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NO-SCALE STRUCTURE FROM THE SUPERSTRING

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ABSTRACT

We present a general method using topological arguments to derive symmetries of the low energy effective Lagrangian extracted from the generating functional for the superstring after compactification. This method is extended to show how some of these symmetries are broken in the loop expansion for the superstring. A simple algebraic argument shows that the tree level effective Lagrangian for small field fluctuations can only be of the no-scale $SU(n,1)/SU(n) \times U(1)$ form.

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ADDENDUM

As should be clear from the context of the first part of the paper, the result of pp. 7 et seq. concerning the uniqueness of the $SU(n,1)/SU(n) \times U(1)$ form is supposed to be relevant and applicable only when $|\phi^i| \ll 1$. The non-perturbative effects discussed earlier are of course crucial when $|\phi^i| = O(1)$. This point will be stressed more clearly in the published version of our paper.

We thank S. Ferrara for suggesting the desirability of this clarification.

The extraction of an effective four-dimensional field theory from the superstring requires an Ansatz for the compactification of the six extra dimensions. Prime candidates for compactification are Calabi-Yau¹⁾ manifolds, although other possibilities should also be considered. Assuming that some such manifold is a good ground state for the superstring, purely topological arguments provide knowledge of the field content of the low energy four-dimensional effective theory²⁾, on Yukawa couplings^{3),4)} and on the vacuum expectation values of some fields. Such an analysis has been developed⁵⁾ starting from the modified ten-dimensional supergravity theory which is the low energy limit of the superstring, and imitating a Calabi-Yau compactification. The resulting effective four-dimensional theory was found⁵⁾ to have the no-scale structure of a non-linear σ -model defined on a non-compact Kähler manifold, which had been advocated previously as the basis of phenomenological supergravity models in which the gauge hierarchy $m_W/m_P \ll 1$ could be generated dynamically⁶⁾.

In this paper we describe a general method for extracting the symmetries of the Kähler potential of the low energy effective theory directly from general properties of the string generating functional defined on a general $M_4 \times K_6$ ⁷⁾ background, without invoking an intermediate $D = 10$ supergravity stage. We show how the $d = 4$ no-scale Kähler structure emerges from the superstring at the tree level. Our method also shows how such a tree level symmetry of the Kähler potential can be broken at the level of one or more loops, in contrast to what has been found⁸⁾ for superpotential terms in the effective low energy theory. Although we develop our method in the context of Calabi-Yau compactification¹⁾, it would also be applicable to other compactification schemes.

First we review briefly the field content of the low energy effective theory. Massless fields in four dimensions are associated¹⁾ with zero modes on the six-dimensional manifold of compactification. For a Calabi-Yau ground state with the observable E_8 broken to $SU(3) \times E_6$, the number of gauge non-singlet massless fermions transforming as $(3, 27)$ and $(\bar{3}, \bar{27})$ can be deduced from the index theorem for the corresponding Dirac operator. On a Kähler manifold this index reduces to the Dolbeault index⁹⁾:

$$\#(3, 27) - \#(\bar{3}, \bar{27}) = \int_{K_6} \text{td}(T(K_6)) \wedge \text{ch } V = \sum_{q=0}^3 (-1)^q \dim H_{q,1} \quad (1)$$

which is the following combination of Betti numbers $b_{p,q} \equiv \dim H_{p,q}$:

$$\#(3,2\bar{7}) - \#(\bar{3},2\bar{7}) = b_{2,1} - b_{1,1} \quad (2)$$

if the scalar curvature is zero so that $b_{0,2} = b_{2,0} = 0$. Massless gauge singlet superfields are associated with fluctuations of the metric of the compactification manifold which are solutions to the Lichnerowicz equation¹⁾. We must consider two different kinds of deformations of this internal metric: deformations that change the metric but not the Kähler structure of the manifold, and complex deformations that change the Kähler structure. The former can be visualized as dilatations of K_6 and are expressible as forms in $H_{1,1}$. The dimension $b_{1,1}$ of this cohomology group gives the number of real massless singlet fields associated with such deformations. Notice also that zero modes of the antisymmetric tensor field $B_{\mu\nu}$ are associated with elements of $H^2(K_6)$, which is isomorphic to $H_{1,1}(K_6)$ if K_6 is a Ricci flat Kähler manifold [recall that $H^2(K_6) = \bigoplus_{p+q=2} H_{p,q}(K_6)$ and $b_{2,0} = b_{0,2} = 0$]. Thus the number of zero modes of $B_{\mu\nu}$, namely b_2 , is equal to the number of (1,1) deformations, namely $b_{1,1}$. This equality is necessary if these zero modes are to combine in complex superfields which we call T^α : $\alpha = 1, 2, \dots, b_{1,1}$. Secondly, the number of massless fields associated with deformations of the Kähler structure, which we call C^a , is given by the dimension of $H_1(T)$, the first cohomology group with coefficients in the holomorphic tangent bundle T on K_6 . For a Kähler manifold, $H_1(T)$ is isomorphic to $H_{2,1}(K_6)$, so the number of such singlets C^a is equal to $b_{2,1}$. It has recently been pointed out⁸⁾ that an additional set of massless fields E transforming in the (8,1) representation of $SU(3) \times E_6$ will exist if the dimension of $H_1(\text{End}T)$ is non-zero. This is not yet known for any realistic manifold, and we will disregard E fields in the rest of this paper.

The dynamics of the low-mass superfields (T^α, C^a) will be a non-linear σ -model defined on a Kähler manifold whose symmetries we now investigate. We start from the string generating functional in a generic background $M_4 \times K_6$ with metric G^{ij} , whose bosonic part^{10),7)} is:

$$Z(G^{ij}) = \int \mathcal{D}g \, \mathcal{D}X \exp - \int_{\Sigma} (\sqrt{g} g_{\mu\nu} G^{ij}(X(\sigma, \tau)) \partial^\mu X_i \partial^\nu X_j) d\sigma d\tau \quad (3)$$

where Σ is the two-dimensional string world sheet: Σ is S^2 at the tree level, a torus in one loop, etc. Symmetries of (3) are characterized as transformations: $G^{ij} \rightarrow G'^{ij}$ which leave $Z(G^{ij}) = Z(G'^{ij})$. We use the conformal invariance $g_{\mu\nu} \rightarrow e^{\sigma(x)} g_{\mu\nu}$ of (3) to work in the conformal gauge. Then a sufficient condi-

tion for the transformation $G^{ij} \rightarrow G^{ij'}$ to be a symmetry (recall that we are in the critical dimension for the superstring) is that

$$\int_{\Sigma} (G^{ij'} - G^{ij}) \partial^i X_i \partial^j X_j d\sigma d\tau = 0 \quad (4)$$

for any string configuration. In principle, there could be additional transformations which do not satisfy (4), but do become symmetries when the remaining integrations in (3) are performed: we will not consider this possibility in the rest of this paper. Our Ansatz (4) can be extended to any possible set of background fields for the string, for instance $B_{\mu\nu}$ or the dilaton field $\phi^{(1)}$.

We can get interesting results in the simplest case (4) using purely topological arguments. Clearly (4) will be satisfied if the transformation $G^{ij} \rightarrow G^{ij'}$ reduces to a pure change of co-ordinates on the string world sheet. The fact that $G^{ij} \partial^i X_i \partial^j X_j$ is the induced metric on Σ means that the integrand in (4) is the pull-back on Σ of the deformation $G^{ij'} - G^{ij}$. If we take $G^{ij'}$ to be a complex deformation of G^{ij} then the integrand in (4) becomes the pull-back on Σ of an element of $H_1(T)$, the space of complex deformations of K_6 . Thus the integrand in (4) corresponds to some element of the space of complex deformations of the string world sheet. In this particular case it is clear what is a necessary and sufficient condition for (4) to be satisfied, namely that the space of complex deformations of Σ has dimension zero. The set of deformations of the string world sheet is given by the "moduli"¹²⁾ of the Riemann surface Σ , which are elements of $H_1(T_{\Sigma})$ where T_{Σ} is the holomorphic tangent bundle on Σ . This cohomology group is given by the corresponding Teichmüller space $T(\Sigma)$, which is defined by¹³⁾⁻¹⁵⁾:

$$T(\Sigma) = \mathcal{M}(\Sigma) / \text{Diff}_0(\Sigma) \cdot \text{Conf}(\Sigma) \quad (5)$$

where $\mathcal{M}(\Sigma)$ is the space of metrics on Σ , $\text{Diff}_0(\Sigma)$ is the set of diffeomorphisms connected to the identity, and $\text{Conf}(\Sigma)$ is the set of conformal transformations. The dimension of $T(\Sigma)$ depends on the genus as shown in the Table. From this we deduce that any change of the background metric G^{ij} which corresponds to a complex deformation is a good symmetry at the tree level: $\Sigma \sim S^2$. In this case, the fact that $\dim T(S^2) = 0$ tells us that the pull-back by any X of $G^{ij'} - G^{ij}$ is necessarily cohomologous to zero, i.e., zero up to a topologically trivial diffeomorphism. At the one loop level, the fact that $\dim T(T^2) = 2$ tells us that (4) will be non-zero for some configurations X , and similarly for higher loop contributions.

This argument has interesting physical consequences. In particular, we see that the set of ground states related by complex deformations is degenerate at the tree level of the string. This degeneracy is broken by one-loop string configurations X with $\int_{\Sigma} (G' - G) \partial X \bar{\partial} X \neq 0$, though the order or magnitude of this non-perturbative effect depends on the value of the string action for X . In particular, configurations X with infinite (large) actions will contribute zero (a small amount) to the functional integral. In order to visualize symmetries at the four-dimensional level we must represent the changes in the background fields as transformations on the four-dimensional fields. This can easily be done for the C^a fields associated with deformations of the metric of K_6 . These deformations can be parametrized²⁾ by polynomials

$$P(\{z\}; C^a) = 0 \quad ; \quad a = 1, 2, \dots, b_{2,1} \quad (6)$$

where P is the most general polynomial function of several complex co-ordinates $\{z\}$ which defines a Ricci flat Kähler manifold K_6 with $SU(3)$ holonomy. The parameters C^a are now seen as co-ordinates in the moduli space of K_6 . These parameters will depend on $x \in M_4$ when we have different manifolds at different points in M_4 . The fields $C^a(x)$ are the set of $b_{2,1}$ fields associated with the deformations that we described before. The above analysis tells us that the potential for these fields is flat at the tree level of the string, but is modified by non-perturbative one-loop string effects, so that some C^a fields can acquire a mass. Thus the known no-renormalization theorems⁸⁾ cannot be extended to include these non-perturbative effects.

We can now use (6) to look for additional symmetries of the C^a fields, which may persist to some order in the loop expansion for the string. Not all the different values of the C^a characterize topologically inequivalent structures on K_6 . The description of the moduli of K_6 in terms of these fields is redundant in the sense that there are some changes of the co-ordinates $\{z\} \rightarrow \{z'\}$ which are equivalent to changes $C^a \rightarrow C'^a$:

$$P(\{z\}; C^a) = P(\{z'\}; C'^a) \quad (7)$$

This kind of symmetry is called the modular symmetry of K_6 . If we take the $C^a(x)$ to be constants independent of space-time x , corresponding to vacuum expectation values $\langle C^a \rangle$, the modular symmetry (7) induces a global symmetry in the non-linear σ -model for the C^a fields. We can deduce this symmetry at each order in the

string loop expansion if we remember that, as seen earlier, deformations of the metric defined by different values of the C^a can be pulled back to deformations of the induced metric on Σ . These deformations will be equivalent if they are related to each other by a change of co-ordinates on Σ , which is the equivalent of equation (7) on the string. The pull-backs of C^a and $C^{a'}$ will be two points τ and τ' in $T(\Sigma)$, and these will be equivalent if $\tau' = g\tau$ for some g in the modular group of Σ : $\text{Diff}(\Sigma)/\text{Diff}_0(\Sigma)$ ¹⁵⁾. At the one-loop level, where Σ is the torus T^2 , the modular symmetry is given by elements of $SL(2, \mathbb{Z})$, which is the same as that found in Ref. 2), up to a Z_5 identification. This modular symmetry is reduced to smaller subgroups of $SL(2, \mathbb{Z})$ as the genus of the string world sheet is increased. Thus the tree level flatness of the potential for the non-linear σ -model for the C^a fields is expected to be broken to the modular symmetry $SL(2, \mathbb{Z})$ at the one-loop level, and further broken by two and higher loop effects. This loss of symmetry could have important phenomenological consequences, in that tree level symmetries of the couplings in the effective low energy theory can be broken by string loop effects.

We now consider symmetries of the set of fields T^α associated with deformations in $H_{1,1}$ that do not change the Kähler structure of K_6 . To do this, we must extend our previous analysis of Eq. (3) to include the antisymmetric tensor field B^{ij} in the string generating functional⁷⁾:

$$Z(G^{ij}, B^{ij}) = \int \mathcal{D}g \int \mathcal{D}X \exp - \int [\sqrt{g} g_{\mu\nu} G^{ij} + \epsilon_{\mu\nu} B^{ij}] \partial^\mu X_i \partial^\nu X_j d\sigma d\tau \quad (8)$$

Now symmetries are changes $(G^{ij}, B^{ij}) \rightarrow (G^{ij'}, B^{ij'})$ such that $Z(G^{ij'}, B^{ij'}) = Z(G^{ij}, B^{ij})$. In the conformal gauge this condition reduces to (4) and

$$\int_\Sigma \epsilon_{\mu\nu} (B^{ij'} - B^{ij}) \partial^\mu X_i \partial^\nu X_j = 0 \quad (9)$$

for any X . Conditions (4) and (9) for (1,1) deformations can again be interpreted in simple topological terms as pull-backs of forms in $H_{1,1}(K_6)$ and $H_2(K_6)$ respectively. Thus we will have a symmetry when the pull-back by any string configuration Σ is in a trivial cohomology class of $H_{1,1}(\Sigma)$ and $H_2(\Sigma)$ respectively, in other words when the change $(G^{ij}, B^{ij}) \rightarrow (G^{ij'}, B^{ij'})$ can be reduced on Σ to a pure change of co-ordinates. If the changes $(G^{ij}, B^{ij}) \rightarrow (G^{ij'}, B^{ij'})$ are non-trivial deformations in $H_{1,1}(K_6)$ and $H_2(K_6)$ respectively, obedience of the conditions (4) and (9) depends not only on $H_{1,1}(\Sigma)$ and $H_2(\Sigma)$, but also on the homotopy class of the string world sheet. For example, for the tree level $\Sigma \sim S^2$

we have $H_{1,1}(\Sigma)$ and $H_2(\Sigma) \neq 0$, so that if X is non-contractible, as can happen if $\pi_2(K_6) \neq 0$, then the conditions (4) and (9) will not be obeyed. This situation corresponds to the world sheet instantons discussed in Ref. 16). Then the only symmetries preserved by such world sheet instantons are trivial ones where $G^{ij'} - G^{ij}$ and $B^{ij'} - B^{ij}$ are in the trivial classes of $H_{1,1}(K_6)$ and $H_2(K_6)$.

As was pointed out in Ref. 17), the effects of these instantons on the vacuum energy of the bosonic string in a background (G^{ij}, B^{ij}) is suppressed at the tree level by the infinite volume of the conformal Killing vectors^{14),18)}. The group of conformal Killing vectors ξ on Σ , which are defined by changes of world sheet co-ordinates $(\sigma, \tau) \rightarrow (\sigma', \tau')$ such that¹³⁾

$$P_1(\xi)_{ab} = \nabla_a \xi_b + \nabla_b \xi_a - (\nabla_c \xi^c) g_{ab} = 0 \quad (10)$$

can play for the (1,1) deformation a rôle similar to that of the modular group for complex deformations. In fact, the group of conformal Killing vectors, which is $SL(2, \mathbb{C})$ at the tree level, can be interpreted as a kind of modular group in the sense that the conformal transformation $(\nabla_c \xi^c) g_{ab}$ is equivalent to a diffeomorphism. If the change $G^{ij} \rightarrow G^{ij'}$ induces a conformal transformation of the two-dimensional metric on Σ , this will be a symmetry if it corresponds to a conformal Killing vector of Σ . The simplest case is when this change can be compensated by an overall rescaling of the co-ordinates on Σ , which corresponds to the non-compact dilatation subgroup of the group of Killing vectors. One of the complex superfields T^α corresponds to these dilatations: it was called simply T in previous literature. The tree level string generating functional is invariant under dilatations, which means that the potential for T is flat at the tree level, and zero because of the division by the infinite volume of $SL(2, \mathbb{C})$, the group of conformal Killing vectors of $\Sigma \sim S^2$. We expect the flatness of this potential to be lost at the one-loop level because on the torus the conformal Killing vectors are reduced to pure translations. The symmetries associated with the other conformal Killing vectors distinct from pure rescalings of the co-ordinates would have been realized as symmetries of the four-dimensional fields only if these fields parametrized the full set of (1,1) deformations including those which are equivalent to diffeomorphisms when projected onto Σ . This is not the case since the T^α are defined on the moduli of dilatations.

Thus we have seen that at the string tree level the $b_{1,1}$ fields T^α and the $b_{2,1}$ fields C^a have a flat potential. This fact will now be used in a purely

algebraic argument which fixes their Kähler potential uniquely at this level, showing that it has the $SU(n,1)/SU(n) \times U(1)$ no-scale structure which was proposed previously for phenomenological reasons⁶⁾. It is well known that the only Kähler potential which gives a flat potential for a single complex superfield has a non-linear $SU(1,1)/U(1)$ symmetry¹⁹⁾, and we now give an extension of this result to many complex superfields.

The set of flat structures in $d = 4$ supergravity has been characterized as follows in Ref. 20). One introduces

$$\phi \equiv \exp \left(-G/3 \right) \quad (11)$$

where G is the Kähler potential of the non-linear σ -model for the $d = 4$ fields, so that

$$G = -3 \ln \phi \quad (12)$$

and the matrix θ of second derivatives of ϕ :

$$\theta^i_j \equiv \frac{\partial^2 \phi}{\partial z^i \partial \bar{z}^j} \quad (13)$$

The $d = 4$ potential vanishes if and only if

$$\det \theta = 0 \quad (14)$$

and we now show that after field reparametrization, all models with the property (14) can be written in $SU(n,1)/SU(n) \times U(1)$ form. The matrix θ has a set of $(n-m)$ zero eigenvalues corresponding to the $(n-m)$ eigenvectors $z^{\beta,0}$, and a set of m non-zero eigenvalues λ_b (which could a priori be either positive or negative) corresponding to m eigenvectors z^b . We analyze starting in this eigenvector basis, so that

$$\theta = \text{diag} (0, \dots, 0; \lambda_1, \dots, \lambda_m) \quad (15)$$

Then we rescale each of the eigenvectors z^b :

$$\hat{z}^b \equiv \sqrt{|\lambda_b|} z^b \quad (16)$$

so that now θ takes the form

$$\theta = \text{diag} (0 \dots, 0; \pm 1, \pm 1, \dots, \pm 1) \quad (17)$$

where the $\pm$ sign applies to $\lambda_b > (<) 0$. Henceforth we drop the carets on the variables z^b . Thus we have for example

$$\frac{\partial^2 \phi}{\partial z^m \partial z_m^*} = \pm 1 \quad (\text{no sum over } m) \quad (18a)$$

and

$$\frac{\partial^2 \phi}{\partial z^p \partial z_m^*} = 0 \quad (p \neq m) \quad (18b)$$

We can now try to reconstruct the possible forms of ϕ compatible with Eq. (18). Integrating once we get

$$\frac{\partial \phi}{\partial z_m^*} = \pm z^m + f(z^{b \leq m}, z_{b \leq m-1}^*) \quad (19)$$

where f is a constant of integration depending on $z^{1,2,\dots,m}$ and $z_{1,2,\dots,m-1}^*$. Integrating again over z_m^* we get

$$\phi(z^{\beta^0}, z^b; z_{\beta^0}^*, z_b^*) = \pm z^m z_m^* + F(\hat{z}^m, \hat{z}_{\leq m-1}^*) + g(\hat{z}^{\leq m-1}, \hat{z}_{\leq m-1}^*) \quad (20)$$

where F is an integral over f , and g is another integration constant. Note now that $F+g$ must be Hermitian, which is only possible if F is actually independent of z^m , and hence we can write

$$\phi(z^{\beta^0}, z^b; z_{\beta^0}^*, z_b^*) = \pm z^m z_m^* + \hat{\phi}(z^{\beta^0}, z^{b \leq m-1}; z_{\beta^0}^*, z_{b \leq m-1}^*) \quad (21)$$

Clearly we can proceed by iteration until we reach

$$\phi(z^{\beta^0}, z^b; z_{\beta^0}^*, z_b^*) = \sum_{b=1}^m (\pm) z^b z_b^* + \hat{\phi}(z^{\beta^0}, z^{\leq 0}) \quad (22)$$

But now we know

$$\frac{\partial^2 \phi}{\partial z^{\beta^0} \partial z_{\beta^0}^*} = 0 \quad (23)$$

so that ϕ must be a holomorphic function plus its Hermitian conjugate:

$$\hat{\phi}(z^{\beta,0}, z^{\alpha,*}) = h(z^{\beta,0}) + h^+(z^{\alpha,*}) \quad (24)$$

Finally we define

$$z^0 \equiv h(z^{\beta,0}) \quad z_0^* \equiv h^+(z^{\alpha,*}) \quad (25)$$

in terms of which

$$\phi = \sum_{b=1}^m (\pm) z^b z_b^* + z^0 + z_0^* \quad (26)$$

The final step involves calculating the kinetic energy terms derived from ϕ (26):

$$\mathcal{L} \ni \frac{\partial^2 G}{\partial z^i \partial z_j^*} \partial_r z^i \partial^r z_j^* \quad (27)$$

where

$$\begin{aligned} \frac{\partial^2 G}{\partial z^0 \partial z_0^*} &= 3/\phi^2 \\ \frac{\partial^2 G}{\partial z^0 \partial z_b^*} &= 3/\phi^2 [(\pm) z^b] \\ \frac{\partial^2 G}{\partial z^b \partial z_c^*} &= \left(-\frac{3}{\phi}\right) (-1) \delta_b^c + \frac{3}{\phi^2} (\pm z_b^*)(\pm z^c) = \\ &= \frac{3[\delta_b^c (z^0 + z_0^* + \sum_d (\pm) z^d z_d^*) + (\pm z_b^*)(\pm z^c)]}{\phi^2} \end{aligned} \quad (28)$$

It is evident that for the matrix $\partial^2 G / \partial z^i \partial z_j^*$ to be positive definite for all values of the field variables - so that we avoid ghosts - all the signs $(\pm)$ must be chosen negative. Hence the unique flat potential for many fields is

$$G = -3 \ln \phi \quad \phi = z^0 + z_0^* - \sum_{b=1}^m z^b z_b^* \quad (29)$$

which is of course the original $SU(m+1,1)/SU(m) \times U(1)$ Kähler potential.

We conclude this letter with some general comments. First, our programme for extracting low energy symmetries directly from general properties of the string can clearly be extended to more general variations in the background

fields. In particular, we can study changes in the dilaton field and, for the heterotic string, changes in the background gauge connection. The latter case allows our arguments to be extended to gauge non-singlet fields. Secondly, our type of analysis allows us to deduce some non-trivial string loop effects from purely topological considerations. Thirdly, we have shown that knowledge of the set of fields which have a flat potential at the tree level is sufficient to fix uniquely the Kähler potential of the four-dimensional theory, which must be of the no-scale $SU(n,1)/SU(n) \times U(1)$ type^{19),6)}. However, in our approach this extended non-compact symmetry appears accidental, and is presumably not a symmetry of the massive string modes, since it is not a subgroup of the Virasoro algebra, and the previous topological arguments have indicated that the potential is no longer flat when string loops are taken into account. It seems in any case likely that the flat $SU(n,1)/SU(n) \times U(1)$ structure is not maintained by radiative corrections in the low energy effective theory, although the masslessness of gauge non-singlet fields is apparently maintained²¹⁾ unless effects of the non-trivial gauge kinetic function are included.

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Σ	genus	dim T	order of perturbation theory
Sphere	0	0	tree level
Torus	1	2	one loop
$n \geq 2$ handles	$n \geq 2$	$6n - 6$	$n \geq 2$ loops

Table

Note also that by the Riemann-Roch theorem
 $\dim (\text{conformal Killing vectors}) = \dim (T(\Sigma)) + 6(1-n).$

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