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WHY THE MIXED SIX-QUARK SPATIAL SYMMETRY IS ESSENTIAL
FOR NUCLEON-NUCLEON REPULSION

F. Myhrer

Department of Physics and Astronomy,
University of South Carolina, Columbia S.C. 29208, USA

and

J. Wroldsen

CERN--Geneva

A B S T R A C T

We show in a bag model that the six-quark [42] spatial symmetry part of the asymptotic NN wave function is approximately energy degenerate with the [6] spatial symmetry part. This explains the importance of incorporating the spatial [42] symmetry in resonating group method calculations which give hard core-like NN S-wave phase shifts.

During the last few years there has been a considerable effort to explain the NN scattering data from an underlying quark model. The first attempts to understand the NN short range repulsion from quark degrees of freedom was made by Libermann^[1], DeTar^[2], Harvey^[3,4] and others^[5] using the adiabatic approximation. Oka and Yazaki^[6,7], Harvey et al.^[8], Faessler et al.^[9] and others^[10,11,12] refined the six-quark calculations above using the resonating group method (RGM) or the generator coordinate method (GCM) to avoid the adiabatic approximation. Since a nucleon consists of a quark core surrounded by a pionic cloud^[13], it is reasonable to assume that the short range NN interactions are given by the quark degrees of freedom and the long range ones are given by the meson exchanges between the nucleons. Including meson exchanges for the longer ranges in the calculations above one is now able to explain the observed NN S-wave phase shift behaviour due mainly to the substructure of the nucleons^[7,9,12]. The mechanism for this effective NN repulsion is similar to the one responsible for the α - α repulsion which originates from the nucleon substructure of the α -particle where the dominant effect comes from the antisymmetrization requirement of the eight-nucleon wave function^[14]. However, to explain the two-nucleon hard core like S-wave and 1P_1 ^[7,9,11] phase shifts found in the RGM six-quark calculations (no long range meson exchanges included), antisymmetry arguments are not sufficient due to more degrees of freedom available in the three-quark nucleon substructure as compared to the case of four nucleons in an α -particle.

The starting point for the NN six-quark calculations is the assumption that the totally antisymmetric six-quark wave function can be written as a product of two three-quark cluster wave functions and a relative wave function^[3],

$$\Psi_6 = \mathcal{A} \left\{ \sum_{BB'} \Psi_B(1,2,3) \Psi_{B'}(4,5,6) \chi_{BB'}(r) \right\} , \quad (1)$$

where asymptotically Ψ_B (or $\Psi_{B'}$) is a well defined wave function for a three-quark baryon B (or B'), $\chi_{BB'}(r)$ is the relative BB' wave function and the antisymmetry operator $\mathcal{A}$ ensures that the total six-quark wave function is antisymmetric under the interchange of any two quarks. Since the spatial part of the three-quark cluster wave function is totally symmetric ($\square\square\square$), the two clusters have the following possible spatial symmetries:

$$\begin{array}{c} \boxed{} \times \boxed{} = \boxed{} + \boxed{} + \boxed{} + \boxed{} \\ [3] \times [3] = [6] + [51] + [42] + [33] \end{array} \quad (2)$$

The SU(4) spin-isospin symmetries for the three-quark baryon cluster are also symmetric, i.e. {3}, and thus decompose as in eq.(2). We here use square and curly brackets to denote the spatial and spin-isospin symmetries respectively. The SU(3) colour wave function for the six-quark state must be the colour singlet $\psi_{[2^3]}^c = \boxed{}$. A possible six-quark wave function being antisymmetric under the interchange of any two quarks and asymptotically describing two baryon clusters in relative S-wave is

$$\psi_6(r) = a \left(\psi_{[2^3]}^c \cdot \left\{ a[6]\{33\} + b[42]\{33\} + c[42]\{51\} \right\} \right), \quad (3)$$

where a, b and c are constants. As shown by Harvey^[3] when $r \rightarrow \infty$ we have two free nucleons in relative S-state if (for the NN isospin T and spin S channels TS=01 or 10)

$$\psi_6(r \rightarrow \infty) = \psi_{[2^3]}^c \cdot \left\{ \frac{1}{3}[6]\{33\} + \frac{2}{3}([42]\{33\} - [42]\{51\}) \right\} \quad (4)$$

It is now well known that the [42] spatial part of the six-quark wave function is important in explaining the short range repulsion in NN S-wave scattering (as is also the mixed spatial symmetry for $\alpha\alpha$ ^[14] and K^+N ^[15] scattering). In this work we will show, using a chiral bag model, that the mixed spatial symmetry six-quark state [42] is responsible for the calculated hard core-like NN phase shifts (1S_0 , 3S_1 and 1P_1).

Earlier Obukhovsky et al.^[16] looked at this problem using the MIT bag model^[17]. They showed for TS=01 that the six-quark state with colour-spin $\{2^3\}_{cs}$ and with all quarks in the lowest $S_{1/2}$ state, i.e. in the spatial symmetry [6], is almost energy degenerate with the state in which four quarks are in the $S_{1/2}$ and two are in the $P_{1/2}$ state forming the spatial mixed symmetry [42] with colour-spin symmetry $\{42\}_{cs}$. In ref.16 it was found that the hyperfine colour-magnetic interaction almost cancels the kinetic excitation energy of the two P-state quarks. However, for the excited states Obukhovsky et al.^[16] only discussed the state with colour-spin $\{42\}_{cs}$ and with spatial symmetry [42]. Using standard methods of inner products one finds that there are several other colour-spin representations that might be

used to build an antisymmetric six-quark state with spatial symmetry [42]. These are $\{42\}_{cs}, \{321\}_{cs}, \{222\}_{cs}, \{3111\}_{cs}$ and $\{21111\}_{cs}$ for $TS=01$ and $\{411\}_{cs}, \{33\}_{cs}, \{2211\}_{cs}, \{2211\}_{cs}$ and $\{111111\}_{cs}$ for $TS=10$ (see, e.g., ref.7). As long as one does not have the transformation matrix between the colour-spin basis and the spin-isospin basis (which is used to express the two-baryon cluster wave functions), it is not obvious how to interpret the significance of the calculation done by Obukhovsky et al. [16].

We will use the chiral bag model to calculate both the expectation values of the colour-magnetic quark-quark interaction operator [17]

$$H_{cm} = - \sum_{i < j} c_{ij} \lambda_i^a \sigma_i \cdot \lambda_j^a \sigma_j \quad (5)$$

and the quark-quark interaction due to the presence of Goldstone pions in the chiral bag model [13],

$$H_{\pi} = \sum_{i,j} d_{ij} \vec{\tau}_i \sigma_i \cdot \vec{\tau}_j \sigma_j \quad (6)$$

for the six-quark states.

First, let us define the following mixed spatial symmetry state:

$$\Psi_{[42]} = \sqrt{1/2} \Psi_{[2^3]}^c \cdot [42] \cdot (\{33\} - \{51\}) \quad , \quad (7)$$

which together with the totally symmetric spatial state

$$\Psi_{[6]} = \Psi_{[2^3]}^c \cdot [6] \cdot \{33\} \quad (8)$$

allow us to write Ψ_6 of eq.(4) as

$$\Psi_6 = \Psi_{[2^3]}^c \cdot (1/3 \Psi_{[6]} + \sqrt{8/9} \Psi_{[42]}) \quad . \quad (9)$$

The excitation energies of the $S^4 P^2$ $\Psi_{[42]}$ state (four quarks in $S_{1/2}$ state and two quarks excited to the $P_{1/2}$ state) and the S^6 $\Psi_{[6]}$

state are given by the bag states $S_{1/2}$ and $P_{1/2}$ eigen-energies $\omega_S=2.04/R^{[17]}$ and $\omega_P=3.81/R^{[18]}$ respectively (R =bag radius). The two $P_{1/2}$ quarks in the $S^4P^2 \Psi_{[42]}$ state therefore make this state ($\Delta\omega \equiv \omega_P - \omega_S$)

$$\Delta E = 2\Delta\omega \quad (10)$$

more energetic than the $S^6 \Psi_{[6]}$ state. This energy we will compare with the quark-quark interaction energies of eqs.(5) and (6).

We will calculate the expectation values of H_{cm} and H_π for the $S^6 \Psi_{[6]}$ and $S^4P^2 \Psi_{[42]}$ in a "mode-symmetric" approximation. This means we assume that the numbers c_{ij} and d_{ij} entering the operators H_{cm} and H_π can be approximated by constants independent of the mode of the quarks i and j . We know that these numbers really depend on the quark modes as follows^[19] (m =nucleon mass, α_s =strong coupling constant):

$$c_{SS}=\alpha_s \ 0.177/4R \ , \ c_{SP}=\alpha_s \ 0.112/4R \ , \ c_{PP}=\alpha_s \ 0.162/4R \ , \quad (11)$$

and

$$d_{SS}=0.440/m^2R^3 \quad d_{SP}=0.178/m^2R^3 \ , \ d_{PP}=0.072/m^2R^3 \ . \quad (12)$$

For the S^4P^2 state a reasonable "average" two-quark interaction strength is estimated from eqs.(11) and (12) to be

$$c_{ij} \approx c = \alpha_s \ 0.15/4R \ , \ d_{ij} \approx d = 0.3/m^2R^3 \ , \quad (13)$$

which gives

$$c = \begin{cases} 13\text{MeV} & , \alpha_s=2.2 \\ 9\text{MeV} & , \alpha_s=1.55 \end{cases} \quad \text{and} \quad d = 1.25\text{MeV} \quad (14)$$

for a typical six-quark bag radius of $R=6.5\text{GeV}^{-1}$. For the S^6 state the average two-quark interaction is simply given by $c_{ij}=c'=c_{SS}$ and $d_{ij}=d'=d_{SS}$ as given by eqs.(11) and (12). This gives (for $R=6.5\text{GeV}^{-1}$):

$$c' = \begin{cases} 15\text{MeV} , \alpha_s=2.2 \\ 11\text{MeV} , \alpha_s=1.55 \end{cases} \quad \text{and } d' = 1.8\text{MeV} . \quad (15)$$

Obukhovskiy et al.^[16] avoided this "mode-symmetric" approximation above by using a fractional parentage expansion of the orbital part of the six-quark wave function.

To proceed we calculate the expectation values of H_{cm} using a fractional parentage expansion in colour and spin space. The necessary tables for performing these calculations are given by Harvey^[3]. We find for TS=01 (and TS=10 in parenthesis below) that (in the rest of this paper the colour singlet six-quark state $\Psi_{[2^3]}^C$ is suppressed. It should multiply all six-quark wave functions):

$$\begin{aligned} \langle [42]\{51\} | H_{cm} | [42]\{51\} \rangle &= 16c/9 \quad (16c/9) , \\ \langle [42]\{51\} | H_{cm} | [42]\{33\} \rangle &= 160c/9 \quad (160c/9) , \\ \langle [42]\{33\} | H_{cm} | [42]\{33\} \rangle &= -26c/9 \quad (-38c/9) . \end{aligned} \quad (16)$$

From DeGrand et al.^[17] we have for TS=01 (and TS=10 in parenthesis):

$$\langle [6]\{33\} | H_{cm} | [6]\{33\} \rangle = 8c'/3 \quad (8c') . \quad (17)$$

The operator H_π is diagonal in the spin-isospin basis, and we find for both TS=01 and 10 that

$$\begin{aligned} \langle [f]\{33\} | H_\pi | [f]\{33\} \rangle &= -72d , \\ \langle [f]\{51\} | H_\pi | [f]\{51\} \rangle &= -124d , \end{aligned} \quad (18)$$

independent of the spatial symmetry $[f]$.

From eqs.(16) and (17) we find for TS=01 (TS=10) that

$$\langle \Psi_{[42]} | H_{cm} | \Psi_{[42]} \rangle = -55c/3 \quad (-19c) \quad , \quad (19)$$

$$\langle \Psi_{[6]} | H_{cm} | \Psi_{[6]} \rangle = 8c'/3 \quad (8c') \quad . \quad (20)$$

From eq.(18) we similarly find for both TS=01 and 10 that

$$\langle \Psi_{[42]} | H_{\pi} | \Psi_{[42]} \rangle = -98d \quad , \quad (21)$$

$$\langle \Psi_{[6]} | H_{\pi} | \Psi_{[6]} \rangle = -72d' \quad . \quad (22)$$

As seen from the calculations above, the excitation energy ΔE , eq.(10), of the [42] state is partly cancelled by the difference of the colour magnetic interaction energies, eqs.(19) and (20). We find for TS=01 that

$$E_{[42]} - E_{[6]} = 2\Delta\omega - 55c/3 - 8c'/3 - 98d + 72d' \quad (23)$$

and for TS=10 that

$$E_{[42]} - E_{[6]} = 2\Delta\omega - 19c - 8c' - 98d + 72d' \quad . \quad (24)$$

We immediately see that the contribution from the d-terms almost vanishes. To estimate the energy differences in eqs.(23) and (24) we may take the original MIT approach^[17], i.e. $\alpha_s=2.2$ and neglect the d-terms. This gives $E_{[42]} - E_{[6]} \approx 0.27\text{GeV}$ for TS=01 and $E_{[42]} - E_{[6]} \approx 0.18\text{GeV}$ for TS=10. In a chiral bag calculation^[20], i.e. using $\alpha_s=1.55$ and including the d-terms, we find $E_{[42]} - E_{[6]} \approx 0.36\text{GeV}$ for TS=01 and $E_{[42]} - E_{[6]} \approx 0.26\text{GeV}$ for TS=10. Thus we see that the energy gap caused by exciting two quarks to a $P_{1/2}$ state is almost cancelled by the colour-magnetic interactions, whether we include the pion-mediated quark-quark interactions or not.

In summary, the partial cancellation between the kinetic and colour-magnetic energy splittings are reasonable since for a two-baryon system the colour-magnetic (and the pionic) interactions give an energy difference of $2(m_{\Delta} - m_N) \approx 600\text{MeV}$ which is roughly equal to the kinetic energy excitation $2\Delta\omega \approx 550\text{MeV}$ ($R=6.5\text{GeV}^{-1}$). Keep in mind that we have used $\omega_p=3.81/R$ which is correct for a $P_{1/2}$ bag state, whereas

the $P_{3/2}$ bag state has a lower kinetic energy, $\omega_{P_{3/2}} = 3.20/R^{[18]}$, which gives a smaller kinetic excitation energy in eq.(10), $\Delta E \approx 360 \text{ MeV}$.

For $TS=01$ and 10 we do a similar calculation for the $\Delta\Delta$ channel. For both $TS=01$ and 10 the wave functions are given by^[3]

$$\Psi'_s(r) = \sqrt{4/45} \Psi_{[6]} + \sqrt{41/45} \Psi'_{[42]} \quad (25)$$

where

$$\Psi'_{[42]} = \sqrt{1/41} \cdot [42] \cdot (4 \{33\} + 5 \{51\}) \cdot \quad (26)$$

We find that the expectation values of H_{cm} and H_π increase the energy difference between $S^6 \Psi_{[6]}$ and the $S^4 P^2 \Psi'_{[42]}$ states. The conclusion is thus that the spatial $[42]$ symmetry state contributes much less in the $\Delta\Delta$ channel compared to the NN channel.

We have argued that the mixed spatial symmetry state $[42]$ is of little importance in the $\Delta\Delta$ (and also for the hidden colour CC) channel. Only in the NN channel, defined by eq.(7), do we find almost energy degeneracy between the two spatial states $[6]$ and $[42]$. If the two spatial symmetry states $\Psi_{[42]}$ (eq.(7)) and $\Psi_{[6]}$ (eq.(8)) are energy degenerate we know from Harvey's work^[3] that for two free nucleons ($r \rightarrow \infty$ in eq.(6)) the $\Psi_{[42]}$ dominates, see eq.(9) (However, for finite r we cannot uniquely determine the relative ratio $\Psi_{[42]} / \Psi_{[6]}$ ^[21], and thus we cannot assume $[42]$ dominance). We should here stress that the largest component of the cancellation (eqs.(23) and (24)) is the non-diagonal contribution in the spin-isospin basis, eq.(16). In conclusion the antisymmetry requirements of the six-quark wave function together with the strongly attractive colour-magnetic interactions of the $[42]$ state (and repulsive for $[6]$) of the NN channel is sufficient to understand the short-range hard core-like S -wave NN phase-shift behaviour, a behaviour which requires the presence of the mixed symmetry spatial state $[42]$.

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