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QUASI-YANG-MILLS STRUCTURE FOR THE OPEN BOSONIC STRING

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A B S T R A C T

Using the description of bosonic open strings by a double p-form, we derive algebraically the ghost-auxiliary field spectrum, as well as the BRS symmetry, of the free bosonic string theory. A quasi-Yang-Mills structure emerges, where Lagrangians and possible anomalies are related to invariant exterior forms of higher rank. The passage from the free case to the interacting one necessitates the understanding of vertex operators defined so as to ensure some geometrical Jacobi-Bianchi identities.

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In a recent paper¹⁾, Neveu, Nicolai and West have found that the fundamental objects of the bosonic string theory are built with double K-form gauge fields, ψ_K^K and ϕ_K^{K+1} , where K stands for any positive or null integer and indicates the rank of the forms associated with the up or down antisymmetrized Virasoro indices [see Refs. 1) and 2) for the string excitation contents of ψ_K^K and ϕ_K^{K+1}]. A covariant formulation of the free open string theory has been obtained in this way, altogether with some additional features concerning the ghost sector of the theory^{1),2)}.

In this letter we start from the hypothesis that ψ_K^K and ϕ_K^{K+1} are actually the fundamental classical fields of the bosonic string theory. Since these fields are K-form gauge fields, it is thus possible to use geometrical methods³⁾ to build directly the BRS symmetry of the theory, and derive at once its ghost and antighost spectra, as well as the associated Stueckelberg auxiliary fields. Once this is done, a Lagrangian can be built, including BRS invariant gauge fixing terms, and a classification of anomalies becomes possible by solving Wess-Zumino consistency conditions. In this approach, Lagrangians and anomalies are simply related with invariant exterior forms of higher rank.

As a first step, we consider the free case. Then we shift to the unified notations of Ogawa and Kato⁴⁾, and analyze the interacting case. The passage from the free case to the interacting one necessitates the understanding of vertex operators defined as to ensure some Bianchi identities. In this way the uniqueness of the string interaction appears as an equivalent to the uniqueness of the Chern-Simons form of rank 3. More precisely, the definition of the way two strings interact together point by point will emerge from the requirement that geometrical identities such as Jacobi identities must hold true. This allows in turn for the notion of a generalized Chern-Simons formula, leading to the determination of the interacting theory from the free one. We will see that the structure which emerges is a quasi-Yang-Mills one, and that the determination of Lagrangians and anomalies will be algebraically trivial. Furthermore, it will be possible to introduce p-form gauge string objects, the lowest excitations of which are pointlike p-form gauge fields. The latter are known to play a central role in the mechanism of anomaly cancellation⁵⁾.

Let us briefly recall the notations of Ref. 1). The variances of ψ_K^K and ϕ_K^{K+1} are determined by the action of the operators d, D, K and η which are built from the structure of the Virasoro algebra and act on products of fields as

differential operators. Given a tensor C_b^a , with a upper and b lower antisymmetrized Virasoro indices, one has, by definition²⁾:

$$\begin{aligned}
 [dC]_{[m_1 \dots m_{b+1}]}^{[m_1 \dots m_a]} &= L_{[m_1} C_{m_2 \dots m_{b+1}]}^{[m_1 \dots m_a]} + a W_{p[m_1} C_{m_2 \dots m_{b+1}]}^{p[m_1 \dots m_a]} \\
 &\quad - \frac{1}{2} b V_{[m_1 m_2}^p C_{p \dots m_{b+1}]}^{[m_1 \dots m_a]} \\
 [DC]_{[m_1 \dots m_b]}^{[m_1 \dots m_{a-1}]} &= L_{-p} C_{[m_1 \dots m_b]}^{[p m_1 \dots m_{a-1}]} + b W_{[m_1 p}^q C_{q m_2 \dots m_b]}^{[p m_1 \dots m_{a-1}]} \\
 &\quad - \frac{1}{2} (a-1) V_{k\ell}^{[m_1} C_{[m_1 \dots m_b]}^{k\ell m_2 \dots m_{a-1}]} \\
 [KC]_{[m_1 \dots m_b]}^{[m_1 \dots m_a]} &= (L_0 + m + m - 1) C_{[m_1 \dots m_b]}^{[m_1 \dots m_a]} \\
 m &= \sum m_i \quad n = \sum m_i \\
 [\mathcal{Z}C]_{[m_1 \dots m_{b+1}]}^{[m_1 \dots m_{a-1}]} &= \mathcal{Z}_{[m_1} C_{m_2 \dots m_{b+1}]}^{[q m_1 \dots m_{a-1}]}
 \end{aligned} \tag{1}$$

where the L_n are the generators of the Virasoro algebra, which reads:

$$\begin{aligned}
 [L_m, L_m] &= V_{mm}^p L_p \\
 [L_{-m}, L_{-m}] &= -V_{mm}^p L_{-p} \\
 [L_m, L_{-m}] &= W_{mm}^p L_p + W_{mm}^p L_{-p} + \mathcal{Z}_{mm} L(m) \\
 L(m) &\equiv 2L_0 + \frac{1}{6} (m^2 - 1)
 \end{aligned} \tag{2}$$

The properties which express that the Virasoro algebra is a Lie algebra are the commutation relations:

$$d^2 = D^2 = dD - Dd - 2\mathcal{Z}K = 0 \tag{3}$$

We define now ϕ_K^K and ϕ_K^{K+1} to be normal-ordered products of creation a^+ and annihilation a operators. The ϕ_K^K 's and ϕ_K^{K+1} 's of Ref. 1) are simply given by the action of these operators on the vacuum $|0\rangle$.

Ghost spectrum

The ghost spectrum is obtained by adding ghost and antighost degrees of freedom to the classical indices of ψ_K^K and ϕ_K^{K+1} . We proceed in analogy with the device used to introduce ghosts and antighosts for a p-form gauge field in Ref. 3). ψ_K^K and ϕ_K^{K+1} become:

$$\begin{aligned}\tilde{\psi}_K^K &\equiv \psi_K^K + \psi_{K-1}^{K+1}(1,0) + \psi_K^{K+1}(0,1) + \psi_{K-2}^{K+2}(2,0) + \psi_{K-1}^{K+2}(1,1) + \psi_K^{K+2}(0,2) \\ &\quad + \sum_{g=3}^K \sum_{q=0}^g \psi_{K-g+q}^{K+q}(g-q,q) \\ \tilde{\phi}_K^{K+1} &\equiv \phi_K^{K+1} + \phi_{K-1}^{K+1}(1,0) + \phi_K^{K+2}(0,1) + \phi_{K-2}^{K+2}(2,0) + \phi_{K-1}^{K+2}(1,1) + \phi_K^{K+3}(0,2) \\ &\quad + \dots\end{aligned}\tag{4}$$

In any object $\chi_{K''-g+q}^{K'+q}(g-q,q)$, q and $g-q$ stand respectively for the antighost and ghost numbers. The ghosts $\psi_{K-g}^K(g,0)$ and $\phi_{K-g}^{K+1}(g,0)$ are those introduced by Neveu, Nicolai and West¹⁾. The other ghosts ($q > 1$), are of a new type. The compatibility of this new set of ghosts with unitarity will be shortly verified by writing a BRS invariant and gauge fixed Lagrangian with ghost number zero, function of the above fields, and reproducing the known classical, gauge fixed Lagrangian of Ref. 1). A definition of the statistics of all fields must be given. This is achieved through the prescription that when g is even (respectively odd) $\chi_{K''-g+q}^{K'+q}(g-q,q)$ has the physical, respectively unphysical, statistics. In other words, the grading is defined as the sum of the ghost and antighost numbers.

Free theory

d and D change the ranks of the form on which they act. Following the general construction of gauge theories, we must construct the field strengths of ψ_K^K and ϕ_K^{K+1} . They must be of the form $G_{K+1}^K = d\psi_K^K + D\phi_{K+1}^{K+1} + \dots$, and $F_K^K = d\phi_{K-1}^{K+1} + D\psi_K^{K+1} + \dots$, where the dots stand for terms needed to ensure that some Bianchi identities hold true, i.e., that $(d+D)G$ and $(d+D)F$ vanish when $F = G = 0$. The solution is:

$$\begin{aligned} G_{K+1}^K &= d\psi_K^K + D\psi_{K+1}^{K+1} + 2\mathcal{Z}\phi_K^{K+1} \\ F_{K+1}^{K+1} &= d\phi_K^{K+1} - D\phi_{K+1}^{K+2} - K\psi_{K+1}^{K+1} \end{aligned} \quad (5)$$

and the Bianchi identities read:

$$\begin{aligned} dG_{K+1}^K &= DG_{K+2}^{K+1} - 2\mathcal{Z}F_{K+1}^{K+1} \\ dF_{K+1}^{K+1} &= -DF_{K+2}^{K+2} - KG_{K+2}^{K+1} \end{aligned} \quad (6)$$

BRS symmetry

We introduce the BRS operators s and $\bar{s}$, with ghost numbers $(1,0)$ and $(0,1)$ and define:

$$\begin{aligned} \tilde{d} &\equiv d + s & \tilde{\mathcal{Z}} &\equiv \mathcal{Z} \\ \tilde{D} &\equiv D + \bar{s} & \tilde{K} &\equiv K \end{aligned} \quad (7)$$

s and $\bar{s}$ act as differential operators on all the fields introduced above, their grading being equal to one. To determine the action of the operators s and $\bar{s}$ on the fields, we impose the following horizontally conditions on the field strengths³⁾:

$$\begin{aligned} \tilde{G}_{K+1}^K &\equiv \tilde{d}\tilde{\psi}_K^K + \tilde{D}\tilde{\psi}_{K+1}^{K+1} + 2\tilde{\mathcal{Z}}\tilde{\phi}_K^{K+1} = G_{K+1}^K \\ \tilde{F}_{K+1}^{K+1} &\equiv \tilde{d}\tilde{\phi}_K^{K+1} - \tilde{D}\tilde{\phi}_{K+1}^{K+2} - \tilde{K}\tilde{\psi}_{K+1}^{K+1} = F_{K+1}^{K+1} \end{aligned} \quad (8)$$

Expansion in ghost number of (8) determines directly the action of s and $\bar{s}$ on the classical ghost and antighost fields in $\tilde{\phi}_K^K$ and $\tilde{\phi}_K^{K+1}$. The parts with ghost number $(1,0)$ and $(0,1)$ read:

$$\begin{aligned}
 s \psi_K^K &= -d \psi_{K-1}^K(1,0) - D \psi_K^{K+1}(1,0) - 2\gamma \phi_{K-1}^{K+1}(1,0) \\
 s \phi_K^{K+1} &= -d \phi_{K-1}^{K+1}(1,0) + D \phi_K^{K+2}(1,0) + K \psi_K^{K+1}(1,0)
 \end{aligned}
 \tag{9a}$$

and

$$\begin{aligned}
 \bar{s} \psi_K^K &= -D \psi_K^{K+1}(0,1) - d \psi_{K-1}^K(0,1) - 2\gamma \phi_{K-1}^{K+1}(0,1) \\
 \bar{s} \phi_K^{K+1} &= -D \phi_K^{K+2}(0,1) + d \phi_{K-1}^{K+1}(0,1) - K \psi_K^{K+1}(0,1)
 \end{aligned}
 \tag{9b}$$

For the parts (2,0) and (1,1), one has

$$\begin{aligned}
 s \psi_{K-1}^K(1,0) &= -d \psi_{K-2}^K(2,0) - D \psi_{K-1}^{K+1}(2,0) - 2\gamma \phi_{K-2}^{K+1}(2,0) \\
 s \phi_{K-1}^{K+1}(1,0) &= D \phi_{K-2}^{K+2}(2,0) - d \phi_{K-1}^{K+1}(2,0) + K \psi_{K-1}^{K+1}(2,0)
 \end{aligned}
 \tag{10a}$$

and

$$\begin{aligned}
 s \psi_{K(0,1)}^{K+1} + \bar{s} \psi_K^{K+1}(1,0) &= -d \psi_{K-1}^{K+1}(1,1) - D \psi_K^{K+2}(1,1) - 2\gamma \phi_{K-1}^{K+2}(1,1) \\
 s \phi_{K(0,1)}^{K+2} - \bar{s} \phi_K^{K+2}(1,0) &= -d \phi_{K-1}^{K+2}(1,1) + D \phi_K^{K+3}(1,1) + K \psi_{K-1}^{K+2}(1,1)
 \end{aligned}
 \tag{10b}$$

which give the variations of the primary ghosts, and so on. The s -variations in (9) and (10) are identical to the ones derived by Neveu, Nicolai and West, when one replaces ghosts by infinitesimal parameters¹⁾. However, the BRS symmetry is not yet entirely determined by (8): the action of s on the ghosts $(g-q, q)$ with $q > 1$ or of $\bar{s}$ on the ghost $(g-q, q)$ with $q < g-1$, is degenerate [see, for example, (10b)]. One can raise this degeneracy by introducing auxiliary fields for ψ_K^K and ϕ_K^{K+1} , which will also play the role of Lagrange multipliers of the relevant gauge fixing conditions for the classical and ghost fields in BRS invariant Lagrangians. These fields are defined as:

$$\begin{aligned}\tilde{b}_K^{K+2} &= b_K^{K+2}(1,1) + b_{K-1}^{K+2}(2,1) + b_K^{K+2}(1,2) + \sum_{g=3}^K \sum_{q=1}^g b_{K-g+q}^{K+2}(1+g-q, q) \\ \tilde{c}_K^{K+2} &= c_K^{K+2}(1,1) + c_{K-1}^{K+2}(2,1) + c_K^{K+2}(1,2) + \dots\end{aligned}\quad (11)$$

and the above-mentioned degeneracy is lifted by defining, for $g > q > 1$:

$$\begin{aligned}s \psi_{K-g+q}^{K+q}(g-q, q) &= b_{K-g+q}^{K+q}(1+g-q, q) \\ s \phi_{K-g+q}^{K+1+q}(g-q, q) &= c_{K-g+q}^{K+1+q}(1+g-q, q)\end{aligned}\quad (12)$$

with

$$\begin{aligned}s b_{K-g+q}^{K+q}(1+g-q, q) &= 0 \\ s c_{K-g+q}^{K+1+q}(1+g-q, q) &= 0\end{aligned}\quad (13)$$

At this point, the action of s and $\bar{s}$ on all fields has been fully determined. The existence of Bianchi identities implies that the horizontality condition (8) defines s and $\bar{s}$ such that

$$\tilde{d}^2 = \tilde{D}^2 = \tilde{d} \tilde{\eta} - \tilde{D} \tilde{K} - 2 \tilde{\eta} \tilde{K} = 0 \quad (14)$$

and since $\tilde{\eta} = \eta$ and $\tilde{K} = K$, one has finally

$$s^2 = s \bar{s} - \bar{s} s = \bar{s}^2 = 0 \quad (15)$$

which ensures that s and $\bar{s}$ are nilpotent operators, as they should. Note that the introduction of the anti-BRS operator $\bar{s}$ is tightly linked to the existence of the operator D , since $\tilde{D} = D + \bar{s}$, and not from an a priori mere doubling of the BRS operator s associated to d .

Finally, a classical BRS invariant density Lagrangian, function of the classical gauge fields and of the field strengths can be written as the following sum of ordered terms:

$$\mathcal{L} = \frac{1}{2} \sum_K (-)^K (G_{K+1}^{\dagger} \phi_K^{K+1} - \psi_K^{\dagger} F_K^K) + h.c. \quad (16)$$

This Lagrangian is degenerate and one has to fix the gauge. This gauge fixing problem is trivial since we know the BRS symmetry and the ghost and auxiliary field sectors of the theory. The gauge fixed Lagrangian can be written as:

$$\begin{aligned} \mathcal{L}_{GF} = & \sum_{\substack{0 \leq q \leq K \\ 1 \leq q \leq g}} s \left(\psi_{K-q+q}^{K+q} \overset{\dagger}{\eta}_{K+q}(q, g-q, q) \phi_{K+q}^{K-g+q}(q, g-q+1) \right) \\ & + \frac{\alpha}{4} \sum_{\substack{1 \leq g \leq K \\ 1 \leq q \leq g}} b_{K+q-g}^{K+q+1}(1+q, 1+g-q) \eta^{g+1} b_{K-g+q}^{K+q+1}(1+g-q, 1+q) \end{aligned} \quad (17a)$$

where η^n means that the operator η is applied n times. The first term determines a ghost interaction, of the form $\psi s \phi$ (respectively $\psi s \phi'$) and a gauge fixing term. Indeed the action of s on the ghosts ψ and ϕ provides terms of the type b_0 and c_0' , which thus play the role of gauge fixing terms, as can be seen after the elimination of b and c through their equation of motion. When $\alpha = \beta = 0$, one has a Landau gauge, i.e., a gauge with $\delta(0)$ and $\delta(0')$ in front of the functional integral measure. For $\alpha = \beta = 1$, one has a Feynmann gauge. The gauge used in Ref. 1) is a Landau one and is obtained by taking

$$\mathcal{L}_{GF} = \sum_{\substack{0 \leq q \leq K \\ 1 \leq q \leq g}} s \left(\psi_{K+q-g}^{K+q+1}(q, g-q+1) \eta^{g+1} \phi_{K-g+q}^{K+q+1}(g-q, q) \right) \quad (17b)$$

This gauge implies that all the ϕ 's vanish. If furthermore we want $\phi_K^K = 0$ for $K > 1$, we add simply

$$\mathcal{L}'_{GF} = \sum_{K \geq 0} s \left(\psi_{K+1}^{K+1} \overset{\dagger}{\eta}_{K+1}(0,0) \phi_K^{K+2}(0,1) \right) \quad (17c)$$

When the gauge (17b) is used, the classical Lagrangian reduces to $\mathcal{L} = -\frac{1}{2} \sum (-1)^K \phi_K^{K+1} K \phi_K^K + h.c.$, which shows that our system of ghosts, albeit different from the one of Ref. 1), describes the same physics.

To proceed further, it is convenient to use the unified notations introduced by Kato and Ogawa⁴⁾. From now on, all the indices in ϕ_K^K and ϕ_K^{K+1} are contracted by ghost creation and annihilation operators β^+ and β and one can define a gauge field χ by

$$\begin{aligned} \chi = \sum_{n,m} (& \psi_{[m_1 \dots m_a]}^{[m_1 \dots m_a]} \beta^{m_1+} \dots \beta^{m_a+} \bar{\beta}_{m_1}^+ \dots \bar{\beta}_{m_a}^+ \bar{\beta}_0 \\ & + \phi_{[m_1 \dots m_{a+1}]}^{[m_1 \dots m_{a+1}]} \beta^{m_1+} \dots \beta^{m_a+} \bar{\beta}_{m_1}^+ \dots \bar{\beta}_{m_{a+1}}^+) \end{aligned} \quad (18)$$

One has $\{\bar{\beta}_0 \beta_0\} = 1$, $\delta_{nm} = \{\bar{\beta}_n^+, \beta_m\} = \{\bar{\beta}_n, \beta_m^+\}$ for $n, m > 1$ and all the other anticommutators vanish. Observe that χ has an odd grading in the ghost operators β^+ and β . We call it a one-form since it starts with $\phi_0 \bar{\beta}_0$. One can also define¹⁾:

$$\begin{aligned} d & \equiv \beta^{a+} (L_n + W_{mn}^P \bar{\beta}_P^+ \beta^m + \frac{1}{2} \beta^{m+} V_{mn}^P \bar{\beta}_P) \\ D & \equiv \beta^n (L_{-n} + W_{mn}^P \beta^{m+} \bar{\beta}_P + \frac{1}{2} \bar{\beta}_P V_{mn}^P \beta^m) \\ M & \equiv 2 \bar{\beta}_0 \sum_{n=1}^{\infty} n \beta_n^+ \beta_n \\ K & \equiv \beta_0 (L_0 - 1 + \sum_n n \beta_n^+ \bar{\beta}_n + n \bar{\beta}^{a+} \beta^a) \\ Q & \equiv d + D + K + M \end{aligned} \quad (19)$$

and then

$$Q^2 = 0 \quad (20)$$

The advantage of this notation is that the rules of exterior calculus can now be systematically applied. $\tilde{\chi}$ and $\tilde{Q}$ are defined to be

$$\begin{aligned} \tilde{\chi} & \equiv \chi(\tilde{\psi}, \tilde{\phi}) \\ \tilde{Q} & \equiv \tilde{d} + \tilde{D} + \tilde{K} + \tilde{M} \quad \tilde{d} \equiv d + s \quad \tilde{D} \equiv D + \bar{s} \end{aligned} \quad (21)$$

where $\tilde{K}$ and $\tilde{M}$ are as previously horizontal. The field strength of χ is simply

$$F = Q\chi \quad (22)$$

and the Bianchi identities in Eq. (6) become

$$Q F = 0 \quad (23)$$

Furthermore, the BRS symmetry follows directly from

$$\tilde{F} = F \quad (24)$$

once expanded on ghost number.

To proceed, one must be able to consider products of functionals. Given two functional operators χ and χ' , they are two ways of defining the product of χ by χ' . The first one corresponds to the usual "trace" operation which will be denoted by $\chi \cdot \chi'$: it corresponds to the product $\chi^\dagger \chi'$ used above. The second operation, denoted by $[\chi, \chi']$, means that some non-trivial overlapping function is used to convert two strings into a third one, in order to describe possible interactions. Both products are bilinear and must obey the graded Leibnitz rule under the action of the operators d and D . Q , χ and χ' have an odd number of β^+ operators. This means that their grading is one. Now Q is self-adjoint. It follows that the action of Q on $\chi \cdot \chi'$ or $[\chi, \chi']$ must follow the rule

$$\begin{aligned} Q(\chi \cdot \chi') &= (Q\chi) \cdot \chi' - \chi \cdot (Q\chi') \\ Q[\chi, \chi'] &= [Q\chi, \chi'] - [\chi, Q\chi'] \end{aligned} \quad (25)$$

and in particular $\frac{1}{2}Q[\chi, \chi] = -[\chi, Q\chi]$.

χ now looks like an usual Abelian Yang-Mills one-form gauge field and Q as the usual differential operator. Besides $F = Q\chi$ is a two-form (with grading 2). An invariant Lagrangian should be associated to a Q exact form, function only of F . Indeed, suppose the existence of $I(F)$ such that

$$I(F) = QL \quad (26)$$

F being horizontal, (26) implies that

$$\begin{aligned} S L &= - Q L_{(1,0)} \\ \bar{S} L &= - Q L_{(0,1)} \end{aligned} \quad (27)$$

and the action $\langle 0 | L | 0 \rangle$ will then be BRS invariant, since Q annihilates the vacuum. The Bianchi identity $QF = 0$ and the fact that we restrict ourselves to a bilinear Lagrangian in the fields determine the simplest choice

$$I = F \cdot F = Q(\chi \cdot F) = Q(\chi \cdot (Q\chi)) \quad (28)$$

which in turn implies that $L = (\chi \cdot F)$, which is in fact equivalent to (16) when all contractions are made on β^+ operators. We thus recover the result found in Ref. 1), in a purely geometrical manner.

If we count that χ and Q have rank one, and F has rank two, L is of rank three, with ghost number zero. To investigate the possibility of anomalies³⁾, one looks for the solutions $\Delta_{(1,0)}$, with ghost number one, of the Wess-Zumino consistency condition:

$$S \Delta_{(1,0)} = Q \Delta_{(2,0)} \quad (29)$$

Such solutions, whenever fermions were to be coupled to the string theory, could correspond to string field theory anomalies. Solutions to (29) do exist in relation with Q exact forms, and a possible candidate is the obvious generalization of the Abelian anomaly, associated with the six-form $F \cdot (F \cdot F)$:

$$\begin{aligned} F \cdot (F \cdot F) &= Q(\chi \cdot (F \cdot F)) \\ \leadsto \Delta_{(1,0)} &= \chi_{(1,0)} \cdot (F \cdot F) \end{aligned} \quad (30)$$

More generally, one could consider any Q -exact functional of F .

Interacting case

We have seen that an Abelian Yang-Mills like structure is hidden in the covariant formulation of the free bosonic string field theory with gauge field χ , curvature $F = Q\chi$, Bianchi identity $QF = 0$, Lagrangian $\chi \cdot Q\chi$ and possible anomaly associated to $\chi \cdot (F \cdot F)$.

To introduce interactions, the problem consists in generalizing Q into a covariant derivative Q_c . One possibility is to use the product $[,]$ introduced in (25) and covariantize the Q operator by means of

$$Q_c = Q + [\chi,] \quad (31)$$

The covariant field strength is defined as

$$F_c = Q\chi + \frac{1}{2} [\chi, \chi] \quad (32)$$

One has

$$Q F_c = - [\chi, Q\chi] \quad (33)$$

One sees that in order to get for the Bianchi identity

$$Q_c F_c = 0 \quad (34)$$

the following Jacobi identity must hold true

$$[\chi, [\chi, \chi]] = 0 \quad (35)$$

The geometrical requirement (34) thus severely constrains the possible interaction described by the overlapping function corresponding to the bracket $[\chi, \chi]$, in the same way as the Lie algebra structure restricts the structure functions of a Lie group.

The BRS symmetry can be now derived from the horizontality of $\tilde{F}_c$ defined by

$$\tilde{F}_c = \tilde{Q} \tilde{\chi} + \frac{1}{2} [\tilde{\chi}, \tilde{\chi}] = F_c \quad (36a)$$

This yields in particular

$$S \chi = - Q \chi_{(1,0)} - [\chi, \chi_{(1,0)}] \quad (36b)$$

Replacing χ by the ϕ_K^K 's and the ϕ_K^{K+1} 's, and $\chi_{(1,0)}$ by the corresponding infinitesimal parameters one gets the gauge transformations on the classical fields in the interacting case.

An invariant action should be derived from a Q exact four-form, containing the free action defined in (27) and function only of the field strengths, as in the Abelian case. Thinking of the Chern-Simons formula in the Yang-Mills case, one is tempted to introduce

$$I_4 = Q \left(\chi \cdot (Q\chi) + \frac{1}{3} \chi \cdot [\chi, \chi] \right) \quad (37)$$

Indeed, computing the right-hand side of (37) and assuming that the definition of the bracket is compatible with

$$\chi \cdot [\chi', \chi''] = \chi'' \cdot [\chi, \chi'] \quad (38)$$

one gets

$$I_4 = F_c \cdot F_c$$

I_4 is indeed only a function of the field strengths F_c and the invariant Lagrangian density reads

$$L = \chi \cdot Q\chi + \frac{1}{3} \chi \cdot [\chi, \chi] \quad (39)$$

In this construction, the key assumption is in the existence of a vertex operator such that the products $[,]$ and $(.)$ satisfy the Jacobi condition (35) and the cyclicity condition (38). The simplicity of the description of the interacting string theory is then really striking: the algebra determines everything up to the overlapping structure of two interacting strings, and leads to an interaction governed by the Chern-Simons form of rank 3.

Knowing the BRS symmetry from (36), it is then obvious to gauge fix the Lagrangian (39) in a BRS invariant way, as in (17). Besides, as in the free case, anomalies may exist in the interacting theory in association with Q -exact forms

made from F_c , since (35) and (38) ensure the validity of the Chern-Simons formula.

Further string symmetries

The string field χ , built with ϕ_K^K and ϕ_K^{K+1} has been interpreted as a one-form since its β^+ expansion starts with a linear term. One can also consider an object χ_p associated to a system ϕ_K^{K+p-1} and ϕ_K^{K+p} . The expansion of χ_p starts with terms of the form $(\beta^+)^p$ and we call it a p-form gauge string field. It is not difficult to determine the symmetries acting on $\chi_p^{(3)}$, by means of building field strengths satisfying some Bianchi identities. The simplest example is for $p = 2$ when one considers interactions with the basic string field χ :

$$\begin{aligned}\tilde{G}_3 &= \tilde{Q} \tilde{\chi}_2 + \lambda (\tilde{\chi} \cdot (\tilde{Q} \tilde{\chi}) + \frac{1}{3} \tilde{\chi} \cdot [\tilde{\chi}, \tilde{\chi}]) \\ \tilde{F}_c &= \tilde{Q} \tilde{\chi} + \frac{1}{2} [\tilde{\chi}, \tilde{\chi}]\end{aligned}\tag{40}$$

Here λ is an arbitrary parameter. Setting $\tilde{G}_3$ and $\tilde{F}$ to be horizontal defines the symmetry. One has, for instance

$$s \chi_2 = -Q \chi_{2(1,0)} + \lambda \chi_{(1,0)} \cdot Q \chi\tag{41}$$

The introduction of p-forms χ_p in interaction with the fundamental one-form χ should in general modify the cohomology of the BTRS operator s , a phenomenon which should play a role when supersymmetry is introduced, in view of the anomaly cancellation mechanism.

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NOTE ADDED

While completing this letter we received a paper by E. Witten⁶⁾ where some geometrical considerations for the interacting case leading to results analogous to ours are made.

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