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THE NUCLEON IN THE COLOUR DIELECTRIC MODEL

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A B S T R A C T

With parameters of the colour dielectric model obtained from fitting pure gluonic properties of QCD, we obtain a good description of the three-quark system. The colour dielectric field at the centre of the hadron is not very different from its vacuum value. The properties of the nucleon are $(m_N + m_\Delta)/2 = 1040$ MeV, $\langle r^2 \rangle^{1/2} = 1.2$ fm, $g_A/g_V = 1.32$, $\mu_P = 2.66 \mu_N$ for a current quark mass $m_q = 20$ MeV.

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Quantum chromodynamics plays a central role in the description of hadron interactions at a fundamental level. In the lattice regularization, QCD has produced new and important results during the last years. On the other side there has been a growing phenomenology using simple models of QCD. It looks advantageous to us to combine both developments. We have therefore proposed the colour dielectric model¹⁾⁻³⁾, the input parameters of which are related to the original QCD-action. At the same time, this model can be applied to hadron spectroscopy and reactions. In a previous publication⁴⁾, we have described the pure gluonic sector of this model; in this paper we calculate nucleon properties with the same form of effective action.

The colour dielectric model proposes a local order parameter $\chi(x)$ which is related to the dielectric constant $\epsilon = \chi^4$ and can be calculated by averages over gluon fields in a volume $L^4 < 1/\Lambda_{\text{QCD}}^4$:

$$\chi_L(x_0) = \frac{1}{N} \text{Tr} \text{av}_{L^4} P \exp(i \oint_{x_0} A_\mu dx_\mu) \quad (1)$$

Here $A_\mu = \sum_a \lambda^a A_\mu^a$ are the gauge fields. The average of Wilson loops through x_0 is taken in the volume L^4 , the trace is performed in $SU(N)$ space and $N = 3$ is the number of colours. The continuum limit of the final effective action contains besides the colour dielectric neutral scalar field χ eight average gauge fields B_μ^i which are defined as:

$$B_\mu(x) = \lim_{x' \rightarrow x} \partial_\mu' \text{av}_{L^4} P \exp i \int_x^{x'} A_\mu dx_\mu \quad (2)$$

The quarks can be included in the effective Lagrangian using local gauge invariance. The lattice action for quarks is given with a as lattice constant and e_μ as unit vector in direction μ ($\mu=0,1,2,3$) in the following form:

$$S_q = \frac{1}{2} i a^3 \sum_{i,j} \bar{\psi}_i \gamma_\mu e_\mu \exp i \int_{x_i}^{x_j} A_\mu dx_\mu \psi_j \quad (3)$$

It averages using Eq. (2) to

$$\text{av}_{L^4} S_q = i a^3 \sum_i \bar{\psi}_i \gamma_\mu e_\mu (K^i + i a e_\lambda B_\lambda^i) (\psi_i + e_k a \partial_k \psi_i) \quad (4)$$

with $K^i = K(x_i) = \text{av}_{L^4} P \exp i \int A_\mu dx^\mu$. The main approximation in the fermion

action consists of replacing $K(x_i)$ by $\chi(x_i)$, the trace of K in colour space. In colour $SU(2)$ this would present no approximation, since an average of unitary matrices can always be represented as a scalar field $0 < \chi < 1$ and a new unitary matrix. The same statement is, however, not true in $SU(3)$ where in general the average can be represented by a Hermitian 3×3 matrix times a unitary matrix. For the colour dielectric effective action we only keep the trace of K . With Eq. (1), we obtain in the continuum limit

$$\lim_{L^4} S_q \Rightarrow i \int d^4x \bar{\psi}(x) \chi(x) \left(\gamma^\mu \partial_\mu + i \gamma^\mu B_\mu / \chi \right) \psi(x) \quad (5)$$

Using the covariant derivative

$$D_\mu = \partial_\mu + i B_\mu / \chi$$

the total effective Lagrangian has the form

$$\mathcal{L} = \mathcal{L}(\chi) + i \chi(x) \bar{\psi} \gamma^\mu (\partial_\mu + i B_\mu / \chi) \psi - \bar{\psi} \psi m_q - \frac{1}{4g^2} \chi^4 F_{\mu\nu} F_{\mu\nu} \quad (6)$$

where

$$F_{\mu\nu} = (\partial_\mu + i B_\mu / \chi) B_\nu / \chi - (\partial_\nu + i B_\nu / \chi) B_\mu / \chi$$

and $\mathcal{L}(\chi) = T - V$ is parametrized with

$$V(\chi) = B \chi^2 (A+6 - 2(A+4) \chi + (A+3) \chi^2) \quad (7)$$

and

$$T(\chi) = \frac{1}{2} \sigma_v^2 (\partial_\mu \chi)^2 \quad (8)$$

The form of $V(\chi)$ is chosen in accordance with our previous work⁴⁾ on the pure gluon theory, where we set $\chi = (\sigma_v - \sigma) / \sigma_v$. Note that the effective pure gluonic Lagrangian of Eq. (6) is similar to the Friedberg-Lee soliton model^{5),6)} in this case. The dielectric constant is defined in the Lagrangian of Eq. (6) as $\epsilon = \chi^4$.

The fermion part of the colour dielectric model in Eq. (6), however, is totally different from the Friedberg-Lee model. In particular we note that the colour dielectric field leads to absolute quark confinement, since with $\langle \chi \rangle = 0$ in the vacuum the quarks can no longer propagate, i.e., their effective mass

becomes infinite. To demonstrate this we introduce a new effective fermion field $\tilde{\psi}$ by rescaling the original field ψ

$$\tilde{\psi} = \sqrt{\chi} \psi \quad (9)$$

which shifts the χ -dependence from the kinetic term in Eq. (6) to the mass term $(mq/\chi)\tilde{\psi}\tilde{\psi}$. Also new gauge fields $\tilde{A}_\mu^a$ can be defined according to

$$\tilde{A}_\mu^a = \frac{1}{g} B_\mu^a / \chi \quad (10)$$

As a first approximation, we shall consider the $\tilde{A}_\mu$'s as perturbative gluon fields, dropping the non-linear parts of the Lagrangian. Phenomenologically we consider the χ -part as sufficient to describe the 0^{++} glueball and associate the χ -propagator with the glueball mass, i.e., $V(x)$ at $\chi = 0$ can be expanded as

$$\frac{1}{2} \frac{\partial^2 V}{\partial \chi^2} \Big|_{\chi=0} \chi^2 = \frac{1}{2} m_{GB}^2 \chi^2 = B(A+6) \chi^2 \quad (11)$$

In the previous paper we have found that $A = -2$, $\sigma_V = 160$ MeV and $B^{1/4} = 203$ MeV reproduces rather well the following physical observables: the transition temperature $T_c = 160$ MeV to the gluon plasma, a glueball mass $m_{GB} = 720$ MeV and a string tension $\sqrt{E/l} = 450$ MeV. In the calculation of the nucleon we use the same parameters A , $B^{1/4}$, σ_V . We note in passing that we also tried to calculate the nucleon with these parameters in the Friedberg-Lee soliton bag model and always obtained nucleon masses $m_N > 2$ GeV. This is related to a general problem of the original bag model which must use different bag constants B for the heavy and light quark systems, typically $B^{1/4}(c\bar{c}) = 200$ MeV⁷⁾ and $B^{1/4}(qqq) \approx 150$ MeV⁸⁾. We will show how different colour dielectric fields in the interior describe both systems with the same effective Lagrangian.

The only remaining input parameter is the current quark mass on a hadronic size scale. We have argued³⁾ that the average light quark mass should lie between 20 MeV and 30 MeV, i.e., it is somewhat larger than in the estimates based at $Q^2 = 1$ GeV².

The equations of motion for the χ -field and the rescaled ψ -field (now written without $\sim$), are derived from the effective action for N quarks of equal mass m_q

$$(\vec{\alpha} \cdot \vec{p} + \frac{m_q}{\chi}) \psi = E \psi \quad (12)$$

and

$$-\epsilon_V^2 \vec{\nabla}^2 \chi + \frac{\partial V}{\partial \chi} = N \bar{\psi} \psi \frac{m_q}{\chi^2} \quad (13)$$

Numerically, we solve the equations for the potential $\tilde{V}(\chi) = V(\chi - \delta)$, which has a minimum at $\chi = \delta \approx 10^{-3}$ such that $m_q/\delta > 20$ GeV. Decreasing δ even more we do not find any variation of the results. Qualitatively, one understands the solution of the χ -field in the presence of the quark fields by regarding the effective potential

$$V_{\text{eff}}(\chi) = V(\chi) + N \bar{\psi} \psi \frac{m_q}{\chi} \quad (14)$$

which has a minimum away from $\chi = 0$.

Approximating the single particle densities as

$$\bar{\psi} \psi \approx \frac{1}{2} \bar{\psi} \gamma_0 \psi = \frac{1}{2} \frac{1}{\frac{4\pi}{3} R^3} \quad (15)$$

we estimate the scalar density $\bar{\psi} \psi = 0.12 \text{ fm}^{-3}$ for a hadron size of $R = 1 \text{ fm}$. Then we get for $\bar{\chi}$ as the minimum of $V_{\text{eff}}(\chi)$ ($m_q = 20 \text{ MeV}$, $m_{GB}^2 \sigma_V^2 = 0.013 \text{ GeV}^4$)

$$\chi(0) \approx \bar{\chi} = 0.16 \quad (16)$$

In Fig. 1 we show the exact solutions of the coupled equations for $\chi(r)$. Indeed the inside value of χ is not very different from zero [$\chi(0) = 0.17$]. Comparing to the $3\bar{3}$ flux tube⁴⁾ solution [$\chi(0) = 0.6$], we see even less change of the vacuum value of χ . The corresponding quark wave functions (Fig. 2) have a reduced small component compared to the MIT bag model since the effective quark mass inside the bag amounts to $m_q/\bar{\chi} \approx 120 \text{ MeV}$. In effect the colour dielectric model interpolates between the constituent quark model $m_{\text{eff}} \approx 300 \text{ MeV}$ and the original MIT bag model $m_q = 0$.

The total energy of the χ -bag is composed of three parts: the energy of N quarks, and kinetic and potential energy of the χ -field

$$E_{bag} = N \cdot E + \int \left\{ \frac{1}{2} (\vec{\nabla} \chi)^2 \sigma_V^2 + V(\chi) \right\} d^3r \quad (17)$$

We found for the nucleon ($N=3$) $E = 317$ MeV, a kinetic energy of 23 MeV and a potential energy of 179 MeV. Approximately correcting for spurious centre-of-mass energy by the expectation value of $\langle \vec{p}^2 \rangle$ for the quarks, i.e.,

$$\langle \vec{p}^2 \rangle = N \int \psi^\dagger \vec{p}^2 \psi d^3r \quad (18)$$

we get, for the mass of the nucleon (note the mass of the nucleon and delta are degenerate in this approximation):

$$\frac{m_N + m_\Delta}{2} = \sqrt{E_{bag}^2 - \langle \vec{p}^2 \rangle} = 1040 \text{ MeV}$$

The rms radius of the quark distribution is 1.2 fm which is larger than the experimental rms radius of 0.83 fm. The magnetic moment of $2.66 \mu_N$ ($\mu_p = 2.79 \mu_N$) and the ratio of $g_A/g_V = 1.32$ are excellent results. Since $\bar{\chi}$ inside the bag is rather small it is clear that the shape of the effective potential $V(\chi)$ near $\chi = 1$ does not influence the nucleon mass. Calculations with different sets of parameters B and A do not change the result as long as $m_{GB}^2 \sigma_V^2$ is kept constant. The shape of the effective potential V around $\chi = 0$ determines the dynamics.

In order to understand the results of the numerical calculation we can give an approximation for the energy of the colour dielectric bag

$$E_{bag}(N) = N \cdot \sqrt{\left(\frac{\omega}{R}\right)^2 + \left(\frac{m_q}{\bar{\chi}}\right)^2} + \frac{4\pi}{3} R^3 \left(\frac{1}{2} m_{GB}^2 \sigma_V^2 \right) \bar{\chi}^2 \quad (19)$$

We use $\omega \approx 2$ and $\bar{\chi}$ is the minimum of the effective potential in the vicinity of $r = 0$

$$V_{eff}(\chi) = \frac{m_q N \bar{\chi}^2}{\chi} + \frac{1}{2} m_{GB}^2 \sigma_V^2 \chi^2 \quad (20)$$

Here R is the size of the equivalent MIT bag. We dropped the derivative terms of χ in the field equation derived from Eq. (20) and energy expression (19), because the kinetic energy of the χ field (23 MeV) contributes little to the energy of the colour dielectric bag. The minimum $\bar{\chi}$ has the value

$$\bar{\chi} = \left(\frac{N m_q \bar{q} \bar{q}}{m_{GB}^2 \sigma_V^2} \right)^{1/3} \quad (21)$$

For $\bar{\phi}\phi$ we use the approximation given before Eq. (15). One finds that the energy E_{bag} and the radius R can be rescaled to dimensionless quantities ε and r

$$\varepsilon = E_{\text{bag}} / \lambda, \quad r = R \cdot \lambda \quad (22)$$

with $\lambda = (m_{GB} \sigma_V m_q)^{1/3}$. Then Eq. (19) simplifies to the following universal energy formula

$$\varepsilon \approx \frac{2N}{r} + N^{1/3} r^3 + 0.5 N^{2/3} r \quad (23)$$

This energy assumes its minimum for $r = N^{1/6} \times 0.86$ and gives a product $\varepsilon \cdot r = E_{\text{bag}} \cdot R = 2.9 N$, which can be compared to the MIT-bag model of $E_{\text{bag}} \cdot R = 2.67 N$. The ratio

$$\frac{m_q}{\bar{\chi}} / \sqrt{\left(\frac{\omega}{R}\right)^2 + \left(\frac{m_q}{\bar{\chi}}\right)^2} = m_{\text{eff}} / \Omega \approx 0.6$$

is independent of N or the choice of m_{GB} , σ_V , m_q . This explains a value of $g_A/g_V = 1.32$ between the $m_q = 0$ result of $g_A/g_V = 1.1$ and the $m_q \rightarrow \infty$ result of $g_A/g_V = 5/3$. It is not sensitive to the input parameters. To illustrate the analytical calculation we compare it to the numerical solution. For $N = 3$ we find $R = 1.03 / (m_{GB} \sigma_V m_q)^{1/3} = 1.5 \text{ fm}$ or $\langle r^2 \rangle^{1/2} = 0.73 R = 1.1 \text{ fm}$ and $E_{\text{bag}} = 1144 \text{ MeV}$ compared to a value of $E_{\text{bag}}(\text{exact}) = 1153 \text{ MeV}$ and a $\langle r^2 \rangle_{\text{exact}}^{1/2} = 1.2 \text{ fm}$. The approximate magnetic moment of the proton

$$\begin{aligned} \mu &= \frac{e}{2m_{\text{eff}}} \cdot \frac{1}{3} (m_{\text{eff}} R) \frac{4\Omega R + 2m_{\text{eff}} R - 3}{2(\Omega R)^2 - 2\Omega R + m_{\text{eff}} R} = \\ &= 2.7 \mu_N \left(\frac{R}{1.5 \text{ fm}} \right). \end{aligned} \quad (24)$$

is in agreement with the numerical value of $\mu = 2.66 \mu_N$. The mass formula of Eq. (23) also allows to estimate exotic $6q$ -configurations. Experiments do not show any dibaryons, so we would expect that already on the level of the (ϕ, χ) dynamics without perturbative gluon fields the six-quark system is not or only weakly bound. We find numerically

$$\Delta E = E_{6q} - 2E_{3q} = -264 \text{ MeV} \quad (25)$$

and Eq. (23) also gives an estimate of $\Delta E \approx -244 \text{ MeV}$. This result can be compared to the MIT bag result $\Delta E = -622 \text{ MeV}$ and the soliton bag model⁶⁾ where $\Delta E = -580 \text{ MeV}$. Hence in the colour dielectric model the 6q-system is much less bound. This may help to generate a realistic baryon-baryon interaction at short distances, where parallel work has shown the importance of a not very bound 6q configuration in the adiabatic potential⁹⁾.

Future work on the colour dielectric model has to progress on two levels. On the fundamental level, "measurements" of $\langle \chi^2 \rangle$ and $\langle (\partial \mu \chi)^2 \rangle$ can be made by lattice simulations to investigate the effective potential $V(\chi)$. Phenomenologically one wants to pursue the calculation of gluon effects on the nucleon and in the 6q-system. Work on both ends of the problem will help to disentangle the complex structures of hadronic interactions.

REFERENCES

- 1) H.B. Nielsen and A. Patkos - Nucl.Phys. B195 (1982) 137.
- 2) G. Mack - Nucl.Phys. B235 (1984) 197.
- 3) G. Chanfray, O. Nachtmann and H.J. Pirner - Phys.Lett. 147B (1984) 249.
- 4) M. Rosina, A. Schuh and H.J. Pirner - Nucl.Phys. A448 (1986) 557.
- 5) R. Friedberg and T.D. Lee - Phys.Rev. D15 (1977) 1964.
- 6) R. Goldflam and L. Wilets - Phys.Rev. D25 (1982) 1951.
- 7) J. Baacke, Y. Igarashi and G. Kasperidus - Z.Phys. C113 (1982) 119.
- 8) T. DeGrand, R.L. Jaffe, K. Johnson and J. Kiskis - Phys.Rev. D12 (1975) 2060.
- 9) L. Wilets, A. Schuh and H.J. Pirner - Proc. of the Workshop on Nuclear Chromodynamics, Santa Barbara (1985).

FIGURE CAPTIONS

Fig. 1 The colour dielectric field $\chi(r)$ for the nucleon.

Fig. 2 The big ($u(r)$) and small (v) components of the quark wave function in the nucleon.

- Figure 1 -

- Figure 2 -