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DECONFINEMENT IN THE FUNDAMENTAL SU(2) HIGGS MODEL

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A B S T R A C T

The SU(2) Higgs model with a Higgs doublet in the fundamental representation of SU(2) is investigated at finite temperature by Monte Carlo methods. In addition to the phase transition lines of the zero temperature theory, we find rapid crossovers for the Polyakov line $\langle W \rangle$ in the confinement sector of the theory. In the deconfined phase of this sector we find good agreement with an Ising-like behavior and a critical exponent for the magnetization of approximately $1/3$.

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The classic perturbative treatment of the standard model of weak and electromagnetic interactions at finite temperature¹⁾ has indicated that at a certain temperature T_c a phase transition occurs. At this temperature the theory is found perturbatively to move from a broken "Higgs" phase to a "symmetry restored" phase.

This standard picture is, however, not completely free of conceptual problems. In a manifestly gauge invariant treatment (e.g. on the lattice) it has been shown that no local gauge invariant order parameter exists which can distinguish between the two phases^{2,3)}. In fact, $\langle\phi(x)\rangle$ is strictly zero for all non-zero values of the couplings and mass parameters, despite its apparent non-vanishing value inferred from the tree-level effective potential in the unitary gauge. One may then naively wonder to what extent a determination of the phase transition temperature T_c from the finite temperature effective potential for $\phi(x)$ will be meaningful. Furthermore, and this is a purely technical difficulty, the perturbative treatments of ref. 1 break down close to T_c due to infrared divergences arising from long wavelength fluctuations as T approaches T_c . This latter problem can, to some extent, be circumvented by ϵ -expansion techniques⁴⁾.

The obvious way to settle (albeit only in the form of numerical evidence) some of the questions left unanswered by the standard perturbative treatment of the Weinberg-Salam model's finite temperature phase structure, is to study by Monte Carlo techniques the model on the lattice. Considerable progress has already been made toward understanding the zero temperature phase structure of the SU(2) gauge-Higgs system with the Higgs field in the fundamental representation^{5,6)}. This corresponds to the Weinberg-Salam model without the extra U(1) gauged hypercharge, and without the fermion sector. Although seemingly a drastically simplified version of the Weinberg-Salam model, it nevertheless contains all the features we believe to be important for understanding the gauge-Higgs interactions, as well as all the important qualitative aspects of the full phase diagram associated with the Weinberg-Salam model. This is confirmed by preliminary Monte Carlo studies of the full SU(2)⊗U(1) gauge group⁷⁾.

In this letter we wish to report on a Monte Carlo study of the fundamental SU(2) Higgs model at finite temperature. Our emphasis here

will be on the aspect of deconfinement in this model, as observed when starting in the "confinement" or "Higgs" phases, respectively. A more detailed analysis of the phase structure, including Monte Carlo results for various gauge invariant observables in this finite temperature theory, will be reported elsewhere⁸⁾.

We use the by now standard parametrization of the lattice action:

$$S = S_g - \lambda \sum_x [\Phi(x) \Phi^\dagger(x) - 1]^2 - \sum_{x,y,\mu} \Phi^\dagger(x) Q(x,y) \Phi(y) \quad (1)$$

with the pure gauge piece

$$S_g = \frac{1}{g^2} \sum_p \text{Tr}[U_p + U_p^\dagger] \quad (2)$$

(the sum on p running over elementary plaquettes), and the Higgs "hopping" matrix

$$Q(x,y) = \delta_{x,y} - \kappa [\delta_{x,y-\mu} U_\mu(x) + \delta_{x,y+\mu} U_\mu^\dagger(x)] \quad (3)$$

In the naive continuum limit these parameters are related to the continuum action⁵⁾

$$S_{\text{cont.}} = - \int d^D x [1/2 D_\mu \tilde{\Phi}(x)^2 + m^2 \tilde{\Phi}(x)^2 + \tilde{\lambda} \tilde{\Phi}(x)^4] \quad (4)$$

through the relations

$$\tilde{\Phi}(x) = \frac{\sqrt{\kappa}}{a} \Phi(x), \quad \tilde{\lambda} = \frac{1}{\kappa^2} \lambda, \quad m^2 = \frac{1-2\lambda-8\kappa}{\kappa a^2} \quad (5)$$

with a denoting the lattice spacing.

Conforming with the standard notation of the pure gauge sector, one defines $\beta \equiv 4/g^2$. Finite temperature is, as usual in this euclidean formulation, introduced by making the lattice periodic in the time direction with period N_t :

$$\Phi(\vec{x}, t+N_t) = \Phi(\vec{x}, t), \quad U_\mu(\vec{x}, t+N_t) = U_\mu(\vec{x}, t) \quad (6)$$

The temperature in units of a^{-1} is then simply $T = 1/N_t$.

It is convenient to rewrite the measure of integration as

$$d\phi d\phi^\dagger dU = g^3 dg d\Omega dU, \Omega \in SU(2) \quad (7)$$

by separating the Higgs measure into an integration over the real and positive radial part $\rho(x)$, and the angular piece $\Omega(x)$, which is just an element of $SU(2)$. We then finally arrive at the action

$$S = S_g - \lambda \sum_x [g(x)^2 - 1]^2 - \sum_x [g(x)^2 - \frac{3}{2} \ln g(x)^2] \\ + \kappa \sum_{x,\mu} g(x) g(x+\mu) \text{Tr}[\Omega^\dagger(x) U_\mu(x) \Omega(x+\mu)] \quad (8)$$

The action (8) in conjunction with the measure $dp d\Omega dU$ was the starting point of our Monte Carlo calculation. As can be seen, in the limit $\lambda \rightarrow \infty$ the $\rho(x)$ field freezes into $\rho(x) = \pm 1$. Although our Monte Carlo routines hence would be greatly simplified in this limit, we would lose important information about the phase structure. Thus, whereas at $\lambda = \infty$ the Higgs and confinement phases are analytically connected^{9,10)}, at sufficiently small fixed λ one of the two phases can only be reached from the other by crossing a line of phase transitions. This follows from a mean field analysis¹¹⁾, and has also been found numerically⁶⁾. We have therefore kept λ finite throughout the present calculation.

For the integration over the Haar measure for $SU(2)$, both for the gauge field $U_\mu(x)$ and the angular part $\Omega(x)$, we have used Y_{120} , the 120-element icosahedral subgroup. We have employed the standard Metropolis algorithm with proposed changes of the group elements to one of the 12 nearest neighbors in the icosahedral subgroup. The proposed changes in the radial part of the Higgs field $\rho(x)$ was taken randomly in the interval $[-\Delta\rho, \Delta\rho]$ with $\Delta\rho$ adjusted to obtain a 50% acceptance rate of the changes. A Monte Carlo iteration consisted of a sweep through the links of the lattice, with 3 hits per link, followed by a sweep through the sites of the lattice, with 3 hits per site on both the radial and angular part of the Higgs field. We have studied lattices of sizes $8^3 \times 2$ and $8^3 \times 4$, and will here present data from both. All results were obtained from random starts, with 1000 iterations for thermalization. Data were collected from the following 24000 iterations for the $8^3 \times 2$ lattice, and 12000 iterations for the $8^3 \times 4$ lattice.

We probe deconfinement in this model in a standard way by measuring the excess free energy of a static colored source in the fundamental representation, as it is given by the Polyakov line expectation value:

$$\langle W(x) \rangle = e^{-(E-E_{\text{vac}})/T}, \quad W(x) = \frac{1}{2} \text{Tr} \left[\prod_{t=1}^{N_t} U_0(\vec{x}, t) \right] \quad (9)$$

Of course, only in the limit $\kappa = 0$, when the gauge and Higgs fields are decoupled, is $W(x)$ a good order parameter for deconfinement. Then the global $Z(2)$ symmetry $U_\mu(\vec{x}, t^*) \rightarrow z U_\mu(\vec{x}, t^*)$, $z \in Z(2)$ (for all $\vec{x}$ with t^* fixed), which leaves the action (8) invariant for $\kappa = 0$, protects the Polyakov line from acquiring a vacuum expectation value since $W(x) \rightarrow zW(x)$ under the $Z(2)$ transformation. At the temperature where this global $Z(2)$ symmetry is spontaneously broken $W(x)$ gets a vacuum expectation value, and we have deconfinement. We therefore already know that on the line $\kappa = 0$ in the phase diagram there is a well defined (2nd order) phase transition at some intermediate temperature¹²⁾.

When $\kappa \neq 0$ one would expect the system to behave in many respects like an $SU(2)$ gauge theory with dynamical fermions, a system which has recently been studied extensively^{13,14)}. Thus, since κ plays the role of a "hopping parameter" in this scalar gauge theory, we would expect that for κ non-zero but small ($\kappa < 0.15$), one should still witness a sharp rise in $\langle W(x) \rangle$ at a certain temperature. The sharp cross-over in $\langle W(x) \rangle$ for $\kappa \neq 0$ should presumably not be associated with a phase transition in the strict sense of a singularity in the partition function.

These expectations are confirmed by our Monte Carlo data. In Fig. 1. we show $|\langle W \rangle|$ as a function of β on a $8^3 \times 2$ lattice for $\lambda = 0.1$ and $\kappa = 0.15$ as well as $\kappa = 0.35$. According to ref.6., the former should at zero temperature correspond closely to the confinement region, whereas in the latter case one should already have passed into the Higgs phase. Indeed, for $\kappa = 0.15$ we find that $|\langle W \rangle|$ stays close to zero up to $\beta \approx 1.85$ where it starts rising smoothly, indicating that at this coupling the excess free energy of a colored source is substantially lowered. In contrast, if we start in the Higgs phase, here at $\lambda = 0.1$ and $\kappa = 0.35$, we see that $|\langle W \rangle|$ is practically flat over the whole range of couplings shown. In fact, the curve now crosses the axis at $\beta = 0$ with $|\langle W \rangle| = 0.47$, showing that when starting in the Higgs phase the excess free energy of a colored source is finite in the complete range of β from strong to weak coupling. In this regime interactions between the link variables $U_\mu(x)$ and the $\rho(x)$ field play a crucial role.

It is interesting to observe that the original zero temperature phase transition lines can still be felt, even here in the finite temperature system. This can be seen, for example, by staying at fixed λ and β , and then moving upwards in the phase diagram from the small- κ (confinement) to the large- κ (Higgs) region. The results for $|\langle W \rangle|$ obtained in this way as a function of κ are shown in fig. 2. The couplings β and λ were fixed at $\beta = 1.25$ and $\lambda = 0.1$. We see a clear rise in $|\langle W \rangle|$ around $\kappa \approx 0.25$, which we interpret simply as the finite temperature remnant of the Higgs-confinement phase transition line in the zero temperature theory. There, for $\beta = 1.25$ and $\lambda = 0.1$ the phase transition was found to occur for $\kappa \approx 0.26$ ⁵⁾ in nice agreement with our value quoted above.

In ref. 13 it was noted that for the SU(2) gauge theory with dynamical fermions there was a clear change in the qualitative behavior of $|\langle W \rangle|$ when going from a lattice of size $8^3 \times 2$ to one of size $8^3 \times 4$. A very smooth behavior in $|\langle W \rangle|$ on the $8^3 \times 2$ lattice was there found to turn into quite a sharp cross-over when going to the $8^3 \times 4$ lattice. This result, which simply shows that a time extent of $N_t = 2$ is too small to allow for a precise determination of a possible phase transition, is also found in the present data. This is clearly demonstrated in fig. 3, where we show Monte Carlo measurements for $\lambda = 0.1$ and $\kappa = 0.15$ as a function of β . Two things are observed by comparison with the corresponding data from the $8^3 \times 2$ lattice (fig.1): the cross-over shifts toward a larger β -value (from $\beta \approx 1.85$ to $\beta \approx 2.27$), and $|\langle W \rangle|$ now shows such a steep rise at $\beta \approx 2.27$ that it is almost indistinguishable from a 2nd order phase transition. As we shall discuss briefly below, the shift in β is actually just an indication of approximate scaling. The very sharp rise in $|\langle W \rangle|$ on the $8^3 \times 4$ lattice suggests a measurement of an at least approximately "critical exponent" for the magnetization.

In the case of the pure SU(2) gauge theory the prediction¹⁵⁾, based on Z(2) universality arguments, of a magnetization exponent of approximately 1/3 (as in the 3-D Ising model) has already been quite conclusively confirmed by Monte Carlo measurements¹⁶⁾. More important for our purposes is the fact that even in the presence of fermions one has been able to extract the same critical exponent within the accuracy of the Monte Carlo data^{17,14)}.

If this universality is approximately obeyed for the data shown in fig. 3, then we should expect

$$|\langle W \rangle| \propto |T - T_c|^{1/3} \quad (10)$$

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FIGURE CAPTIONS

- Figure 1: The Polyakov line expectation value $\langle W \rangle$ as a function of β on the $8^3 \times 2$ lattice. With fixed $\lambda = 0.1$ we show here results for $\kappa = 0.15$ (the confinement region at zero temperature) and $\kappa = 0.35$ (the Higgs phase).
- Figure 2: Crossing the zero temperature phase boundary between Higgs and confinement behavior by keeping $\beta = 1.25$ and $\lambda = 0.1$ fixed while varying κ . The zero temperature phase transition at $\kappa \approx 0.26$ [ref. 6] is clearly seen also in these finite temperature data.
- Figure 3: The smooth rise in $\langle W \rangle$ found on the $8^3 \times 2$ lattice becomes far more steep when going to the $8^3 \times 4$ lattice. Results are here shown for $\lambda = 0.1$ and $\kappa = 0.15$ as a function of β .
- Figure 4: A test of $Z(2)$ universality by displaying $\langle W \rangle^3$ versus $\exp[12\pi^2\beta/43] \propto T$.
- Figure 5: Comparison of our present Monte Carlo data with the "universality prediction" $\langle W \rangle^3 = c^3(\beta - \beta_c)$, $c = 0.53$ of refs. 14 and 17.

- Figure 1 -

- Figure 2 -

- Figure 3 -

- Figure 4 -

- Figure 5 -