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LOW ENERGY REMNANTS OF SUPERSTRING
ANOMALY CANCELLATION TERMS

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ABSTRACT

The local one-loop terms which cancel anomalies in superstring theories are known to lead to axion- $F\tilde{F}$ couplings after truncation to four dimensions. The supersymmetrization of these four-dimensional axion couplings seems to imply the existence of further (both bosonic and fermionic) $D = 10$ local terms, supersymmetric counterparts of the "Wess-Zumino" terms. The truncation to four dimensions of these extra terms leads to modifications of the tree level truncated $N = 1$ supergravity Lagrangian. We extract the form of the modified Kähler potential as well as gauge functions of the low energy theory. Phenomenological conclusions extracted from the tree level truncation get modified. In particular gaugino masses $M \approx (2\pi)^{-5} m_{3/2}$ do appear.

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Attempts to construct a unified theory including gravity usually lead to higher dimensions $d > 4$, a generalization of field theory or both simultaneously, while particle physics up to energies of one hundred GeV can be very well described by four-dimensional field theory. We have then to face the question whether and how we can test a possible departure of $d = 4$ field theory at low energies. "Low" refers to the intrinsic scale of, for example, compactification of extra dimensions or string tension. Since these scales are believed to be given by the Planck mass $M_p \sim 10^{19}$ GeV and since the unified field theories usually do not lead to very definite predictions, these questions will be difficult to answer. Nonetheless one should investigate whether these questions could be answered in principle. Let us therefore look at the unique properties that these unified theories are supposed to have. One of them is finiteness in perturbation theory¹⁾, believed to be present in $d = 10$ superstring theory. A low energy approximation to such a theory is usually non-renormalizable, i.e., definite predictions of string theory can be simulated in the low energy approximation by a special choice of counterterms in the process of regularization and renormalization. This also applies to the second special property of string theory-anomaly freedom²⁾. While superstring theory is anomaly-free, this might, a priori, not be true for any field theoretic approximation involving only the massless fields. In such a situation, direct influence of excited string modes at low energies are expected to occur. Unfortunately, this situation is not realized; the effects of heavy string modes are screened and remnants of anomaly cancellation can at best be seen in an indirect way. Nonetheless let us have a closer look at this mechanism. The "naive" ten-dimensional field theoretic approximation was plagued by anomalies, i.e., a calculation of the one-loop effective potential reveals the existence of non-local non-invariant terms that cannot be made invariant by a choice of a special coefficient of the also generated local terms. This situation changes dramatically²⁾ through the addition of a new term to the field strength H_{MNP} of the antisymmetric tensor field B_{MN} . While the influence on the existence of the non-local terms is only minor, this change implies a different transformation behaviour of B_{MN} and the generated local terms can now combine with the non-local terms to give an invariant result which gives anomaly freedom³⁾. The coefficient of these terms, however, has (in the process of regularization and renormalization) to be carefully chosen and is, of course, non-vanishing. In the theory under consideration it is given by

$$\frac{+1}{18 (2\pi)^5} \left[4 (\omega_{3L} - \frac{1}{30} \omega_{3Y}) X_7 - 6 B X_9 \right] \quad (1)$$

where $X_8 = dX_7$ and

$$X_8 = \frac{1}{24} \text{Tr} F^4 - \frac{1}{7200} (\text{Tr} F^2)^2 - \frac{1}{240} \text{Tr} F^2 \text{Tr} R^2 + \frac{1}{8} \text{Tr} R^4 + \frac{1}{32} (\text{Tr} R^2)^2 \quad (2)$$

in the notation of Ref. 3). The coefficient is dictated by gauge and Lorentz invariance of the field theory itself and the influence of string theory is screened. What about the effects of such terms on a four-dimensional theory after compactification? This question is interesting since in $d = 4$ gravitational anomalies do not exist and also the cancellation of gauge anomalies is much easier to achieve than in ten dimensions. Remnants of such terms in the $d = 4$ theory would then give us a hint about the existence of higher dimensions.

A discussion of these terms needs a specification of the mechanism of compactification. To be explicit we will use the prescription of Refs. 4) and 5), although our results will hold also in a more general situation. A first possible effect in the $d = 4$ theory has been observed in Refs. 6) and 7) and involves couplings of pseudoscalar particles to the gauge fields. The field B_{MN} ($M = 1, \dots, 10$) contains (at least) two pseudoscalars: θ obtained from $B_{\mu\nu}$ ($\mu, \nu = 1, \dots, 4$) by a duality transformation and $\eta = \epsilon^{mn} B_{mn}$ ($m, n = 5, \dots, 10$) which have scalar partners in the supergravity multiplet

$$\begin{aligned} S &= \varphi^{-3/4} e^{3\sigma} + i\theta \\ T &= \varphi^{3/4} e^{\sigma} + i\eta \end{aligned} \quad (3)$$

The last term in (1) BX_8 now indicates that η also has axion-like couplings⁸⁾, e.g., the term

$$\frac{1}{720 (2\pi)^5} \epsilon^{MNPQRSTUUVW} B_{MN} (\text{Tr} F^2)_{PQRS} (\text{Tr} R^2)_{TUVW} \quad (4)$$

leads to $\eta \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$ in $d = 4$ where $\text{Tr} R^2$ is replaced by its vacuum expectation value. Taking the $E_8 \times E_8$ model and using $\text{Tr} R^2 = (1/30) \text{Tr} F^2$, one finds that the two axions couple differently to observable and hidden sector^{6), 7)}

$$\begin{aligned} &(\theta + \eta \epsilon) F_{\mu\nu}^8 \tilde{F}^{8\mu\nu} \\ &(\theta - \eta \epsilon) F_{\mu\nu}^6 \tilde{F}^{6\mu\nu} \end{aligned} \quad , \quad \epsilon \sim 10^{-5} \quad (5)$$

thus allowing a solution of the strong CP problem in the observable sector. It was speculated⁶⁾ that through supersymmetry the corresponding scalar dilatons also couple differently to E_6 and E_8 . Since these vacuum expectation values correspond to the inverse of the coupling constants, this might have interesting consequences⁶⁾.

In this note we want to explore more systematically the implications of supersymmetry. All known string theories need supersymmetry for consistency. One therefore obviously expects supersymmetry to be present in any reliable low energy approximation. Even including the process of compactification, we do not expect a hard breakdown whereas a soft breakdown (probably spontaneous) will occur. The supersymmetric counterparts of the terms in (1) shall therefore exist. In principle, they could be deduced by a calculation of anomalies of the supercurrent in the same way as the original terms come from cancellation of gauge and gravitational anomalies. Such a calculation is at the moment not possible since the complete classical supersymmetric theory is not known. We will therefore try to find the additional terms by imposing supersymmetry. Even here it is a complicated task to calculate the full supersymmetric theory at the one-loop level, but we can isolate a few terms that are absolutely required by supersymmetry. Among other things they will give us new information on the structure of corrections to the classical Kähler potential of the four-dimensional theory and might give us a hint for the possible existence of extra dimensions.

As discussed above, the truncation of the BX_8 couplings to four dimensions leads to axion couplings which by themselves are not supersymmetric. Formula (5) suggests that the tree level + one-loop gauge kinetic functions for the hidden E_8 and the observable E_6 sectors are

$$f_{ab}^8 = \delta_{ab} (S + \varepsilon T) \quad (6a)$$

$$f_{ab}^6 = \delta_{ab} (S - \varepsilon T) \quad (6b)$$

Other four-dimensional couplings which supersymmetrize those in Eq. (5) should also be present if the compactification process consistently preserves one unbroken supersymmetry. Thus the gauge kinetic functions (6) imply that, e.g., the following fermionic terms should exist

$$i(\partial_\mu \text{Im} f_{ab}) (\bar{\chi}^a \gamma_5 \gamma^\mu \chi^b) \longrightarrow i \varepsilon (\partial_\mu \eta) (\bar{\chi} \gamma_5 \gamma^\mu \chi) \quad (7)$$

where the χ fields are gauginos and we use the notation of Ref. 9). This term is a fermionic counterpart of the terms in Eq. (5) and presumably should be obtained after truncation of some local one-loop operator which is in turn a supersymmetric counterpart of the BX_8 coupling. We will show that it is easy to identify the $d = 10$ operator which leads to the term (7) after truncation. There will be in general many other ten-dimensional local operators whose truncation will contribute to the effective four-dimensional theory and we will discuss some of them later on. Before doing that, let us give some details about the truncation procedure, particularly the one involving fermionic terms. This we will need later when truncating some fermionic $D = 10$ local operators.

In Ref. 5), a truncation of the $D = 10$ supergravity + Yang-Mills Lagrangian preserving one unbroken supersymmetry was presented. One assumes an ansatz for the 10-dimensional metric

$$g_{MN} = \begin{pmatrix} \exp(-3\sigma) g_{\mu\nu} & 0 \\ 0 & \exp(\sigma) \delta_{mn} \end{pmatrix} \quad \begin{matrix} \mu, \nu = 1-4 \\ m, n = 5-10 \end{matrix} \quad (8)$$

which leads to a canonical Einstein Lagrangian. Counting the number of internal and external contractions of each term in the $D = 10$ Lagrangian gives immediately the dependence on σ of each of them. One can then reconstruct the dependence on the complex fields S and T defined above. As remarked in Ref. 6), the $d = 4$ Kähler potential is most easily obtained from the fermionic (gravitino bilinear) sector and one gets

$$G = -\log(S+S^*) - 3\log(T+T^* - 2|C_x|^2) + \log|W|^2 \quad (9)$$

where W is the E_6 superpotential and C_x are the fields in the 27 of E_6 . The tree level gauge kinetic functions are obtained upon truncation of the $D = 10$ gauge field strengths and one gets $f_6 = f_8 = S$, as remarked above. In order to put the truncated fermionic sector into standard $N = 1$ supergravity form, one needs some redefinitions. Thus in order to obtain from the gaugino kinetic term $E(\bar{\chi} \Gamma^M \partial_M \chi)$ the $d = 4$ supergravity term $(e \text{Ref}_{ab})(\bar{\chi}'_a \gamma^\mu \partial_\mu \chi'_b)$, one has to rescale the gaugino $d = 4$ fields

$$\chi' = e^{-\frac{9}{4}\sigma} \gamma^{\frac{3}{8}} \chi \quad (10)$$

The same happens with the gravitino field ψ_a which requires a rescaling (a is a

flat index):

$$\psi'_a = e^{-3/4\sigma} \psi_a \quad (11)$$

in order to have a minimal kinetic term. One can check that the above redefinitions put all the truncated fermionic terms into standard $N = 1$ supergravity form. The truncated Lagrangian possesses a classical invariance⁵⁾ inherited from the one already existing in $D = 10$, corresponding to rescalings of the string dilaton ϕ . Thus, the Lagrangian scales as¹⁰⁾

$$\left. \begin{aligned} S &\rightarrow \lambda S \\ g_{\mu\nu} &\rightarrow \lambda g_{\mu\nu} \\ \psi_\mu &\rightarrow \lambda^{1/4} \psi_\mu \\ \chi &\rightarrow \lambda^{-3/4} \chi \end{aligned} \right\} \rightarrow e^{-1} \mathcal{L} \rightarrow \lambda^{-1} e^{-1} \mathcal{L} \quad (12)$$

This is thus a classical scale invariance. There is also another scale invariance¹⁰⁾ in the tree level Lagrangian which is a consequence of Calabi-Yau compactification

$$\left. \begin{aligned} T &\rightarrow \lambda' T \\ C_x &\rightarrow (\lambda')^{1/2} C_x \\ S &\rightarrow S \end{aligned} \right\} \rightarrow \mathcal{L} \rightarrow \mathcal{L} \quad (13)$$

This is obvious from Eq. (9). Notice that both invariances of the tree level Lagrangian are violated by the one-loop axion couplings since they scale like

$$\eta F_{\mu\nu} \tilde{F}^{\mu\nu} \rightarrow \lambda' \lambda^{-2} \eta F_{\mu\nu} \tilde{F}^{\mu\nu} \quad (14)$$

under the symmetries (12) and (13). These symmetries will however be useful in order to identify possible supersymmetrizations of the axion couplings. Notice for example that the term in Eq. (7) scales like $(\lambda' \lambda^{-2})$ under the above dilations, i.e., just like the axion couplings, as it should be.

We can now search for supersymmetric counterparts of the BX_8 local operators. In particular, the four-dimensional term (7) suggests the existence of a term obtained from $B \text{tr}(F^2) \text{tr}(F^2)$ by the substitution:

$$\text{tr}(F_{[MN} F_{PA]}) \rightarrow \text{tr}(\partial_M \bar{\chi} \Gamma_{NPA} \chi) \quad (15)$$

Such a term is

$$\varepsilon^{MNPQRSTUUVW} \varphi^{3/4} (\partial_M B_{NP}) T_V (\bar{\chi} \Gamma_{QRS} \chi) T_V (F_{TU} F_{VW}) \quad (16)$$

The factor $\phi^{3/4}$ is fixed by imposing that this term transforms under the $D = 10$ scale invariance⁵⁾

$$\begin{aligned} g_{MN} &\rightarrow \lambda g_{MN} & \psi_A &\rightarrow \lambda^{-1/4} \psi_A \\ \varphi &\rightarrow \lambda^{-4/3} \varphi & \chi &\rightarrow \lambda^{-1/4} \chi \end{aligned} \quad M, N = 1-10 \quad (17)$$

in the same way as the BX_8 terms (i.e., like λ^{-5}). It is easy to check that, setting the indices as follows:

$$\varepsilon^{\mu\nu\rho\sigma} \varepsilon^{ijk} \varepsilon^{\bar{i}\bar{j}\bar{k}} \varphi^{3/4} (\partial_\mu B_{i\bar{i}}) T_V (\bar{\chi} \Gamma_{\nu\rho\sigma} \chi) \langle T_V F_{j\bar{j}} F_{k\bar{k}} \rangle \quad (18)$$

and recalling the ansatz $B_{i\bar{i}} = \eta \varepsilon_{i\bar{i}}$, one obtains precisely the term in Eq. (7). In particular, all explicit dependence on σ and ϕ gets cancelled due to the field redefinitions in Eq. (10). In Eqs. (16) and (18), one can also substitute F^2 by R^2 , in both cases a v.e.v. in extra dimensions $\langle F^2 \rangle = 30 \langle R^2 \rangle$ leaves a residual term in (7). The fact that the truncation of this $D = 10$ local term produces a supersymmetric counterpart of the η -axion coupling, reinforces the belief that this is one of the terms which supersymmetrize the BX_8 coupling.

There are other obvious candidates for extra local $D = 10$ terms. We know that the bilinear $(\bar{\chi} \Gamma_{QRS} \chi)$ usually appears with other fermionic bilinears like $(\bar{\psi}_Q \Gamma_R \psi_S)$ in the $D = 10$ supergravity Lagrangian^{11),12)} [they are inside the same "auxiliary field" t_{QRS} ¹¹⁾]. Thus another Susy-counterpart of the BX_8 term should be

$$\varepsilon^{MNPQRSTUUVW} \varphi^{3/4} (\partial_M B_{NP}) (\bar{\psi}_Q \Gamma_R \psi_S) T_V (F_{TU} F_{VW}) \quad (19)$$

which also scales like λ^{-5} under the $D = 10$ symmetry (17). The truncation of this term, when a v.e.v. for $\text{tr}(F_{ij} \bar{F}_{k\bar{k}})$ [or $\text{tr}(R_{j\bar{j}} \bar{R}_{k\bar{k}})$] is present, leads to the following coupling in four dimensions

$$\frac{i}{\text{Re } S} (\partial_\mu \eta) \varepsilon^{\mu\nu\rho\sigma} (\bar{\psi}_\nu \gamma_\rho \psi_\sigma) \quad (20)$$

If we now compare this coupling to the corresponding general coupling appearing in the Cremmer et al. Lagrangian

$$(G'^T D_\mu T) \varepsilon^{\mu\nu\sigma} (\bar{\Psi}_\nu \gamma_\sigma \Psi_\sigma) + h.c. \quad (21)$$

we conclude that the Kähler potential G is modified when including the effect of the one-loop local terms in such a way that $\delta G'^T \sim \varepsilon/\text{Re}S$. Recalling that the one-loop terms considered should scale as the $\eta\tilde{F}\tilde{F}$ couplings under the four-dimensional scale invariances (12) and (13), one obtains a modified Kähler potential

$$G_v \equiv G + \delta G \quad ; \quad \delta G = -\alpha \varepsilon \frac{(T+T^*)}{(S+S^*)} \quad (22)$$

where α is a number $O(1)$ and G is the tree level Kähler potential in (9). We must remark here that in order to identify the coefficient in (20) with G'^T , the Lagrangian should be already in standard $N = 1$ supergravity form (i.e., canonical Einstein and kinetic gravitino Lagrangians). There could exist one-loop terms such that those gravitational terms get corrections once F^2 or R^2 get a v.e.v. However, the changes induced in (20) by the corresponding Weyl rescalings will be of order of ε^2 (i.e., two-loop) and thus will be negligible to the order we are working. Thus simple arguments of $d = 4$ and $d = 10$ supersymmetry have allowed us to calculate the form of the corrections to both the Kähler potential and gauge kinetic functions of the truncated theory.

With the extra term δG in (22) the kinetic terms of the S and T fields are no longer diagonal. Interestingly enough, one can define a new field S'

$$S' = S + \alpha \varepsilon T \quad (23)$$

such that, to first order in ε (one-loop) one has:

$$G_v = -\log(S' + S'^*) - 3\log(T + T^* - 2|C \times l|^2) + \log|W|^2 \quad (24)$$

i.e., the structure in Eq. (9) is maintained. This redefinition diagonalizes the kinetic terms of S' and T . It seems then that the effects of the extra one-loop local terms in the gravitational sector of the truncated theory amount to a field-dependent finite wave function renormalization of S . However, the effects on the gauge non-singlet sector are more relevant since, as we discussed above, the gauge kinetic functions get the modifications in Eqs. (6). In terms of the new variable S' one has:

$$f_{ab}^8 = \delta_{ab} (S' + \varepsilon(1-\alpha)T) \quad (25a)$$

$$f_{ab}^6 = \delta_{ab} (S' - \varepsilon(1+\alpha)T) \quad (25b)$$

Equations (23)-(25) imply certain modifications to the low energy phenomenology obtained from the tree level truncation. It seems that no essential modification appears for the purely gravitational or chiral $d = 4$ sectors, due to Eq. (24). However, the situation changes concerning the gauge sector, in particular regarding the possibility of residual $d = 4$ supersymmetry breaking through hidden gaugino condensation^{13),14)}. In the presence of hidden E_8 gaugino condensation the low energy scalar potential is¹⁵⁾:

$$V = (G''^{-1})_i^j F_j F^i - 3e^G + (D^2\text{-term}) \quad (26)$$

where

$$F_j \equiv e^{G/2} G'_j + \frac{f_j^{8'}}{4} \langle \bar{\chi} \chi \rangle \quad (27)$$

and $j = S, T, C_x$. It is well known¹⁴⁾ that for $f_T^{8'} = 0$ (i.e., for $\alpha = 1$) the scalar potential is positive definite, $V = F_{S'}^2$, and it is minimized for $F_{S'} = 0$. Then, through Eq. (27), $\langle S' \rangle$ is fixed to cancel $F_{S'}$. However, one has for the gaugino masses in the present case

$$M^8 = f_{S'}^{8'} F_{S'} = 0 \quad (28a)$$

$$M^6 = f_{S'}^{6'} F_{S'} + f_T^{6'} F_T = -\varepsilon(1+\alpha) F_T \quad (28b)$$

Thus if the coefficient of the operators (21) is such that $f^8 = S'$ (i.e., $\alpha = 1$), we obtain vanishing cosmological constant, scalar masses and E_8 gaugino mass but a mass for observable gauginos:

$$M^6 \sim \varepsilon m_{3/2}, \quad \varepsilon \sim 10^{-5} \quad (29)$$

where $m_{3/2}$ is the residual gravitino mass. This is to be contrasted with the situation one gets from tree level truncation¹⁴⁾ in which the observable sector is supersymmetric. For other values of "a" for which f^8 explicitly depends on T,

the scalar potential is no longer positive definite since the $-3e^G$ term is not exactly cancelled by the contributions from F_T and F_{C_X} but a term linear in ϵT remains. In this situation, the potential obtained from (26) is rather complicated and the results of the minimization hardly reliable since, e.g., for $\epsilon T \sim S'$ the redefinition in (24) is no longer valid [we used $a\epsilon \sim \log(1+a\epsilon)$]. In any case, if one could find an interesting minimum for such a case one expects both $F_{S'} \neq 0$ and $F_T \neq 0$ in general so that observable gaugino masses $\sim m_{3/2}$ will still appear. Furthermore, in this more general case one also expects $A \neq 0$ and scalar masses since the result $A = m = 0$ obtained from the tree level truncation was a direct consequence of the positive definiteness of the scalar potential, which no longer holds if $f_T^{8'} \neq 0$. From this discussion and Eq. (29), we conclude that in order to have gaugino masses no larger than ~ 1 TeV (so that Higgs doublets do not get too massive) one has

$$m_{3/2} \lesssim \epsilon^{-1} \text{ TeV} \sim 10^{7-8} \text{ GeV} \quad (30)$$

If supersymmetry breaking is driven by gaugino condensation, this requires a condensation scale $\Lambda \lesssim 10^{14}$ GeV and the v.e.v. of the antisymmetric tensor in extra dimensions $H_{ijk} = c\epsilon_{ijk}$ needs also to be $c \lesssim (10^{14} \text{ GeV})^3$ if we insist in cancelling the cosmological constant by this mechanism. Notice that the result (30) can be made compatible with cosmological bounds from the gravitino abundance.

The phenomenological situation is somewhat brighter if one includes the terms we are discussing since we have now an explicit mechanism for transmitting the supersymmetry breaking from the hidden to the observable sector. If, furthermore, the coefficient in the one-loop $D = 10$ operators is such that $f_T^{8'} = 0$, we could still maintain a vanishing cosmological constant at this level. In this particular situation, we would have as a starting point for the effective "low energy supergravity model" $M \sim m_{3/2}$, $m \approx A \approx 0$ and the expected supersymmetric spectrum would be quite heavy. In a more general case ($a \neq 1$) the cancellation of the cosmological constant is not guaranteed at this level and non-vanishing m and A values are also expected.

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