CERN-TH.4317/85

VACUUM-INDUCED FRIEDEL-TYPE OSCILLATIONS
IN RELATIVISTIC MANY-FERMION SYSTEMS

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ABSTRACT

It is shown that generalized Atiyah-Patodi-Singer η -invariants naturally appear in the thermodynamic potential describing the equilibrium properties of a localized system of relativistic interacting fermions. External $U(1)$ gauge fields induce oscillating charge distributions which should be experimentally detectable for strong Coulomb fields produced by highly stripped large- Z atoms. Casimir energy shifts result by the same mechanism.

CERN-TH.4317/85

December 1985

1. Introduction

Atiyah-Patodi-Singer η -invariants¹⁾, generalised to open spaces²⁾, have been shown³⁾ to play a generic role in the phenomenon of fermion fractionisation⁴⁾. The discovery⁴⁾ of fractional fermion charges has given rise to the discussion of a vast range of new physical phenomena: spin and statistics of Skyrmeons⁵⁾ in the realm of current algebra soliton models of hadrons and their phenomenological reverberations, the role of fractional fermionic charges of Dirac and 't Hooft-Polyakov magnetic monopoles in the catalysis of reactions in grand unified models etc.

In our context, the η -invariant describes the spectral asymmetry of the Dirac operator containing an external field, a $U(1)$ gauge field, for instance, which eventually is promoted to a dynamical degree of freedom in order to introduce a fermion-fermion interaction. Formally, the η -invariant is expressed as the zeroth order weight of the difference of the densities of positive and negative energy states. In this paper we are going to consider the first order weight and the response to a variation of the gauge field, the charge density induced by the spectral asymmetry. The first order weight is to be interpreted as a Casimir energy and has been discussed elsewhere⁷⁾. It is interesting to observe that this quantity automatically appears in the grand canonical potential for a relativistic interacting many-fermion system and, hence, partly determines its equilibrium properties.

In this paper we are exploiting this fact in a simple, topologically trivial case: interacting fermions in an external potential. The latter can be practically realised in highly stripped large Z atoms, so there is the chance of an experimental study of vacuum-induced charge distributions. Under circumstances to be specified we shall show that these charge distributions yield long-range oscillatory potentials of an order of magnitude which should allow for experimental detection if Z is chosen sufficiently large.

As far as the many-body aspects of a localised thermodynamic equilibrium state of a $U(1)$ gauge-theory with fermions, i.e. spinor QED, are concerned, we have shown⁸⁾ that a Landau-type basis⁹⁾ can be constructed such that the corresponding spectrum yields the exact partition function. In our context this means that e.g. the contribution of the first order weight of the spectral asymmetry to the total energy, the Casimir shift, can be unambiguously identified. It is quite obvious that this fact is instrumental

for its experimental identification⁷⁾. If we compute the charge density as the linear response of the first order weight of the spectral asymmetry versus the gauge-field in this Landau-type basis it is clear that contributions from correlations appear which, however, have been found to be small in characteristic cases¹⁰⁾.

The existence of an exact Landau-type basis is equivalent to the existence of an optimal mean-field which generates a spectrum such that the exact partition function for our system of interacting fermions, e.g. for spinor QED obtains. Hence, our representation starts with the case of fermions in an external field (chapter 2 and 3) which then immediately generalises to the case with interaction (chapter 4).

2. Generalised η -invariants in equilibrium states of many-fermion systems

In this chapter we should like to discuss the connection of equilibrium properties of many-fermion systems and their vacuum structure. We shall show, in particular, that the energy of the "occupied Dirac sea" combines with the so-called zero-point vibration energy to an expression which is recognised as a natural generalisation of the APS η -invariant¹⁾. The latter is well-known to have interesting applications in various physics problems⁵⁾¹¹⁾, in particular for fermion fractionisation⁴⁾; Dirac sea contributions to atomic states have been interpreted⁷⁾ as Casimir potentials.

The connection we are looking for appears in a transparent way if we work in the Schrödinger picture of QED. States are represented by functionals the time-dependence of which is dictated by the Schrödinger equation

$$(i\partial_t - \int d^3x \left\{ -\frac{\delta^2}{\delta\varphi \delta\varphi^*} + V(\varphi, \varphi^*) \right\}) \Psi[\varphi, \varphi^*; t] = 0 \quad (2.1)$$

$\varphi(x)$, $\varphi^*(x)$ are the Schrödinger field and its conjugate. To begin with the potential $V(\varphi, \varphi^*)$ is chosen as

$$V(\varphi, \varphi^*) = \varphi^* A^2 \varphi \quad (2.2)$$

where A is a differential operator describing the single particle dispersion in an external field. At a later stage, the latter will be promoted to a dynamical degree of freedom. An immediate generalisation of the Feynman-Hibbs¹²⁾ result for the harmonic oscillator yields¹³⁾ the Green's functional of equation (2.1) ($T:=t-t'$)

$$\begin{aligned} G[\varphi, \varphi'; T] &= \det \left(\frac{A}{2\pi \hbar i \sin AT} \right) \cdot \\ \exp i \int d^3x \left\{ \frac{1}{2} (\varphi^* A \cotg AT \varphi)(\vec{x}) + \frac{1}{2} (\varphi'^* A \cotg AT \varphi')(\vec{x}) \right. \\ &\quad \left. - \left(\varphi^* \left(\frac{A}{\sin AT} \right) \varphi' \right)(\vec{x}) \right\}. \end{aligned} \quad (2.3)$$

Analytic continuation $T \rightarrow T'=i\beta$ leads to the partition function of our system, a Fermi gas in an external potential

$$Z^{-1} = \left(\text{tr } G \Big|_{T=i\beta} \right)^{1/2} \quad (2.4)$$

For fermions, the trace has to be computed with antiperiodic boundary conditions

$$\text{tr } G \Big|_{\text{fermions}} = \int \mathcal{D}\varphi \int \mathcal{D}\varphi^* G[\varphi, -\varphi; T]; \quad (2.5)$$

for bosons, of course, we would have to calculate

$$Z = \text{tr } G \Big|_{\text{bosons}} = \int \mathcal{D}\varphi \int \mathcal{D}\varphi^* G[\varphi, \varphi; T]. \quad (2.6)$$

The slightly queer-looking relation (2.4) for the partition function of

fermions in an external field obviously guarantees that the thermodynamical potential receives the correct sign; introducing the external field as a dynamical degree of freedom leads to fermion loops obtained as response functions which in turn have the correct sign. This formal construction avoids the introduction of Grassmannian integrations. The result of a straightforward calculation is

$$\mathbb{Z}|_{\text{fermions}} = \det \left(e^{\frac{\beta A}{2}} + \ln(1 + e^{-\beta A}) \right). \quad (2.7)$$

Clearly, the same result obtains if the usual ζ -function regulated fermionic determinant is computed. We calculate the energy

$$\begin{aligned} E &= - \frac{\partial}{\partial \beta} \ln \mathbb{Z} \\ &= - \frac{1}{2} \text{tr} A + \text{tr} \frac{A}{1 + e^{-\beta A}} \end{aligned} \quad (2.8)$$

and, after having introduced an (external) gauge field A_μ , the charge density

$$n(\vec{x}) = -\beta^{-1} \frac{\delta \ln \mathbb{Z}}{\delta A_0(\vec{x})}. \quad (2.9)$$

In terms of the spectrum of A

$$A \gamma_\alpha(\vec{x}) = \varepsilon_\alpha \gamma_\alpha(\vec{x}) \quad (2.10)$$

we have

$$E = - \frac{1}{2} \sum_\alpha \varepsilon_\alpha + \sum_\alpha n_\alpha \varepsilon_\alpha \quad (2.11)$$

where

$$n_\alpha = \left(e^{\beta \varepsilon_\alpha} + 1 \right)^{-1} \quad (2.12)$$

is the Fermi-distribution and

$$n(\vec{x}) = \sum_{\alpha} n_{\alpha} |\psi_{\alpha}(\vec{x})|^2 = \frac{1}{2} \sum_{\alpha} |\psi_{\alpha}(\vec{x})|^2. \quad (2.13)$$

For the sake of clarity, we enclose our system in a finite volume V ; the spectrum is then discrete. The limit $V \rightarrow \infty$ needs special consideration²⁾, peculiarities of the continuum limit pertaining to the specific questions we are treating in this paper will be dealt with in the following chapter.

The structure of a relativistic spectrum - a mass gap $[-m, m]$ separating particle and antiparticle scattering states and containing a bound-state point spectrum - leads after the well-known charge conjugation to the following interpretation of these expressions: introducing the particle and antiparticle Fermi distributions

$$n_{\alpha}^{(+)} = \frac{1}{e^{\beta \varepsilon_{\alpha}} + 1} \quad \varepsilon_{\alpha} > 0 \quad (2.14)$$

$$n_{\alpha}^{(-)} = \frac{1}{e^{-\beta \varepsilon_{\alpha}} + 1}, \quad \varepsilon_{\alpha} < 0$$

the total energy E can be written as $(n_{\alpha}^{(-)} = 1 - n_{\alpha}^{(+)})$

$$E = \sum_{\varepsilon_{\alpha} > 0} n_{\alpha}^{(+)} \varepsilon_{\alpha} - \sum_{\varepsilon_{\alpha} < 0} n_{\alpha}^{(-)} \varepsilon_{\alpha} - E_c \quad (2.15)$$

$$E_c := \frac{1}{2} \left(\sum_{\varepsilon_{\alpha} > 0} \varepsilon_{\alpha} - \sum_{\varepsilon_{\alpha} < 0} \varepsilon_{\alpha} \right).$$

Similarly, the charge density is then given by

$$n(\vec{x}) = \sum_{\varepsilon_{\alpha} > 0} n_{\alpha}^{(+)} |\psi_{\alpha}(\vec{x})|^2 - \sum_{\varepsilon_{\alpha} < 0} n_{\alpha}^{(-)} |\psi_{\alpha}(\vec{x})|^2 + n_c(\vec{x}) \quad (2.16)$$

$$n_c(\vec{x}) = \frac{1}{2} \left(\sum_{\varepsilon_{\alpha} > 0} |\psi_{\alpha}(\vec{x})|^2 - \sum_{\varepsilon_{\alpha} < 0} |\psi_{\alpha}(\vec{x})|^2 \right);$$

in a self-explanatory notation we indicated positive and negative energy branches of the spectrum and the corresponding Fermi-distributions over which the spectral sums are to be performed.

The energy E_c has been interpreted⁷⁾ as a Casimir energy shift of, for instance, atomic levels and its eventual experimental detection has been discussed. In this paper we shall attempt to get a qualitative understanding and an order of magnitude estimate of the induced charge density $n_c(x)$.

The first two terms in (2.15) and (2.16) have an immediate interpretation: $n^{(\pm)}$ denote the occupation probability of particle and antiparticle states with energies $|\epsilon_\alpha|$ which is controlled by the Fermi energy brought into the game by the replacement $\epsilon_\alpha \rightarrow \epsilon_\alpha - \epsilon_F$.

The induced terms E_c and $n_c(x)$ are obtained as the difference of two terms which, taken by themselves, are certainly divergent; a regularisation procedure has to be introduced to control the convergence properties of their difference. Following earlier proposals¹⁴⁾ we choose a ζ -function type regularisation and define

$$H(\vec{x}, s) = \frac{1}{2} \left(\sum_{\epsilon_\alpha > 0} \epsilon_\alpha^s |\psi_\alpha(\vec{x})|^2 - \sum_{\epsilon_\alpha < 0} \epsilon_\alpha^s |\psi_\alpha(\vec{x})|^2 \right); \quad (2.17)$$

in physically interesting cases the spectral sums should converge for sufficiently negative s , analytic continuation in s defines the sums (2.15) and (2.16).

Since the eigenfunctions $\psi_\alpha(\vec{x})$ are normalised in a box of finite volume we have

$$E_c = \int d^3x \left[H(\vec{x}, s) \right]_{s \rightarrow 1} = \left[\int d^3x H(\vec{x}, s) \right]_{s \rightarrow 1} \quad (2.18)$$

$$n_c(\vec{x}) = \left[H(\vec{x}, s) \right]_{s \rightarrow 0} ,$$

for $s=0$, we obtain the APS η -invariant $\eta(0)$, where

$$\eta(s) = \int d^3x \ H(\vec{x}, s) . \quad (2.19)$$

The continuum version of this equation is amply discussed in the literature, our explicit calculations described in the following chapter set the scope of conditions under which the continuum version of $H(x;s)$

$$H(\vec{x}, s) = \int_{-\infty}^{+\infty} dE \ \rho(E) \ E^s \ |\psi_E(\vec{x})|^2 \ \text{sign} \ E \quad (2.20)$$

$$\rho(E) = \frac{1}{2\pi i} \int_{-i\infty+c}^{+i\infty+c} d\beta \ e^{\beta E} \ Z(\beta)$$

is a well-defined quantity.

3. Oscillations of the Friedel type induced by vacuum deformations

In this chapter we should like to present an explicit calculation of the induced charge density $n_c(x)$ which results from the deformation of the Dirac sea by an external field; mechanisms by which the same effect is produced dynamically will be discussed in the next chapter.

It is quite clear that in order to obtain non-vanishing effects, i.e. a non-vanishing $H(x;s)$ in (2.17), the external fields have to distinguish between particles and antiparticles. Non-trivial global topological properties, as e.g. in the case of fermions in a monopole field¹⁵⁾, give rise to interesting effects but are not pertinent to the point we want to make.

Typical external fields leading to non-vanishing $H(x;s)$ are $U(1)$ -gauge fields among which the electromagnetic case is of particular interest since strong Coulomb fields can be experimentally provided in the form of highly ionised large- Z atoms. Effects produced by $H(x;s)$ turn out to be of an order of magnitude which should allow for their experimental detection for large Z .

We start our calculation by taking for A the Dirac operator

$$A = \frac{1}{i} \vec{\alpha} \cdot \vec{\partial} + \beta m + V(\vec{x}) \quad (3.1)$$

where $V(x)$ is a suitably chosen external potential fulfilling the conditions implicitly (or explicitly) used in the course of our deductions; for instance, we are going to require the validity of Levinson's theorem and have thus to postulate that $|V|$ and $|rV|$ are integrable. As we already stated, our main example will be a strong Coulomb field which we take as screened in order not to get into the intricacies¹⁶⁾ of the Coulomb problem.

The spectrum (2.10) is then determined by solving

$$(-i\vec{\alpha} \cdot \vec{\partial} + \beta m + V(\vec{x})) \psi = E \psi ; \quad (3.2)$$

the Dirac spinor ψ is decomposed in the usual manner¹⁷⁾¹⁸⁾ (our notation is taken from ref.(19)) into partial waves

$$\psi_{jm}^e(r, \theta, \varphi) = \begin{pmatrix} i \frac{G_{je}}{r} \varphi_{jm}^e \\ \frac{F_{je}}{r} \hat{\sigma} \cdot \hat{r} \varphi_{jm}^e \end{pmatrix} \quad (3.)$$

$$\varphi_{jm}^e = (2\ell+1)^{-1/2} \begin{pmatrix} (\ell \pm m + 1/2)^{1/2} Y_{\ell m - 1/2}(\theta, \varphi) \\ \pm (\ell \mp m + 1/2)^{1/2} Y_{\ell m + 1/2}(\theta, \varphi) \end{pmatrix}, \quad j = \ell \pm 1/2$$

where the normalisation of ψ translates into

$$\int_0^\infty dr (F_{je}^2 + G_{je}^2) = "1", \quad (3.4)$$

for continuum states, of course, "1" has to be taken as $\delta(0)$. Equation (3.2) reads in terms of F_{j1} and G_{j1} as

$$(E + V - m) G_{je}(r) = - \left(\frac{d}{dr} + \frac{1}{r} (j + 1/2) \right) F_{je} \quad (3.5)$$

$$(E + V + m) F_{je}(r) = \left(\frac{d}{dr} - \frac{1}{r} (j + 1/2) \right) G_{je}$$

Employing a well-known trick²⁰⁾²¹⁾ and multiplying these equations for F_{j1} , G_{j1} at energy E and for F'_{j1} , G'_{j1} at energy E' by F'_{j1} , G'_{j1} and F_{j1} , G_{j1} respectively we get

$$(E - E') (G_{je} G'_{je} + F_{je} F'_{je}) = \frac{d}{dr} (G_{je} F'_{je} - F_{je} G'_{je}).$$

Let

$$k' = k + \delta k \quad \text{or} \quad E - E' = - \delta k \frac{k}{E};$$

comparison of the order δk and integration then yields

$$\int_0^R dr (F_{je}^2 + G_{je}^2) = \frac{E}{k} \left(F_{je} \frac{d}{dk} G_{je} - G_{je} \frac{d}{dk} F_{je} \right) \Big|_R \quad (3.6)$$

since $F_{j1}(0) = G_{j1}(0) = 0$.

Physically speaking this equation relates the charge in a sphere of radius R per channel $([k, k + \delta k]; j, \ell)$ to a surface term corresponding to the electric flux (again per channel $([k, k + \delta k]; j, \ell)$). The point now is that at sufficiently large distances from the interaction region given by the range r_0 of the potential $V(x)$ this flux is given by the asymptotic scattering solutions of (3.2) or (3.5). With proper normalisation, i.e. in order to comply with (3.4), we have for $j = \ell \pm 1/2$

$$\psi_{jm}^e \sim e^{-i\delta_{je}} \begin{pmatrix} i \left(\frac{E+m}{4\pi E} \right)^{1/2} \varphi_{jm}^e \left(h_e^{(2)}(kr) + e^{2i\delta_{je}} h_e^{(1)}(kr) \right) \\ - \left(\frac{E-m}{4\pi E} \right)^{1/2} \varphi_{jm}^e \left(h_{e\pm 1}^{(2)}(kr) + e^{2i\delta_{je}} h_{e\pm 1}^{(1)}(kr) \right) \end{pmatrix} \quad (3.7)$$

where $E=(k^2+m^2)^{1/2}$ denotes the energy, δ_{je} the scattering phase shift and $h_e^{(1)}$, $h_e^{(2)}$ are the usual spherical Hankel functions. Employing age-old asymptotic formulae and inserting into (3.6) we get for $R \gg r_0$ and $kR \gg 1$

$$\int_0^R dr (F_{je}^2 + G_{je}^2) = \frac{1}{\pi} \left\{ R + \frac{\partial \delta_{je}}{\partial k} + (-1)^{e+1} \frac{m}{2Ek} \sin 2(kR + \delta_{je}) + O(R^{-1}) \right\}. \quad (3.8)$$

This relation has a clear-cut physical interpretation: the first term guarantees the proper normalisation (3.4), the next term is the density of states and yields Levinson's theorem after spectral integration; the last term, finally, gives the effect which we are interested in and corresponds to an oscillating charge distribution. More precisely speaking, the induced charge density per wave number interval $[k, k+\delta k]$ at the distance $R=|\vec{x}|$ expressed in terms of the partial waves (3.3)

$$\rho(\vec{x}, k) = \frac{1}{4\pi R^2} \left\{ \sum_j 2(e+1) (F_{je}^2 + G_{je}^2)_{j=e+1/2} + \sum_j 2e (F_{je}^2 + G_{je}^2)_{j=e-1/2} \right\} \quad (3.9)$$

is obtained by subtracting positive and negative energy branches. Therefore, for sufficiently large R we get from (3.8) by differentiation

$$q(\vec{x}, k) = \frac{1}{4\pi R^2} \frac{2m}{E}.$$

$$\sum_e (-1)^{e+1} \left\{ (e+1) \left(\cos 2(kR + \delta_{e+\frac{1}{2}e}^{(+)}) - \cos 2(kR + \delta_{e+\frac{1}{2}e}^{(-)}) \right) \right. \\ \left. + e \left(\cos 2(kR + \delta_{e-\frac{1}{2}e}^{(+)}) - \cos 2(kR + \delta_{e-\frac{1}{2}e}^{(-)}) \right) \right\} \quad (3.10)$$

where $\delta_{je}^{(\pm)}$ denote the positive and negative energy phase shifts respectively which are straightforwardly related to the particle and antiparticle scattering phase shifts. The first two terms in (3.8) cancel and the induced charge density $n_c(x)$

$$n_c(\vec{x}) = \int_0^\infty dk q(\vec{x}, k) \quad (3.11)$$

appears as a spectral integral over oscillating terms. More generally, the regularised density defined in (2.20) now results as

$$H(\vec{x}, s) = \frac{1}{4\pi R^2} \int dE \frac{2m}{k} E^s. \\ \cdot \sum_e (-1)^{e+1} \left\{ (e+1) \left(\cos 2(kR + \delta_{e+\frac{1}{2}e}^{(+)}) - \cos 2(kR + \delta_{e+\frac{1}{2}e}^{(-)}) \right) \right. \\ \left. + e \left(\cos 2(kR + \delta_{e-\frac{1}{2}e}^{(+)}) - \cos 2(kR + \delta_{e-\frac{1}{2}e}^{(-)}) \right) \right\}. \quad (3.12)$$

The detailed convergence properties, of course, depend on the particular model, i.e. the specific form of the external potential. We shall restrict our comments to a few general remarks. First of all, for small phase shifts $\delta_{je}^{(\pm)}$ the contributions to a channel $(j, e, [k, k + \delta k])$ are proportional to $\delta_{je}^{(+)} - \delta_{je}^{(-)}$ (because of the opposite sign the two phases add), if this difference vanishes e.g. as $O(E^{-1})$ the integral converges already for $s=0$, i.e. $n_c(x)$ is well-defined. In general it is not possible to formulate precise mathematical conditions under which the spherically symmetric charge distribution show oscillatory behaviour or exponential decrease, the complex

singularity structure of the integrand is responsible for either alternative. Physically speaking, it is clear that sharp resonances lead to an oscillating contribution: the steep rise of the scattering phase-shift by π produces an edge in the integrand of (3.11) or (3.12) and, hence, an oscillatory contribution to the induced charge density or the density $H(x,s)$. This mechanism for the generation of an oscillating behaviour is rather similar to the one responsible for the formation of Friedel oscillations²²⁾ where the edge is given by the Fermi energy.

In summary, we have seen that in systems which show sharp resonances the "Dirac sea" of its Fermion constituents is deformed such that oscillatory charge distributions are built up the periods of which are given by the resonance positions. The strength of the effect is roughly determined by the magnitude of the phase shift. For atoms the latter is given by $Z\alpha$, the effect of this charge distribution on the spectrum of large Z atoms should be appreciable.

The formalism described up to now is not suitable to describe many-body systems with interacting constituents. To discuss the vacuum effects in this substantially more complicated situation, we have to extend it. The following chapter shortly introduces our ideas on this problem.

4. Vacuum effects for localised systems of interacting fermions in equilibrium

The first step in relating this problem to the case of free fermions in an external field is, of course, to promote the latter to a dynamical degree of freedom. This is formally done by writing the partition function $Z(\beta)$ in the interacting case as

$$Z(\beta) = \int \mathcal{D}A_\mu \, Z[A_\mu; \beta] \, e^{-S[A]} \quad (4.1)$$

where $Z[A_\mu; \beta]$ is the partition function (2.4) in which the external field is taken to be a $U(1)$ -gauge field; $S[A]$ is the Euclidean action of the latter.

The stationary point of (4.1) has been shown²³⁾ to be non-unique, it is given as a one-parameter set of time-independent solutions of (2.10)

where the potential - the g-Hartree potential -

$$A_{\mu}^{stat}(\vec{x}) = \int d^3x' D_{\mu\nu}^{(0)}(\vec{x}, \vec{x}') \sum_{\alpha} n_{\alpha} \left\{ g |\psi_{\alpha}(\vec{x}')|^2 - (1-g) \psi_{\alpha}^*(\vec{x}') \psi_{\alpha}(\vec{x}') \right\} \quad (4.2)$$

has to be determined self-consistently. In deriving this result a ghost-free covariant gauge has been chosen.

As usual we expand around the stationary point and obtain a perturbation series for $Z(\beta)$ or the grand canonical potential

$$K = \beta^{-1} \ln Z = K_{g-H} + K_{corr} \quad (4.3)$$

where K_{g-H} is given in terms of the g-Hartree basis obtained from (2.10) with (4.2) and K_{corr} subsumes a perturbation expansion in terms of the coupling strength. Obviously K does not depend on g , this parameter therefore controls the relative magnitude of the two contributions on the right-hand side of (4.3). For QED we have shown, in particular, that g can be chosen such that

$$K_{corr}|_{g=g_0} = 0 \quad (4.4)$$

which means that for $g=g_0$ the thermodynamic potential is exactly given in terms of a single particle basis. This is equivalent to showing the existence of a mean-field acting as an external field such that the corresponding single particle spectrum yields the exact partition function. It is exactly at this point that the case of interacting fermions connects up with what we have said in the preceding chapters. The vacuum contribution E_c to the total energy is identified as before, the induced charge density $n_c(x)$ receives additional correlations from g_0 ⁷⁾. Our main conclusions concerning the oscillatory character of $n_c(x)$ remain the same, complex systems like atoms provide an ample choice of narrow resonances giving rise via the mechanism discussed below to a complex and interesting pattern of oscillations in the charge density originating from a deformation of the "Dirac sea".

References

- 1) M. Atiyah, V. Patodi, and J. Singer, Bull.London Math.Soc.5(1973) 42.
- 2) C. Callias, Comm. Math. Phys. 62 (1978) 213;
J. Lott, Comm. Math. Phys. 93 (1984) 533.
- 3) M. Paranjape and C. Semenoff, Phys. Lett. B132 (1983) 369;
A. Niemi and G. Semenoff, Phys. Lett 135 (1984) 121.
- 4) R. Jackiw and G. Rebbi, Phys. Rev. D13 (1976) 3398.
- 5) A.P. Balachandran, V.P. Nair, S.G. Rajeev, and A. Stern, Phys.Rev.D27
(1983) 1153;
J. Goldstone and R.L. Jaffe, Phys. Rev. Lett. 51 (1983) 1518.
- 6) V.A. Rubakov, Nucl. Phys. B203 (1982) 311;
F. Wilczek, Phys. Rev. Lett. 48 (1982) 1144;
C.G. Callan, Phys. Rev. D25 (1982) 2141.
- 7) K. Dietz and G. Weymans, J. Phys. B17 (1984) 4801.
- 8) K. Dietz and G. Weymans, Physica 131A (1985) 363.
- 9) L.D. Landau, Sov. Phys. JETP 3 (1956) 920.
- 10) J.P. Connerade, K. Dietz, M.W.D. Mansfield, and G. Weymans, P.Phys. B17
(1984) 1211.
- 11) A.J. Heeger, Comm.in Solid State Phys. 10 (1981) 53;
W.P. Su, J.R. Schrieffer and A.J. Heeger, Phys.Rev. B22 (1980) 2099;
R. Jackiw and J.R. Schrieffer, Nucl. Phys. B190 (1981) 253.
- 12) R.P. Feynman and A.R. Hibbs, "Quantum Mechanics and Path Integrals",
McGraw-Hill, New York, 1965.
- 13) K. Dietz and Th. Millack, CERN preprint 1985 TH.-4316.
- 14) A.J. Niemi and G.W. Semenoff, Ann. of Phys., to be published.
- 15) H. Yamagishi, Phys. Rev. D27 (1983) 2383 and references given there.
- 16) J.D. Dollard, J. Math. Phys. 5 (1964) 729;
V.G. Gorshkov, JETP (Sov. Phys.) 13 (1961) 1037; ZhETF 40 (1961) 1481.
- 17) C.G. Darwin, Proc. Roy. Soc. London Ser. A118 (1928) 654.
- 18) H.A. Bethe and E.E. Salpeter, "Handbuch der Physik XXXV, Springer,
Berlin Göttingen Heidelberg, 1957.
- 19) C.Itzykson and J.B. Zuber, "Quantum Field Theory", McGraw-Hill, 1980.
- 20) A. Martin, Comptes rendus des séances de l'Académie des Sciences, t.243,
p. 22, R9 Séance du 2 juillet 1956.
- 21) C. Kittel, "Quantum Theory of Solids", Chap.18, John Wiley, New York
London, 1963.
- 22) J. Friedel, Nuovo Cim. 7 Suppl. (1958) 287.
- 23) K. Dietz, O. Lechtenfeld, and G. Weymans, J.Phys. B15 (1983)4301, 4315.