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ANALYSIS OF THE ENERGY DEPENDENCE OF PROTON-PROTON TOTAL CROSS-SECTIONS IN TERMS OF $O(4)$ EXPANSIONS

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ABSTRACT

The energy dependence of the spin dependent proton-proton total cross-sections from threshold to 800 MeV laboratory energy is described using $O(4)$ expansions. No statistical evidence for dibaryons is found.

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INTRODUCTION

A recent article^[1] was devoted to the analysis of proton-proton scattering data between 0 and 800 MeV. Use was made of two-variable expansions of scattering amplitudes, provided by the representation theory of the rotation group $O(4)$. For a discussion of these expansions we refer to previous articles (Ref. 1-3 and references therein). Here let us just mention that the angular dependence is contained in $O(3)$ spherical harmonics or rotation matrices and its treatment coincides with that of partial-wave analysis. The energy dependence is contained in the reduced $O(4)$ group rotation matrices $d_{j_1 j_2}^{n\nu}(\alpha)$. The $O(4)$ expansions have many desirable physical features [1-3] and can be considered to be a tool for energy dependent phase-shift analysis over reasonably large regions of energy, e.g. from threshold to $T_L = 800$ MeV for nucleon-nucleon scattering^[1]. The approach of our previous article^[1] was a hybrid one, in that we did not return all the way to the large amount of experimental data on proton-proton scattering but instead, made use of the results of available phase-shift analyses^[4]. The $O(4)$ expansions were used to fit the energy dependence of the pp partial-wave amplitudes $A_{\ell s \ell' s'}^j(\alpha)$.

The purpose of this article is to perform a direct analysis of pp experimental data, namely to fit the low and intermediate energy spin-dependent total cross-sections from 0 to 800 MeV and to comment on fits over a larger region, from threshold to 1.5 GeV.

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2. THE O(4) EXPANSIONS OF TOTAL CROSS-SECTIONS

In previous articles [1-3] we have presented and used O(4) expansions of scattering amplitudes $M_{s\sigma's'}(\alpha, \delta, \phi)$ in the initial spin-final spin formalism [5-7]. The energy variable α is defined by the relation

$$\sin \alpha = \sqrt{\frac{w-4m^2}{w_M-4m^2}}, \quad w = (p_1 + p_2)^2, \quad (2.1)$$

$$0 \leq w \leq w_M, \quad 0 \leq \alpha \leq \frac{\pi}{2}$$

where m is the proton mass and w_M is some suitably chosen energy cut-off (the energy up to which we wish to fit the data). Restricting to forward scattering and performing elementary simplifications, we write the O(4) expansion as

$$M_{s\sigma's'}(\alpha, 0, 0) = \sum_{j=0}^{\infty} \sum_{n=\max(j,s)}^{\infty} \frac{\min(j,s)}{[(n+1)^2 - v^2]^{n/2}} T_{jss'\sigma'}^{nv} d_{j\sigma}^{nv*}(\alpha) \quad (2.2)$$

where s, σ (s', σ') are the final (initial) state spin and spin projection; j is the total angular momentum and the O(4) d-functions $d_{j\sigma}^{nv}(\alpha)$ are given in the literature [1,3,8]. The singlet and two triplet total cross-sections are given by the formulas

$$\begin{aligned} \sigma_S(\alpha) &= \frac{4\pi}{k} \operatorname{Im} M_{SS}(\alpha, 0), \\ \sigma_{T0}(\alpha) &= \frac{4\pi}{k} \operatorname{Im} M_{00}(\alpha, 0), \\ \sigma_{T1}(\alpha) &= \frac{4\pi}{k} \operatorname{Im} M_{11}(\alpha, 0) \end{aligned} \quad (2.3)$$

where we have put $M_{SS} \equiv M_{0000}$, $M_{\sigma\sigma} \equiv M_{1\sigma 1\sigma}$ ($\sigma = 0$ or 1) and the c.m.s. three momentum is

$$k = \frac{1}{2} \sqrt{w_M - 4m^2} \sin \alpha. \quad (2.4)$$

At threshold $\sigma_S(0)$ is some nonzero constant, whereas $\sigma_{T0}(\alpha)$ and $\sigma_{T1}(\alpha)$ vanish like k for $\alpha \rightarrow 0$.

The one-variable expansion (2.2) cannot be directly used since the set of functions $d_{j\sigma}^{nv}(\alpha)$ is overcomplete. To remedy this we reexpand these O(4) functions in terms of a suitable basis, with j so chosen as to provide the correct threshold behavior, and then absorb the j summation in (2.2).

Let us first consider proton-proton singlet scattering. Expansion (2.2) reduces to

$$M_{SS}(\alpha, 0) = \sum_{j=0,2,\dots}^{\infty} \sum_{n=j,j+2,\dots}^{\infty} \frac{(n+1)^2 m_0}{j_0 0 0 d_{j00}^{n0*}}(\alpha). \quad (2.5)$$

The proper threshold behavior of $\sigma_S(\alpha)$ is assured by reexpanding as follows

$$d_{j00}^{n0*}(\alpha) = \sum_{k=0}^{\infty} B_k^{n,2k+1} d_{100}^{2k+10*}(\alpha) \quad (2.6)$$

Finally, we obtain the singlet expansion formulas:

$$\sigma_S(\alpha) = \frac{4\pi}{k} \sum_{N_0=1}^{\infty} \operatorname{Re} F_S^{N_0} i d_{100}^{N_0 0}(\alpha) \quad (2.7)$$

$$\operatorname{Re} F_S^{N_0} = \sum_{j_e=0}^{\infty} \sum_{n_e=j_e}^{\infty} \frac{(n_e+1)^2}{i B_{j_e}^{n_e 0}} \operatorname{Im} S_{j_e}^{n_e} \quad (2.8)$$

$$B_{j_e}^{n_e 0} = \frac{4(N_0+1)^2}{3\pi} \int_0^{\pi/2} d_{j_e 0 0}^{n_e 0}(\alpha) d_{100}^{N_0 0}(\alpha) \sin^2 \alpha d\alpha \quad (2.9)$$

where we use the $O(4)$ singlet amplitudes^[1] $S_j^n \equiv \tau_{j00}^{n0}$ and the subscript e or 0 indicates whether a given label takes on even or odd values. Notice that $d_{100}^{n0}(\alpha)$ and B_j^{nN} are pure imaginary.

For proton-proton triplet scattering we use the $O(4)$ triplet amplitudes $\tau_{j\sigma}^{n\nu} = \tau_{j11\sigma}^{n\nu}/\sqrt{2j+1}$. The corresponding cross-sections σ_{T0} and σ_{T1} both vanish at threshold like $k \sim \sin \alpha$, hence we shall reexpand in terms of the basis functions $d_{200}^{n0}(\alpha)$. We drop all details of calculations which are straightforward and involve the use of recursion relations developed earlier^[1] both for the $O(4)$ d-functions and the $O(4)$ amplitudes $\tau_{j\sigma}^{n\nu}$. The final expansions of the triplet proton-proton total cross-sections are

$$\sigma_{T0}(\alpha) = \frac{4\pi}{k} \sum_{N_e=2}^{\infty} (\text{Im } F_{T0}^{N_e}) d_{200}^{N_e}(\alpha), \quad (2.10)$$

$$\sigma_{T1}(\alpha) = \frac{4\pi}{k} \sum_{N_e=2}^{\infty} (\text{Im } F_{T1}^{N_e}) d_{200}^{N_e}(\alpha). \quad (2.11)$$

The phenomenological parameters $F_{TK}^{N_e}$ ($K = 0, 1$) are related to the $O(4)$ amplitudes $\tau_{j\sigma}^{n\nu}$ as follows:

$$F_{T0}^{N_e} = \sum_{n_0=1}^{\infty} \sum_{n_e=1}^{\infty} (n_0+1)^2 B_{010}^{n_0} e_{T0}^{n_0} + \sum_{j_e=2}^{\infty} \sum_{n_e=j_e}^{\infty} \frac{\sqrt{2j_e+1}}{2j_e(j_e+1)} \times \quad (2.12)$$

$$\times \{ \sigma_{n_e}^{j_e} B_{j10}^{n-2} e_{j10}^{n-2} + \sigma_{n_e}^{j_e} B_{j10}^{n+2} e_{j10}^{n+2} + \rho_{n_e}^{j_e} B_{j10}^{n1} e_{j10}^{n1} \}$$

$$F_{T1}^{N_e} = \sum_{j_0=1}^{\infty} \sum_{n_0=j_0}^{\infty} \sqrt{2j_0+1} n_0(n_0+2) B_{j011}^{n_0} e_{T1}^{n_0} + \sum_{j_e=2}^{\infty} \sum_{n_e=j_e}^{\infty} \frac{\sqrt{2j_e+1}}{4j_e(j_e+1)} \times \quad (2.13)$$

$$\times \{ \sigma_{n_e}^{j_e} B_{j11}^{n-2} e_{j11}^{n-2} + \sigma_{n_e}^{j_e} B_{j11}^{n+2} e_{j11}^{n+2} + \rho_{n_e}^{j_e} B_{j11}^{n1} e_{j11}^{n1} \}$$

The subscripts e and 0 again indicate whether a certain label takes even or odd values, respectively. The auxiliary quantities, figuring in (2.12) and (2.13) are

$$\rho_n^j = 2(n+1)^2 [n(n+2)+j(j+1)] \quad (2.14)$$

$$\sigma_n^j = -[(n-2)(n-1)(n+1)(n+2)(n-j-1)(n-j)(n+j)(n+j+1)]^{1/2}$$

and

$$B_{010}^{n0} e = \frac{4(N+1)^2}{5\pi} \int_0^{\pi/2} d_{010}^{n,0*}(\alpha) d_{200}^{n,0}(\alpha) \sin^2 \alpha$$

$$B_{j10}^{n1N} e = \frac{4(N+1)^2}{5\pi} \int_0^{\pi/2} d_{j10}^{n,1*}(\alpha) d_{200}^{n,0}(\alpha) \sin^2 \alpha = B_{j,11}^{n,0N} e \quad (2.15)$$

$$B_{j,11}^{n,1N} e = \frac{4(N+1)^2}{5\pi} \int_0^{\pi/2} (d_{j,11}^{n,1*}(\alpha) - d_{j,11}^{n,1*}(\alpha)) d_{200}^{n,0}(\alpha) \sin^2 \alpha.$$

Notice that all the coefficients $B_{j1\sigma}^{n\nu N}$ are pure imaginary. Notice also that only even values of j contribute to σ_{T0} and thus only coupled triplet states contribute, as it should be. The expansion formulas for the total cross-sections are quite simple, namely (2.7), (2.10) and (2.11). The only expansion functions needed are $d_{100}^{n0}(\alpha)$ and $d_{200}^{n0}(\alpha)$. We have

$$d_{j00}^{n0}(\alpha) = e^{-i\frac{\pi}{2}j} \frac{j!}{2^j j!} \frac{(2j+1)(n-j)!}{(n+1)(n+j+1)!} \frac{1}{2} (\sin \alpha)^{j+1} C_{n-j}^{j+1}(\cos \alpha) \quad (2.16)$$

where $C_{n-j}^{j+1}(\cos \alpha)$ are Gegenbauer polynomials.

3. NUMERICAL ANALYSIS AND RESULTS

The experimentally measured spin-dependent pp total cross-sections $\sigma_0, \sigma_1 = -\Delta\sigma_T/2$, and $2(\sigma_1 + \sigma_2) = -\Delta\sigma_L$ are related to the singlet-triplet cross-sections by the relation

$$\begin{pmatrix} \sigma_0 \\ \sigma_1 \\ -\Delta\sigma_L \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 & 1 & 2 \\ -1 & 1 & 0 \\ -2 & -2 & 4 \end{pmatrix} \begin{pmatrix} \sigma_S \\ \sigma_{T0} \\ \sigma_{T1} \end{pmatrix}. \quad (3.1)$$

This formula, together with the expansions (2.7), (2.10) and (2.11), forms the basis of a fit of observable quantities in terms of the phenomenological expansion coefficients $\text{Re } F_S^N$, $\text{Im } F_{T0}^N$ and $\text{Im } F_{T1}^N$. Two avenues are open to us. The first is to perform a pure O(4) fit to the data and to calculate the expansion coefficients on the basis of best fit criteria. The second possibility is to consider the O(4) expansions as describing a background, upon which we superimpose a resonant amplitude, parametrized by Breit-Wigner resonances. A comparison of the two approaches (quality of fit, number of parameters needed, stability, etc.) should then provide us with an objective and model independent evaluation of the role of resonances, in particular dinucleons, in the considered energy regions. Once we obtain the expansion coefficients from a fit, we can invert formula (3.1) and calculate the physically more directly interpretable singlet and triplet cross-sections.

The first type of fit performed is illustrated on Fig. 1a-1f. The three observables σ_0 , σ_1 and $-\Delta\sigma_L$ were fitted simultaneously in the energy region 0-800 MeV, using the O(4) expansions (2.7), (2.10) and (2.11) alone. The same value of the cut-off n_{max} was chosen for each of the three observables. The data

in this region consists of 84 experimental points (30 for σ_0 , 24 for σ_1 , and 30 for $-\Delta\sigma_L$). The criteria used to determine n_{max} (i.e. the total number of parameters $N_T = 3n_{\text{max}}$) are the usual statistical ones. These are the requirement that χ^2/DF (DF is the number of degrees of freedom) should have an acceptably small value and should stop decreasing as n_{max} is increased, that the global "visual" fit to the data should be a good one, that the values of the lower parameters in the fits, in particular $\text{Re } F_S^1$, $\text{Im } F_{T0}^2$ and $\text{Im } F_{T1}^2$, should remain stable when n_{max} is further increased, etc. The full curves on Fig. 1 correspond to a pure O(4) fit with $n_{\text{max}} = 5$. In this case we have $\chi^2/\text{DF} = 1.61$ (for $n_{\text{max}} = 4$ the value is unacceptable, namely $\chi^2/\text{DF} = 6.30$).

In the third column of Table 1 we give the best fit values of the parameters $\text{Re } F_S^N$, $\text{Im } F_{T0}^N$ and $\text{Im } F_{T1}^N$ for $n_{\text{max}} = 5$. The values of the coefficients decrease with N , the statistical errors in the determination of these parameters are, in general, much smaller than the values. Notice the correct threshold behavior of the total cross-sections on Figs. 1d-1f, namely σ_{T0} and σ_{T1} vanish like $k \sim \sqrt{T_{\text{lab}}}$ for $T_{\text{lab}} \rightarrow 0$ whereas σ_S tends to a large positive value (presumably due to the presence of a quasibound pp state at $T_{\text{lab}} \sim 0$, the isospin $I=1$ companion of the deuteron, where T_{lab} is the kinetic energy of the incident proton in the laboratory system.

The dashed curves on Fig. 1 and the numbers in column 4 of Table 1 do not correspond to direct data fits. These curves and numbers were obtained from the O(4) amplitudes, reconstructed previously^[1] from an analysis of the energy dependence of pp partial-wave amplitudes in the same energy region. The corresponding O(4) amplitudes $\text{Im } S_j^N$ and $\text{Re } T_{j\sigma}^N$ are given in Table 2. Inserted into (2.8), (2.12) and (2.13), they provide "predictions" for the total cross-sections. In other words, the closeness of the two curves on Fig. 1 provides

a test of the compatibility of the total cross-section data with the data previously used in phase-shift analyses. On the other hand, measurements of the total cross-sections are seen to provide equations and compatibility conditions for the original $O(4)$ amplitudes S_j^N and T_{jo}^{NV} .

A second type of fit to the data was attempted, namely one introducing a Breit-Wigner resonance at $T = 580$ MeV in the singlet amplitude $M_{SS}(\alpha, 0)$, corresponding to a possible 1D_2 dinucleon resonance that has been proposed^[9] and discussed^[10] in the literature. The width and the height of the resonance were left as free parameters, along with the $O(4)$ parameters $\text{Re } F_S^N$, $\text{Im } F_{T0}^N$ and $\text{Im } F_{T1}^N$ for the background. The amount of experimental points turned out to be insufficient to permit a determination of the free parameters and the fit was in no way better than a pure $O(4)$ fit. The evidence for a resonance obtained in this model independent manner is thus at best inconclusive. Further improvements in this direction can only be made once more data on the three independent proton-proton total cross-sections become available.

A similar fit was performed using the same $O(4)$ expansion formulas in a larger energy region, namely from threshold to $T_M = 1.5$ GeV. All together 157 data points are available in this region, about evenly distributed between σ_0 , σ_1 and $-\Delta\sigma_L$, very few of them above 1.1 GeV. The value of χ^2/DF reached its minimum ($\chi^2=2.13$) in a pure $O(4)$ fit for $n_{\max} = 11$, i.e. 33 free parameters. The visual fit to the data is quite good, but the $O(4)$ expansions introduce physically unjustified oscillations in regions where data are scarce (specially at the upper end of the energy region). Several dibaryons have been proposed in this energy region^[9-11], in particular the 1D_2 , 3F_3 and 1G_4 diproton states at masses 2.15, 2.22 and 2.44 GeV, respectively. We have tried to obtain a fit to the data using an $O(4)$ background plus Breit-Wigner resonance at 580 and 760 MeV, leaving the

widths and amplitudes of the resonances as free parameters. The fit was no better than a pure $O(4)$ fit and the amplitudes of the resonances were compatible with zero. Thus again no evidence in favour of the existence of resonances is obtained. The existing data (and probably any set of data involving only total cross-sections) is insufficient to provide conclusions about the presence or absence of dinucleon resonances. We do not present the fits nor the obtained expansion coefficients here. For further details and discussions see Ref. 12.

4. CONCLUSIONS

The $O(4)$ expansions used alone provide a relatively good fit to the total cross-section data from 0 to 800 Mev and an acceptable one from 0 to 1500 Mev. In the larger region, and even more so, in the region from 0 to 3 GeV, the expansions begin to introduce nonphysical oscillations between the rather sparse data points and at the ends of the intervals. The reason for these oscillations is rather obvious: for $0 \leq \alpha \leq \pi/2$ the $O(4)$ functions $d_{100}^{NO}(\alpha)$ pass through zero $(N+1)/2$ times, the functions $d_{200}^{NO}(\alpha)$, $N/2$ times.

The fits to the data are not improved by adding Breit-Wigner resonances to the $O(4)$ amplitudes. Moreover, the parameters of the resonances are very poorly defined. No conclusions on the existence of diprotons can be obtained from the total cross-sections alone. The other measured quantities do not, as yet, provide too much insight either [10].

The problem of oscillations in experimental quantities, introduced by expansions in terms of $O(4)$ d-functions, will be discussed in a future publication. We recall here that the original expansions were performed in terms of representations of the Lorentz group $O(3,1)$ [2]. The expansions involved an integral, in addition to discrete sums. The replacement of $O(3,1)$ expansions (for all energies) by $O(4)$ expansions for a finite energy region leads to a "discretization" [3] of the expansions. The alternative approach that we are presently considering involves an imbedding of $O(3,1)$ into an $SU(3,1)$ group. This provides expansions in terms of Laguerre polynomials depending on the energy. These polynomials are orthogonal over an infinite half line, hence no energy cut-off is involved and the n oscillations in each basis function occur over an infinite region. The formalization of the above considerations is in progress.

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TABLE 1. Expansion coefficients obtained from present fit to total cross-sections and from the O(4) amplitudes of Ref. 1.

	N	FIT	O(4) PREDICTION
$\text{Re } F_S^N$	1	3.1495	3.0496
	3	-1.1740	-1.0689
	5	-.058474	.057300
	7	1.1794	1.4465
	9	1.2184	1.8804
$\text{Im } F_{T0}^N$	2	-2.8385	-2.8442
	4	.63529	.68748
	6	-.0030353	-.219600
	8	-.38367	-.16606
	10	.12863	.01388
$\text{Im } F_{T1}^N$	2	-4.8317	-4.3355
	4	.22587	.22896
	6	-.46112	-.41912
	8	-.49261	-.73590
	10	.56243	.53740

TABLE 2. The $O(4)$ amplitudes used in Table 1, obtained from the analysis of Ref. 1.

Number of $O(4)$ amplitudes for $J=0$ equal 8	
$\text{Im } S(0,0) = .31004615 \pm .00262730$	
$\text{Im } S(2,0) = -.01115200 \pm .00075737$	
$\text{Im } S(4,0) = .03395264 \pm .00039785$	
$\text{Im } S(6,0) = .02286615 \pm .00027020$	
$\text{Im } S(8,0) = .01578368 \pm .00020363$	
$\text{Im } S(10,0) = .00918582 \pm .00014866$	
$\text{Im } S(12,0) = .00269824 \pm .00008968$	
$\text{Im } S(14,0) = .00309195 \pm .00005480$	
Number of $O(4)$ amplitudes for $J=2$ equal 6	
$\text{Im } S(2,2) = -.11212876 \pm .00095642$	
$\text{Im } S(4,2) = -.00914926 \pm .00039038$	
$\text{Im } S(6,2) = .00993293 \pm .00024520$	
$\text{Im } S(8,2) = -.00223705 \pm .00016521$	
$\text{Im } S(10,2) = -.00093672 \pm .00009374$	
$\text{Im } S(12,2) = .00062671 \pm .00002820$	
Number of $O(4)$ amplitudes for $J=4$ equal 5	
$\text{Im } S(4,4) = .01988410 \pm .00048376$	
$\text{Im } S(6,4) = -.00540836 \pm .00017403$	
$\text{Im } S(8,4) = .00180425 \pm .00007021$	
$\text{Im } S(10,4) = -.00047070 \pm .00002575$	
$\text{Im } S(12,4) = .00005920 \pm .00000541$	
Number of $O(4)$ amplitudes for $J=6$ equal 3	
$\text{Im } S(6,6) = -.00010062 \pm .00000921$	
$\text{Im } S(8,6) = -.00003651 \pm .00000377$	
$\text{Im } S(10,6) = -.00000525 \pm .00000154$	

Number of $O(4)$ amplitudes for $J=0$ equal 6	
$\text{Re } T(1,0,0,0) = .12305846 \pm .00120154$	
$\text{Re } T(3,0,0,0) = -.02055011 \pm .00044585$	
$\text{Re } T(5,0,0,0) = .00183883 \pm .00025476$	
$\text{Re } T(7,0,0,0) = .00056612 \pm .00016259$	
$\text{Re } T(9,0,0,0) = .00080273 \pm .00008443$	
$\text{Re } T(11,0,0,0) = .00035092 \pm .00003625$	
Number of $O(4)$ amplitudes for $J=2$ equal 5	
$\text{Re } T(2,1,2,0) = .01341150 \pm .00030902$	
$\text{Re } T(4,1,2,0) = -.00006075 \pm .00015601$	
$\text{Re } T(6,1,2,0) = .00024325 \pm .00008779$	
$\text{Re } T(8,1,2,0) = .00012419 \pm .00004405$	
$\text{Re } T(10,1,2,0) = -.00000383 \pm .00001583$	
Number of $O(4)$ amplitudes for $J=4$ equal 5	
$\text{Re } T(4,1,4,0) = -.00064655 \pm .00002171$	
$\text{Re } T(6,1,4,0) = -.00010145 \pm .00001288$	
$\text{Re } T(8,1,4,0) = -.00004371 \pm .00000835$	
$\text{Re } T(10,1,4,0) = -.00000398 \pm .00000467$	
$\text{Re } T(12,1,4,0) = .00000054 \pm .00000177$	
Number of $O(4)$ amplitudes for $J=6$ equal 5	
$\text{Re } T(6,1,6,0) = .00006364 \pm .00000368$	
$\text{Re } T(8,1,6,0) = .00001042 \pm .00000219$	
$\text{Re } T(10,1,6,0) = .00000208 \pm .00000144$	
$\text{Re } T(12,1,6,0) = .00000110 \pm .00000082$	
$\text{Re } T(14,1,6,0) = -.00000009 \pm .00000031$	
Number of $O(4)$ amplitudes for $J=2$ equal 5	
$\text{Re } T(2,0,2,1) = .01794177 \pm .00030523$	
$\text{Re } T(4,0,2,1) = .00182048 \pm .00015729$	
$\text{Re } T(6,0,2,1) = .00079868 \pm .00009049$	
$\text{Re } T(8,0,2,1) = .00045562 \pm .00004612$	
$\text{Re } T(10,0,2,1) = .00000075 \pm .000001670$	

Number of O(4) amplitudes for J=4 equal 3	
Re T(4,0,4,1) = -.00031603 ± .00001861	
Re T(6,0,4,1) = -.00001676 ± .00000763	
Re T(8,0,4,1) = -.00000281 ± .00000266	
Number of O(4) amplitudes for J=6 equal 4	
Re T(6,0,6,1) = .00003075 ± .00000341	
Re T(8,0,6,1) = -.00000533 ± .00000181	
Re T(10,0,6,1) = .00000196 ± .00000097	
Re T(12,0,6,1) = .00000029 ± .00000036	
Number of O(4) amplitudes for J=1 equal 5	
Re T(1,1,1,1) = .09047773 ± .00038187	
Re T(3,1,1,1) = -.00088195 ± .00012687	
Re T(5,1,1,1) = -.00103094 ± .00006883	
Re T(7,1,1,1) = .00028529 ± .00004063	
Re T(9,1,1,1) = -.00034192 ± .00001562	
Number of O(4) amplitudes for J=3 equal 6	
Re T(3,1,3,1) = -.00963015 ± .00007621	
Re T(5,1,3,1) = .00193039 ± .00003392	
Re T(7,1,3,1) = -.00009062 ± .00002073	
Re T(9,1,3,1) = -.00027720 ± .00001266	
Re T(11,1,3,1) = .00014145 ± .00000712	
Re T(13,1,3,1) = -.00002260 ± .00000235	
Number of O(4) amplitudes for J=5 equal 6	
Re T(5,1,5,1) = .00000996 ± .00000065	
Re T(7,1,5,1) = .00001595 ± .00000066	
Re T(9,1,5,1) = .00000310 ± .00000046	
Re T(11,1,5,1) = -.00000258 ± .00000037	
Re T(13,1,5,1) = .00000035 ± .00000022	
Re T(15,1,5,1) = .00000067 ± .00000019	
Number of O(4) amplitudes for J=7 equal 4	
Re T(7,1,7,1) = -.00000245 ± .00000050	
Re T(9,1,7,1) = -.00000248 ± .00000031	
Re T(11,1,7,1) = -.00000116 ± .00000020	
Re T(13,1,7,1) = -.00000009 ± .00000016	

FIGURE CAPTIONS.

Fig. 1. The $O(4)$ fits to total cross-sections for $n_{\max} = 5$. The solid lines correspond to direct fits to the data (84 experimental points). The dashed lines were calculated on the basis of $O(4)$ amplitudes reconstructed from a phase shift analysis (see ref. 1). T is the laboratory energy in Mev.

(a) The unpolarized cross-section σ_0 . (b) The "transverse" polarized cross-section $(-\Delta\sigma_T/2)$. (c) The "longitudinal" polarized cross-section $(-\Delta\sigma_L)$. (d) The singlet cross-section σ_s . (e) The triplet cross-section σ_{T0} . (f) The triplet cross-section σ_{T1} .