess-Zumino term, for which the radius of the group manifold m^{-1} is quantized in units of $\alpha'^{-\frac{1}{2}}$

$$p m^2 \alpha' = 1. \quad (4)$$

The appearance of the integer p follows from the quantization of the coefficient of the Wess-Zumino term. In what follows, we restrict our attention to the case $p = 1$, for which the non-linear σ model is equivalent on the one hand to a theory of free fermions¹⁶⁾, and on the other to a theory of free bosons defined on $M_n \times T^r$, where r is the rank of G ¹⁵⁾. We shall return to this point later when we compare our results with those of Casher et al.¹⁷⁾.

Since the isometry group of the group manifold is $G_L \times G_R$, and since the vevs of $\hat{\phi}$ and $\hat{H}_{MNP}$ given in Ref. 1) are also $G_L \times G_R$ -invariant, the n -dimensional theory will contain the Yang-Mills gauge bosons of $G_L \times G_R$. For generic Kaluza-Klein theories, however, a consistent Kaluza-Klein ansatz for the massless sector requires that we keep only a subgroup of the full isometry group⁵⁾. We recall that a "consistent" ansatz is one for which all solutions of the n -dimensional theory are solutions of the original d -dimensional theory. For a generic Kaluza-Klein theory with homogeneous extra dimensions, a consistent ansatz retains all those fields and only those fields invariant under a transitively-acting subgroup K of the isometry group. In particular, for group manifolds the gauge bosons are

only those of G_L and not those of the full isometry group $G_L \times G_R$. Moreover, the G_L ansatz is in general consistent only if we include Kaluza-Klein scalars in the symmetrized adjoint $\times$ adjoint representation of G_L . Experience with $d = 11$ supergravity, however, teaches us that there may exist certain exceptional theories where a consistent ansatz can be achieved without demanding this "K-invariance"⁵⁾. This can happen either by including fields which are not K-invariant [e.g., the $SO(8)$ ansatz for the S^7 compactification of $d = 11$ supergravity] or by omitting fields which are [e.g., the omission of Kaluza-Klein scalars in the $SO(3)$ ansatz on S^7]. In the present context, an example of the latter phenomenon was provided in Ref. 1), where we showed that the G_L ansatz for the bosonic string was consistent in spite of omitting Kaluza-Klein scalars. In this paper we go one step further and show that even the full $G_L \times G_R$ ansatz is consistent, provided we pay the price of including scalars S^{ij} in the (adjoint G_L , adjoint G_R) representation. This somewhat confusing situation is summarized in the Table. In presenting these results, we shall first give the ansatz for $G_L \times G_R$ without including the scalars and then indicate how their inclusion solves the problem of inconsistency^{*)}.

Let us introduce the Killing vectors K^I on the group manifold G

$$K^I = (L^i, R^i) , \quad (5)$$

where L^i are the generators of left translations

$$[L^i, L^j] = m c^{ij}{}_k L^k , \quad (6)$$

and R^i are the generators of right translations

$$[R^i, R^j] = -m c^{ij}{}_k R^k , \quad (7)$$

and

$$[L^i, R^j] = 0 . \quad (8)$$

*) Since there seems to be some confusion on this point, we emphasize that the omission of scalars will not involve any linearization in the gravity or Yang-Mills sector.

The corresponding Yang-Mills gauge potentials are denoted by

$$A^I = (A^i, B^i) , \quad (9)$$

where A^i are the gauge bosons of G_L and B^i the gauge bosons of G_R . The corresponding field strengths are given by

$$F^I = (F^i, G^i) , \quad (10)$$

where

$$F^i = dA^i + \frac{1}{2} C^i_{jk} A^j \wedge A^k , \quad (11)$$

$$G^i = dB^i - \frac{1}{2} C^i_{jk} B^j \wedge B^k . \quad (12)$$

The Killing vector components L_a^i and R_a^i satisfy the Cartan-Maurer equations

$$\nabla_a L_b^i = -\frac{m}{2} C^i_{jk} L_a^j L_b^k , \quad (13)$$

$$\nabla_a R_b^i = \frac{m}{2} C^i_{jk} R_a^j R_b^k , \quad (14)$$

and we also introduce the notation

$$\begin{aligned} C_{abc} &= C_{ijk} L_a^i L_b^j L_c^k = \\ &= C_{ijk} R_a^i R_b^j R_c^k . \end{aligned} \quad (15)$$

We are now in a position to state the ansatz for $\hat{\phi}$, $\hat{g}_{MN}$ and $\hat{B}_{MN}$ and to calculate the corresponding curvatures and field strengths. For the scalar, we write

$$\hat{\phi}(x, y) = \text{constant} + \phi(x) . \quad (16)$$

For the metric ansatz, we employ the one-forms $\hat{e}^A = \hat{e}_M^A dx^M$, for which

$$\hat{g}_{MN} = \hat{e}_M^A \hat{e}_N^B \eta_{AB} , \quad (17)$$

and write

$$\hat{e}^\alpha(x, y) = e^\alpha(x) , \quad (18)$$

$$\hat{e}^a(x, y) = e^a(y) - \frac{1}{m} \left(L^{ia}(y) A^i(x) + R^{ia}(y) B^i(x) \right). \quad (19)$$

For the antisymmetric tensor ansatz, we write

$$\begin{aligned} \hat{B} = B - \frac{1}{2m^2} (A^i L_a^i + B^i R_a^i)_{, \lambda} (A^j L_a^j - B^j R_a^j) \\ - \frac{1}{m} (A_{\lambda}^i L^i - B_{\lambda}^i R^i) + m b , \end{aligned} \quad (20)$$

and where

$$B = \frac{1}{2} B_{\alpha\beta}(x) e^\alpha_{\lambda} e^\beta \quad (21)$$

and

$$b = \frac{1}{2} b_{ab}(y) e^a_{\lambda} e^b , \quad (22)$$

such that

$$db = c = \frac{1}{6} c_{abc} e^a_{\lambda} e^b_{\lambda} e^c . \quad (23)$$

It is important to note that (α, a) correspond to tangent space indices and (μ, m) to world indices. The ansatz of Ref. 1) is recovered by setting $B_{\mu}^i = 0$.

The quantities $\phi(x)$, $g_{\mu\nu} = e_{\mu}^{\alpha} e_{\nu}^{\beta} \eta_{\alpha\beta}$ and $B_{\mu\nu}$ will be interpreted as the scalar, metric and antisymmetric tensors in n -dimensional space-time, and the quantities $A_{\mu}^i(x)$ and $B_{\mu}^i(x)$ will be the Yang-Mills gauge bosons for G_L and G_R respectively. Equations (17)-(19) are just the "standard ansatz" for the metric tensor familiar to Kaluza-Klein theories⁵⁾. The novel feature of the bosonic string is the Kaluza-Klein ansatz for $\hat{B}_{MN}$, which also involves the Yang-Mills gauge bosons. In this Kaluza-Klein interpretation, the gauge symmetry $G_L \times G_R$ is

just a subgroup of the d-dimensional general co-ordinate group. To see this in more detail, consider a general co-ordinate transformation

$$x^M \rightarrow x^M - \hat{\xi}^M \quad (24)$$

and the corresponding transformations of ϕ , $\hat{g}_{MN}$ and $\hat{B}_{MN}$. Then focus one's attention on the very special transformation

$$\hat{\xi}^M(x,y) = (0, \alpha^i(x) L^{im}(y) + \beta^i(x) R^{im}(y)) \quad (25)$$

with α^i and β^i arbitrary. Then from the Kaluza-Klein ansätze (16)-(23), we may compute the transformation rules for the n-dimensional fields. We find not only the usual Yang-Mills transformation rules for ϕ , $g_{\mu\nu}$, A_μ^i and B_μ^i , but also that the $B_{\mu\nu}$ field transforms as

$$\delta B_{\mu\nu} = A_\mu^i \partial_\nu \alpha^i - A_\nu^i \partial_\mu \alpha^i - B_\mu^i \partial_\nu \beta^i + B_\nu^i \partial_\mu \beta^i. \quad (26)$$

It is now tedious but straightforward to compute the curvature $\hat{R}_{ABCD}$ and the field strength $\hat{H}_{ABC}$, which will be published in detail elsewhere¹⁸⁾. In particular the n-dimensional field strength H is not just dB but rather

$$H = dB - m^{-2} (\Omega_A - \Omega_B), \quad (27)$$

where

$$\Omega_A = F^i \wedge A^i - \frac{1}{6} C^{ijk} A^i \wedge A^j \wedge A^k, \quad (28)$$

$$\Omega_B = G^i \wedge B^i + \frac{1}{6} C^{ijk} B^i \wedge B^j \wedge B^k, \quad (29)$$

i.e., we have acquired n-dimensional Chern-Simons terms, even though there were no such terms in d dimensions where $\hat{H} = dB$. The non-covariance of Ω_A and Ω_B under Yang-Mills transformations is exactly cancelled by the unusual transformation rule for $B_{\mu\nu}$ of (26), ensuring that H does not transform. Hence, although in d dimensions $d\hat{H} = 0$, in n dimensions we have (on using $\alpha'^m{}^2 = 1$)

$$dH + \alpha' (F^i \wedge F^i - G^i \wedge G^i) = 0. \quad (30)$$

Note that the left- and right-handed gauge fields enter with opposite sign, and that this equation is exact to all orders in α' !

The d-dimensional $\hat{\beta}$ functions^{4),1)} are now related to the n-dimensional β functions by

$$\hat{\beta}^\phi(x, y) = \beta^\phi(x), \quad (31)$$

$$\hat{\beta}_{\alpha\beta}^g(x, y) = \beta_{\alpha\beta}^g(x), \quad (32)$$

$$\hat{\beta}_{\alpha\beta}^B(x, y) = \beta_{\alpha\beta}^B(x), \quad (33)$$

$$\hat{\beta}_{\alpha b}^g(x, y) = (\beta_{\alpha i}^A(x) L_b^i + \beta_{\alpha i}^B(x) R_b^i) m^{-1}, \quad (34)$$

$$\hat{\beta}_{\alpha b}^B(x, y) = (\beta_{\alpha i}^A(x) L_b^i - \beta_{\alpha i}^B(x) R_b^i) m^{-1}, \quad (35)$$

$$\hat{\beta}_{ab}^g(x, y) = \beta_{ij}^S(x) (L_a^i R_b^j + L_b^i R_a^j) m^{-2}, \quad (36)$$

$$\hat{\beta}_{ab}^B(x, y) = \beta_{ij}^S(x) (L_a^i R_b^j - L_b^i R_a^j) m^{-2}, \quad (37)$$

where

$$\begin{aligned} 16\pi^2 \beta^\phi &= 4(\partial\phi)^2 - 4\Box\phi - R + \frac{1}{12}H^2 \\ &+ \frac{1}{2}\alpha' (F_{\alpha\beta}^i F^{i\alpha\beta} + G_{\alpha\beta}^i G^{i\alpha\beta}) + \dots \end{aligned} \quad (38)$$

$$\beta_{\alpha\beta}^g = R_{\alpha\beta} - \frac{1}{4} H_{\alpha\gamma\delta} H_{\beta}{}^{\gamma\delta} + 2 \nabla_{\alpha} \nabla_{\beta} \phi + \alpha' (F_{\alpha\gamma}^i F_{\beta}{}^{\gamma\delta} + G_{\alpha\gamma}^i G_{\beta}{}^{\gamma\delta}) + \dots, \quad (39)$$

$$\beta_{\alpha\beta}^B = \nabla^{\gamma} H_{\alpha\beta\gamma} - 2 H^{\gamma}{}_{\alpha\beta} \partial_{\gamma} \phi + \dots, \quad (40)$$

$$\beta_{\alpha}^{gi} = D_{\beta} F_{\alpha}{}^{i\beta} - \frac{1}{2} H_{\alpha\gamma\delta} F^{\gamma\delta i} - 2 F_{\alpha}{}^{i\beta} \partial_{\beta} \phi + \dots, \quad (41)$$

$$\beta_{\alpha}^{Bi} = D_{\beta} G_{\alpha}{}^{i\beta} + \frac{1}{2} H_{\alpha\gamma\delta} G^{\gamma\delta i} - 2 G_{\alpha}{}^{i\beta} \partial_{\beta} \phi + \dots. \quad (42)$$

Hence we appear to obtain sensible n-dimensional equations for the gravity fields $\phi(x)$, $g_{\mu\nu}(x)$, and $B_{\mu\nu}(x)$ interacting with the Yang-Mills field $A_{\mu}^i(x)$ and $B_{\mu}^i(x)$, in which all cross terms have cancelled. Note that in the above β functions and in the Chern-Simons equation (30), all the y-dependence of the original ansatz has disappeared without imposing any inconsistent constraints on the n-dimensional fields. The problem arises when we consider the $\hat{\beta}_{ab}^g$ and $\hat{\beta}_{ab}^B$ components which yield

$$\beta_{ij}^g = F_{(i}{}^{\alpha\beta} G_{j)}{}_{\alpha\beta} + \dots, \quad (43)$$

$$\beta_{ij}^B = F_{(i}{}^{\alpha\beta} G_{j)}{}_{\alpha\beta} + \dots, \quad (44)$$

whose vanishing would obviously lead to unacceptable constraints on F and G , not required by the higher-dimensional field equations, but imposed by an inconsistent Kaluza-Klein ansatz.

Fortunately, an inconsistency in the ab components is a rather mild one and can be cured by the introduction of scalar fields S_{ij} in the (adjoint G_L , adjoint G_R) representation into the Kaluza-Klein ansatz, so that β_{ij}^g and β_{ij}^B acquire

extra terms of the form $\square S_{(ij)}$ and $\square S_{[ij]}$ respectively, where $\square = \otimes^{\alpha} \otimes_{\alpha}$. [This is to be contrasted with inconsistencies in the $\alpha\beta$ components which would require massive spin-2 fields or $\alpha\beta$, which would require massive spin-1, and hence give rise to the reintroduction of an infinite tower of massive states⁵⁾.] This is reminiscent of the $SO(8)$ ansatz in the S^7 compactification of $d = 11$ supergravity. A naive K-invariance argument would suggest that all the components of the Einstein equation would be inconsistent, whereas one finds that the problem resides only in the ab components and that this is cured by the introduction of Kaluza-Klein scalars. However, whereas the gravity-Yang-Mills ansatz can easily be written down exactly, it is much more difficult to construct the exact scalar ansatz. Similarly, in the bosonic string the exact ansatz involving $S_{ij}(x)$ is rather complicated, and here we restrict ourselves to the linearized results which we now quote without proof. Define the fluctuations $\hat{h}_{MN}(x,y)$, $\hat{k}_{MN}(x,y)$ and $\hat{\phi}(x,y)$ by

$$\hat{g}_{MN} = \langle \hat{g}_{MN} \rangle + \hat{h}_{MN} , \quad (45)$$

$$\hat{B}_{MN} = \langle \hat{B}_{MN} \rangle + \hat{k}_{MN} , \quad (46)$$

$$\hat{\phi} = \langle \hat{\phi} \rangle + \hat{\varphi} , \quad (47)$$

then

$$\hat{h}_{ab} + \hat{k}_{ab} = S_{ij}(x) \left[L_b^i R_a^j - \frac{2(k+2)}{m^2 C_A (3k-4)} D_a D_b (L_c^i R^{jc}) \right] \quad (48)$$

$$\hat{\varphi} = \frac{1}{4} S_{ij}(x) L_a^i R^{ja} , \quad (49)$$

where

$$D_a V_b = \nabla_a V_b + \frac{1}{2} H_{ab}{}^c V_c . \quad (50)$$

It should be emphasized that the above scalar ansatz is valid only at the linearized level, but one can confirm using standard techniques⁵⁾ that the scalars

$S^{ij}(x)$ are indeed massless as expected.

So far, we have isolated the massless sector of the bosonic string propagating on $M_n \times G$, and determined the β -functions to one-loop order, and we now turn our attention to the heterotic string. To go beyond the Chapline-Manton equations obtained in Ref. 1), however, it is also necessary to include the two-loop contributions to $\hat{\beta}_{MN}^g$ and $\hat{\beta}_{MN}^B$. When $\hat{H}_{MNP}$ is neglected, the contribution to $\hat{\beta}_{MN}^g$ is given by

$$\alpha' \hat{R}_{MPQR} \hat{R}_N{}^{PQR} . \quad (51)$$

Combined with the Yang-Mills terms of (39), this yields the order α' contribution to $\beta_{\mu\nu}^g$

$$\alpha' (R_{\mu\varrho\sigma\tau} R_{\nu}{}^{\sigma\tau} - F_{\mu\varrho}^i F_{\nu}{}^{i\varrho} - G_{\mu\varrho}^i G_{\nu}{}^{i\varrho}) . \quad (52)$$

Note that F^2 and G^2 both enter with a sign opposite to R^2 .

In Ref. 1), the heterotic string β functions were simply obtained by choosing $n = 10$, $G = SO(32)$ or $E_8 \times E_8$ and by setting B_{μ}^i (and the scalars S^{ij}) to zero in (38)-(42), but this naive ansatz breaks down at two loops because the correct contribution to $\beta_{\mu\nu}^g$ in the heterotic string is

$$\alpha' \left(\frac{1}{2} R_{\mu\varrho\sigma\tau} R_{\nu}{}^{\sigma\tau} - F_{\mu\varrho}^i F_{\nu}{}^{i\varrho} \right) . \quad (53)$$

Comparison of (52) and (53) tells us that what is required is an ansatz for which (with $H_{\mu\nu\rho} = 0$)

$$\frac{1}{2} R_{\mu\varrho\sigma\tau} R_{\nu}{}^{\sigma\tau} = G_{\mu\varrho}^i G_{\nu}{}^{i\varrho} . \quad (54)$$

In order to achieve this, we consider the decomposition $G_R \supset SO(8)_{\text{internal}}$ and equate the corresponding spin connection $B_{\mu}^{[IJ]}$ ($I, J = 1, \dots, 8$) with the spin connection ω_{μ}^{IJ} of the transverse $SO(8)_{\text{space-time}}$. It is here that we make contact with the work of Casher et al.¹⁷⁾ who consider the derivation of the heterotic

string from T^{16} compactification of the bosonic string in $d = 26$. In order to account for the fermions in 10 dimensions, they identify the space-time group of the heterotic string with

$$SO(8)_{\text{diagonal}} \subset SO(8)_{\text{internal}} \times SO(8)_{\text{spacetime}}, \quad (55)$$

and choose the embedding of $SO(8)_{\text{internal}} \subset G_R$ to be such that the adjoint representation of G_R contains the 8_s representation of $SO(8)_{\text{internal}}$. [Since, in contrast to the bosonic sector of $d = 11$ supergravity⁵⁾, the bosonic string admits a sensible Euclideanization, it may be possible to interpret the covariant generalization of (55) as an embedding of the full space-time $SO(10)$ in G_R .]

The crucial point is that (54) now provides us with the desired Lorentz Chern-Simons term when substituted into (30). Before doing so, however, we should restore the dependence on $H_{\mu\nu\rho}$ neglected in (51)-(54). As discussed by Hull^{19),20)}, this is achieved by replacing ω_μ by $\omega_\mu^\pm$ in (53), where $\omega_\mu^\pm$ is the spin connection with torsion given by $\pm \frac{1}{2} H_{\mu\nu\rho}$. Noting the sign change in (41) and (42), this corresponds to replacing ω_μ by ω_μ^+ in (54)*). Hence

$$dH = \frac{\alpha'}{2} (R_+^{ab} \wedge R_+^{ab} - 2 F^i \wedge F^i). \quad (56)$$

We note that this equation is now non-linear in H but exact to all orders in α' .

It should be admitted, however, that in common with Casher et al.¹⁷⁾, we have no dynamical understanding of the decomposition $G_R \supset SO(8)$ and the identification (55). Nor do we see any justification for the truncation of the string spectrum which seems to be entailed in reproducing that of the heterotic string. The idea is that the G_R index m on the creation operators $\tilde{\alpha}_m^{(n)}$ ($m = 1, \dots, 496$) is truncated to $\tilde{\alpha}_I^{(n)}$ ($I = 1, \dots, 8$) where I is the 8_s spinor of $SO(8)_{\text{internal}}$, and hence the particle states with I indices transform as fermion representations of $SO(8)_{\text{diag}}$. Hence, in some sense, the G_R Yang-Mills boson $B_{\mu I}$ is really the gravitino $\phi_{\mu I}$, and the scalars S_{II} are really the gauginos χ_{II} . In our group manifold approach, however, the origin of fermion statistics is very obscure. One can ask, however, whether this truncation of the string spectrum is consistent, and here we can invoke the criterion of "K-invariance"⁵⁾, where $K \subset G$. Since we are interested here in the entire infinite string spectrum, moreover, K need not

*) This implies a different bosonic string $\hat{\beta}$ from that calculated by Fridling and van der Ven²¹⁾ when $\hat{H} \neq 0$. See Hull and Townsend (to be published).

act transitively on the manifold. This will be discussed elsewhere¹⁸⁾, but it suffices to say that, in contrast with Casher et al.¹⁷⁾, we do not require the $SO(8)$ to be embedded regularly in G_R . This is just as well, since we are forced on the group manifold to choose either $G_L = G_R = E_8 \times E_8$ or $G_L = G_R = SO(32)$ and there are no regular embeddings of $SO(8)$ in $SO(32)$ containing the 8_s . This circumvents the problem of Casher et al. who, in obtaining the $SO(32)$ string, were forced to choose $G_R = SO(32)$ but $G_L = E_8 \times E_8$.

To obtain the Type II strings, we repeat the above procedure for both G_R and G_L , choosing the embeddings leading to either $(8_c, 8_s)$ in the case of Type IIA or $(8_s, 8_s)$ in the case of Type IIB. The extra 64 bosonic degrees of freedom are then provided by $S_{I,J}$ or S_{IJ} respectively. Vanishing two-loop β functions are obtained from (53) and (54) by setting $A_\mu \sim \omega_\mu^-$ and $B_\mu \sim \omega_\mu^+$ which then yields the Chern-Simons relation $dH = (\alpha'/2)(R_+^{ab} \wedge R_+^{ab} - R_-^{ab} \wedge R_-^{ab})$.*

Finally we note that the derivation of ten-dimensional superstring properties from the bosonic string in $d = 506$ holds to all orders in α' , as may be seen by substituting the Kaluza-Klein ansatz directly into the string Lagrangian. Neglecting the dilaton, we have

$$L = \partial_+ X^M \partial_- X^N (\hat{g}_{MN} + \hat{B}_{MN}) \quad (57)$$

on using the orthonormal gauge for the world-sheet metric. Substituting (16) to (23), we obtain

$$\begin{aligned} L = & \partial_+ X^\mu \partial_- X^\nu [g_{\mu\nu} + B_{\mu\nu} + \\ & + (A_\mu^i L_i^m A_\nu^j L_j^n + B_\mu^i R_i^m B_\nu^j R_j^n + A_\mu^i L_i^m B_\nu^j R_j^n) g_{mn}] \\ & + \partial_+ X^\mu A_\mu^i L_i^n \partial_- y^m g_{mn} \\ & + \partial_- X^\mu B_\mu^i R_i^n \partial_+ y^m g_{mn} \\ & + (\nabla_+ y^m \nabla_- y^n) g_{mn} + (\partial_+ y^m \partial_- y^n) B_{mn}, \end{aligned} \quad (58)$$

where $\nabla_+ y^m = \partial_+ y^m + A_{\mu 1}^{i m} \partial_+ X^\mu$ and $\nabla_- y^m = \partial_- y^m + B_{\mu 1}^{i m} \partial_- X^\mu$. By the identification $B_\mu \sim \omega_\mu^+$, and invoking non-Abelian fermionization¹⁶⁾, we recover the corresponding action for the heterotic string. [This differs from the manifestly gauge-invariant action given by Hull and Witten^{19,20)}, since in our action $B_{\mu\nu}$ already has the

*) Note the departure from the naive result $dH = 0$.

non-trivial Yang-Mills and Lorentz transformation rules (26).] The Type II string actions may also be derived in this way.

The identification of Yang-Mills gauge potentials with gravitational spin connections, first proposed in a rather different context²²⁾ and employed here in going from 506 to 10 dimensions, has also been employed by Candelas et al. in going from 10 to 4²³⁾. This is suggestive of some grander scheme which would combine the two into some purely gravitational phenomenon in $d = 506$. Indeed, the ultimate utility of our Kaluza-Klein approach may be to throw light on the correct compactification from 10 to 4, and we note in this connection the new exact Chern-Simons relation (56). Note, incidentally, that although the coefficient of the torsion is not fixed uniquely by anomaly cancellations and supersymmetry alone²⁰⁾, it is fixed by Kaluza-Klein. It is $\sqrt{3}$ times the torsion introduced by Scherk and Schwarz²⁴⁾ and Nepomechie²⁵⁾. Moreover, as discussed in Ref. 1), there seems to be no compelling reason to insist that $H_{\mu\nu\rho}$ be zero in the true ground state. As emphasized by Hull²⁰⁾, this might have the phenomenological advantage of replacing E_6 by $SO(10)$ as the effective symmetry in four dimensions. We also note the following practical advantages of the $d = 506$ approach. Once the higher-loop $\hat{\beta}$ functions of the bosonic string are known, it is then a straightforward matter of substituting in the Kaluza-Klein ansatz to obtain the ten-dimensional β functions of the heterotic string. Furthermore, if the trick $A_\mu \sim \omega_\mu^-$ is to solve the heterotic string equations to all orders in α' (or if the vanishing of the higher-loop Type II β functions is to hold to all orders in α'), then the Yang-Mills field strengths coming from the $(\alpha')^n$ terms in the $d = 506$ β functions must conspire to cancel the Riemann terms coming from the $(\alpha')^{n+1}$ terms. Thus the fact that our Kaluza-Klein ansatz itself involves α' may help us to deduce $(n+1)$ -loop β functions from n -loop β functions.

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	With KK scalars	Without KK scalars
Generic KK theory	$G_L : (\text{adj}_L, \text{adj}_L)_{\text{symm}}$	-
Bosonic string theory	$G_L \times G_R : (\text{adj}_L, \text{adj}_R)$	G_L

Table

Gauge groups surviving in consistent truncations of theories compactified on the group manifold G , and the corresponding scalar representations.

Note added

After the completion of this work, we became aware of a paper by Nepomechie, Seattle preprint 40048-23 P5, who also considers the $G_L \times G_R$ ansatz but sets $B_\mu \sim \omega_\mu$ rather than $B_\mu \sim \omega_\mu^+$. He does not comment on the ensuing inconsistency of (43)-(44) and its cure by Kaluza-Klein scalars. Furthermore, he does not discuss our result that the $R_{\mu\rho}^{ab} R_\nu^{\rho ab}$ contribution to $\beta_{\mu\nu}^g$ comes not only from the one-loop term $G_{\mu\rho}^{i\rho i} G_\nu^{i\rho i}$ in $\beta_{\mu\nu}^g$, but also from the two-loop term $\hat{R}_{MP}^{AB} \hat{R}_N^{PAB}$ in $\hat{\beta}_{MN}^g$.

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