CERN-TH.4363/86

ON THE EFFECTIVE POTENTIAL AND GRAVITINO
MASS DETERMINATION IN COMPACTIFIED SUPERSTRING MODELS

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ABSTRACT

We compute the one-loop effective potential in a class of no-scale models proposed as the low-energy limit of ten-dimensional superstrings compactified on a Calabi-Yau manifold. We use as regularization prescriptions both a cut-off over integration momenta and n subtractions ($n > 2$). In all cases the effective potential is a monotonically decreasing function of the gravitino mass. It is suggested that massive string modes and/or non-perturbative effects on the string could stabilize the potential.

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Ten-dimensional superstring theories¹⁾ with gauge group $E_8 \times E_8'$ and compactified on a six-dimensional Calabi-Yau manifold²⁾ are, nowadays, the best candidates to unify all known interactions in Nature and to describe a realistic low energy supersymmetric theory.

An effective four-dimensional low energy theory has been obtained by Witten³⁾ through a truncation^{*)} of the field theory limit (ten-dimensional $N=1$ supergravity) of superstrings. It corresponds to the old no-scale models⁴⁾ with a symmetry $SU(1,1)/U(1) \times SU(n,1)/U(n)$ and the following content of chiral superfields: two gauge singlets S and T and the matter fields ϕ^{λ} contained in 27 and $\overline{27}$ representations of $E_6 \subset E_8$. The problem of supersymmetry breaking is taken care of by the hidden E_8' (or better the biggest subgroup into which E_8' is broken at the compactification scale) which becomes strong at the scale Λ_c and triggers the condensation of the E_8' gluinos⁵⁾. This results in a supersymmetry breaking minimum with zero cosmological constant. However, since the theory has an $SU(1,1) \subset SU(n,1)$ symmetry, the gravitino mass remains undetermined at the tree level.

Two problems, related to the low energy phenomenology of the model, must now be solved: i) The dynamical determination of $m_{3/2}$; ii) The communication of supersymmetry breaking from the hidden to the observable sector, since neither scalars nor gauginos feel the supersymmetry breaking at tree level⁶⁾⁻⁷⁾.

In this paper we will be concerned with the issue i) above. Radiative corrections near the Planck or compactification scale are good candidates to determine dynamically the gravitino mass^{6),7)}. We have analyzed the one-loop effective potential and found that, unlike previous hopes^{6),7)}, it does not determine the vacuum expectation value of the T -field and, thus, the gravitino mass. We have compared different regularizations of the effective potential and found, as physically required, that the qualitative features of the regularized effective potential do not depend on the particular prescription chosen.

At energies below the condensation scale Λ_c , the tree level potential is given by^{3),5)}:

*) Although the correct prescription should be to integrate out all massive string excitations over the Calabi-Yau manifold.

$$V_0 = \frac{1}{2S_R} \left\{ e^G |\Gamma|^2 + \frac{1}{6} e^{\frac{2}{3}G} |f_e|^2 \right\} + \frac{1}{2} D^\alpha D^\alpha \quad (1)$$

where $S_R = \text{Re}S$,

$$G = -3 \log (T + T^\dagger - 2 \phi^e \phi_e^\dagger) \quad (2)$$

$$\Gamma = \omega(S) + f(\phi) - 2 S_R \omega_S$$

$f(\phi) = d_{lmn} \phi^l \phi^m \phi^n$ is the superpotential for matter fields and $\omega(s) = c + h \exp [-8\pi^2 S / C_2(G)]$ effective superpotential induced by non-perturbative effects in the hidden sector. G is the biggest subgroup into which the hidden E_8 is broken down during compactification, and responsible for gaugino condensation.

The potential (1) is positive definite so that its vacuum state is determined by

$$\Gamma = f_R = D^\alpha = 0 \quad (3)$$

providing expectation values to $\langle S \rangle \equiv g_{\text{GUT}}^{-2}$ and $\langle \phi^l \rangle = 0$. However, since $V_0(\langle S \rangle, \langle \phi^l \rangle) = 0$, the direction (3) is flat and $T_R \equiv \text{Re}T$ is undetermined at the tree level. On the other hand, the determination of $\langle T_R \rangle$ is of paramount importance for the low energy phenomenology that can be deduced from the theory, since many relevant quantities ($m_{3/2}$, M_{GUT} , Λ_c) depend upon it^{3),5)}. The hope is that radiative corrections to (1) will generate a non-trivial dependence on T_R and fix its vacuum expectation value.

We will compute the effective potential in $N = 1$ supergravity with the Kähler potential⁵⁾

$$G = -\log (S + S^\dagger) + G + \log |f(\phi) + \omega(S)|^2 \quad (4)$$

and the gauge kinetic function^{3),5)}

$$f_{\alpha\beta} = \delta_{\alpha\beta} S \quad (5)$$

along the flat direction defined by (3).

The one-loop contribution to the effective potential is given by

$$V_{\text{eff}} = \frac{1}{32\pi^2} \int^{\Lambda^2} dK^2 K^2 \text{Str} \log (K^2 + M^2) \quad (6)$$

where M^2 is the squared mass matrix for normalized fields. The cut-off Λ , which is an artifact in renormalizable theories ($\Lambda \rightarrow \infty$), has a physical meaning if the theory is non-renormalizable. In our case, since the theory defined by (4) and (5) is an effective one for scales below Λ_c , which is given by [using the condition (3)]

$$\Lambda_c^2 = 4\pi \alpha_{\text{GUT}} e^{-\alpha/3} (T_R)^{-1} \quad (7)$$

with

$$\alpha = \frac{4\pi}{\alpha_{\text{GUT}} C_2(G)} \quad (8)$$

we should take $\Lambda \equiv \Lambda_c$.

A straightforward calculation shows that the integral (6) is given by

$$V_{\text{eff}} = \frac{1}{64\pi^2} \left\{ \Lambda_c^2 \text{Str} M^2 + \text{Str} M^4 \log \frac{M^2}{\Lambda_c^2} + \right. \\ \left. + \Lambda_c^4 \text{Str} \log \left(1 + \frac{M^2}{\Lambda_c^2} \right) - \text{Str} M^4 \log \left(1 + \frac{M^2}{\Lambda_c^2} \right) \right\} \quad (9)$$

If $M^2 \ll \Lambda_c^2$, we can expand (9) around M^2/Λ_c^2 and write⁷⁾

$$V_{\text{eff}} = \frac{1}{32\pi^2} \left\{ \Lambda_c^2 \text{Str} M^2 + \frac{1}{2} \text{Str} M^4 \left[\log \frac{M^2}{\Lambda_c^2} - \frac{1}{2} + O\left(\frac{M^2}{\Lambda_c^2}\right) \right] \right\} \quad (10)$$

This expansion is valid for a renormalizable theory, where $\Lambda \rightarrow \infty$ and only the quadratic and logarithmic divergences are kept in the effective potential. However, in the case of a non-renormalizable theory the minimization of (9) is expected to give $M^2 \simeq O(\Lambda_c^2)$ and the expansion (10) cannot be justified "a priori".

The states that contribute to M^2 in (9) are: the scalar S , the chiral fermions $\tilde{S}$ and $\tilde{T}$ and the gravitino. The normalization matrix along the flat direction (3) is given by

$$\left(\mathcal{G}^{-1} \right)^I_J = \begin{pmatrix} (2S_R)^2 & 0 & 0 \\ 0 & \frac{1}{3} e^{-\frac{2}{3}G} & 0 \\ 0 & 0 & 6 e^{-G/3} \delta^2_m \end{pmatrix} \quad (11)$$

where $I, J = S, T, \ell$.

After diagonalization, the normalized mass matrices are found to be $M_j^2 = m_{3/2}^2 \tilde{M}_j^2$ ($j=0, 1/2$), where⁷⁾

$$\tilde{M}_0^2 = \begin{pmatrix} (1+\alpha)^2 & 0 \\ 0 & (1-\alpha)^2 \end{pmatrix}, \quad \tilde{M}_{1/2}^2 = \begin{pmatrix} \alpha^2 & 0 \\ 0 & 4 \end{pmatrix} \quad (12)$$

and

$$m_{3/2}^2 = \frac{4 \alpha_{GUT}^2 C_2(G)}{h} X^{3/2} \quad (13)$$

We have introduced the variable $x \equiv m_{3/2}^2 / \Lambda_c^2$ related to T_R by

$$X = \left(\frac{h}{4} \right)^2 \alpha^2 e^{-\frac{2}{3}G} (T_R)^{-2} \quad (14)$$

Using (12) and (13), the effective potential (9) is found to be

$$V_{eff} = \left(\frac{\alpha_{GUT}^2 C_2(G)}{2\pi h} \right)^2 v(x) \quad (15)$$

with

$$v(x) = -2x^2 + g_1 x^3 \log x + g_2 x^3 + g_3 x + g_4 x^3 \quad (16)$$

and the functions g_i defined by

$$g_1(\alpha) = 12\alpha^2 - 2$$

$$g_2(\alpha) = (1+\alpha)^4 \log(1+\alpha)^2 + (1-\alpha)^4 \log(1-\alpha)^2 - 2\alpha^4 \log \alpha^2$$

$$g_3(x, \alpha) = \log [1 + (1+\alpha)^2 x] + \log [1 + (1-\alpha)^2 x] - \\ - 2 \log (1 + \alpha^2 x) - 4 \log (1 + x)$$

$$g_4(x, \alpha) = - (1+\alpha)^4 \log [1 + (1+\alpha)^2 x] - \quad (17) \\ - (1-\alpha)^4 \log [1 + (1-\alpha)^2 x] + \\ + 2\alpha^4 \log (1 + \alpha^2 x) + 4 \log (1 + x)$$

We have to observe here that the limit $M^2/\Lambda_c^2 \ll 1$ in (10) corresponds to the limit $x^2 \ll 1$ in (16), and the expansion

$$v(x) = -4x^2 + g_1 x^3 \left(\log x - \frac{1}{2} \right) + g_2 x^3 + O(x^4) \quad (18)$$

would reproduce the expansion (10). However, as we have stated above, the expansion (18) cannot be justified "a priori" and, instead, we have to study the vacuum structure of the whole potential (16). This will show a dramatic change with respect to the results⁷⁾ obtained from (18). In particular there are cancellations in the asymptotic expansion of (16) that will prevent the appearance of any local minimum. Actually we find that

$$v(x) \underset{x \gg 1}{\simeq} -4x \log x \quad (19)$$

for all values of α . Of course, the potential (16) is not reliable for arbitrarily large values of x . In fact the upper bound $M_j < M_P$ for the massive states of the theory, Eq. (12), translates into the bound $x \lesssim 0(1)$, where we have used $h = 0(1)$ and $G \leq E_8'$.

In Fig. 1 we show the potential (16) for $x < 1$ and two extreme values of α : $\alpha = 2$ and 20. Using (8) and $2 < C_2(G) < 30$, all possible physical values of α_{GUT} can be easily accommodated for $2 < \alpha < 20$. We see that the dependence on α is very slow and $v(x)$ is a monotonically decreasing function of x . The potential is unbounded from below, though for $x \gg 1$ the string excitation modes cannot be neglected and could stabilize the total potential. In any way the tendency of the theory would be to get a gravitino much heavier than the Planck scale, which should be considered as a phenomenological disaster.

A different regularization procedure to evaluate the integral (6) is, instead of cutting off the momenta, making subtractions at the physical cut-off $\kappa^2 = \Lambda_c^2$ to cancel the divergences. This regularization has been shown recently⁸⁾ to be useful to deal with effective non-renormalizable theories. The minimal number of subtractions needed is two, since the third divergence is taken care of by the supersymmetric property

$$\text{STr } \mathbb{1} = 0 \quad (20)$$

We define the doubly-subtracted effective potential by⁸⁾

$$V_{\text{reg}}^{(2)} \equiv V_{\text{eff}}(M^2) - 2 V_{\text{eff}}(M^2 + \Lambda_c^2) + V_{\text{eff}}(M^2 + 2\Lambda_c^2) \quad (21)$$

where $V_{\text{eff}}(M^2)$ is given by (6). Using (20), (21) can be computed as

$$V_{\text{reg}}^{(2)} = \frac{1}{64\pi^2} \text{STr} \left\{ M^4 \log \frac{M^2}{\Lambda_c^2} - 2(M^2 + \Lambda_c^2)^2 \cdot \log \left(1 + \frac{M^2}{\Lambda_c^2} \right) + (M^2 + 2\Lambda_c^2)^2 \log \left(2 + \frac{M^2}{\Lambda_c^2} \right) \right\} \quad (22)$$

whose behaviour for $M^2 \ll \Lambda_c^2$ is

$$V_{\text{reg}}^{(2)} \simeq \frac{1}{16\pi^2} \Lambda_c^2 \log 2 \text{STr } M^2 \quad (23)$$

similar to (10).

The evaluation of (22) requires the use of some subtleties as we will explain. If $F(0) \neq 0$, then $\text{STr } F(M^2) = \text{STr}^{(m)} F(M^2) + \text{STr}^{(0)} F(M^2)$, where $\text{STr}^{(m)}$ ($\text{STr}^{(0)}$) takes into account all massive (massless) states of the theory. In our case $\text{STr}^{(m)}$ will sum over the masses given in (12). Therefore, $\text{STr}^{(0)} F(M^2) = F(0) \text{STr}^{(0)} \mathbb{1} = -F(0) \text{STr}^{(m)} \mathbb{1}$, where the property (20) has been used. In our case $\text{STr}^{(m)} \mathbb{1} = -4$. This leads to $V_{\text{reg}}^{(2)}$ given by (15) with

$$v_{\text{reg}}^{(2)}(x) = (f_0 - 2f_1 + f_2 + 16 \log 2) x \quad (24)$$

where

$$f_n(x, \alpha) = [n + (1+\alpha)^2 x]^2 \log [n + (1+\alpha)^2 x] + \\ + [n + (1-\alpha)^2 x]^2 \log [n + (1-\alpha)^2 x] - \\ - 2 [n + \alpha^2 x]^2 \log [n + \alpha^2 x] - 4 [n+x]^2 \log [n+x] \quad (25)$$

with $n = 0, 1, 2, \dots$

The shape of the regularized potential (24) is similar to the cut-off potential (16). Actually the behaviour of (24) for large values of x is

$$v_{\text{reg}}^{(2)}(x) \underset{x \gg 1}{\simeq} -12x \log x \quad (26)$$

and $v_{\text{reg}}^{(2)}$ is a monotonically decreasing function of x so that no minimum appears, as shown in Fig. 2.

These features are shared by the m -subtracted regularized potential, for $m > 3^*$). A straightforward calculation shows that

$$V_{\text{reg}}^{(m)} = \frac{1}{64\pi^2} \text{STr} \left\{ \sum_{n=0}^m (-1)^n \binom{m}{n} (M^2 + n \Lambda_c^2)^2 \log \left(n + \frac{M^2}{\Lambda_c^2} \right) \right\} \quad (27)$$

whose behaviour for $M^2 \ll \Lambda_c^2$ is given by

$$V_{\text{reg}}^{(m)} \simeq \frac{1}{32\pi^2} A(m) \Lambda_c^2 \text{STr} M^2 \quad (28)$$

with

$$A(m) = \sum_{n=0}^m (-1)^n \binom{m}{n} n \log n \quad (29)$$

On the other hand, using again the comments following Eq. (23), the potential (15) can be written as

*) Although it would be difficult to understand the physical meaning⁹⁾ of $v_{\text{reg}}^{(m)}$ with $m > 2$.

$$v_{\text{reg}}^{(m)}(x) = \sum_{n=0}^m (-1)^n \binom{m}{n} [f_n(x, \epsilon) + 4n^2 \log n] x \quad (30)$$

The behaviour of $v_{\text{reg}}^{(m)}(x)$ for very large values of x can be found to be

$$v_{\text{reg}}^{(m)}(x) \underset{x \gg 1}{\simeq} -B(m)x, \quad m \geq 3 \quad (31)$$

with

$$B(m) = 4 \sum_{n=0}^m (-1)^{n+1} \binom{m}{n} n^2 \log n \quad (32)$$

It is easy to prove that $A(m) > 0$ and $B(m) > 0$ for $m \geq 3$ so that the qualitative features of $v_{\text{reg}}^{(m)}(x)$ are the same as those of the cut-off potential (16) and the doubly-subtracted potential (24). In particular, all of them have a maximum at the origin ($T_R \rightarrow \infty$), no local minima and are monotonically decreasing (unbounded from below) functions. In particular for $m = 3$ the potential (30) is shown in Fig. 2.

In conclusion, we have studied the effective potential in a class of no-scale $N = 1$ supergravity models which have been proposed as the four-dimensional effective field theory of ten-dimensional superstrings compactified on a Calabi-Yau manifold. Out of the scalar fields of the theory, S and ϕ^{λ} get a tree-level vacuum expectation value while the tree-level potential is flat along the direction of the T -field, leaving $m_{3/2}$ undetermined. The one-loop contribution, evaluated at the tree-level vacuum, shows no local minimum along the T -direction so that, in spite of previous hopes^{6),7)}, the gravitino mass cannot be determined by radiative corrections near the Planck or compactification scale. A possible way out, which is now under investigation¹⁰⁾, is that non-perturbative effects of the string configuration would break explicitly the $SU(1,1)$ symmetry, or no-scale structure, of the theory lifting the flatness along the T_R -direction and fixing the value of $m_{3/2}$.

NOTE ADDED

After completion of this paper we received a preprint (Y.J. Ahn, J.D. Breit and G. Segré, University of Pennsylvania preprint, December 17, 1985) where the effective potential using a cut-off regularization prescription is computed. Their results agree with the similar calculation contained in the first part of this paper.

ACKNOWLEDGEMENTS

I would like to thank M.K. Gaillard for useful discussions and suggestions concerning the subtraction regularization prescription of the effective potential. I acknowledge discussion with J. Ellis, C. Gomez and D.V. Nanopoulos, and J. Ellis for the careful reading of the manuscript.

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FIGURE CAPTIONS

Fig. 1 The effective potential (16), where the cut-off regularization prescription has been used, for two extreme values of α , $\alpha = 2$ and 20. The corresponding curve for any intermediate value of α , $2 < \alpha < 20$, lies in between.

Fig. 2 The same as in Fig. 1 for the effective potential computed using a doubly-subtracted regularization prescription, Eq. (24), and a triple-subtraction, Eq. (30).

Fig. 1

Fig. 2