FOUR JET PRODUCTION AT HADRON COLLIDERS

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ABSTRACT

We examine some of the characteristic features of four jet production at hadron colliders given by QCD ($2 \rightarrow 4$) subprocesses. We find that the universal behaviour of parton scattering subprocesses - which is a good approximation for ($2 \rightarrow 2$) subprocesses and to a lesser extent for ($2 \rightarrow 3$) subprocesses - is violated more significantly in the case of the ($2 \rightarrow 4$) parton scattering processes. We suggest methods for testing if the four jet data sample receives additional contributions from double hard parton scattering.

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Higher order QCD processes give important measurable signatures at hadron colliders. For example two, three and four jet production has been observed by the UA1 and the UA2 experiments at the CERN SppS [1,2]. Unfortunately the higher order QCD calculations for jet physics are rather cumbersome and require non-trivial computational effort. Recently, however, some progress has been reported. The cross-sections of all $(2 \rightarrow 4)$ parton subprocesses have been calculated at the leading $O(\alpha_s^4)$ order [3-8]. In addition the matrix elements squared for all $(2 \rightarrow 2)$ and $(2 \rightarrow 3)$ parton subprocesses have been obtained at $O(\alpha_s^3)$ in n -dimensions by Ellis and Sexton [9]. Therefore the analysis of the complete $O(\alpha_s^3)$ QCD corrections to jet production as well as realistic estimates of four jet production rates can now be attempted.

In this letter we shall examine some of the characteristic features of the production of four hard jets in $p\bar{p}$ collisions at CERN SppS and Tevatron energies. Four jet production rates have already been estimated [10]. However, this calculation was based on the assumption of approximately universal behaviour of $(2 \rightarrow 4)$ subprocesses. We shall test the validity of this assumption by comparing the approximate calculation with the exact result.

Four jets can also be produced by double parton scattering [11-12]. We shall investigate ways of distinguishing this production mechanism from the standard QCD $(2 \rightarrow 4)$ mechanism.

Our analysis will include the contributions of all the QCD $(2 \rightarrow 4)$ subprocesses. We shall use the analytic expressions for the subprocess cross-sections obtained in refs. [3-6]. It is worth mentioning that each of the subprocess cross-sections has been calculated by at least two different methods and the results have been confronted with many consistency checks.

We shall only consider the production of well separated hard jets, specified by appropriate cuts. In order to bring the calculation close to the experimental situation we shall use a set of cuts which are typical of the jet measurements in the UA2 and UA1 experiments. In particular we shall assume for the final jets

the minimum transverse momentum

$$p_{\perp}^i > p_{\perp}^c \quad i=1,2,3,4 \quad (1a)$$

the maximum pseudorapidity

$$|\eta_i| < \eta_c \quad i=1,2,3,4 \quad (1b)$$

the minimum total transverse energy

$$\sum_{i=1}^4 E_{\perp}^i > E_{\perp}^c \quad (1c)$$

and the minimal angular separation

$$\cos(\theta_{ij}) < \omega_c \quad i,j=1,2,3,4 \quad (1d)$$

where θ_{ij} is the opening angle between the three momentum of the jets i,j . We shall assume that the effects due to the fragmentation of the final partons into hadrons are negligible and furthermore we assume perfect detection efficiency for jets. All the cross-sections in this letter are calculated using the

Duke & Owens structure functions (set I, $\Lambda = 200$ MeV) and are given in nanobarns. We shall use for the scale of the evolution of the structure functions and the strong coupling constant $\alpha_s(Q^2)$ the average transverse energy

$$Q^2 = (E_{\perp}/n)^2 \quad (2)$$

where n is the total number of large transverse energy jets in the final state and $E_{\perp}$ denotes the total transverse energy.

Let us consider first the relative magnitudes of the contributions of the various QCD subprocesses. In Table 1 we have listed the individual contributions of all the 28 distinct subprocesses. The first column of cross-section values are for pure parton processes at total incoming energy $\sqrt{s} = 100$ GeV. The second column of cross-section values are contributions to the physical $p\bar{p}$ collision cross-section at $\sqrt{s} = 630$ GeV. Summation over the allowed final light quark flavours has been performed with $n_F = 5$. All the entries of Table 1 have been calculated using the cuts of eqs.(1a - 1d) with the cutoff values

$$p_{\perp}^c = 15 \text{ GeV}, \quad \eta_c = 0.8, \quad E_{\perp}^c = 70 \text{ GeV} \quad \text{and} \quad \omega_c = 0.643 \quad (3)$$

We have found orders of magnitude differences in the values of the individual subprocess contributions. Note in particular the significant dominance of the six gluon subprocess and the relative suppression of the six quark as well as the annihilation type subprocesses.

In Table 2 we have listed cross-section values for two, three and four jet production using the jet cuts of eqs.(1a - 1d). At $\sqrt{s} = 630$ GeV the cutoff values of eq.(3) have been used while at Tevatron energy $\sqrt{s} = 1.6$ TeV we have used

$$p_{\perp}^c = 30 \text{ GeV}, \quad \eta_c = 1.6, \quad E_{\perp}^c = 140 \text{ GeV} \quad \text{and} \quad \omega_c = 0.643 \quad (4)$$

At $\sqrt{s} = 630$ GeV the cross-section ratios of two, three and four jet production are in qualitative agreement with the measured values [2]. Fig.1 shows the inclusive jet transverse momentum distributions for the two, three and four jet cross-sections. As expected, the distributions are similar in shape.

In Table 2 we have also listed cross-section values obtained in the approximation of assuming that the scattering subprocesses behave universally and that their relative normalisation is given by the naive color factor $9/4$ [13].

$$d\sigma_{gg} : d\sigma_{gq} : d\sigma_{q\bar{q}} = (9/4)^2 : (9/4) : 1 \quad (5)$$

Assuming the validity of the relations in eq.(5), a universal parton structure function can be defined (the so-called UA1 structure function)

$$F(x) = G(x) + (4/9) \cdot (Q(x) + \bar{Q}(x))$$

and the physical cross – section is obtained as

$$d\sigma_{PP} \approx (9/4)^2 \int F(x_1)F(x_2)dx_1dx_2 d\hat{\sigma}^{q\bar{q}}(\hat{s})/(x_1x_2) , \quad (6)$$

where $G(x)$, $Q(x)$ denote gluon and quark structure functions, respectively. For two and three jet production at collider energies this approximation is acceptable¹ [12]. For four jet production we see (Table 2) that the approximation is valid at best to within a factor of 2. In more detailed distributions the discrepancy may become even more severe. This stronger violation of the universality approximation in the case of $(2 \rightarrow 4)$ subprocesses can also be seen directly from the entries of Table 1.

The $p_{\perp}$ distributions of jets produced via $(2 \rightarrow 2)$, $(2 \rightarrow 3)$ and $(2 \rightarrow 4)$ subprocesses are very similar in shape (see Fig.1). To illustrate some more detailed correlations in the four jet final states we can consider for example the p_{out} and acoplanar momentum p_{acop} distributions (Figs.2 and 3). p_{out} is defined [10] as the transverse momentum flow out of the plane given by the beam direction and the direction of the transverse momentum vector of the jet with the largest transverse momentum:

$$p_{out} = \sum_{i=1}^3 |(\vec{n} \cdot \vec{p}_i)|/2 \quad (7)$$

where $\vec{n}$ is the vector normal to the plane of the vectors $\vec{p}_{beam}$ and $\vec{p}_{\perp}^{max}$. Fig.2 shows the p_{out} distribution for the four jet final jet, with the three jet contribution also shown for comparison. The p_{out} distribution of four jet final states has its maximum at a somewhat larger p_{out} value than the maximum of the p_{out} distribution for three jet production. Acoplanar momentum has been invented as a topological variable for quantifying four jet final states [14], and is related to the minimum momentum flowing out of a plane:

$$p_{acop} = \min_{\{\vec{n}\}} \{ \sum_i |(\vec{p}_i \cdot \vec{n})|/2 \} \quad (8)$$

where $\vec{n}$ is a unit vector chosen to minimise the right – hand – side in the above expression. Fig.3 shows the acoplanarity distribution for the four jet final state. Note that at hadron colliders it is necessary to boost each event to the parton centre of mass system. This introduces an inherent ambiguity due to the minimum bias contribution, and therefore the measurement of the acoplanarity distribution may have practical difficulties. The curves in Fig.2 and Fig.3 are smooth. This is to be

¹ If in eq.(6) the gluon – gluon scattering cross – section $d\hat{\sigma}^{gg}(\hat{s})$ is used instead of the quark – quark cross – section, the approximation becomes better due to the dominance of the gluon – gluon subprocess.

expected since we are considering the phase space region of four hard, well – separated jets. Note that the acoplanar momentum is strongly limited by the small value of the pseudo rapidity cut η_c (see eqs.(1b) and (3))

At very high hadron collision energies relatively hard events can be produced with small x – values where the density of the partons is very high. If the relative density of the partons [11,12]

$$W(x, Q^2) = \alpha_s \cdot xG(x, Q^2) / (Q^2 R^2), \quad (9)$$

where R is the hadron radius, is close to one factorisation breaks down and multiple hard scattering will occur. With the cutoff values (3), $W(x, Q^2) \ll 1$, and therefore we do not expect much deviation from the perturbative QCD calculation. However with the increase of the collision energy of the incoming hadrons at fixed parton collision subenergy, the probability that two hard scatterings will occur becomes larger.¹ Unfortunately, quantitative estimates of the production rate of four jet events by this mechanism require knowledge of the double parton wave function and are therefore rather uncertain. Although model calculations have been attempted [12], it is perhaps more natural to test for the presence of double scattering experimentally by comparing the predictions of the $O(\alpha_s^4)$ perturbative QCD calculation with the data.

If the double hard scattering mechanism does give a sizeable contribution it can easily be distinguished from the leading order perturbative QCD contribution. The angular correlations of the jets and the x – dependence are significantly different in the two cases. For example, the p_{out} distribution above some p_{out}^{min} threshold value will fall very steeply because of the steep x – dependence of double hard scattering and the events will be dominated by configurations where [12]

$$\begin{aligned} \vec{p}_{1\perp} + \vec{p}_{2\perp} &= 0, \quad \vec{p}_{3\perp} + \vec{p}_{4\perp} = 0 \quad \text{and} \quad \vec{p}_1 \cdot \vec{p}_3 \approx 0 \\ \min(|\vec{p}_{1\perp}|, |\vec{p}_{3\perp}|) &\approx p_{out}^{min}. \end{aligned} \quad (10)$$

It may be useful to consider also the azimuthal distribution of the jets. For example, Fig.4 shows the distribution in the angle ϕ_{min} , defined as the minimum azimuthal angle between the three momentum of the jet with largest transverse momentum and the three momenta of the remaining jets. In the region $\phi_{min} > 90^\circ$ the double hard scattering mechanism cannot contribute for kinematical reasons (up to jet broadening and "intrinsic" transverse smearing corrections). Systematic errors could in principle be reduced by adjusting the normalization of the $O(\alpha_s^4)$ QCD calculation to fit the data in the region $\phi_{min} > 90^\circ$, allowing a possible excess of events to be identified in the complementary region.

We have demonstrated that the complete evaluation of all $(2 \rightarrow 4)$ parton subprocess cross – sections can provide a useful link between the theory and the data. It appears that the contributions from double hard scattering can be readily separated from the $O(\alpha_s^4)$ QCD contributions. We note that the quantitative experimental study of three jet production rates has

¹ In a more ambitious use of multiple hard scattering it has been suggested that minimum bias events could be described by this mechanism [15].

required the knowledge of some properties of four jet final states [2]. The computer program used to calculate the four jet cross-sections has required ≈ 0.1 sec IBM 3090 CPU time to generate one event. Thus for a preliminary analysis the approximate formula given by the universality assumption may be useful because of the much shorter computer time involved.

Finally, we remark that in future hadron collider experiments it may be very important to be able to estimate reliably the QCD four jet background, in particular in considering the production of new heavy bosons (Higgs bosons, technihadrons, composite gauge bosons etc.) [16].

Acknowledgement. We are grateful for helpful conversations with K. Einsweiler, P. Jenni and W. Scott.

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FIGURE CAPTIONS

- Fig.1 Inclusive jet transverse momentum distribution for two, three and four jet production at $\sqrt{s} = 630\text{GeV}$ with the cutoff values of eq.(3).
- Fig.2 p_{out} distribution in inclusive production of three and four jets at $\sqrt{s} = 630\text{GeV}$ with the cutoff values of eq.(3).
- Fig.3 p_{acop} distribution in four jet production at $\sqrt{s} = 630\text{GeV}$ with the cutoff values of eq.(3).
- Fig.4 ϕ_{min} distribution in four jet production at $\sqrt{s} = 630\text{GeV}$ with the cutoff values of eq.(3).

TABLE CAPTIONS

- Table 1 Cross-section values of parton subprocesses at $\sqrt{s} = 100\text{GeV}$ and at cutoff values of eq.(3) (first column) and individual parton subprocess contributions to the physical pp cross-section at $\sqrt{s} = 630\text{GeV}$ with the same cuts (second column).
- Table 2 Two, three and four jet cross-section values at $Spp\bar{S}$ and Tevatron energies using the cutoff values of eqs.(3) and (4) respectively. Cross-section values obtained with use of the approximate formula of eq.(6) are also given. The error bars represent the errors of numerical integration.

subprocesses	$\Delta\sigma^{\text{parton}}_i(\text{nb})$ $\sqrt{s}=100\text{GeV}$	$\Delta\sigma^{\text{pp}}_i(\text{nb})$ $\sqrt{s}=630\text{GeV}$
1. $g + g \rightarrow g + g + g + g$	1.12	0.434
2. $g + g \rightarrow g + g + q + \bar{q}$	0.251	0.101
3. $g + g \rightarrow q + \bar{q} + q_1 + \bar{q}_1$	0.152E-01	0.607E-02
4. $g + g \rightarrow q + \bar{q} + q + q$	0.264E-02	0.998E-03
5. $g + q \rightarrow g + g + g + q$	0.287	0.162
6. $g + q \rightarrow g + q + q_1 + \bar{q}_1$	0.425E-01	0.234E-01
7. $g + q \rightarrow g + q + q + q$	0.844E-02	0.479E-02
8. $q + g \rightarrow g + g + g + q$	0.275	0.160
9. $q + g \rightarrow g + q + q_1 + \bar{q}_1$	0.401E-01	0.232E-01
10. $q + g \rightarrow g + q + q + q$	0.811E-02	0.475E-02
11. $q + \bar{q} \rightarrow g + g + g + g$	0.127E-01	0.440E-02
12. $q + \bar{q} \rightarrow g + g + q + \bar{q}$	0.374E-01	0.115E-01
13. $q + \bar{q} \rightarrow g + g + q_1 + \bar{q}_1$	0.157E-01	0.500E-02
14. $q + \bar{q} \rightarrow q + \bar{q} + q + q$	0.180E-02	0.554E-03
15. $q + \bar{q} \rightarrow q + \bar{q} + q_1 + \bar{q}_1$	0.687E-02	0.218E-02
16. $q + \bar{q} \rightarrow q_1 + \bar{q}_1 + q_1 + \bar{q}_1$	0.358E-03	0.110E-03
17. $q + q \rightarrow q_1 + \bar{q}_1 + q_2 + \bar{q}_2$	0.187E-02	0.580E-03
18. $q + q \rightarrow g + g + q + \bar{q}$	0.727E-01	0.415E-02
19. $q + q \rightarrow q + q + q + \bar{q}$	0.139E-02	0.803E-04
20. $q + q \rightarrow q + q + q_1 + \bar{q}_1$	0.484E-02	0.291E-03
21. $q + q_1 \rightarrow g + g + q + \bar{q}_1$	0.805E-01	0.110E-01
22. $q + q_1 \rightarrow q + q_1 + q + \bar{q}_1$	0.134E-02	0.197E-03
23. $q + q_1 \rightarrow q + q_1 + q_1 + \bar{q}_1$	0.127E-02	0.185E-03
24. $q + q_1 \rightarrow q + q_1 + q_2 + \bar{q}_2$	0.387E-02	0.580E-03
25. $q + q_1 \rightarrow g + g + q + \bar{q}_1$	0.991E-01	0.351E-01
26. $q + q_1 \rightarrow q + \bar{q}_1 + q + \bar{q}_1$	0.158E-02	0.570E-03
27. $q + q_1 \rightarrow q + \bar{q}_1 + q_1 + \bar{q}_1$	0.149E-02	0.542E-03
28. $q + q_1 \rightarrow q + \bar{q}_1 + q_2 + \bar{q}_2$	0.452E-02	0.167E-02
29. total $gg \rightarrow 4 \text{ jets}$	1.39	0.542
30. total $gq \rightarrow 4 \text{ jets}$	0.338	0.190
31. total $qg \rightarrow 4 \text{ jets}$	0.323	0.188
32. total $q\bar{q} \rightarrow 4 \text{ jets}$	0.166	0.165E-01
33. total $qq \rightarrow 4 \text{ jets}$	0.183	0.622E-01

Table 1

	$\sqrt{s} = 630 \text{ GeV}$		$\sqrt{s} = 1.6 \text{ TeV}$	
	$\Delta\sigma(\text{nb})$	$\Delta\sigma^{\text{aPP}}(\text{nb})$	$\Delta\sigma(\text{nb})$	$\Delta\sigma^{\text{aPP}}(\text{nb})$
2jet	71.1 ± 1.2	59 ± 1.1	60.3 ± 1.2	51.5 ± 1.1
3jet	19.7 ± 0.4	14.8 ± 0.8	25.4 ± 0.8	19.6 ± 0.8
4jet	1.00 ± 0.04	0.54 ± 0.02	$2.22 \pm .25$	1.2 ± 0.1

Table 2

Fig. 1

Fig. 2

Fig. 3

Fig. 4