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STOCHASTIC EQUATIONS FOR GRAVITY

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ABSTRACT

The Langevin equations governing stochastic quantization of gravity are established. These equations hold true independently of the value of the torsion. They are covariant under the BRS symmetry associated to local diffeomorphisms and Lorentz transformations. The method determines simultaneously the Langevin equations of the diffeomorphisms and Lorentz ghosts, and of the classical vielbein and spin connection.

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Stochastic quantization is a method which is formally equivalent to the path integral formalism or to the canonical one for building quantum scalar field theories¹⁾. This technique was applied first by Parisi and Wu to the case of Yang-Mills theories²⁾. In their original paper, the stochastic dynamics of the Yang-Mills field is described by means of a Langevin equation. By transposing this equation into a Fokker-Planck equation, one can prove in a rather general way, formally true for any type of gauge theory, that the stochastic quantization method and the usual methods based on a gauge-fixed and BRS-invariant Lagrangian predict the same values for any gauge-independent quantity³⁾. Recently, the stochastic description of the ghost sector of Yang-Mills theories has been derived from the principle that the evolution along the stochastic time must be compatible with the BRS symmetry^{4),5)}. The method is purely geometrical, since the BRS symmetry⁶⁾ can be constructed independently of the notion of a Lagrangian, and it turns out that the system of Langevin equations describing the whole set of classical and ghost fields of a Yang-Mills theory, including the possible gauge-restoring forces, can be determined from the sole knowledge of the cohomological properties of the BRS operator^{4),5)}. The method has also been applied to determine the stochastic formulation of these gauge theories involving interacting p-form gauge fields⁵⁾.

The aim of this letter is to prove that stochastic quantization can be used also in curved space, when gravity is coupled. We will in fact derive the Langevin equations of gravity, with a trivial generalization to gauge systems coupled to gravity. We choose to work in the vielbein formalism. Given any quantum action coupled to gravity, the fields describing the gravitational sector are⁷⁾⁻⁹⁾

$$\begin{array}{ccc} e^a & = & e^a_\mu dx^\mu \\ \xi^\mu & & \bar{\xi}^\mu \\ & \backslash & / \\ & b^\mu & \end{array} \qquad \begin{array}{ccc} \omega^{ab} & = & \omega^{ab}_\mu dx^\mu \\ \Omega^{ab} & & \bar{\Omega}^{ab} \\ & \backslash & / \\ & d^{ab} & \end{array}$$

(1)

Here a and b stand for the Lorentz indices of D-dimensional Lorentz symmetry. e^a_μ and ω^{ab}_μ denote respectively the vielbein and the spin connection. ξ^μ and $\bar{\xi}^\mu$ on the one hand, and Ω^{ab} and $\bar{\Omega}^{ab}$ on the other hand, are respectively the ghosts and antighosts of local diffeomorphisms and Lorentz transformations. b^μ and d^{ab} are

the corresponding Stückelberg auxiliary fields. The latter permit one to define the BRS variations of antighosts. The action of the gauge symmetry on all the fields, i.e., that of the BRS operator, can be geometrically determined, and any admissible action, possibly gauge-fixed, must be invariant under s transformations. The action of s on the fields (1) is as follows⁷⁾⁻⁹⁾

$$\begin{aligned}
 s e^a &= -\Omega^{ab} e^b - (i_\xi \omega^{ab}) e^b + L_\xi e^a \\
 s \omega^{ab} &= -D \Omega^{ab} + i_\xi R^{ab} \\
 s \Omega^{ab} &= -\frac{1}{2} [\Omega, \Omega]^{ab} + \frac{1}{2} i_\xi i_\xi R^{ab} \\
 s \xi^\mu &= \frac{1}{2} \{ \xi, \xi \}^\mu = \xi^\alpha \partial_\alpha \xi^\mu \\
 s \bar{\Omega}^{ab} &= d^{ab} \\
 s \bar{\xi}^\mu &= b^\mu \\
 s d^{ab} &= 0 \\
 s b^\mu &= 0
 \end{aligned} \tag{2}$$

where $D = d + \omega$ is the Lorentz-covariant derivative. $R = d\omega + \omega\omega$ is the Lorentz curvature two-form. The scalar Lorentz ghosts Ω^{ab} , $\bar{\Omega}^{ab}$ and the vector field ghosts ξ^μ and $\bar{\xi}^\mu$ are anticommuting objects. i_ξ is the contraction operator along the anticommuting vector field ξ^μ , and $L_\xi \equiv i_\xi d - d i_\xi$ is the graded Lie derivative along ξ^μ . The grading is defined as the sum of the Lorentz degree and ghost number. Therefore s is a graded differential operator with grading equal to one, i_ξ has grading 0, and L_ξ has grading 1. One can verify that $s^2 = 0$ on all quantities. If one introduces the following change of variables,

$$\begin{aligned}
 \hat{s} &\equiv s - L_\xi \\
 \hat{\Omega}^{ab} &\equiv \Omega^{ab} - i_\xi \omega^{ab} = \Omega^{ab} - \xi^\mu \omega_\mu^{ab} \\
 \hat{b}^\mu &\equiv b^\mu - \{ \xi, \bar{\xi} \}^\mu = b^\mu - \xi^\alpha \partial_\alpha \bar{\xi}^\mu - \bar{\xi}^\alpha \partial_\alpha \xi^\mu \\
 \hat{d}^{ab} &\equiv d^{ab} - L_\xi \bar{\Omega}^{ab} = d^{ab} - i_\xi d \bar{\Omega}^{ab}
 \end{aligned} \tag{3}$$

one can rewrite Eqs. (2) under the equivalent and simplest form

$$\begin{aligned}
 \hat{s} \omega^{ab} &= -D \hat{\Omega}^{ab} & \hat{s} e^a &= -\hat{\Omega}^{ab} e^b \\
 \hat{s} \hat{\Omega}^{ab} &= -\frac{1}{2} [\hat{\Omega}, \hat{\Omega}]^{ab} & \hat{s} \xi^\mu &= -\frac{1}{2} \{ \xi, \xi \}^\mu \\
 \hat{s} \hat{\Omega}^{ab} &= \hat{d}^{ab} & \hat{s} \bar{\xi}^\mu &= \hat{b}^\mu \\
 \hat{s} \hat{d}^{ab} &= 0 & \hat{s} \hat{b}^\mu &= 0
 \end{aligned} \tag{4}$$

Further, one can combine the classical and ghost quantities into extended objects compatible with the existence of the above-defined grading. One defines

$$\begin{aligned}\tilde{d} &\equiv d + s \\ \hat{d} &\equiv d + \hat{s} = (\exp -i_\xi)(d+s) \exp i_\xi\end{aligned}\quad (5)$$

$$\begin{aligned}\tilde{\omega}^{ab} &\equiv \omega^{ab} + \Omega^{ab} \\ \hat{\omega}^{ab} &\equiv \omega^{ab} + (\Omega^{ab} - i_\xi \omega^{ab}) = (\exp -i_\xi) \tilde{\omega}^{ab}\end{aligned}\quad (6)$$

Then one finds that Eqs. (2) or (4) are equivalent to the following constraints on generalized curvatures, once expanded in ghost number^{7),8)}.

$$\begin{aligned}\hat{R} &\equiv \hat{d} \hat{\omega} + \hat{\omega} \hat{\omega} = R \equiv d\omega + \omega\omega \\ \hat{T} &\equiv \hat{d} \hat{e} + \hat{\omega} \hat{e} = T = de + \omega e \\ \hat{e} &\equiv e\end{aligned}\quad (7)$$

Equations (7) are also equivalent to

$$\begin{aligned}R &\equiv \tilde{d} \tilde{\omega} + \tilde{\omega} \tilde{\omega} = (\exp i_\xi) R \\ \tilde{T} &\equiv \tilde{d} \tilde{e} + \tilde{\omega} \tilde{e} = (\exp i_\xi) T \\ \tilde{e} &\equiv e + i_\xi e = (\exp i_\xi) e\end{aligned}\quad (8)$$

The constraint that $\hat{e}$ has no component with non-zero ghost number is geometrically crucial. It enforces the invertibility of the vielbein e_μ^a , and determines the link between the diffeomorphisms and the translation sector of the Poincaré algebra⁷⁾. The Bianchi identities $DR = 0$, $DT = Re$ ensure that $\hat{d}^2 = \tilde{d}^2 = 0$, which means in fact that the constraints (7,8) truly determine s and $\hat{s}$ with $s^2 = \hat{s}^2 = sd + ds = \hat{s}d + d\hat{s} = 0$ ^{7,8)}. It is most convenient to introduce the following "Lorentz-covariant" operators, constructed from the "Lorentz-scalar" operators s , $\hat{s}$ and $L_\xi = [i_\xi, d]$

$$\begin{aligned}S &\equiv s + \Omega \\ \tilde{D} &\equiv D + S = d + s + \omega + \Omega\end{aligned}$$

$$\begin{aligned}
 d_\xi^\omega &\equiv L_\xi + i_\xi \omega = i_\xi D - D i_\xi \\
 \hat{S} &\equiv \hat{s} + \hat{\omega} = S - d_\xi^\omega \\
 \hat{D} &\equiv D + \hat{S} = d + \hat{s} + \omega + \hat{\omega}
 \end{aligned}$$

(9)

These operators satisfy the commutation properties

$$\begin{aligned}
 S^2 &= \frac{1}{2} i_\xi i_\xi R & S D + D S &= i_\xi R \\
 \hat{S}^2 &= 0 & \hat{S} D + D \hat{S} &= 0
 \end{aligned}$$

(10)

Equations (10) are trivial to verify by making use of the identities $\tilde{R} = \tilde{D}\tilde{D}$ and $\hat{R} = \hat{D}\hat{D}$ together with the BRS equations (7) and (8). This yields $\tilde{D}\tilde{D} = \exp i_\xi R$, $\hat{D}\hat{D} = R$ and thus Eq. (10) by expansion in ghost number.

In the stochastic approach, all fields and ghosts (1) depend on the stochastic time t in addition to the space-time variable x^μ (1). Besides, one introduces a noise source for each field. We denote as η^{ab} , γ^{ab} , $\bar{\gamma}^{ab}$, β^{ab} , ρ_μ^a , K^μ , $\bar{K}^\mu$, β^μ the noise sources of ω_μ^{ab} , Ω^{ab} , $\bar{\Omega}^{ab}$, d^{ab} , e_μ^a , ξ^μ , $\bar{\xi}^\mu$, b^μ respectively. We want to determine the action of $\partial/\partial t$ on all these fields, and also the action of the BRS operator on the noise sources. The requirement is that the stochastic evolution along t must be compatible with the action of s , defined in Eq. (2), or equivalently in Eqs. (7) or (8). This means that $\partial/\partial t$ must commute with s

$$\frac{\partial}{\partial t} s = s \frac{\partial}{\partial t}$$

(11)

It is obvious that the space-time derivative $d = dx^\mu \partial_\mu$ must also commute with $\partial/\partial t$. Therefore one requires

$$\frac{\partial}{\partial t} \hat{d} = \hat{d} \frac{\partial}{\partial t} \quad (12)$$

or equivalently

$$\frac{\partial}{\partial t} \hat{d} = \hat{d} \frac{\partial}{\partial t} - L_{\left(\frac{\partial}{\partial t} \xi\right)} \quad (13)$$

From now on, we adopt the notation $\dot{\cdot}$ for the operator $\partial/\partial t$. By applying $\partial/\partial t$ to the BRS equation (7) and by using Eq. (13), one gets that

$$\dot{\hat{R}} = (d + s - L_{\xi}) \dot{\hat{\omega}} - L_{\dot{\xi}} \hat{\omega} + [\omega, \hat{\omega}] = \dot{\hat{R}} \quad (14)$$

This can be written as

$$\dot{\hat{R}} = \hat{D} \dot{\hat{\omega}} - L_{\dot{\xi}} \hat{\omega} = \dot{\hat{R}} \quad (15)$$

We shall expand Eq. (15) in ghost number. Then, assuming that the stochastic time derivatives of fields are linear functions of the noises, we shall solve the resulting equations, and find the expressions of $\dot{\hat{\omega}}$, $\dot{\hat{\Omega}}$, $\dot{\hat{\xi}}$, i.e., the Langevin equations of ω , Ω , ξ . We begin with the part with ghost number 2 in Eq. (15):

$$\hat{S} \dot{\hat{\Omega}} = L_{\dot{\xi}} \hat{\Omega} = i_{\dot{\xi}} d \hat{\Omega} = -i_{\dot{\xi}} \hat{S} \omega \quad (16)$$

Since $\hat{S}^2 = 0$, one must have $\hat{S} \dot{\hat{\xi}} = 0$. This implies

$$\dot{\hat{\xi}}^\mu = \hat{S} Z^\mu + K^\mu = \hat{S} Z^\mu + K^\mu \quad (17)$$

and the $\hat{S}$ variation of the source K^μ must be

$$\hat{S} K^\mu = 0 \quad \Leftrightarrow \quad S K^\mu = \{ \xi, K \}^\mu \quad (18)$$

Z^μ is an arbitrary vector field, with ghost number 1, which can be a function of all the fields ω , e , $\hat{\xi}$. We combine Eq. (17) with Eq. (16) and find

$$\widehat{S}(\widehat{\Omega} + i\dot{\xi}\omega) = 0 \quad (19)$$

This implies

$$\begin{aligned} \dot{\widehat{\Omega}}^{ab} &= -i\dot{\xi}\omega^{ab} + \widehat{S}V^{ab} + \gamma^{ab} \\ &= -i\dot{\xi}_Z\omega^{ab} + \widehat{S}V^{ab} + \gamma^{ab} - i_K\omega^{ab} \end{aligned} \quad (20)$$

provided that one has

$$\widehat{S}\gamma^{ab} = 0 \quad \Leftrightarrow \quad \widehat{S}\gamma^{ab} = -[\widehat{\Omega}, \gamma]^{ab} \quad (21)$$

V^{ab} , which is valued in the Lie algebra $\mathcal{G}_d$ of the Lorentz group and has ghost number 0, plays a role analogous to K^μ in Eq. (17) and can depend on all the fields. In the stochastic process, when the sources K^μ and γ^{ab} are specified as Gaussian ones, the functions Z^μ and V^{ab} are expected to stabilize the t-evolution of the unphysical modes in ω^{ab} , corresponding respectively to the local diffeomorphisms and Lorentz invariances.

To determine the Langevin equation which expresses the form of $\dot{\omega}$, we isolate the part with ghost number 1 in Eq. (15), and we use the already-found expressions of $\widehat{\Omega}$ and $\dot{\xi}$. One finds

$$\widehat{S}\dot{\omega} + D(-i\dot{\xi}\omega + \widehat{S}V + \gamma) - L_{\dot{\xi}}\omega = 0 \quad (29)$$

Using the commutation property $\widehat{S}D + D\widehat{S} = 0$, one gets from this equation

$$\widehat{S}(\dot{\omega} - DV) - i\dot{\xi}R + D\gamma = 0 \quad (30)$$

Equation (30) is clearly consistent, since $\widehat{S}\gamma = 0$, $\widehat{S}R = 0$ and $\widehat{S}\dot{\xi} = 0$. We insert the expression (17) of $\dot{\xi}$ into (30) and get

$$\widehat{S}(\dot{\omega} - DV - i_ZR) - i_KR + D\gamma = 0 \quad (31)$$

The solution of this equation is

$$\dot{\omega}^{ab} = D V^{ab} + i_{\xi} R^{ab} + \frac{\delta \mathcal{L}}{\delta \omega^{ab}} + \eta^{ab} \quad (32)$$

where the BRS variation of the source η^{ab} of the spin connection is

$$\hat{S} \eta^{ab} = -D \delta^{ab} + i_{\kappa} R^{ab} \quad (33)$$

In Eq. (32), we have used the fact that the cohomology of $\hat{S}$ is non-trivial for $G_{\mathbb{R}}$ -valued quantities with ghost number 0. Indeed, $\hat{S}$ -invariant but not $\hat{S}$ -exact Lagrangians exist, such as the Einstein Lagrangian, the Weyl Lagrangian and the cosmological constant^{8),9)}. BRS-invariant gauge-fixing terms which are of the type $s(\bar{M})$ can be included in $\mathcal{L}$. However, they correspond to mere redefinitions of V^{ab} by terms at least quadratic in the ghosts and antighosts, and one can restrict $\mathcal{L}$ in Eq. (32) to be a classical gauge-invariant Lagrangian.

The stochastic equation for the vielbein is obtained from the BRS equation $\hat{T} = T$. From $\hat{T} = \dot{T}$, $\hat{e} = e$, one gets

$$\hat{D} \dot{e} + \dot{\omega} e - L_{\dot{\xi}} e = \dot{T} \quad (34)$$

where we have already computed $\dot{\omega}$ and $\dot{\xi}^{\mu}$. The part with ghost number 1 yields

$$\hat{S} e + (-i_{\xi} \omega - L_{\dot{\xi}} + \hat{S} V + \gamma) e = 0 \quad (35)$$

or

$$\hat{S} (e + V e) - \mathcal{L}_{\dot{\xi}}^{\omega} e + \gamma = 0 \quad (36)$$

Here we have used $\hat{S} e = 0$ and the definition (9) of the covariant Lie derivative $\mathcal{L}_{\dot{\xi}}^{\omega}$. Further, one has

$$\mathcal{L}_{\dot{\xi}}^{\omega} e = \mathcal{L}_{\hat{S} \xi}^{\omega} e + \mathcal{L}_{\kappa}^{\omega} e = \hat{S} (\mathcal{L}_{\xi}^{\omega} e) + \mathcal{L}_{\kappa}^{\omega} e$$

so that one gets finally

$$\dot{e}^a = -V^{ab}e^b + d_Z e^a + \frac{\delta \mathcal{L}}{\delta e^a} + \rho^a \quad (37)$$

where the noise source ρ^a for the vielbein undergoes the following BRS transformation law:

$$\hat{S} \rho^a = -\gamma^{ab}e^b + d_K^\omega e^a \quad (38)$$

In the same way as $\dot{\omega}$ contains $\partial \mathcal{L} / \partial \omega$, $\dot{e}$ contains $\partial \mathcal{L} / \partial e$ as a consequence of the non-triviality of the cohomology of $\hat{S}$ for objects with ghost number zero and valued in the fundamental representation of the Lorentz algebra.

We can now summarize the results. The Langevin equation for the classical and ghost fields of gravity are simply

$$\begin{aligned} \dot{e}^a &= \frac{\delta \mathcal{L}}{\delta e^a} - V^{ab}e^b - d_Z^\omega e^a + \rho^a \\ \dot{\omega}^{ab} &= \frac{\delta \mathcal{L}}{\delta \omega^{ab}} + D V^{ab} + i_Z R^{ab} + \gamma^{ab} \\ \hat{S} \omega^{ab} &= \hat{S} V^{ab} - i_{\hat{S} Z} \omega^{ab} + \gamma^{ab} - i_K \omega^{ab} \\ \dot{Z}^\mu &= \hat{S} Z^\mu + K^\mu \end{aligned} \quad (39)$$

Besides, the BRS variations of the sources are

$$\begin{aligned} \hat{S} \rho^a &= -\gamma^{ab}e^b + d_K^\omega e^a \\ \hat{S} \gamma^{ab} &= -D \gamma^{ab} + i_K R \\ \hat{S} K^\mu &= 0 \\ \hat{S} \gamma^{ab} &= 0 \end{aligned} \quad (40)$$

The derivation of these equations has been purely geometrical. The dynamics is specified by imposing that all sources are Gaussian ones. In Eqs. (39), the functions V^{ab} and Z^μ characterize the possible gauge-restoring forces of the stochastic process corresponding to the diffeomorphism and Lorentz invariances. Physics is expected to be independent of the choice of these functions³⁾. In

contrast with Eq. (39), Eqs. (40), which express the BRS variations of noises, are independent of the gauge-restoring forces. These features are similar to those encountered in flat space^{4),5)}. Notice that the stochastic equations (39) and (40) hold true for any admissible gravitational Lagrangian $\mathcal{L}$, including these cases where the torsion is non-vanishing.

There is no difficulty in obtaining the other stochastic equations for the antighost sector of the theory, using the anti-BRS operator of gravity⁷⁾. The method is detailed in Refs. 4) and 5) in the case of flat space gauge symmetries. One gets $\dot{\bar{\xi}}^\mu = \hat{S}Z^\mu + \bar{K}^\mu$, and $\dot{\bar{b}}^\mu$ is obtained from the computation of $\dot{b}^\mu = s_{\bar{\xi}}^\mu$, using the chain rule. In turn, one finds $\dot{\bar{\Omega}} = \hat{S}V^{ab} - i\hat{S}_Z\omega + \bar{\gamma} - i\bar{K}\omega$. Finally, $\dot{\bar{b}}^{ab} = s_{\bar{\Omega}}^{ab}$ can also be computed from the chain rule.

It is now rather simple to derive the stochastic equations for a scalar field ϕ and a Yang-Mills system coupled to gravity, with gauge field $A_\mu dx^\mu$, valued in the Lie algebra $\mathcal{G}$ of a given Lie group, and ghost c . One can apply the same method as we have done for gravity, starting from the BRS equations $\hat{F} = \hat{d}A + AA = F$ and $\hat{G} = \hat{D}\phi = G$, or equivalently $\tilde{F} = \exp -i_\xi F$ and $\tilde{G} = \exp i_\xi G$ ^{7),8)}. One gets the following Langevin equation for A , c , ϕ :

$$\begin{aligned} \dot{A} &= DV^\xi + i_Z F + \eta^\xi + \frac{\delta \mathcal{L}}{\delta A} & D &\equiv d + \omega + A \\ \dot{\hat{c}} &= \hat{S}V^\xi - i_{\hat{S}_Z} A + \gamma^\xi - i_K A & \hat{S} &\equiv \hat{S} + \hat{\mathcal{R}} + \hat{c} \\ \dot{\phi} &= -V^\xi \phi + \lambda + \frac{\delta \mathcal{L}}{\delta \phi} \end{aligned} \quad (41)$$

V^ξ characterizes the gauge-restoring force in the Yang-Mills sector, Z^μ and K^μ have been defined above, and η^ξ , γ^ξ , λ are respectively the noise sources for the Yang-Mills field A , the ghost c , and the scalar field λ . The BRS transformation laws of sources are

$$\begin{aligned} \hat{S} \eta^\xi &= -D\gamma^\xi + i_K F \\ \hat{S} \gamma &= 0 \\ \hat{S} \lambda &= -\gamma^\xi \varphi + \mathcal{L}_K^{A+\omega} \varphi \end{aligned}$$

(42)

To conclude, we have shown that stochastic quantization can also be applied to the cases of gravity and of gauge systems coupled to gravity. We find it interesting that the derivation of the stochastic equations of gravity is directly linked to the cohomological properties of the BRS operator of the corresponding symmetries. As in the case of gauge symmetries in flat space, one retains at once the Langevin equations of classical and ghost fields. The dynamics of the latter fields is therefore defined in a non-perturbative way, independently of the usual field theory formalisms. Besides, the method provides in a systematic way the most general form of the possible gauge-restoring forces which stabilize the stochastic evolution of unphysical modes of gauge fields, while preserving the gauge independence of physical quantities. It is an open question whether one could use the above stochastic equations to explore non-perturbative aspects of gravity. Using an Einstein-Hilbert Lagrangian for $\mathcal{L}$ will certainly produce difficulties, since the corresponding action is not positive definite. These problems have been tackled in Ref. 10).

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