PHYSICS AT LEP

Edited by
John Ellis and Roberto Peccei

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ERRATUM TO
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Please amend your copy according to the following:

On p. 208, in Eq. (2.7), $\alpha_{em}$ should be replaced by $\alpha_{em}^2$.

On p. 221, the ordinate in Fig. 2.16 should be in dimensionless units.

On p. 237, Figures 3.8a and 3.8b should be interchanged.

On p. 241, Eq. (3.16) should read

$$\frac{\sigma_e}{E} = 1.65\% / E, \quad \text{for } 0.02 \text{ GeV} < E < 2.7 \text{ GeV}$$

$$\frac{\sigma_e}{E} = 1\%, \quad \text{for } E > 2.7 \text{ GeV}.$$  

On p. 296, in the definition of $\star(\kappa)$ following Eq. (A.6), $1/2 \kappa$ means $1/(2\kappa)$.

On p. 351, in both Eqs. (3.4) and (3.5), please read $|m_B^2 - m_F^2|$ (instead of $|m_B^2 - m_F^2|$).

On p. 376, in the caption to Fig. 3.8, we have taken a lepton mass of $m_e = 70$ GeV (and not 40 GeV).

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ABSTRACT

This report surveys physics which may be investigated at LEP, the Large Electron-Positron collider under construction at CERN. Five general areas are emphasized, namely: precision measurements at the $Z^0$ peak; studies of toponium; searches for possible new particles; QCD, $\gamma\gamma$, and heavy quark studies; and experiments at the highest LEP energies up to and beyond the $W^+W^-$ pair-production threshold. Wherever possible, full cross-section formulae are given, together with references to the original literature where more details may be found.
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INTRODUCTION

At the time of writing, construction of the LEP machine and of the four large LEP experiments is proceeding apace. Whilst many engineers and physicists are naturally preoccupied with the preparation of the machine and experiments, the time is coming to think again about the physics which can be done with LEP. There are already several reviews of physics with high-energy e⁺e⁻ colliders, including two previous CERN Yellow Reports on LEP physics. However, there have been many developments in both experiment and theory since those reports were written. Most notably, the W⁺ and Z⁰ gauge bosons have been discovered at the CERN p̅p Collider, providing dramatic confirmation of key predictions of the Standard Model. However, e⁺e⁻ collisions at PEP and PETRA have not revealed the top quark, raising the possibility that LEP may be the machine to study it—a possibility that may soon be clarified by experiments at the p̅p Collider. The last few years have also witnessed many theoretical attempts to go beyond the Standard Model. No experimental evidence for any of these ideas has been found, but we now have a much clearer understanding of how they could be tested by experiments at LEP.

For these reasons, it has seemed appropriate to make a new survey of the physics which could be done at LEP. Thanks largely to the initiative of its then Chairman, Günter Wolf, the LEP Experiments Committee asked us, the two theorists on the Committee, to organize this new survey. We identified five principal areas of LEP physics, namely: precision studies at the Z⁰ peak; toponium; searches for new particles; QCD, γγ and heavy quark physics; and high-energy running beyond the W⁺W⁻ threshold. Working groups were set up for each one of these areas. The groups all contained one or more experimentalists from each of the major LEP collaborations, as well as theorists who served as coordinators for the working groups. Preliminary results from this survey were presented at an open session of the LEP Experiments Committee in March 1985, called the LEP Physics Jamboree. This report contains the final written fruits of the working groups' labours. We hope it will serve as a useful handbook for LEP physicists as they prepare to analyse the data which should soon be rolling in. Emphasis has been placed on collecting together as many basic formulæ as
should be useful, and giving references whenever possible to the original literature where more details could be found. Some of the more complicated formulae are also available in the 'Electronic Yellow Book', under CERN IBM account 73.EYB, to which references are given in the text when appropriate.

We are excited by the physics opportunities offered by LEP, and have been impressed by the continued enthusiasm of our experimental colleagues. We would like to pay homage here to the quantity and quality of effort they have devoted to this report, despite their more immediate and time-consuming commitments to tight schedules for preparing their experiments. We would also like to put on record the very harmonious and friendly co-operation that we noticed between members of the different experimental collaborations within the various LEP physics working groups.

All of us physicists who participated in this survey are very grateful to the CERN and DESY staff members who have worked so hard and efficiently on the preparation and presentation of this report. We would like to thank those at CERN involved in its publication for their heroic and cheerful help, in particular Arlette Coudert, Marinette Glomet, Suzy Vascotto and Kitty Wakley, and also thank Marianne Hausser and Werner Knaut of DESY for their essential help. We can only hope that the subsequent value of this report to the LEP physics community will justify the effort they have devoted to its realization.

John Ellis and Roberto Peccei
PRECISION TESTS OF THE ELECTROWEAK THEORY AT THE Z⁻⁰

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* * *

1. INTRODUCTION

The present status of precision tests of the electroweak theory can be summarized by recalling a few significant results. In this article I shall always refer to the definition of the weak mixing angle in terms of the W and Z⁻⁰ mass ratio [1]:

\[
\sin^2 \theta_W = s^2 = 1 - \left( \frac{m_W^2}{m_Z^2} \right). 
\]

(1)

So far, the most precise measurements of \( s^2 \) were obtained from \( (\nu) \)-nucleon deep-inelastic scattering and from the CERN pp Collider. In particular, I quote here the recent results from the CDHS Collaboration at CERN [2] and the CCCFRR Group at FNAL [3]. Their results include radiative corrections corresponding to the definition of \( s^2 \) in Eq. (1) and are respectively:

CDHS:
\[
s^2 = 0.227 \pm 0.008 \pm 0.009 .
\]

(2)

CCCFRR:
\[
s^2 = 0.242 \pm 0.011 \pm 0.005 .
\]

(3)

The complexities of hadronic physics lead to an additional theoretical uncertainty of about \( \delta s^2_{\text{theory}} \approx 0.006 \). Note that the effect of radiative corrections is to decrease \( s^2 \) by \( \delta s^2_{\text{rad}} \approx 0.013 \). The quoted values of \( s^2 \) are in good agreement with the results from other measurements of \( s^2 \) in \( (\nu) \)-nucleon scattering.
They tend, however, to increase the world average with respect to the old value 
\( s^2 = 0.217 \pm 0.014 \) [4]. By combining the latter value with the new ones in
Eqs. (2) and (3), one obtains something like

\[
\left( {\nu_e} \right)\text{-N world average: } s^2 = 0.23 \pm 0.01 .
\] (4)

On the other hand, from the last run at the CERN \( pp \) Collider the following
result was recently reported by the UA2 Collaboration [5] as obtained from the
\( W \) mass including radiative corrections (the UA1 analysis is still in progress):

\[
\text{UA2: } s^2 = 0.227 \pm 0.005 \pm 0.007 .
\] (5)

It is quite remarkable that the accuracy of the Collider result is by now
comparable with that from \( (\nu_e)\text{-N} \) scattering. Even more remarkable is the perfect
agreement of these widely different determinations of \( \sin^2 \theta_W \) from very distant
domains of momentum transfer.

We also recall for comparison the CHARM result [6] on \( s^2 \) from the ratio of
\( v_\mu e \) and \( \bar{\nu}_\mu e \) cross-sections:

\[
\text{CHARM: } s^2 = 0.215 \pm 0.032 \pm 0.012 ,
\] (6)

and the SLAC result on the left-right asymmetry in electron-deuterium scat-
ttering [7]:

\[
\text{SLAC e-D: } s^2 = 0.224 \pm 0.012 \pm 0.008 .
\] (7)

Note that the results in Eqs. (6) and (7) do not include the effect of radiative
corrections, which however is smaller than the errors quoted.

The quantitative level of experimental verification of the electroweak
theory is specified at present by the agreement, within the stated accuracies,
among these and other determinations of \( \sin^2 \theta_W \) [8].

For the near future, before LEP starts operating, one expects the following
improvements from sources other than \( e^+e^- \) annihilation.

The CDHS and CHARM Collaborations plan to bring the experimental error on
\( s^2 \) from \( (\nu_e)\text{-N} \) scattering down to the level of \( \delta s^2 = \pm 0.005 \), when the analysis
will be completed. The CHARM II Collaboration plans to collect enough statistics
on \( v_\mu e \) scattering for the error on \( s^2 \), derived from the ratio of the two cross-
sections, to be brought down to \( \delta s^2 = \pm 0.005 \). The determination of \( s^2 \) from these
purely leptonic channels would be free from hadronic uncertainties.

In \( pp \) colliders the ratio \( m_W/m_Z \) will eventually provide a more precise
determination of \( s^2 \) than the separate measurements of \( m_W \) or \( m_Z \). In fact, the
ratio is free from the systematic error on the energy scale. At the improved
CERN Antiproton Collider (ACOL), it is planned to collect, by the end of 1988, an integrated luminosity $\int L \, dt = 10 \, \text{pb}^{-1}$. This corresponds to about $5 \times 10^3 \, W \to e^- e^+$ and $5 \times 10^2 \, Z \to e^- e^+$. With a realistic improvement of the detectors, such as that planned by the UA1 Collaboration, one can presumably aim at a precision on the ratio $m_W/m_Z$ given by

$$\delta(m_W/m_Z) = \pm 2 \times 10^{-3},$$

which would correspond to

$$\delta s^2 = \pm 0.004.$$  \hspace{1cm} (9)

The first-generation experiments at the Fermilab Tevatron are not expected to do better, because higher energies are no great advantage in this respect.

This precision could possibly be further improved at pp colliders after the year 1989. For example, the ACOL running could continue during the LEP operation. Also, the collaboration working for the second-generation DØ detector (due to start operation in 1992-93) at the Tevatron explicitly includes a precise measurement of $m_W$ and $m_Z$ among its main physics goals.

2. MEASUREMENT OF THE $Z^0$ MASS AND WIDTH AT LEP

In discussing experiments at LEP around the $Z^0$, our working group has always taken the following ideal configuration for reference. A constant luminosity is assumed, given by

$$L = 10^{31} \, \text{cm}^{-2} \, \text{s}^{-1}. \hspace{1cm} (10)$$

It is proposed that data collection be organized in intervals of two days (at 100% efficiency) at each of 13 energies above and below the resonance in 2 GeV steps. For example, if $s^2 = 0.23$ and $m_Z = 92 \, \text{GeV}$, one would cover the energy region from $\sqrt{s} = 80$ GeV up to $\sqrt{s} = 104 \, \text{GeV}$. In addition, a suitable number $N$ of days will be dedicated to data-taking on the $Z^0$. We shall see that as a result of this analysis our working group will recommend $N \geq 100$ days (100%).

At LEP the $Z^0$ mass and width are determined, through the above procedure of data collection, by reconstructing the line shape of the $Z^0$ resonance in the channels $e^+ e^- \to \mu^+ \mu^- X$.

The first conclusion of our analysis [9, 10] was that the errors on $m_Z$ and $\Gamma_{\text{tot}}$ are completely dominated by systematics. With the luminosity of Eq. (10) the statistical error obtained from $e^+ e^- \to \mu^+ \mu^- X$ in about 10 days would be given by
\[(\delta m_z)_{\text{stat.}} \leq \pm 10 \text{ MeV,} \]
\[(\delta \Gamma_{\text{tot}})_{\text{stat.}} \leq \pm 15 \text{ MeV.} \]

There is only a small dependence of these numbers on additional running time.

The list of possible systematic errors includes: a) the machine energy spread; b) the luminosity monitoring; c) the absolute energy scale (which must be stable, reproducible, and known); d) the QED radiative corrections.

It is planned that the energy spread of the LEP beams will be given by

\[\frac{\Delta E}{E} = 0.78 \times 10^{-3} \text{ (E/50 GeV)},\]

which nominally corresponds to

\[\Delta W = 70 \text{ MeV},\]

where \( W \leq m_z \) is the total centre-of-mass energy. However, on the one hand, one obtains directly by symmetry \( \Delta m_z \leq 0 \); on the other hand, it is also true that \( \Delta \Gamma_{\text{tot}} \ll \Delta W \). The important point is that the spread is known and can be taken into account rather precisely. Thus the spread of the beam energy is not a problem at LEP.

The luminosity monitoring can certainly be realized to a level of accuracy of 3%, as is the case at PETRA. However, for the purpose of measuring the \( z^3 \) shape, one only needs a precise monitoring over the relevant energy interval of a few GeV. Thus a realistic figure of 2% over a 10 GeV range can be assumed. For example, the induced error on \( m_z \) and \( \Gamma_{\text{tot}} \) would be \( \Delta m_z \leq \pm 10 \text{ MeV,} \Delta \Gamma_{\text{tot}} \leq 0 \) for a linear variation of 2%/10 GeV, and \( \Delta m_z \leq 0, \Delta \Gamma_{\text{tot}} = \pm 10 \text{ MeV} \) for a quadratic variation of the same amount.

The main limitation on \( \delta m_z \) appears to arise from the determination of the absolute energy scale. Without a measured polarization, the energy determination is made by measuring the line integral of the magnetic field over the magnet length \( l_{\text{mag}} \):

\[E = \int B(I) \, dl \leq B(I) l_{\text{mag}}.\]

One can realistically obtain

\[\frac{\delta E}{E} = \frac{\delta B}{B} + \frac{\delta l_{\text{mag}}}{l_{\text{mag}}} = \pm (2 + 1) \times 10^{-4},\]

which leads to
\( (\delta E/E) \leq 3 \times 10^{-4} \) \hspace{1cm} (16)

or

\[ \delta m \leq 28 \text{ MeV} . \] \hspace{1cm} (17)

On the other hand, by implementing a precisely measurable beam polarization, the energy scale determination can be improved by electron spin-resonance calibration (transverse polarization is adequate for this purpose). With this method, one can, in principle, obtain

\[ (\delta E/E)_{\text{abs}} \leq 1 \times 10^{-5} \] \hspace{1cm} (18)

or

\[ \delta m_Z \leq 1 \text{ MeV} . \] \hspace{1cm} (19)

However, the reproducibility of the beam energy is in any case dominated by the magnetic field stability. Over a short period of time, the relative stability of the beam energy can be estimated to be

\[ (\delta E/E)_{\text{rel}} \leq 1 \times 10^{-4} . \] \hspace{1cm} (20)

Thus, in practice, this is the precision that can be obtained. The corresponding distortion of the shape implies that

\[ \delta m_Z \leq \delta \Gamma_{\text{tot}} \leq 10 \text{ MeV} . \]

It is well known that the line shape is greatly affected by the QED radiative corrections. In particular, the main effect is due to bremsstrahlung from the initial electron-positron legs, which shifts the resonance maximum at higher energy and produces a large radiative tail above the resonance. It is easily realized that the complete leading-order QED radiative corrections are not enough. From Figs. 1a,b it can be seen that the effect of the leading-order QED correction on the \( Z^0 \) mass and width is of order

\[ \delta \Gamma_{\text{tot}} / \Gamma_{\text{tot}} \leq \delta m_Z / \Gamma_{\text{tot}} \leq 10\% , \]

which implies that

\[ \delta \Gamma_{\text{tot}} \approx \delta m_Z \approx 250 \text{ MeV} . \] \hspace{1cm} (21)

The origin of this relatively large effect is easily understood in terms of large leading logarithms. The line shape is most conveniently observed in an
Fig. 1 Total cross-sections for $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ in Born approximation (solid line) and including the complete first-order QED corrections (dashed line). The experimental cuts were assumed as in Eqs. (67)-(69). The parameters were taken as follows: in Fig. 1a, $m_\mu = 90$ GeV, $\sin^2 \theta = 0.2435$, $\Gamma_{tot} = 2.26$ GeV; in Fig. 1b, $m_\mu = 94$ GeV, $\sin^2 \theta = 0.2157$, $\Gamma_{tot} = 2.67$ GeV. These figures are taken from R. Kleiss, Ref. [35].

Inclusive channel such as $e^+e^- \rightarrow \mu^+\mu^-X$ or $e^+e^- \rightarrow X$. The large logarithms are associated with real and virtual radiation from the initial legs. Note that

$$\ln \left( \frac{m_Z^2}{m_e^2} \right) \approx t = 24.2,$$

so that $(a/s)t \lesssim 6\%$. In an inclusive experiment there are no similar logarithms associated with the final state, such as $\ln \left( \frac{m_Z^2}{m_\mu^2} \right)$ or $\ln \left( \frac{m_Z}{\Delta m} \right)$, where $\Delta m$ is the maximum energy of an undetected photon. We recall that there cannot be mass singularities when all degenerate states are summed up [11] (i.e. states with the same mass: $\mu, \mu +$ collinear $\gamma$, etc.). Thus the only mass singularities are those connected with the initial state and which contribute corrections of order $a^n(\alpha_s)t^n$ with $m, n \geq 0$. It is simple to realize that, for our purpose, the only interesting terms beyond those of order $(a/s)t \lesssim 6\%$, are the contributions of order $[(a/s)t]^2 \lesssim 0.3\%$ and $a/s \lesssim 0.2\%$, which are expected to lead to additional shifts $\delta m_Z, \delta \Gamma_{tot} \lesssim 0(10)$ MeV. Once these terms are included, the remaining ambiguity on $\delta m_Z$ and $\delta \Gamma_{tot}$ is well below 10 MeV, and the accuracy could, if necessary, be further improved without much effort.

In fact the sums of whole towers of leading and next-to-leading logarithms of all orders are known. The related formalism has been extensively developed in the context of the QCD-improved parton model [12]. One defines a scale-dependent probability density of finding an electron of energy fraction $x$ in a parent electron,
\begin{equation}
e(x, \frac{x_2^2}{x}) = \delta(1 - x) + \frac{a}{2\pi} T \rho(x) + \frac{1}{2} \left( \frac{a T}{2\pi} \right)^2 \int_0^1 \frac{dy}{x} P(y) P(y) \sum \ldots,
\end{equation}

where \( T = t + O(\alpha^2) \), and in the non-singlet leading logarithmic approximation,

\begin{equation}
P(x) = \left( \frac{1 + x^2}{1 - x} \right) \ldots.
\end{equation}

The corrected cross-section is then given by the convolution (Fig. 2):

\begin{equation}
\sigma_c(S) = \int_0^1 dx_1 dx_2 e(x_1, S) e(x_2, S) \sigma(x_1 x_2 S),
\end{equation}

where \( \sigma(S) \) includes the Born approximation and the remaining terms of order \( \alpha \) without logs. In the Breit-Wigner denominator the \( S \)-dependence of \( \Gamma_{\text{tot}} \) can also be taken into account (see Wetzel [13]).

A detailed account of this treatment of QED corrections is given elsewhere in this report [14]. The final results are summarized in Fig. 3. Thus the QED radiative corrections to the \( g^0 \) shape can be computed to the required accuracy and do not pose any difficult problem.

---

**Fig. 2** General structure of diagrams for initial-state bremsstrahlung together with their gauge completions.

**Fig. 3** Total cross-section for \( e^+e^- + \mu^+\mu^- (\gamma) \) without cuts. The dotted line is the resonant Born cross-section (i.e. the Born cross-section minus the absolute square of the photon term). The dashed line includes the leading logarithmic first-order QED correction. The solid line is obtained by adding the second-order leading logarithms [i.e. terms of order \((\alpha/\pi)t\)^2, see text] and the finite first-order terms (of order \( \alpha/\pi \) with no logarithms). The arrows indicate the peak position. The values of the parameters are \( m_\mu = 92 \text{ GeV} \), \( \sin^2 \theta_w = 0.23 \), \( \Gamma_{\text{tot}} = 2.628 \text{ GeV} \) (in the Breit-Wigner denominators, the \( s \)-dependent width was inserted: \( \Gamma_{\text{tot}}(S) = \Gamma_{\text{tot}}(m_\mu)/(S/m_\mu) \)). Figure taken from Ref. [14].
The overall conclusion is that at LEP the $Z^0$ mass and width can be measured with relative ease down to $\Delta m_Z = \Delta \Gamma_{\text{tot}} = \pm 50$ MeV. A factor of 2-3 improvement can be reached with a determined effort, and for this a measurable beam polarization [15] (e.g. transverse) is needed in order to improve the energy calibration. Since $m_Z$ is a fundamental constant (like $\alpha$, $G_F$, etc.), the achievement of maximum of precision on $m_Z$ is certainly worth the effort. As $e^+e^-$ annihilation is unchallenged for this type of measurement, we think that the LEP community should make a dedicated drive in this direction.

3. **The $Z^0$ Mass Versus $\sin^2 \theta_W$**

The precise measurement of $m_Z$ discussed in the previous section fixes $\sin^2 \theta_W$, with an accuracy which is difficult to match in other experiments. At least a second precise measurement is needed for a meaningful and accurate test of the electroweak theory [16]. In this section we discuss the precision needed in order to probe the really interesting dynamical aspects of the theory.

The quantity $\sin^2 \theta_W$ can be obtained in the standard model either from the $W$ and $Z^0$ mass ratio or from the current couplings. At the tree level we have the two basic relations:

$$s_{\text{Born}}^2 = 1 - \frac{m_W^2}{m_Z^2}$$  \hspace{1cm} (27)

$$s_{\text{Born}}^2 = \frac{\frac{\alpha(m_Z)}{\mu}}{72 G_F} \frac{1}{m_W^2}$$  \hspace{1cm} (28)

The equality at the tree level of these two (in principle, different) quantities is the essence of the standard electroweak theory. This equality would be broken by the presence of higher-than-doublet Higgs multiplets [17], of heavier $W$ and $Z$ bosons (due for example to compositeness [18] and/or new gauge degrees of freedom [19]), of new effective four-fermion couplings [20], and so on.

In general, when the electroweak radiative corrections are implemented, the two relations (27) and (28) are modified separately. Alternatively, in agreement with the convention spelled out in Eq. (1), one can use the first relation, Eq. (27), as a definition of $s^2$ and transfer all the effects of radiative corrections to the second relation, Eq. (28). The corrected expression of Eq. (28) can be conveniently written in the form

$$s^2 = \frac{\frac{\alpha(m)}{\mu}}{72 G_F} \frac{1}{m_W^2} (1 + \delta)$$  \hspace{1cm} (29)

Note that the couplings are now taken at the scale $\mu = m_W, m_Z$. This way of parametrizing the corrections is particularly useful, because once the couplings
have been made to run up to $m$ then $\delta$ is 'small' in the sense that it contains no leading logs [e.g. $(\alpha_s/\pi) \ln (m^2/m_F^2)$, where $f$ is a fermion]. The intuitive reason is that the masses are defined from the pole position in the corresponding propagator. Thus the relevant couplings are those which appear in the residues of the poles [21].

The factor

$$\alpha(m)/\alpha(m_e) < 1.073 + 0.003$$  \hspace{1cm} (30)

corresponding to [8]

$$\alpha^{-1}(m) = 127.70 \pm 0.3 + 8/9\pi \ln (m_e/36 \text{ GeV})$$  \hspace{1cm} (31)

differs relatively much from unity because of the large logarithms which are determined by the QED $\beta$-function. Their dynamical content is rather uninteresting. In any case, the corresponding physics, which is contained in the photon vacuum polarization (Fig. 4), can be directly studied by measuring the total $e^+e^-$ annihilation cross-section from threshold up to the $Z^0$ resonance region.

Fig. 4 The photon vacuum polarization diagrams

There are no large logarithms in $G_F(m)/G_F(m_e)$. This is due to the vanishing of the relevant anomalous dimension, a fact which is indirectly related to charge conservation [22]. As a consequence, the scale dependence can be omitted in $G_F$, by redefining $\delta$ in Eq. (29):

$$s^2 = \frac{\alpha(m)}{\gamma} \frac{1}{2} \Gamma \frac{m_e^2}{m^2} (1 + \epsilon).$$  \hspace{1cm} (32)

It follows from Eqs. (1) and (32) that

$$m^2_Z = \frac{\mu^2}{s^2 c^2},$$  \hspace{1cm} (33)

where $c^2 = 1 - s^2$ and

$$\mu^2 = \frac{\mu^2}{1 - \Delta \Gamma} = \frac{\alpha(m)}{\alpha(m_e)} (1 + \epsilon).$$  \hspace{1cm} (34)
By writing \( 1/(1 - \Delta r) \) instead of \( 1 + \Delta r \), one takes into account the resummation of a whole series of large logs of all orders. One has

\[
\nu^2_{\text{Born}} = \nu a(m_e)/(2 G_F) = (37.281 \text{ GeV})^2. \quad (35)
\]

Most of the interesting physics is contained in \( \epsilon \). If one makes the set of assumptions

\[
N_{\text{families}} = 3
\]

\[
\mu^2_H = \mu^2_Z
\]

\[
\mu^2_t \leq 36 \text{ GeV ,}
\]

where \( \mu^2_H, \mu^2_t \) are the Higgs and top masses respectively, then one obtains [8, 21, 23, 24]

\[
\Delta r = 0.070 \pm 0.002 \quad (37)
\]

which, by comparison with Eq. (30), means that \( \epsilon \) is of order of a few per mille. The quoted theoretical error on \( \Delta r \) arises mainly from the contribution of \( u \) and \( d \)-quarks in the vacuum polarization diagram of Fig. 4, which enters into the determination of the factor in Eq. (30). Intuitively this error is large because \( \log \frac{m_Z^2}{m_{u,d}^2} \) is greatly affected by the ambiguities in the meaning and the value of \( m_{u,d} \). In principle these ambiguities can be resolved by relating the vacuum polarization integral to the measured cross-section for \( e^+e^- \to \) hadrons [25, 26]. The remaining error given in Eq. (37) is mainly due to the present experimental errors on \( \sigma(e^+e^- \to \) hadrons) in the low energy region. From Eq. (33) and \( s^2 = 0.23 \), one obtains

\[
\delta s^2 = 3.7 \times 10^{-4} \left( \frac{m_Z}{50 \text{ MeV}} \right) + 0.35 \delta(\Delta r). \quad (38)
\]

We thus see that

\[
\delta m^2_Z = \pm50 \text{ MeV} \pm \delta s^2 = \pm0.0004 \quad (39)
\]

and

\[
\delta(\Delta r) = \pm0.002 \pm \delta s^2 = \pm0.0007. \quad (40)
\]

Clearly this last figure is at present the limit on the precision that one can envisage.

Even if the standard electroweak theory is correct and we assume that the top-quark mass will be precisely known by the time of LEP operation, with
\( m_t = 40 \text{ GeV}, \) still the value of \( s^2 \) derived from a given value of \( m_Z \) depends on the number of families and on the Higgs mass, which in Eqs. (36) were purely guesswork. It is well known that \( m_H \) is almost completely undetermined in the standard model [17]. The natural domain of values for \( m_H \) is given by

\[
10 \text{ GeV} \leq m_H \leq 1 \text{ TeV}.
\]

If we assume that \( N_{\text{families}} = 3 \) and \( m_t = 40 \text{ GeV}, \) and let \( m_H \) vary in the range given in formula (41), then as seen from Table 1 [23] or Fig. 5, the induced variation of \( \Delta r \) corresponds to:

\[
10 \text{ GeV} \leq m_H \leq 1 \text{ TeV} + 5s^2 = \pm 0.0025.
\]

Table 1

The quantity \( \sin^2 \theta_w \) for various values of \( m_Z, m_H, m_t \) (all masses in GeV);

\[
\sin^2 \theta_w = 1 - \frac{m_t^2}{m_Z^2} \quad \text{(from Ref. [23])}.
\]

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Fig. 5 Plot of $s^2 = \sin^2 \theta_W$ (defined by $s^2 = 1 - m_Z^2 / m_W^2$) as a function of $m_W$ for different values of the Higgs mass $m_H$ and the top-quark mass $m_t$. This figure was obtained by W. Wetzel from his calculation (Ref. [34]) which is in perfect agreement with the results of Ref. [23], reported in Table 1.

As one of the main tasks of experiments is to throw light on the Higgs sector of the standard theory, it is clear that this last number indicates the accuracy range to be reached.

It is also interesting to recall that the possible presence of a new heavy family of fermions, with a large splitting $m_{t'} - m_{b'}$, could lead to very large effects on $\Delta r$. It is in fact known that the bound [27]

$$m_{t'} - m_{b'} \lesssim 250-300 \text{ GeV}$$

(43)

can be imposed from the observed value near unity of the $\phi$ parameter (defined from the ratio of charged to neutral-current rates in $\nu^+\nu^-N$ scattering). At tree level:

$$\phi_{\text{tree}} = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1.$$  

(44)

The electroweak radiative corrections which shift $\phi$ away from unity depend quadratically on $m_{t'}$. For $m_{t'} \gg m_{b'}$, through the definition of $s^2$ in Eq. (1) this quadratic term propagates into $\Delta r$ and the radiative corrections to other measurable quantities. It is interesting to observe that for sufficiently large $m_{t'} - m_{b'}$, $\Delta r$ is decreased with increasing $m_{t'} - m_{b'}$, and the point $\Delta r = 0$ is reached for $m_{t'} - m_{b'} \lesssim 250 \text{ GeV}$. If instead the heavy-quark isospin doublet is
not split, \( m_t \sim m_b - m_Z \), then typically \( \Delta r \leq +0.003 \) and \( \delta s^2 \leq +0.001 \) [28]. Precisely, one obtains [28]

\[
\begin{align*}
\Delta r_{t,b} & \leq 250 \text{ GeV} + 0.07 \leq \delta s^2 \leq +0.0035.
\end{align*}
\]  

(45)

The large correction, \(-7 \times 10^{-2}\), corresponds to \( \Delta r \sim 0 \). A pictorial summary of the importance of the various effects which we have discussed is presented in Fig. 6.

In conclusion, the accuracy goal to be pursued at LEP must be

\[
\delta s^2 / s^2 \leq \pm 1\% \quad \text{or} \quad \delta s^2 \leq \pm 0.002.
\]  

(46)

This is required in order to reach the most concealed layers of dynamics, as for example the elusive Higgs sector. At this level of precision, stringent limits can be put on new generations of fermions or other forms of heavy matter (e.g. supersymmetric [29] partners of quarks and leptons) as well as on possible deviations from the tree-level relation Eq. (28). In the next section we shall discuss which measurements at LEP can lead to such accurate tests of the electroweak theory.

---

**Fig. 6** Given an experimental value for \( m_t \) (for example \( m_t = 92 \text{ GeV} \)), the corresponding value of \( s^2 = \sin^2 \theta_W \) is shown in the upper part of the figure in the Born approximation and including radiative corrections. One sees that a large contribution to the radiative correction is due to the running of \( \alpha \) from the electron mass up to the \( Z^0 \) mass (the band is due to theoretical errors as described in the text). A widely split \( (m_t \gg m_b) \) new weak-isospin quark doublet with \( m_s \gg m_b \) would tend to compensate the previous effect (in part or even entirely). In the lower part of the figure the corresponding values of \( \Delta A_R \) are shown for each value of \( s^2 \) in the Born approximation or including radiative corrections. The size of the experimental error which it is hoped to achieve at LEP is also displayed.

4. **ASYMMETRIES AT THE $Z^0$**

Here we shall consider the most sensitive experiments at the $Z^0$ peak, other than the measurement of $m_Z$. Some further precision measurements can be done above the resonance. For example, at LEP II one can obtain $m_W$ from $e^+e^- \rightarrow W^+W^-$. Note that from

$$m_W^2 = \mu^2/s^2$$  \hspace{1cm} (47)

[see Eqs. (1) and (33)] it follows that, at fixed $\mu$:

$$\delta m_W = (m_W/2)(\delta s^2/s^2)$$  \hspace{1cm} (48)

and

$$m_Z = m_W/(m_W^2 - \mu^2)^{1/2}$$  \hspace{1cm} (49)

The relation between $m_W$ and $m_Z$ is displayed in Fig. 7 and in Table 2. We see directly from Eq. (48) that for $s^2 < 0.23$ it follows that

$$\delta s^2/s^2 \simeq 0.01 \Rightarrow \delta m_W = \pm 400 \text{ MeV}.$$  \hspace{1cm} (50)

If we fix $m_Z$ rather than $\mu$, the corresponding $\delta m_W$ is smaller, as can be seen from Fig. 7. This precision on $m_W$ is well within the possibilities of LEP

---

**Fig. 7** The $W$ mass versus the $Z$ mass, including the effect of radiative corrections for different values of the Higgs mass and of the t-quark mass. The corresponding numbers are given in Table 2, and are taken from Ref. [23].
experiments. This is discussed at length in the report by the Working Group on High Energies [30].

At resonance the optimal tests are provided by asymmetries. The most important ones are listed below.

a) The forward-backward asymmetry \( A_{FB} \). It can be measured in any channel \( e^+e^- \rightarrow f\overline{f} \), where \( f \) is either a quark or a lepton. For precision tests the most convenient channels are \( f = e^- \), \( \mu^- \) (asymmetries for quarks and effects of \( b\overline{b} \) mixing are discussed in Ref. [31]). If we call 'forward' the initial \( e^- \) direction and \( \sigma_F(f) \) the cross-section for finding the fermion \( f \) in the forward hemisphere (within a given fiducial region), then

\[
\begin{array}{|c|c|c|c|}
\hline
m_t & m_H = 10 & m_H = 100 & m_H = 1000 \\
\hline
30 & 90 & 78.37 & 78.29 & 78.12 \\
 & 92 & 80.88 & 80.80 & 80.64 \\
 & 94 & 83.33 & 83.26 & 83.10 \\
 & 96 & 85.74 & 85.66 & 85.51 \\
 & 98 & 88.11 & 88.03 & 87.88 \\
60 & 90 & 78.36 & 78.27 & 78.11 \\
 & 92 & 80.86 & 80.78 & 80.62 \\
 & 94 & 83.30 & 83.22 & 83.06 \\
 & 96 & 85.69 & 85.62 & 85.46 \\
 & 98 & 88.05 & 87.98 & 87.82 \\
90 & 90 & 78.55 & 78.47 & 78.30 \\
 & 92 & 81.06 & 80.98 & 80.81 \\
 & 94 & 83.50 & 83.43 & 83.27 \\
 & 96 & 85.90 & 85.82 & 85.67 \\
 & 98 & 88.26 & 88.18 & 88.03 \\
130 & 90 & 78.79 & 78.70 & 78.53 \\
 & 92 & 81.30 & 81.22 & 81.05 \\
 & 94 & 83.75 & 83.67 & 83.53 \\
 & 96 & 86.16 & 86.08 & 85.92 \\
 & 98 & 88.53 & 88.45 & 88.29 \\
180 & 90 & 79.14 & 79.06 & 78.88 \\
 & 92 & 81.66 & 81.57 & 81.40 \\
 & 94 & 84.12 & 84.04 & 83.87 \\
 & 96 & 86.54 & 86.46 & 86.29 \\
 & 98 & 88.92 & 88.84 & 88.67 \\
230 & 90 & 79.61 & 79.52 & 79.34 \\
 & 92 & 82.13 & 82.04 & 81.87 \\
 & 94 & 84.61 & 84.52 & 84.34 \\
 & 96 & 87.03 & 86.95 & 86.77 \\
 & 98 & 89.43 & 89.34 & 89.17 \\
\hline
\end{array}
\]
Often $A_{FB}$ is also indicated as the charge asymmetry.

b) (Final) polarization asymmetry $A_{pol}$. In $e^+e^- \rightarrow f \bar{f}$ it is given by

$$A_{pol} = \frac{\sigma(f_L) - \sigma(f_R)}{\sigma(f_L) + \sigma(f_R)},$$

(52)

where $\sigma(f_{L,R})$ are the cross-sections for production of a fermion $f$ with left- or right-handed helicity. In practice, it can only be measured for $e^+e^- \rightarrow \tau^+\tau^-$ by reconstructing the $\tau$ polarization from the pion spectrum in the decay $\tau \rightarrow \pi\nu$.

c) The left-right asymmetry $A_{LR}$. A longitudinal polarization for the $e^-$ beam is needed in this case. The asymmetry $A_{LR}$ is given by

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R},$$

(53)

where $\sigma_{L,R}$ are the cross-sections for $e^-_{L,R} + e^+ + X$, where $X$ can be any channel. In particular, the experiment can be done in a totally inclusive way.

With no polarization one can only use $A_{FB}$ and $A_{pol}$ for precision tests. We shall see that $A_{pol}$ is more sensitive than $A_{FB}$ to $\sin^2 \theta_W$ but can be measured with less accuracy. With longitudinal polarization there are substantial advantages [31]. First, $A_{LR}$ is obviously easier to measure than $A_{pol}$ and carries equivalent information. Secondly, the sensitivity of $A_{FB}$ to $\sin^2 \theta_W$ is much improved by a longitudinal polarization. These statements are easily understood from the tree-level expressions of the asymmetries. At the top of the $Z^0$ resonance the diagram with $Z^0$ exchange is completely dominant at the tree level, the relative corrections to the asymmetries from the photon terms being of order $(\Gamma_{tot}/m_Z)^2$. Neglecting such terms, the expressions of the asymmetries at the resonance (after integration over angles) are given in the Born approximation by

$$|A_{FB}| \leq 3\eta_f \frac{\eta_e + \frac{1}{2} P_e}{1 + 2P_e\eta_e},$$

(54)

$$|A_{LR}| \leq 2P_e\eta_e,$$

(55)

$$|A_{pol}| \leq 2\eta_f,$$

(56)

where $P_e$ is the $e^-$ longitudinal polarization and $\eta_f$ is given by
\[ \eta_f = \frac{V_f A_f}{V_f^2 + A_f^2}, \]

(57)

where \( V_f \) and \( A_f \) are proportional to the vector and axial vector couplings of the \( Z^0 \) to the fermion \( f \). In the standard electroweak theory:

\[ V_f = 1 - 4|Q_f|^2 s^2, \]

(58)

\[ A_f = 1. \]

For charged leptons \( V_L \) is small because \( s^2 \) is close to \( 1/4 \) (see Table 3). For \( Q_e = 0 \) one obtains

\[ |\lambda_{FB}| \lesssim 3\eta_L, \]

(59)

\[ |\lambda_{pol}| \lesssim 2\eta_L. \]

(60)

| Table 3 |
|---|---|---|---|
| \( s^2 \) | \( \eta \) | \( \lambda_{FB} \) \( \text{Born} \) | \( \lambda_{pol} \) \( \text{Born} \) |
| 0.22 | 0.118 | 0.042 | 0.24 |
| 0.23 | 0.079 | 0.019 | 0.16 |
| 0.24 | 0.040 | 0.005 | 0.08 |

This different dependence on the small parameter \( \eta_L \) has a strong effect on the respective sensitivity to \( s^2 \). Since \( \eta_L \sim (1 - 4s^2) \), it follows that

\[ |\delta \eta_L| \lesssim 4\delta s^2, \]

and

\[ |\delta \lambda_{FB}| \lesssim 24 \eta_L \delta s^2, \]

(61)

\[ |\delta \lambda_{pol}| \lesssim 8 \delta s^2. \]

(62)

For \( s^2 \sim (0.22 - 0.24) \), \( \lambda_{FB} \) is very small; and the smaller it is, the less it is sensitive to \( s^2 \). In this range of \( s^2 \) one has

\[ \delta s^2 \leq \begin{cases} 
\frac{1}{2} |\delta \lambda_{FB}| & \text{for } s^2 \leq 0.22, \\
|\delta \lambda_{FB}| & \text{for } s^2 \leq 0.24,
\end{cases} \]

(63)
whilst

\[ \delta s^2 = \frac{1}{8} |\delta A_{\text{pol}}| \quad (64) \]

for all values of \( s^2 \). Alternatively, if a sizeable longitudinal polarization \( P_e \) is present, the dependence of \( A_{\text{FB}} \) on \( \eta_2 \) becomes nearly linear, as is seen from Eq. (54), so that its size and sensitivity to \( s^2 \) are much increased. At the same time \( A_{LR} \), which is nearly as good as \( A_{\text{pol}} \), can also be measured. From Eq. (55) one obtains

\[ \delta s^2 = \frac{1}{8} |\delta A_{LR}| . \quad (65) \]

Clearly \( A_{LR} \) is much easier to measure than \( A_{\text{pol}} \).

In conclusion, Eqs. (63), (64), and (65) make it possible to read the experimental errors \( \delta A \) which would correspond to, say, \( \delta s^2 = \pm 0.002 \).

We now consider the electroweak radiative corrections. In particular we are interested in the size of the purely weak radiative corrections and in their dependence on \( m_H^\prime, m_L^\prime, \) etc. In general the corrected asymmetry can be written down as the sum of three terms:

\[ A = A_{\text{Born}} + \delta A_{\text{QED}} + \delta A_{\text{weak}} = A_{\text{int}} + \delta A_{\text{QED}} . \quad (66) \]

Each of the three terms is not completely unambiguous. For example, I define \( A_{\text{Born}} \) by insertion of the radiatively corrected \( s^2 \) from Eq. (33) (for given \( m_H^\prime, m_L^\prime, \) etc.) in the Born formulae (54) to (56). Thus by using the physical \( s^2 \) in \( A_{\text{Born}} \), some weak effects are included in the variation of \( s^2 \). Other people prefer to define \( A_{\text{Born}} \) in terms of \( s^2_{\text{Born}} \), i.e. the one obtained from \( m_Z^\prime \) via the tree-level relation Eq. (28). Also, in the following we shall use the calculations of Ref. [21], where \( \delta A_{\text{QED}} \) is computed by real and virtual \( \gamma \) emission from the complete Born amplitude (with \( \gamma \) and \( Z^\prime \) exchange), but does not include the one-loop \( \gamma \) vacuum polarization diagrams which contribute to the running of \( \alpha \). In fact, we have seen that the shift from \( \alpha(m_e) \) to \( \alpha(m) \) is a major contribution to \( s^2 - s^2_{\text{Born}} \).

The size of the purely weak correction \( \delta A_{\text{weak}} \) [23, 33, 34] at resonance can be read from Table 4 (based on Refs. [23] and [34]) for a few typical values of the parameters. More complete tables can be found in the article by Consoli [21] and in Ref. [23].

We see that \( \delta A_{\text{weak}} \) is a sizeable fraction of \( A_{\text{Born}} \) at resonance. It is important to remark that also \( \delta A_{\text{weak}} \) becomes large when \( m_L^\prime \) (or \( m_{L^\prime} - m_{b^\prime} \), in the case of a fourth family) increases above \( m_Z^\prime \).
Table 4
Values of $\delta A_{\text{weak}} = A_{\text{int}} - A_{\text{Born}}$ [Eq. (66)] (from Ref. [23]).
Note that the weak radiative corrections are the same for $A_{\text{pol}}$ and $A_{LR}$.

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<th>Born $A_{\text{pol}}$</th>
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<td>0.1400</td>
<td>0.4321</td>
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Although the values of $\delta A_{\text{weak}}/A_{\text{Born}}$ are conceptually interesting, in practice what matters is the relation between $m_z$ and $A$ for a given set of values of the relevant parameters. This relation is obtained by eliminating $s^2$ and directly connecting $A$ to $m_z$. A schematic plot of the general strategy is depicted in Fig. 6 for the particular case of $A_{\text{FB}}$. In order to derive precise quantitative results, we refer to the interesting part of $A$, denoted by $A_{\text{int}}$ in Eq. (66), which is obtained from the measured number by subtracting $\delta A_{\text{QED}}$. As discussed later in this section and in the article by Kleiss [35], a fully quantitative
Fig. 8 The forward-backward asymmetry $A_{FB}$ as a function of $m_z$ for $m_t = 30$ GeV and different values of $m_H$. The QED radiative corrections are subtracted, so that $A_{FB}^\text{int} = (A_{FB}^\text{pol})^\text{int}$ as defined in Eq. (66) of the text [see also the discussion following Eq. (66)]. The corresponding numbers are taken from Ref. [23] (see also Ref. [21]).

Fig. 9 The polarization asymmetry $A_{pol}^\text{int}$ as a function of $m_z$ for $m_t = 30$ GeV and different values of $m_H$. The QED radiative corrections are subtracted, so that $A_{pol} = (A_{pol}^\text{pol})^\text{int}$ as defined in Eq. (66) of the text [see also the discussion following Eq. (66)]. The corresponding numbers are taken from Ref. [23] (see also Ref. [21]).

determination of $\delta A_{QED}$ can only be obtained by carefully taking into account the detailed experimental situation for each apparatus. In Fig. 8, $A_{FB}^\text{int}$ is plotted as a function of $m_z$ for $m_t \leq 30$ GeV and different values of $m_H$. The same is done in Fig. 9 for $A_{pol}^\text{int}$. These figures were obtained from the calculations of Lynn and Stuart [23]. The pair of figures 10a and 10b show a comparison of $A_{FB}^\text{int}$ with and without beam polarization for $m_t = 40$ GeV and different values of $m_H$, as computed by Wetzsel [34]. Also shown in these figures is the huge difference that would arise if $m_z$ and $A$ were related by using only tree-level formulae [i.e., $A_{Born}^\text{Born}$].

The experimental accuracy on $A_{FB}$ and $A_{pol}$ which can be obtained at LEP is discussed in detail in Refs. [36] and [37]. Here I simply give the final conclusions of our working group.

On $A_{FB}$, we concluded that for $s^2 \leq 0.23$ with no beam polarization, it would be relatively easy to obtain $\delta A_{FB} = \pm 0.005$ in 40 days of running time with the luminosity given by Eq. (10). With about 200 days of running time, $\delta A_{FB} = \pm 0.002$ would be possible. It would be an important advantage if the $e^-$ beam were longitudinally polarized, and the polarization known to within $\delta P/P \sim 10\%$.

For $A_{pol}$ measured from $e^- e^- \to \tau^- \tau^-$ by observing the pion energy spectrum in the final state $(\tau \to \nu\pi)$ ($\tau \to N \leq 3$ charged tracks), the statistical precision $\delta A_{pol}^\text{int} \leq \pm 0.050$ can be obtained in 10 days and $\delta A_{pol}^\text{int} \leq \pm 0.016$ in 100 days. The main systematics is from the $\tau \to \nu\tau$ decay mode, which affects $A_{pol}^\text{int}$ by $\delta A_{pol}^\text{int} \leq \pm 0.010$. 
Fig. 10 This is the same plot as in Fig. 8 with (a) or without (b) longitudinal polarization of the e beam, as calculated by W. Wetzel in Ref. [33]. One sees that for a fixed experimental accuracy $\delta A_{FB}$ one would gain more dynamical insight if the polarization was available. Also shown is the value for the asymmetry $A_{FB}$ which would correspond to $m_Z$ if all radiative corrections were neglected (i.e. $A_{Born} = s_{Born}(m_Z)$).

It is seen from Figs. 8 to 10 that $\delta A_{FB} \leq \pm 0.002$ and $\delta A_{pol} \leq \pm 0.016$ are just of the order of the corresponding variation $\delta A$ at fixed $m_Z$ obtained by changing $m_H$ from 10 GeV to 1 TeV at constant $m_t = 30$ GeV. This means that the stated accuracy is at the level of the most delicate and interesting dynamical effects. As illustrated in Figs. 11 to 13 [38], the effects of new physics -- as, for example, of heavy fermions (e.g. a fourth family) and of supersymmetric partners of quarks and leptons -- would be very clearly detectable.

Fig. 11 The variations of $A_{pol}$ and $A_{pol}$ due to one heavy-quark weak-isospin doublet with different ratios of u-quark and d-quark masses. (From Ref. [38].)
Fig. 12 The variations of $A_{\text{pol}}$ and $A_{\text{FB}}$ due to one heavy-squark weak isospin doublet with different ratios of $u$- and $d$-squark masses. (From Ref. [38].)

Fig. 13 The variations of $A_{\text{pol}}$ and $A_{\text{FB}}$ due to one heavy-slepton weak isospin doublet with different ratios of $u$- and $d$-slepton masses. (From Ref. [38].)

It is quite obvious that $\delta A_{\text{QED}}$ must be controlled to a better accuracy than for $A$. Although simple in principle, this problem is quite difficult in practice. The main difficulty is to make sure that the theoretical calculation exactly matches the experimental procedure (acceptance, cuts, and so on). Also, it turns out that the lowest-order QED correction is not enough, and some higher-order corrections must also be included.

A very accurate study of $\delta A_{\text{QED}}$ for $A_{\text{FB}}$ at lowest order was performed in our working group and is reported by Kleiss [35]. The QED radiative corrections for $A_{\text{pol}}$ and $A_{\text{LR}}$ were studied in Refs. [39] and [40]. The radiative corrections for $A_{\text{FB}}$ in $e^+e^-\rightarrow\mu^+\mu^-(\gamma)$ near the resonance were evaluated [35] for different values of $m_Z$ and a definite set of 'realistic' cuts. These cuts were chosen in the following way:

$$E_{\mu^+}, E_{\mu^-} \geq 10 \text{ GeV},$$

$$5^\circ \leq \text{angle (e}^- - \text{e}^+) \leq 175^\circ,$$

$$\mu^+ - \mu^- \text{ acollinearity angle} \leq \zeta.$$  

The results for $m_Z = 90$ GeV and 94 GeV, and $\zeta = 10^\circ$, are plotted in Figs. 14 and 15, respectively. The dependence on the acollinearity cut $\zeta$ is shown in Fig. 16. It can be seen that $\zeta \geq 10^\circ$ is required in order to make the correction stable.

The important point which emerges is that near the resonance $\delta A_{\text{QED}}^{\text{FB}} \approx 0.02$ is much larger than both $\delta A_{\text{FB}}^{\text{weak}}$ and the desired accuracy $(\delta A_{\text{FB}}^{\text{exp}} \pm 0.002$. Thus the problem of controlling the QED correction precisely becomes very acute. It is clear that at least the largest higher-order terms in $a$ have to be taken into
Effect of the complete first-order radiative corrections to $A_{FB}$ near the resonance. The solid line is the Born approximation, the short-dashed line is the pure QED correction, whilst the long-dashed line also includes the first-order weak corrections. The values of the relevant parameters are $m_Z = 90$ GeV, $\sin^2 \theta_W = 0.2435$ and $f_{\pi} = 2.26$ GeV for Fig. 14, and $m_Z = 94$ GeV, $\sin^2 \theta_W = 0.2157$ and $f_{\pi} = 2.67$ GeV for Fig. 15. The 'experimental' cuts are described in Eqs. (67) to (69) of the text (Ref. [35]).

![Diagram](image)

**Fig. 16** The dependence of the radiative corrections to $A_{FB}$ on the acollinearity cut. The values of the parameters are given in Fig. 14. (From Ref. [35].)

account, as was already the case for the radiative corrections to the line shape, discussed in Section 2. The problem of higher-order QED corrections to the $e^+e^- \rightarrow \mu^+\mu^- (\gamma)$ cross-section and the corresponding asymmetries has been studied by Greco [4]. In a not completely inclusive experiment there is an additional series of logarithmic terms (with respect to the totally inclusive case). These terms can also be resummed to all orders. For example, assume that all
photons with energy $E_\gamma$ less than $\Delta \omega$ are included, while those with $E_\gamma > \Delta \omega$ are excluded. Then, going back to Eqs. (24) and (25), it can be seen that double logs of the form

$$
\frac{\alpha}{2\omega} \ln \frac{1}{1-(\Delta \omega/E)} \int dx P(x) = \frac{\alpha}{2\omega} \ln \frac{1}{1-(\Delta \omega/E)} \int dx \frac{1 + x^2}{1 - x},
$$

(70)

are generated. These terms represent the main difference with respect to the inclusive case which we fully discussed in connection with the line-shape problem. Their effect can be taken into account to all orders as described by Greco [41]. The difficulty lies in the fact that, as is clear, these terms depend to some extent on the experimental acceptance, so that a detailed study has to be made for each single experiment. The importance of higher-order terms is shown in Fig. 17, obtained in a representative case.

![Graph showing the effect of higher-order QED corrections on $A_{FB}$ near the resonance](image.png)

**Fig. 17** The effect of higher-order QED corrections on $A_{FB}$ near the resonance. The electroweak parameters are $m_t = 92$ GeV, $m_Z = 2.9$ GeV, $\sin^2 \theta_W = 0.23$. The dot-dashed curve is the first-order correction, whilst the solid curve contains corrections of all orders ($\Delta = \Delta \omega/E$). (From Ref. [40].)
5. $Z^0$ DECAYS

The partial width of $Z^0$ into a massless fermion-antifermion pair, including electroweak and strong radiative corrections, is given by

$$\Gamma(Z^0 \rightarrow f\bar{f}X) = N \frac{G_{F} m_Z^3}{24\pi^2} \left[ 1 + (1 - 4|Q_e|s^2)^2 \right] (1 + \delta_f),$$

(71)

where $X = \gamma, g, \ldots$; $\delta_f$ is the electroweak radiative correction, and

$$N = 1 \quad \text{for leptons,}$$

(72)

$$N = 3 \left( 1 + \frac{\alpha_s(m)}{\pi} + \ldots \right) \quad \text{for quarks.}$$

(73)

Here $\alpha_s$ is the QCD coupling whose value at the scale $m$ can be taken as [8]

$$\frac{\alpha_s(m)}{\pi} = 0.04 \pm 0.01.$$  

(74)

For the $t$-quark and perhaps the $b$-quark the mass cannot be neglected, and an extra factor $\Gamma(m)/\Gamma(0)$ must be included in Eq. (71), which is ($\beta = v/c$):

$$\Gamma(m)/\Gamma(0) = \frac{v^2 q^2 \beta q (3 - 5q^2) + A^2 q^2}{v^2 + A^2 q}$$

(75)

(the QCD corrections, different for the vector and axial vector terms, are neglected). Note that by writing $\Gamma_{\text{Born}} = G_{F} m_Z^3$ instead of $\Gamma_{\text{Born}} = \alpha(m_e) m_Z/s^2$ we make the Born approximation more precise. In fact, as already mentioned, the radiative corrections (which result in a scale dependence) are large on both $\alpha$ and $s^2$. On the contrary, there are no leading logs in the scale dependence of the Fermi coupling. Finally, $\delta_f$ is smaller if the value of $s^2$ in Eq. (71) is obtained from $m_Z$ through the radiatively corrected formula (33). In this case, in the standard theory [21]

$$\delta_f \lesssim 0.1\%$$

(76)

provided that $m_t = m_Z$ and there are no other heavy families. Thus the improved Born approximation, which includes $G_{F} m_Z^3$ and the corrected value of $s^2$, is very precise (if the QCD correction is included for quarks). The electroweak radiative corrections are

---

*) One can define $\alpha_s(m)$ according to the $\overline{\text{MS}}$ prescription [42], the second-order correction in Eq. (73) being also known in this case [43].
tive correction $\delta_f$ can be considerably larger if heavy fermions or widely split weak isospin multiplets of SUSY scalars exist [29].

To a good approximation we have

$$\Gamma_{\text{tot}} = \Gamma(Z^0 \rightarrow f\bar{f}X)$$

(77)

because the contribution to $\Gamma_{\text{tot}}$ of other $Z^0$ decays is well below the interesting level of accuracy if there is no new physics below the $Z^0$, as discussed in the summary by Franco on rare $Z^0$ decays [44]. For $s^2 = 0.23$, $m_Z = 92$ GeV, $\alpha_s/\pi = 0.04$, one has

$$\Gamma_{v\bar{v}} = 170 \text{ MeV},$$

$$\Gamma_{e\bar{e}} = 86 \text{ MeV},$$

(78)

$$\Gamma_{d\bar{d}} = 393 \text{ MeV},$$

$$\Gamma_{u\bar{u}} = 305 \text{ MeV}.$$  

For $m_t = 30$ GeV we also find

$$\Gamma_{\text{tot}} = (2560 - 2680) \text{ MeV}.$$  

(79)

Note that the error on $\alpha_s/\pi$ quoted in Eq. (74) corresponds to an ambiguity $\Gamma_{\text{tot}} \pm 20$ MeV which remains even after $m_t$ is known.

We have already seen that $\Gamma_{\text{tot}}$ can easily be measured at LEP [10] with an error $\delta\Gamma_{\text{tot}} \pm 50$ MeV or $\delta\Gamma_{\text{tot}}/\Gamma_{\text{tot}} \pm 2\%$. A comparable relative accuracy can also be obtained for the partial widths into charged leptons. The partial width $\Gamma_{ee}$ can be directly related to the luminosity monitor. By measuring

$$I = \int \sigma(W) e^+ e^- e^- e^+ dW = \frac{6\pi^2}{m_Z^2} \Gamma_{\text{tot}}^2$$

(80)

over the resonance, we obtain $\Gamma_{ee}^2/\Gamma_{\text{tot}}^2$.

From this we have

$$\frac{\delta\Gamma_{ee}}{\Gamma_{ee}} = \left[ \frac{1}{2} \frac{\delta I}{I} \right]^2 + \left[ \frac{1}{2} \frac{\delta\Gamma_{\text{tot}}}{\Gamma_{\text{tot}}} \right]^2 \right]^{1/2} \leq 1.8\%,$$

(81)

where the last figure is obtained for $\delta I/I \leq 3\%$ and $\delta\Gamma_{\text{tot}}/\Gamma_{\text{tot}} \leq 2\%$. Similarly, the ratio $\Gamma_{\mu\mu}/\Gamma_{ee}$ can be measured with an accuracy
\[ \delta(\frac{\Gamma_{\mu\mu}}{\Gamma_{ee}}) \geq 1.5\% , \] (82)

where 1% is due to statistics and 1% to the muon trigger efficiency [10].

Thus we conclude that \( \Gamma_{tot} \), \( \Gamma_{ee} \), and \( \Gamma_{\mu\mu} \) can all be measured with a relative accuracy of about \( \pm 2\% \). This precision is sufficient for neutrino counting (\( \delta \Gamma_{tot} = \pm 50 \) MeV, whilst one \( v \)-species corresponds to \( \delta \Gamma_{vv} \leq 170 \) MeV) or perhaps for establishing the presence of new channels in \( Z^0 \) decays [e.g. light SUSY particles, light technicolour states, etc. [29]] but is not enough for a significant determination of \( \sin^2 \theta_W \). In fact it turns out from Eqs. (71) and (77) that the absolute error on \( \sin^2 \theta_W \) is of the same order as the relative error on the widths:

\[ \delta \sin^2 \theta_W \leq \frac{\delta \Gamma_{tot}}{\Gamma_{tot}} \geq \frac{\delta \Gamma_{\mu\mu}}{\Gamma_{\mu\mu}} . \] (83)

The absolute accuracy \( \delta \sin^2 \theta_W \leq \pm 0.02 \) that can be obtained from the width is about an order of magnitude worse than what one expects to reach from the asymmetries.

6. HIGGS SEARCHES

On the \( Z^0 \) resonance, one can look for the Higgs produced in the decays

\[ Z^0 \rightarrow H^0 \mu \bar{\nu}_\mu , \] (84)

\[ Z^0 \rightarrow H^0 \gamma . \] (85)

The corresponding decay rates and distributions are discussed in the article by Franco [44]. The relevant branching ratios are plotted in Fig. 18. The charged

![Diagram](image)

Fig. 18 The branching ratios \( \Gamma(Z^0 \rightarrow H^0 \mu \bar{\nu}_\mu) / \Gamma_{\mu\mu} \), and \( \Gamma(Z^0 \rightarrow H^0 \gamma) / \Gamma_{\mu\mu} \) as functions of \( m_H \), for \( \sin^2 \theta_W = 0.23 \). (From Ref. [44].)
lepton mode is the most promising for the Higgs search. The photon mode has a smaller branching ratio than that of the charged lepton mode, except for Higgs masses very close to the \( Z^0 \) mass where both processes are invisible. For \( s^2 \sim 0.23 \) the branching ratio \( B(Z^0 \rightarrow H^0 l^+l^-) \), with \( l = e, \mu, \tau \), varies between \( 7 \times 10^{-5} \) and \( 2 \times 10^{-7} \) for \( m_H = (20-70 \) GeV). With the luminosity given in Eq. (10), \( (10^8-10^9) \) \( Z^0 \)'s are produced in one year. For \( m_H \sim 50 \) GeV, one then expects about \( (10-100) \) \( H^0 l^+l^- \) (\( l = e, \mu, \tau \)) events per \( 10^6 \) \( Z^0 \)'s. Note that the rate \( Z^0 \rightarrow H^0 \nu \bar{\nu} \) is about twice as large.

The problem of identifying the Higgs final state is made more difficult if the decay \( H^0 \rightarrow t \bar{t} \) is forbidden because \( m_H \lesssim 2m_t \), as seems likely to be the case for \( m_H \lesssim 50 \) GeV. As the t-quark is easily recognized the signature \( l^+l^-t\bar{t} \) would be a rather good one, much better than \( l^+l^-b\bar{b} \).

In conclusion, the search for the Higgs boson in \( Z^0 \) decays can only be successful if \( m_H \lesssim 50 \) GeV, and will in any case be difficult. Complementary (and in some cases better) possibilities for Higgs-searching at LEP are offered by toponium decays (see the article by Buchmüller et al. [45]) and by \( e^+e^- \rightarrow H^0 Z^0 \) above the \( Z^0 \) [29].

7. NEUTRINO COUNTING

We have already seen that the measurement of \( \Gamma_{tot} \) can lead us to a determination of the number of neutrinos. A more direct way of neutrino counting is to measure the cross-section for \( e^+e^- \rightarrow \nu \bar{\nu} \gamma \) a few GeV above the \( Z^0 \), and this process has been studied in this context by many authors [46]. The relevant Feynman diagrams are shown in Fig. 19. The photon energy spectrum is plotted in Fig. 20. It is assumed that \( m_Z \sim 92 \) GeV, and that there will be 10 days of running [with the luminosity given in Eq. (10)] at /s between 96 and 104 GeV. On the \( Z^0 \) peak in the photon energy the addition of one neutrino type leads to \( \delta \sigma/\sigma \sim 1/3 \), so that for a 3\( \sigma \) determination of the number of neutrinos, one simply needs a 10\% measurement of the cross-section. Thus the statistics is no

Fig. 19 Feynman diagrams for \( e^+e^- \rightarrow \nu \bar{\nu} \gamma \)
problem. In the 10-day run described above, one can obtain $N_\nu$ to an accuracy level of $(4.5-5)\sigma$. The main problem is the background, particularly from $e^+e^- \rightarrow e^+e^-\gamma$. As discussed in detail in the paper by Simopoulos [47], this background can be effectively controlled if the angular coverage for electron detection can be extended down to 6 degrees.

8. SUMMARY AND CONCLUSION

The main results of our working group are listed below:

- $m_\gamma$ and $\Gamma_{\text{tot}}$ can be measured relatively easily to an accuracy $\delta m_\gamma \approx \pm 50$ MeV.
- Polarization (transverse is enough) and a polarimeter are needed to reach the level of $\pm 20$ MeV, an effort which we recommend as a natural task for LEP.
- Neutrino counting will easily be obtained at LEP.
- A new heavy generation of fermions with $m_t$, $-m_b$, $>m_\gamma$ will produce detectable effects. Similarly, the same is true for other widely split, new weak-isospin multiplets of scalar quarks or scalar leptons.
- The search for the Higgs boson in $Z^0$ decays is a difficult task which can only be successful if $m_H \lesssim 50$ GeV.
Asymmetries are the most suitable for precision studies. With no beam polarization, one can only measure the forward-backward asymmetry $A_{FB}$ and the final $\tau$-lepton helicity asymmetry $A_{pol}$. For these asymmetries one can obtain an experimental accuracy corresponding to fixing $\sin^2 \theta_w$ within $\delta \sin^2 \theta_w \approx \pm 0.002$. This precision is needed in order to be sensitive to the details of the weak radiative corrections. In fact, even if the standard model with three families and one elementary Higgs doublet was true and $m_\tau$ was precisely known, the ignorance of the value of $m_H$ is the range $10$ GeV $\leq m_H \leq 1$ TeV would still make $\sin^2 \theta_w$ uncertain by $\delta \sin^2 \theta_w = \pm 0.0025$ for given $m_Z$. In order to reach this required precision, one needs more than 100 days of running at the $z^0$ with $L = 10^{31}$ cm$^{-2}$ s$^{-1}$ -- and a serious effort.

If a measured longitudinal polarization of the $e^-$ beam is available, the advantage will be great for precision tests of the electroweak theory at the $z^0$. The sensitivity of $A_{FB}$ to $\sin^2 \theta_w$ is greatly increased, and one can also measure the left-right asymmetry $A_{LR}$ with great precision; moreover, $A_{LR}$ is very sensitive to $\sin^2 \theta_w$. The same accuracy as that described in the case of no polarization can be obtained in only 40 days of running with a measured longitudinal polarization.

* * *

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MEASUREMENT OF THE $Z^0$ MASS AND WIDTH AT LEP

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1. INTRODUCTION

The first experiment at LEP will be a scan around the $Z^0$ resonance, followed by a long period of data taking at the pole. The scan will give precise information on the mass $m_Z$ on the total width $\Gamma_{\text{tot}}$, and on partial widths. The precision is likely to be limited by systematic rather than statistical errors.

The most precise and most important experiment planned is the measurement of the $Z^0$ mass, which will set the scale for high-precision tests of the electroweak theory if converted into a precise value of $\sin^2 \theta_W$. The only other experiments which are comparable in precision, and hence will allow a stringent test of the electroweak theory, are the measurement of the forward-backward asymmetry in $e^+e^- + \mu^+\mu^-$, and the measurement of the $\tau$ polarization in $e^+e^- + \tau^+\tau^-$. The measurements of the total and partial widths are less important by-products of the scan. The measurement of the total width will allow a check for unknown weakly coupled neutral objects with mass smaller than $m_Z/2$, for instance further generations of neutrinos. However, this information can also be obtained from a measurement of the cross-section for radiative $Z^0$ production followed by its decay into such weakly coupled objects [1], in a systematically more precise and less model-dependent fashion. The measurements of partial widths will enable a generation universality check: $\Gamma_{ee} = \Gamma_{\mu\mu} = \Gamma_{\tau\tau}$?

The scan experiment will be carried out in 13 equidistant steps of 2 GeV in the centre-of-mass energy, ranging from $\sqrt{s} = 80$ GeV to $\sqrt{s} = 104$ GeV, which matches the possibilities of LEP I. We assume further a constant luminosity $L = 10^{31}$ cm$^{-2}$s$^{-1}$ for 2 days (100%) at each point.

We recall briefly the lowest order formulae for the $Z^0$ mass and widths (for more complete formulae including radiative corrections we refer to Altarelli's report [2]):

$$m_Z = \left[ \frac{4G_F}{\sqrt{\pi}} \right]^{1/2} \frac{1}{\sin \theta_W \cos \theta_W}$$

(1)
\[ \Gamma_{ff} = \frac{G_F^3}{24\pi^2} (v_f^2 + a_f^2) \] (2)
\[ \Gamma_{\text{tot}} = \frac{G_F^3}{24\pi^2} \sum_f (v_f^2 + a_f^2). \] (3)

2. **MEASUREMENT OF \( m_Z \) AND \( \Gamma_{\text{tot}} \) FROM THE LINE SHAPE**

The easiest process to look at is \( e^+e^- \rightarrow \mu^+\mu^- \). This process is free of background after cuts, and is triggered upon with efficiency close to 100%. The scan described above yields a total of about 4500 \( \mu^+\mu^- \) events, 2000 of which are collected at the \( Z^0 \) pole. A fit of the line shape yields the statistical errors

\[ \Delta m_Z \leq 10 \text{ MeV} \]
\[ \Delta \Gamma_{\text{tot}} \leq 15 \text{ MeV}, \]

indicating that all error considerations must focus on systematic error sources. The distribution of running time between different energies is of minor importance.

The main systematic errors which will affect \( m_Z \) and \( \Gamma_{\text{tot}} \) are the knowledge of the beam energy and of the effective luminosity (= machine luminosity \( \times \) apparatus lifetime).

The dominant uncertainty of the beam energy arises from the magnetic field tolerance, whereas the knowledge of the effective bending radius is not considered to be a limiting factor. The latter is derived from the RF acceleration frequency and is known with better precision than the magnetic field strength. The measurement of the field integral of each dipole magnet pair, together with the effect of temperature differences in the LEP tunnel, allows a long-term precision of \( \Delta B/B \leq 3 \times 10^{-4} \), which limits the measurement of the \( Z^0 \) mass to \( \pm 30 \text{ MeV} \). Since this error is worse than the error of the experiment, it is desirable to improve the knowledge of the beam energy.

Transversely polarized beams in LEP would be a means of measuring the beam energy to the level of \( 1 \times 10^{-4} \). This method, pioneered at Novosibirsk, uses an artificially induced depolarization resonance to measure precisely \( (-10^{-5}) \) the spin precession frequency of transversely polarized electrons. The precession frequency effectively determines the magnetic field integral. The precision on the beam energy is then, in practice, determined by the short-term (where 'short-term' refers to one LEP filling), magnetic-field stability, which is \( \leq 1 \times 10^{-4} \). Knowledge of the beam energy to this precision implies \( \Delta m_Z \sim 10 \text{ MeV} \).
As far as the effective luminosity is concerned, we notice that the absolute calibration error drops out in the fit of the line shape. What matters is any systematic error other than a scale error in the effective luminosity across the resonance. Such errors may be monotonic with beam energy (for example, geometrical alignment errors of the luminosity monitors would show up this way), or may take extreme values on the $Z^0$ pole (in a way that, for example, dead-time errors would do).

Such effects would, to first order, produce linear or quadratic changes of the line shape. Linear distortions affect $m_z$, but not $\Gamma_{\text{tot}}$. For quadratic distortions the reverse is true. We have guessed at a linear distortion of 2% per 10 GeV, causing $\Delta m_z \approx 10$ MeV, and a quadratic distortion of 1% at $\pm 5$ GeV, causing $\Delta \Gamma_{\text{tot}} \approx 10$ MeV.

The beam energy spread is estimated to be [3]

$$\frac{\Delta E}{E} \sim 0.7 \times 10^{-3} \quad \text{at the } Z^0 \text{ pole},$$

which has no effect on the determination of $m_z$ and $\Gamma_{\text{tot}}$.

Altogether, a precision on $m_z$ and $\Gamma_{\text{tot}}$ at the level $\pm 20$ MeV can be obtained provided the centre-of-mass energy is known to $\pm 10$ MeV. We understand that efforts will be made in the LEP Machine Division to provide a laser polarimeter to measure the transverse beam polarization. If a significant polarization can be detected, the measurement of the beam energy with the spin-resonance technique will be implemented, which will allow achievement of this precision of the centre-of-mass energy.

In view of the small experimental errors, our colleagues, the theorists, are challenged to calculate the electroweak radiative corrections of the $Z^0$ line shape to a comparable level of accuracy.

According to Eq. (1), an error on $m_z$ translates into an error on $\sin^2 \theta_w$:

$$\frac{\Delta m_z}{150 \text{ GeV}} = \Delta \sin^2 \theta_w.$$

For $\Delta m_z = 20$ MeV we obtain $\Delta \sin^2 \theta_w = 0.00013!$ A measurement of $\sin^2 \theta_w$ with comparable precision from another process would permit a consistency check for radiative corrections and, at the same time, would put stringent limits on 'new physics'.

The expected total $Z^0$ width $\Gamma_{\text{tot}}$ according to Eq. (3) is 2.78 GeV ($\sin^2 \theta_w = 0.23, m_z = 92$ GeV). The sensitivity of $\Gamma_{\text{tot}}$ to $\sin^2 \theta_w$ is rather poor: $\Delta \Gamma_{\text{tot}} = 20$ MeV corresponds to $\Delta \sin^2 \theta_w = 0.007$ which, at the time of LEP operation, will be an uninteresting accuracy.
Each new generation of neutrinos adds ∼ 170 MeV to $\Gamma_{\text{tot}}$. Hence, the existence of, say, a fourth neutrino generation, is detectable with good statistical significance (> 8σ). In practice, however, this measurement will be limited by our understanding of radiative corrections, QCD corrections, and possible t-quark threshold effects.

3. MEASUREMENT OF PARTIAL WIDTHS

The partial width $\Gamma_X$ for the channel $e^+e^- \rightarrow X$ is given by the formula

$$\frac{\Gamma_X}{\Gamma_{\text{tot}}} = \frac{m_e^2}{s^2} \int \sigma(e^+e^- \rightarrow X) \, ds,$$

which involves a cross-section integral over the $Z^0$ resonance. Since this necessitates an absolute normalization of the luminosity, the precision is probably worse (a few %) than for $\Gamma_{\text{tot}}$ (0.7%), which is determined from the line shape.

The systematic error on $\Gamma_{ee}$ (s-channel only; the contribution from the t-channel must be subtracted) is given by

$$\frac{\Delta \Gamma_{ee}}{\Gamma_{ee}} \leq \frac{1}{2} \left[ \left( \frac{\Delta \sigma}{\sigma} \right)^2 + \left( \frac{\Delta \Gamma_{\text{tot}}}{\Gamma_{\text{tot}}} \right)^2 \right]^{1/2},$$

where the cross-section integral contributes the dominant error. The latter may be determined to ∼ 3% (mainly from the luminosity measurement), leading to a total systematic error $\Delta \Gamma_{ee}/\Gamma_{ee} \sim 2\%$.

The systematic precisions $\Delta \Gamma_{e\mu}/\Gamma_{e\mu}$ and $\Delta \Gamma_{\tau\tau}/\Gamma_{\tau\tau}$ are estimated to be at the level of 2.5%, somewhat worse than for $\Delta \Gamma_{e\mu}/\Gamma_{e\mu}$ to account for additional systematic errors in the trigger efficiency and background rejection.

These errors do not seem interesting for high-precision tests of the electroweak theory. Still, sensible tests of the generation universality can be performed, taking advantage of the cancellation of the main error coming from the luminosity measurement: one may expect $\Gamma_{e\mu}/\Gamma_{e\mu}$ and $\Gamma_{\tau\tau}/\Gamma_{\tau\tau}$ to be measured with a systematic error of 1.5% from trigger efficiency and background rejection.

4. CONCLUSION

The mass of the $Z^0$ can be measured with high precision (∼20 MeV), provided the beam energy of LEP can be measured to $1 \times 10^{-4}$, for instance by the measurement of the spin precession frequency of transversely polarized beams. If radiative corrections can be well enough calculated a precision on $\sin^2 \theta_W$ of better than 0.001 can be obtained. This sets the scale for the precision to be obtained in other experiments, to allow stringent tests of the electroweak theory.
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[2] G. Altarelli, Precision tests of the electroweak theory at the $Z^0$, this report.
THE $Z^0$ LINE SHAPE

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ABSTRACT

We discuss the radiative corrections for the $Z^0$ line shape which remain in the standard SU(3) × SU(2) × U(1) model after taking into account electromagnetic bremsstrahlung. Numerical results for the $e^+e^- \rightarrow \mu^+\mu^-$ cross-section are given for unpolarized and longitudinally polarized electron beams.

* * *

Although the $Z^0$ boson width $\Gamma_{\text{tot}}$ is small compared with its mass $m_Z$, the $Z^0$ line shape, which is to be measured in future $e^+e^-$ storage rings, is expected to deviate substantially from a pure Breit-Wigner form. The main correction comes from electromagnetic bremsstrahlung, i.e. to order $\alpha$ one additional real or virtual photon compared with the Born diagrams depicted in Fig. 1. A detailed account of the effect of bremsstrahlung on the line shape can be found elsewhere [1] in this report. The present article is concerned with radiative corrections, other than QED bremsstrahlung, which occur in the standard SU(3) × SU(2) × U(1) model. They include the electromagnetic vacuum polarization, the typically weak corrections involving extra lines with heavy bosons, and QCD corrections for quarks.

In evaluating the non-bremsstrahlung $O(\alpha)$ corrections to the leading amplitude, it is important to realize that in the resonance region, i.e. for

$$|E - m_Z| \leq \Gamma_{\text{tot}},$$

(1)

the counting of powers of coupling constants is different than it is away from the resonance, where all one-loop diagrams represent an $O(\alpha)$ correction relative to the Born diagrams. In the resonance region the $Z^0$ propagator itself is of order $1/\alpha$. As a consequence, the $Z^0$ diagram in Fig. 1 has order 1; the photon

Fig. 1 Born diagrams

*) Supported by the Deutsche Forschungsgemeinschaft.
diagram represents an \(O(\alpha)\) correction, and the remaining \(O(\alpha)\) corrections arise from the one-loop diagrams with a resonating \(Z^0\) line.

Working with definite chiralities, \(h_e^* = \pm 1\) and \(h_\mu = \pm 1\), for the initial and final state, the following expression is obtained [2] for the \(e^+e^- + \mu^+\mu^-\) amplitude \(A(h_e^*, h_\mu)\) as a function of the c.m. energy \(E\):

\[
A(h_e^*, h_\mu) = \frac{g^2}{4c^2} \frac{v(h_e^*)[1 + \Delta F(h_e^*)]v(h_\mu)[1 + \Delta F(h_\mu)]}{E^2 - m_Z^2 + i m_Z \Gamma_{tot}(E)} + \frac{g^2 s^2}{E^2} + O(\alpha^2) .
\]  

(2)

Here

\[
[c^2, s^2, v(1), v(-1)] = [m_\mu/m_Z^2, 1 - c^2, 2s^2, 2s^2 - 1] ,
\]  

(3)

and the normalization of \(A\) is such that the differential cross-section for a definite chirality (longitudinal polarization) in the initial state is given by

\[
\frac{d\sigma(h_e^*)}{d \cos \delta} = \frac{E^2}{32\pi} \sum_{h_\mu} (1 + \cos^2 \delta + 2h_e^* h_\mu \cos \delta)|A|^2  .
\]  

(4)

Radiative corrections of order \(\alpha\) are contained in the SU(2) coupling constant \(g\), the weak angle \((s^2)\), the form factor contributions \(\Delta F\), and the expression to be inserted for \(\Gamma_{tot}(E)\). In terms of \(\alpha\), \(m_Z = 92\) GeV, and the \(\mu\)-decay constant \(G_\mu\), one finds (for details see Ref. [2])

\[
\frac{g^2}{4c^2} = \frac{s}{2} \Gamma_\mu m_Z^2 (1 + \Delta_g) ,
\]  

\[
s^2 = \left[1 - \sqrt{\left[1 - \frac{2\alpha_\pi}{G_\mu m_Z^2}\right]}\right] (1 + \Delta_s)  .
\]  

(5)

The expression to be substituted for the energy-dependent width \(\Gamma_{tot}(E)\) is:

\[
m_Z \Gamma_{tot}(E) = m_Z \Gamma_{tot} + \text{Im} \Gamma_Z(E = m_Z) - \text{Im} \gamma_Z(E) ,
\]  

(6)

where \(\Gamma_{tot}\) is the total \(Z^0\) width evaluated to order \(\alpha^2\), and \(\text{Im} \gamma_Z\) represents the imaginary part of the \(Z^0\) self-energy evaluated to order \(\alpha\). Equation (6) can be rigorously justified by a discussion of the \(Z^0\) propagator at the level of two loops (see Ref. [2]). Here we restrict ourselves to presenting some simple arguments in favour of this particular combination of terms. On the resonance, i.e. for \(E = m_Z\), \(\Gamma_{tot}(E)\) is equal to the radiatively corrected \(Z^0\) width, as required by consistency with the fact that the numerator of the first term in Eq. (2) involves the product of the radiatively corrected electronic and muonic
partial width. For $E \not= m_Z$, but still in the resonance region, $\Gamma_{\text{tot}}(E)$ deviates from $\Gamma_{\text{tot}}$ by an order $a^2$ contribution induced by the $r_Z$ terms. Outside the resonance region where $O(a^2)$ effects are of higher order, we may reduce Eq. (6) to its $O(a)$ content. The first two terms cancel to this order and we are left with $m_Z^{-1} \Gamma_{\text{tot}}(E) = -\text{Im} \ r_Z(E)$, which is exactly the result of the $O(a)$ calculation. Equation (6) thus performs an interpolation between what is needed on the resonance and what is sufficient away from it.

To order $a^2$ the total $Z^0$ width is given by the following sum of partial widths referring to individual fermion flavours $f$: 

$$\Gamma_{\text{tot}} = \sum_f \left( \Gamma_{f \bar{f}} + \Gamma_{Wf \bar{f}} + \Gamma_{Hf \bar{f}} \right).$$  

Here $\Gamma_{f \bar{f}}$ is meant to include the contribution from QED bremsstrahlung. The $W$ channel can be neglected (branching ratio below $10^{-6}$ [3]), and similarly the Higgs channel if the Higgs mass is larger than 10 GeV (the branching ratio ranges from $10^{-7}$ at $m_H = 80$ GeV to $2 \times 10^{-3}$ at $m_H = 10$ GeV [3]). Therefore only $\Gamma_{f \bar{f}}$ really matters. For the charged leptons, $\Gamma_{f \bar{f}}$ can essentially be read off from the numerator of the $Z^0$ term in Eq. (2). Generalizing this calculation, we get for a given massless flavour $f$:

$$m_Z^{-2} \Gamma_{f \bar{f}} = \frac{m_Z^2}{48\pi} \frac{g^2}{4c^2} N_C \left[ 1 + 4s^2|Q_f|^2 \right]$$

$$\times \left[ 1 + \frac{3a}{4\pi} \frac{Q_f^2}{|V_f|^2} + \Delta F_f = \frac{3}{8} \frac{N_C^2 - 1}{N_C} \frac{a_s(m_Z)}{\pi} \right] [1 + O(a, a_s)],$$  

where $Q_f$ measures the electric charge, $N_C$ denotes the number of colours ($N_C = 1$ for leptons, $N_C = 3$ for quarks), and $\Delta F_f$ is a particular combination of the left- and right-handed form-factor corrections for the respective fermion.

Besides the QED bremsstrahlung, the leading contribution from gluon radiation has also been included. Using $\Lambda_{\text{MS}} = 100$ MeV for four flavours [4], the QCD correction is estimated to have the magnitude $a_s/\pi = 0.033$.

As far as the form-factor corrections $\Delta F$ are concerned, Table 1 yields $\Delta F = -0.4\%$ for the charged leptons. For quarks the corresponding numbers have even smaller magnitude, and for neutrinos $\Delta F$ is strictly zero since the SU(2) coupling $g$ has been defined in terms of $\Gamma_{\nu\nu}$.

Equation (8) does not apply to a massive top quark. To leading order the width in this case is given by

$$m_Z^{-2} \Gamma_{tt} = \frac{m_t^2}{48\pi} \frac{g^2}{4c^2} 3 \sqrt{1 - \frac{m_t^2}{m_Z^2}} \left[ 1 - \frac{4m_t^2}{m_Z^2} + \left( \frac{1 - 8/3 s^2}{3} \right)^2 \left( 1 + \frac{2m_t^2}{m_Z^2} \right) \right].$$
At present the uncertainty in \( \Gamma_{\ell \ell} \) is dominated by the uncertainty of the top mass rather than by calculating only to leading order. The calculation of the electroweak radiative corrections may be useful once the top mass is known. The main problem, however, is the QCD corrections. As noted in Ref. [5], they could amount to a 30% correction for \( m_t = 40 \text{ GeV} \) where the \( t\bar{t} \) width is roughly equal to the \( \mu \)-pair width.

From the above results and Table 1 we conclude that in the \( \alpha, G_{\mu}, m_Z \) scheme the leading radiative correction to \( \Gamma_{\ell \ell} \) and therefore \( \Gamma_{\text{tot}} \) comes from the renormalization of \( s^2 \), whereas the next leading correction is due to QCD. All other radiative corrections have negligible effect. Numerical values for the different particle widths are given in Table 2. According to this table the total width changes by only 8 MeV compared with the Born estimate. This small figure results

### Table 1

Typical radiative corrections

<table>
<thead>
<tr>
<th>( s^2 )</th>
<th>( \Delta_s )</th>
<th>( \Delta_g )</th>
<th>( \Delta F(1) )</th>
<th>( \Delta F(-1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_t = 40 \text{ GeV} )</td>
<td>0.229</td>
<td>0.105</td>
<td>&lt; 0.001</td>
<td>-0.004</td>
</tr>
<tr>
<td>( m_H = 100 \text{ GeV} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 2

Radiative corrections to the \( Z^0 \) width

(\( \Gamma_{\ell \ell} \) in MeV)

<table>
<thead>
<tr>
<th>( f )</th>
<th>Born</th>
<th>Electroweak</th>
<th>QCD</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_e, \nu_e, v_t )</td>
<td>170</td>
<td>0</td>
<td>0</td>
<td>170</td>
</tr>
<tr>
<td>( e, \mu, \tau )</td>
<td>88</td>
<td>-1.9</td>
<td>0</td>
<td>86</td>
</tr>
<tr>
<td>( u, c )</td>
<td>307</td>
<td>-12.3</td>
<td>10.1</td>
<td>305</td>
</tr>
<tr>
<td>( d, s, b )</td>
<td>389</td>
<td>-10.5</td>
<td>12.8</td>
<td>391</td>
</tr>
<tr>
<td>( t )</td>
<td>66</td>
<td>-8.4 a)</td>
<td>20 b)</td>
<td>78</td>
</tr>
<tr>
<td>( m_t = 40 \text{ GeV} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2621</td>
<td>-70</td>
<td>78</td>
<td>2629</td>
</tr>
</tbody>
</table>

a) Due to renormalization of \( s^2 \).
b) QCD correction as estimated in Ref. [5].
from the fact that the QCD contribution (+78 MeV) and the SU(2) × U(1) contribution (-70 MeV) largely compensate each other.

Having determined the radiative corrections for $\Gamma_{\text{tot}}$ it remains to incorporate the energy dependence from Eq. (6). The transition from $m_Z \cdot \Gamma_{\text{tot}}$ to $m_Z \cdot \Gamma_{\text{tot}}(E)$ is achieved by replacing $m_Z^2$ by $E^2$ on the right-hand side of Eqs. (8) and (9).

We now present some curves which exhibit the effect of the above radiative corrections on the $Z^0$ line shape. Figure 2a shows the unpolarized $e^+e^-\rightarrow \mu^+\mu^-$ cross-section normalized to the QED cross-section $[4\pi\alpha^2/(3m_Z^2)]$ at the $Z^0$ mass, whereas Figs. 2b and 2c refer to beams with right- and left-handed electrons, respectively. The Born approximation (also with respect to $\alpha_s$) is drawn as a dashed line. To put the size of the radiative corrections into perspective with the precision which presumably can be achieved at LEP, the cross-sections at $E = 88, 90, \ldots, 96$ GeV are given statistical error bars. They correspond respectively to 10 days running at the peak energy and two days running at each of the other energies, with a luminosity of $L = 10^{21}$ cm$^{-2}$ s$^{-1}$. The main features in the curves are due to the behaviour of the left- and right-handed couplings $\nu(\pm 1)$ from Eq. (3) under the change of $s^2$ caused by the radiative corrections.

Fig. 2a The $e^+e^-\rightarrow \mu^+\mu^-$ cross-section normalized to $4\pi\alpha^2/(3m_Z^2)$ for unpolarized beams
Fig. 2b and c The $e^+e^- + \mu^+\mu^-$ cross-section normalized to $4\pi\alpha^2/(3m_e^2)$ for beams polarized b) right-handedly, c) left-handedly.

The left-handed coupling $v(-1)^2$ decreases, whereas $v(+1)^2$ increases. These changes compensate each other in the unpolarized cross-section, for which the effect of the radiative corrections is proportional to $(4s^2 - 1)$.

As a final remark we mention that Eq. (2) also applies to the case $e^+e^- + q^+q^-$, where $q$ represents a light quark. Only the couplings $v$ have to be adjusted, and a charge factor has to be included in the photon term.

* * *
REFERENCES


    See also contributions by B. Lynn, M. Peskin and R. Stuart; M. Consoli and A. Sirlin, in this report.

    Further references for the type of corrections discussed in the present article are:


    See also the contribution by E. Franco in this report.


RADIATIVE CORRECTIONS TO THE Z° LINE SHAPE AT LEP

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1. INTRODUCTION

As discussed in Ref. [1], the line shape of the Z° as measured in e⁺e⁻ → µ⁺µ⁻X or e⁺e⁻ → X is much deformed by the QED radiative corrections. The most important distortions in an inclusive experiment are caused by bremsstrahlung from the initial legs. The maximum of the excitation curve is shifted at energies higher than the physical mass, and the radiative tail above the resonance makes the peak broader than the total width of the Z°. Given the precision required at LEP, the inclusion of the complete radiative corrections at leading order in α/π is not enough, and the dominant terms of higher order have also to be taken into account.

The important terms of higher order are the logarithmically enhanced terms. The main large logarithm is given by

\[ t \equiv \ln \left( \frac{m_Z^2}{\alpha^2} \right) \approx 24.2 . \]  

Actually there are also relatively large logarithms of the form \( \ln \left( \frac{m_Z^2}{\Gamma_{\text{tot}}^2} \right) \approx 7.13 \), but they play a more marginal role in this problem, as we shall see. Corresponding to a small correction to the cross-section near the resonance

\[ \frac{\delta \sigma}{\sigma} = O(\epsilon) , \]  

one expects a correction for the mass and width given by

\[ \frac{\delta m_Z}{m_Z} \leq \frac{\delta \Gamma_{\text{tot}}}{\Gamma_{\text{tot}}} \leq O(\epsilon) . \]  

Thus if we want to control the QED corrections to \( m_Z \) and \( \Gamma_{\text{tot}} \) within an accuracy \( \delta m_Z \leq \delta \Gamma_{\text{tot}} \leq 10 \text{ MeV} \), we need to include all terms of order \( \epsilon \geq 0.5\% \) or so. In the perturbative expansion all terms of order \( \alpha^n (\alpha t)^n \) are present with \( m, n \geq 0 \). We need to consider at least the terms of order \( [(\alpha/\pi)t] \leq 6\% \), \( (\alpha/\pi) \leq 0.2\% \),
\[ (\alpha_s/\pi) t^2 \leq 0.3, \text{ i.e. the complete lowest-order correction plus the largest term of higher order. It is well known that the effect of these large logarithms can be neatly described. Actually the whole sum of the leading and the next-to-leading series of terms } (at)^n \text{ and } a(at)^n \text{ with all } n \text{ is known in principle. The related formalism has been extensively studied in the context of the QCD-improved parton model } [2], \text{ where all orders are important because } (\alpha_s / \pi) \times \ln \left( Q^2 / \Lambda_{\text{QCD}}^2 \right) \leq 1 \text{ for large enough } Q^2. \]

In the QED case the relevant parton density is the electron (or positron) density at the scale } s \leq m_e^2: e(x, s). \text{ The factorization of the electron mass singularities implies that all the logarithmically enhanced terms can be included in the electron density (and in the running coupling). The corrected cross-section } \sigma_c \text{ is obtained by a convolution of the electron and positron densities with a reduced cross-section } \sigma \text{ (Fig. 1):

\begin{equation}
\sigma_c(S) = \int \frac{d x_1}{0} \frac{d x_2}{0} \delta(x_1 x_2 S - S_0) e(x_1, S) e(x_2, S) \sigma(x_1 x_2 S),
\end{equation}

\text{where } \sigma \text{ includes the resonant Born term (with } \alpha \text{ replaced by the running coupling) and the QED corrections without logs, e.g. the terms } 5 \alpha / \sigma \leq 0(\alpha_s / \pi). \text{ The explicit form of } e(x, s) \text{ can be found in Eqs. (22)-(24), whilst } \alpha, \text{ up to and including terms of } O(\alpha_s / \pi), \text{ can be read out from Eqs. (31)-(48).

It is precisely in performing the convolution integral that the other large logarithms } \ln \left( m_e^2 / R_{\text{tot}}^2 \right) \text{ appear. They give rise to a series of doubly logarithmic terms } (at \ln \left( m_e^2 / R_{\text{tot}}^2 \right))^n. \text{ A refinement of the calculation consists in keeping only a few terms in } (at)^n, \text{ but all the double logarithmic terms. Their sum can in fact be easily included.

In the following we first describe the calculation of the electron density } e(x, s). \text{ We then consider the convolution integral (4). After recalling the structure of the complete first-order QED correction and constructing } \sigma(s), \text{ we discuss the origin of the double logs and their resummation. Finally, we list our results and conclusions.

2. THE ELECTRON DENSITY } e(x, s)

The electron density } e(x, s) \text{ and the photon density } \gamma(x, s) \text{ in an electron satisfy the evolution equations:

\begin{equation}
\frac{s}{ds} \rho e(x, s) = a(s) \int \frac{dy}{x y} \left[ e(y, s) P_{ee} \left( \frac{x}{y}, s \right) + \gamma(y, s) P_{ee} \left( \frac{x}{y}, s \right) \right],
\end{equation}

\text{Fig. 1 Structure of diagrams for initial-state bremsstrahlung and their gauge completion.}
\[ s \frac{d}{ds} \gamma(x,s) = \frac{\alpha(s)}{2 \pi} \int_x^1 \frac{dy}{y} \left[ e(y,s) P_{\gamma e} \left( \frac{x}{y}, s \right) + \gamma(y,s) P_{\gamma \gamma} \left( \frac{x}{y}, s \right) \right], \quad (6) \]

with \( P_{\gamma \gamma} = O(\alpha^2) \), and

\[ P_{ee}(x,s) = \frac{1+x^2}{1-x^2} + O[\alpha(s)], \quad (7) \]

\[ P_{e\gamma}(x,s) = \left[ x^2 + (1-x)^2 \right] + O[\alpha(s)], \quad (8) \]

\[ P_{\gamma e}(x,s) = \frac{1 + (1-x)^2}{x} + O[\alpha(s)]. \quad (9) \]

We shall often write \( P(x) \) when the corrections of order \( \alpha(s) \) are dropped. Note that the \( P_{\gamma \gamma} \) term is absent in lowest order because there is no corresponding vertex in QED. Here \( \alpha(s) \) is the QED running coupling:

\[ \frac{\alpha}{\alpha(s)} = 1 + b \alpha + O(\alpha^2), \quad (10) \]

with

\[ b = -1/3 \pi, \quad (11) \]

and

\[ 1 = \sum_f \frac{Q_f^2 \ln \frac{s}{m_f^2}}{s}, \quad (12) \]

where the sum \( \sum_f \) goes over quarks (of all colours) and leptons with mass \( m_f^2 \leq s \) (and charge \( Q_f \)). Note that \( \alpha(m_f^2) = \alpha \). The boundary conditions are

\[ e(x, m_e^2) = \delta(1-x) \quad \gamma(x, m_e^2) = 0. \quad (13) \]

Consider first the non-singlet limit \( \gamma(x,s) = 0 \). The first diagram--of order \( [(\alpha/\pi)]^2 \)--that contributes to our calculation through \( \gamma(x,s) \) is shown in Fig. 2. Thus the non-singlet limit is relevant if we are inclusive in any number of photons but exclude extra electrons in the final state. In any case

Fig. 2 A lowest-order diagram which breaks the valence approximation
we shall give later the interesting results with the $\gamma$ term included. In the 
non-singlet approximation the solution of Eq. (5) with the boundary conditions 
in Eq. (13) has a simple form in terms of moments:

$$
e_n(s) = \prod_f f_s \left[ \frac{\alpha(m_f^2)}{\alpha(m_{f+1}^2)} \right]^{A_{n/2}b_f},
$$

where $f = 1$ corresponds to the electron, $f_s$ to the heaviest fermion with $m_f^2 \leq s$
and $m_{f+1}^2 > s$. Similarly,

$$
b_f = b(Q_1^2 + Q_2^2 + \ldots + Q_{\nu}^2).
$$

The moments $e_n(s)$ and $A_n$ are given by

$$
e_n(s) = \int_0^1 dx x^{n-1} e(x, s),
$$

$$
A_n = \int_0^1 dx x^{n-1} P_{ee}(x) = -\frac{1}{2} + \frac{1}{n(n+1)} - 2 \sum_{j=2}^{\infty} \frac{1}{j}.
$$

By defining

$$
\alpha_T = \sum_f f_s \frac{1}{b_f} \ln \frac{\alpha(m_f^2)}{\alpha(m_{f+1}^2)}
$$

[note that $T$ is of the same order as $t$ or 1 defined in Eqs. (1) and (2)], we

can rewrite Eq. (14) in the form

$$
e_n(s) = \exp \left( A_n T/2x \right),
$$

which in $x$ space is equivalent to

$$
e(x, s) = \delta(1-x) + \frac{\alpha_T}{2x} P_{ee}(x) + \frac{1}{2} \left( \frac{\alpha_T}{2x} \right)^2 \int \frac{dy}{y} P_{ee}(y) P_{ee}(\frac{x}{y})

+ \frac{1}{6} \left( \frac{\alpha_T}{2x} \right)^3 \int \frac{dy}{y} P_{ee}(y) \int \frac{dz}{z} P_{ee}(z) P_{ee}(\frac{x}{yz}) + \ldots.
$$

Neglected terms are of order $\alpha^2 T$ and could be included by taking into account
the contributions at two loops to $\alpha(s)$ and $P_{ee}(x, s)$. 

We now improve the non-singlet approximation. In lowest order we have

$$\gamma(x,s) = \frac{\alpha T}{2\pi} P_{\gamma}(x) + \ldots.$$  \hspace{1cm} (21)

Thus the complete result for e(x,s) up to second order in T is given by

$$e(x,s) = \delta(1-x) + \left(\frac{\alpha T}{2\pi}\right) P_{\gamma e}(x)$$

$$+ \frac{1}{2} \left(\frac{\alpha T}{2\pi}\right)^2 \left[ \int_x^1 \frac{dy}{y} \left[ P_{\gamma e}(y) P_{ee}(x) + P_{ee}(y) P_{\gamma e}(x) \right] + \ldots \right].$$  \hspace{1cm} (22)

The integrals can be explicitly computed:

$$\int_x^1 \frac{dy}{y} P_{ee}(y) P_{ee}(x) = \left(\frac{9}{4} - 2\frac{x^2}{3}\right) \delta(1-x) + 8 \left[ \frac{\ln(1-x)}{1-x} \right]$$

$$- 4 \frac{\ln x}{1-x} + \frac{6}{(1-x)} + (1+x) \left[ 3 \ln x - 4 \ln (1-x) \right] - 5 - x,$$

$$\int_x^1 \frac{dy}{y} P_{\gamma e}(y) P_{\gamma e}(x) = \frac{4}{3x} + 2(1+x) \ln x + 1 - x - \frac{4x^2}{3}.$$  \hspace{1cm} (23)

$$\int_x^1 \frac{dy}{y} P_{ee}(y) P_{\gamma e}(x) = \frac{4}{3x} + 2(1+x) \ln x + 1 - x - \frac{4x^2}{3}.$$  \hspace{1cm} (24)

This completes our discussion of e(x,s).

3. **THE REDUCED CROSS-SECTION FOR e^+e^- + W^+W^-**

We now describe the structure of the reduced cross-section \(\sigma(s)\) which appears in the convolution Eq. (4) with the initial lepton densities. If we were prepared to neglect all non-logarithmic terms, the reduced cross-section would simply be identified with the resonant Born cross-section. In this section we shall closely follow the notation of Ref. [3]. The Born cross-sections are then given by

$$\frac{d\sigma_0}{dQ_\mu} = \frac{\alpha^2}{4s} \left[ W_1 (1+c^2) + W_2 c \right],$$  \hspace{1cm} (25)

$$\sigma_0 = \int \frac{d\sigma_0}{dQ_\mu} dQ_\mu = \frac{4\pi a^2}{3s} W_1,$$  \hspace{1cm} (26)

where

\(c = \cos \theta\) and
\[ W_1 = 1 + \frac{2(s-m^2)sc^2_Y}{\|Z(s)\|^2} + \frac{s^2(c_Y^2 + c_A^2)}{\|Z(s)\|^2}, \]  
\[ W_2 = \frac{4(s-m^2)sc^2_A}{\|Z(s)\|^2} + \frac{8s^2c_Yc_A}{\|Z(s)\|^2}, \]  
where \( m = m_2 \),

\[ Z(s) = s - m^2 + im\Gamma, \]

with \( \Gamma \equiv \Gamma_{\text{tot}} \) and

\[ c_A = \frac{-1}{2 \sin \theta_W}, \quad c_Y = c_A(1 - 4 \sin^2 \theta_W). \]

The first term in \( W_1 \), which is 1, is to be omitted in the following. This is the non-resonant, purely Coulomb part which would lead to problems near \( s = x_1 x_2 s - 0 \) in the convolution integral. In fact, when both the exchanged photon in the Born diagram and the radiative photon are soft, they must be treated on an equal basis. Thus we shall drop the Coulomb term and assume that this continuum background has been subtracted.

We now also want to take the non-logarithmic terms of order \( a/w \) into account. We then go back to the general expression of Ref. [3] for the cross-section, including the complete first-order corrections. When the angular integrations are performed and the artificial separation between soft and hard emission is eliminated (which plays no role in the present context), one obtains a result of the form

\[ \sigma(s) = \sigma_0 + \delta\sigma_{\text{LL}} + \delta\sigma_F, \]

where \( \delta\sigma_{\text{LL}} \) is the leading logarithmic correction,

\[ \delta\sigma_{\text{LL}} = \frac{a}{w} \ln \frac{s}{m^2} \int_{0}^{1} \sigma_0(zs) P_{ee}(z) \, dz, \]

and \( \delta\sigma_F \) is the remaining correction,

\[
\begin{align*}
\sigma_0 + \delta\sigma_F &= \sigma_0(s) \left[ 1 + \frac{2a}{w} \left( \frac{s^2}{6} - \frac{5}{6} \right) \right] + \frac{4wa^2}{3s} \left[ 1 + \frac{(s-m^2)sc^2_Y}{\|Z(s)\|^2} \right] \delta\nu_F \\
&+ \frac{3a}{w} \frac{s(s-m^2)c^2_A}{\|Z(s)\|^2} \ln \frac{\|Z(s)\|^2}{sm^2} + \frac{12a}{w} \frac{s^2c_Yc_A^2}{\|Z(s)\|^2} \ln \frac{\|Z(s)\|^2}{m^2s^2}
\end{align*}
\]
\[
- \frac{\alpha}{\pi} \int_0^{1/s(z)} \frac{1+z^2}{(1-z)^2} \, dz + \frac{2\alpha^3}{s} \int_0^1 [C(s, z)s - D(s, z)s] \\
\times \left[ \frac{2}{(1-z)^2} - 1 \right] \, dz + \int \frac{d\alpha}{\mu} \left[ \frac{d\alpha}{\mu} \delta_s + \frac{d\alpha}{\mu} \delta_{\gamma\gamma} \right] \\
+ \frac{d\alpha}{d\mu} \left( \frac{m^2}{s-m^2} \right) \left[ \delta_{\gamma\gamma} + \delta_{\gamma\gamma} \right] + \left( \frac{d\alpha}{d\mu} \right)^2 \delta_{\gamma\gamma} \right) + O(\alpha^4) .
\]

(33)

The list of symbols is given in the following: $\delta_{\gamma\gamma}$ is the vacuum polarization contribution,

\[
\delta_{\gamma\gamma} = \alpha / \pi \sum_{f} \int \frac{d^2 t}{t} \left( -5/9 + 1/3 \ln \frac{s}{m_f^2} \right) .
\]

(34)

It can be completely omitted by replacing $\alpha$ with the running $\alpha(s)$ in the lowest-order cross-section $\sigma_0$. The functions $C(s, s')$ and $D(s, s')$ are given by

\[
C(s, s') = 1 + \frac{\frac{s(s-m^2)}{|Z(s)|^2} + s'(s'-m^2)}{|Z(s')|^2} \left( \frac{c_v^2}{c_A^2} \right) \\
+ \frac{ss'[(s-m^2)(s'-m^2)+m^4r^2]}{|Z(s)|^2 |Z(s')|^2} \left( \frac{c_v^2}{c_A^2} \right)^2 .
\]

(35)

\[
D(s, s') = 1 + \frac{\frac{s(s-m^2)}{|Z(s)|^2} + s'(s'-m^2)}{|Z(s')|^2} \left( \frac{c_v^2}{c_A^2} \right) \\
+ \frac{ss'[(s-m^2)(s'-m^2)+m^4r^2]}{|Z(s)|^2 |Z(s')|^2} \left( \frac{c_v^2}{c_A^2} \right)^2 + \frac{1}{4c_v^2} .
\]

(35)

We also have

\[
\delta_s = 2\alpha / \pi \left[ 2 \ln^2 \left( \frac{\sin \frac{s}{2}}{2} \right) - 2 \ln^2 \left( \cos \frac{s}{2} \right) - \text{Li}_2 \left( \frac{\sin^2 \frac{s}{2}}{2} \right) + \text{Li}_2 \left( \cos^2 \frac{s}{2} \right) \right] ,
\]

(36)

where $\text{Li}_2(z)$ is the dilogarithm function

\[
\frac{d\alpha}{d\mu} = \frac{\alpha^2}{4s} (1 + c^2) ,
\]

(37)
\[
\delta_{\gamma\gamma}^Q = \frac{-4a}{\pi(1+c^2)} \left[ c \left( \ln^2 \left( \sin \frac{\theta}{2} \right) + \ln^2 \left( \cos \frac{\theta}{2} \right) \right) - \cos^2 \frac{\theta}{2} \ln \left( \sin \frac{\theta}{2} \right) \right. \\
+ \sin^2 \frac{\theta}{2} \ln \left( \cos \frac{\theta}{2} \right) \right], \\
\]  
(38)

\[
\frac{d\delta_{\mu}^I}{d\omega} = \frac{a^2}{4s} \frac{2s(s-m^2)}{|Z(s)|^2} \left[ c_V^2 (1+c^2) + 2c_A^2 c \right], \\
\]  
(39)

\[
\frac{d\delta_{\mu}^Z}{d\omega} = \frac{a^2}{4s} \frac{s^2}{|Z(s)|^2} \left[ (c_V^2 + c_A^2) (1+c^2) + 8c_V^2 c_A^2 c \right], \\
\]  
(40)

\[
\text{Im} \, \Pi = \frac{1}{3} a \sum_{i} Q_i^2, \\
\]  
(41)

\[
\delta_{\gamma\gamma}^I = \frac{a}{2\pi} \left[ V_1 - \frac{m^2}{s-m^2} 2sV_2 + \left( A_1 - \frac{m^2}{s-m^2} 2sA_2 \right) \frac{c_V^2 (1+c^2) + 2c_A^2 c}{c_V^2 (1+c^2) + 2c_A^2 c} \right], \\
\]  
(42)

with

\[
V_1 = -c \left[ \ln^2 \sin \left( \frac{\theta}{2} \right) + \ln^2 \cos \left( \frac{\theta}{2} \right) \right] + \ln \sin \left( \frac{\theta}{2} \right) - \ln \cos \left( \frac{\theta}{2} \right) \\
\]  
(43)

\[
V_2 = 2 \ln \tan \left( \frac{\theta}{2} \right) - \frac{1}{2c} \left[ \ln \sin \left( \frac{\theta}{2} \right) + \ln \cos \left( \frac{\theta}{2} \right) \right] - \frac{c}{1-c^2}, \\
\]  
(44)

\[
A_1 = -c \left[ \ln^2 \sin \left( \frac{\theta}{2} \right) - \ln^2 \cos \left( \frac{\theta}{2} \right) \right] + \ln \sin \left( \frac{\theta}{2} \right) + \ln \cos \left( \frac{\theta}{2} \right) \\
\]  
(44)

\[
A_2 = -\frac{1}{2} c \left[ \ln \sin \left( \frac{\theta}{2} \right) - \ln \cos \left( \frac{\theta}{2} \right) \right] + \frac{1}{1-c^2}. \\
\]  

Finally,

\[
\delta_{\gamma\gamma}^Z = \frac{2a}{\pi} \left[ 2 \ln^2 \left( \cos \frac{\theta}{2} \right) - 2 \ln^2 \left( \sin \frac{\theta}{2} \right) + \text{Li}_2 \left( \sin^2 \frac{\theta}{2} \right) - \text{Li}_2 \left( \cos^2 \frac{\theta}{2} \right) \right], \\
\]  
(45)

\[
\delta_{\gamma\gamma}^I = \frac{1}{2} \delta_{\gamma\gamma}^Z + \frac{4a}{\pi} \frac{m^2}{s-m^2} \left[ \ln \left( \tan \frac{\theta}{2} \right) \right] \delta_{\gamma\gamma}^Z, \\
\]  
(46)
where $\delta_\omega$ is the phase shift of the $Z_0$ resonance,
\begin{equation}
\delta_\omega = \arctan \left( \frac{m^2}{m^2 - s} \right),
\end{equation}
where the arctan is defined in the interval $[0, \pi]$, such that $\delta_\omega \to 0$ and \(w\) when $s \to 0$ and $\infty$, respectively.

This completes the description of the finite corrections. The reduced cross-section is obtained by replacing in $\sigma_{LL}$:
\begin{equation}
\ln \frac{s - m^2}{m_e^2} + \ln \frac{s - m^2}{m_e^2} - T,
\end{equation}
where $T$ is defined in Eq. (18). By this replacement we avoid double counting of the leading logarithms, already included in the initial lepton densities.

We now consider a possible refinement which consists in the resummation of a series of double logarithms. These terms arise from the integral in Eq. (4):
\begin{equation}
I = \frac{\alpha}{\pi} \ln \frac{s - m^2}{m_e^2} \int_0^1 \frac{\alpha_0(zs)}{(1-z)^{\frac{1}{2}} + m^2 r^2 + ...} dz.
\end{equation}

By performing the integral, one obtains
\begin{equation}
I = \sigma_0(s) \frac{\alpha}{\pi} \ln \frac{s - m^2}{m_e^2} \ln \frac{(s-m^2)^2 + m^2 r^2}{m^4} + ...,
\end{equation}
where the dots indicate additional terms without double logarithms. Then $\sigma_0 + I$ contains a factor which can be rewritten in the form
\begin{equation}
\sigma_0 + I \leq \sigma_0(s) \left[ 1 + \frac{\alpha}{\pi} \ln \frac{s - m^2}{m_e^2} \ln \frac{(s-m^2)^2 + m^2 r^2}{m^4} + ... \right]
\end{equation}
\begin{equation}
\leq \sigma_0(s) \left[ \frac{(s-m^2)^2 + m^2 r^2}{m^4} \right]^{2\beta} + ... ,
\end{equation}
with
\begin{equation}
\beta = \frac{\alpha}{2\pi} \ln \frac{s - m^2}{m_e^2}.
\end{equation}

This resummation can be implemented by subtracting, at the same time, from the cross-section the corresponding first-order terms in order to avoid double counting. The numerical importance of this refinement will be discussed later in the next section.
4. RESULTS AND CONCLUSION

The main objective of our work was to describe the general method for computing the logarithmically enhanced contributions of the QED radiative corrections to the line shape beyond the leading order. In addition, we have discussed in detail the case of the inclusive \(e^-e^- + \mu^+\mu^-X\) cross-section generalization to any final state of the form \(ffX\) with \(f \neq e^-\) is trivial, where \(X\) represents any number of photons (extra charged leptons may or may not be included). In Fig. 3 we have plotted a general view of the results. The three curves are the Born approximation (no QED corrections), the first-order leading logarithmic approximation (LL1) [where \(e(x,s)\) includes the terms of order \((\alpha/s)t\), and \(\sigma_0\) is given in lowest order] and the complete result (LL1 + LL2 + F in Table 1) [where \(e(x,s)\) also includes terms of order \([(\alpha/s)t]^2\) and \(\sigma_0\) contains

![Graph](image)

**Fig. 3** Total cross-section for \(e^-e^- + \mu^+\mu^- (\gamma)\) as a function of \(s/s\). The values of the relevant parameters are \(m_e = 92\) GeV, \(\Gamma = 2.628\) GeV, \(\sin^2 \theta_c = 0.23\). Dotted line: Born approximation; dashed line: first-order leading-logarithmic approximation; solid line: complete treatment of radiative corrections in leading order plus leading logarithms of next order. The arrow indicates the position of the peak.

<table>
<thead>
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<th>Table 1</th>
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<td>Born</td>
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non-logarithmic terms of order $\alpha/\pi$. The delicate effects we have studied are responsible for the small difference between the two nearby corrected curves. The solid line can be called the complete result because the difference, induced by adding the non-singlet terms of order $[(\alpha/\pi)t]^3$ and the resummation of the double logs to all orders, needs enlarging (Fig. 4) in order to be discerned. Note that the terms of order $[(\alpha/\pi)t]^3$ are evaluated as an estimate of the magnitude of the neglected effects. For a complete treatment at that level of accuracy, one should also evaluate the two-loop contributions to $P_{ee}$ [of order $\alpha(\alpha/\pi)t]$.

The position of the maximum is very sensitive to the details of the corrective terms. In Table 1 we report the values of the cross-section at the peak and the position of the maximum for different cases. The first (second) line (both for the peak position and the corresponding cross-section) is without (with) resummation of double logs. An error of $\pm 20$ MeV on the position of the maximum is unavoidable owing to the relative flatness of the curve around the peak. This error on the mass value can only be reduced by a fit of the whole line shape around the resonance in terms of the complete formulae described in the previous section. The same fit is also necessary in order to obtain the value of $\Gamma_{tot}$.

* * *

REFERENCES

Elsewhere in this report the important role of polarization at LEP has again been emphasized [1]. Transverse polarization together with its measurement is necessary in order to measure the LEP beam energies with extremely good accuracy using a depolarizer. However, transverse polarization may be of limited use for physics [2], while a well-measured longitudinal polarization at the interaction points can have great potential, for the following reasons:

- a better accuracy on the $A_{\text{ch}}$ measurement and the $\tau$ polarization, for example, could be obtained;
- there would be the possibility of measuring the spin-flip asymmetry simply by changing the helicity in one beam -- the systematic experimental errors from the detector side would then, in principle, be much smaller (an accuracy of $\pm 5\%$ in the polarization measurement should be aimed at);
- detailed studies of $W^+W^-$ production mechanism could be carried out.

The plans for the implementation of polarized beams up to $\sqrt{s} = 50$ GeV in LEP Phase I are summarized below.

1. **The Polarization**

There is confidence that highly polarized beams of up to 50 GeV or even higher [3] can be obtained at LEP. The ways of getting and keeping polarization have been successfully tested at PETRA and in other rings. At LEP energies the error correction procedures require further refinement in their application: in particular fast high-resolution polarimeters are needed. In the 50 GeV region, depolarizing effects should be easier to control than at higher energies. On the other hand, the natural polarization rate from the Sokolov-Ternov effect is rather low, and means must be provided to enhance it.

The fight against depolarizing effects is described elsewhere [4]. Figure 1 gives the degree of polarization at LEP in the presence of the linear spin resonances. A polarization of $\pm 50\%$ will only be obtained with a very well corrected vertical closed orbit and at suitably selected energies. Once detected, the polarization can be enhanced by a careful tuning of the relevant closed-orbit harmonics. The influence of non-linear spin resonances should not be dramatic and can be minimized by various correction schemes.

At 50 GeV the natural polarization time is 3 1/2 hours, and this can be reduced to just over 1 hour by the use of eight asymmetric wigglers, in which short, strong magnets alternate with long, weak magnets. With the right choice
of polarity the polarization is strongly enhanced in the right direction and only weakly reduced by the magnets of opposite sign. The wigglers required for radiation damping at injection have been designed to have a useful degree of asymmetry for this purpose. Figure 2 gives the expected build-up time as a function of energy. At 50 GeV the asymptotic polarization level is \( \approx 75\% \). At the

\[
\begin{align*}
Q_x &= 70.35 \\
Q_y &= 78.20 \\
Q_s &= 0.08 \\
\langle Y \rangle &= 0.35 \text{ mm}
\end{align*}
\]

![Fig. 1](image1.png)

![Fig. 2](image2.png)
2. **The Laser Polarimeters** [5]

The principle is to use the spin dependence in the Compton scattering of laser photons. A circularly polarized laser beam incident on a vertically polarized electron beam is back-scattered, with an up/down asymmetry in the cross-section corresponding to an angle of \( \theta = 1/\gamma \) to the electron beam in the laboratory. The method has been used successfully in various rings [6].

The asymmetry in LEP is shown in Fig. 3. Figure 4 shows the location of the laser, the choice of which combines requirements for accessibility during machine operation, minimization of synchrotron radiation background, and minimization of the divergence in the vertical plane of the electron beam at the scattering region.
Two types of laser are under consideration: an argon-ion laser of \( \approx 100 \) W peak, operating at the high repetition rate of individual bunch passages (40 kHz); and a Nd-YAG laser of 80 MW peak pulsed at a slow rate. The latter would produce a high instantaneous photon rate into the calorimeter: one would rely only on the spatial resolution provided by a collimator slit. The former, on the contrary, allows the detector to measure both the position and the energy of individual photons.

Polarization is obtained by measuring the vertical asymmetry shown in Fig. 3. An accuracy of 5% is likely to be obtained.

In summary, with the asymmetric wigglers, appropriate correction schemes, and fast polarimeters, transverse polarization should be obtained, measured, and used successfully at LEP for absolute energy calibration. However, for physics at the \( \sqrt{s} \) we note that the beam lifetime and the time needed to build a substantial amount of polarization would be, unfortunately, of the same order of magnitude, unless one installed highly asymmetric wigglers specifically for polarization enhancement.

3. **SPIN ROTATORS**

For each interaction point requiring longitudinal polarization, a pair of spin rotators must be installed, one each side of the detector region.

In straight sections where there are no RF cavities, sufficient length is available and a conceptual design exists [7, 8].

In straight sections occupied mainly by RF cavities the space is insufficient. A possibility would be to incorporate the rotators into the ends of the bending arcs. But at present, one has still to invent a conceptual design compatible with the constraints of the tunnel installations.

It seems that, unfortunately, the possibility to get longitudinal polarization at all interactions looks like a long-term option depending on further conceptual and technical work and financial means.
REFERENCES AND FOOTNOTES


[2] Although F.M. Renard et al. argue that it could be useful for SUSY searches; see 'New Particles' section of this report.


THE ROLE OF THE ONE-LOOP ELECTROWEAK EFFECTS IN $e^+e^- \rightarrow \mu^+\mu^-$

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1. INTRODUCTION

Present experiments at the CERN pp Collider, and future experiments with the next generation of $e^+e^-$ machines, will open up a new area of research for testing the predictions of the standard model [1]. Weak interaction phenomena will become observable with large statistics, a fact that may well facilitate the study of higher-order effects. Our aim in this report is to discuss the process $e^+e^- \rightarrow \mu^+\mu^-$ at LEP energies in the framework of the standard model, by suggesting a convenient parametrization of the lowest-order amplitude which includes a large part of the one-loop effects.

Electroweak radiative corrections to our process were originally calculated in Ref. [2]. Subsequently, several authors [3-7] performed similar calculations by adopting different strategies and various degrees of accuracy. At the end of our discussion we will compare our simple results with the complete calculation of Ref. [7] in order to point out the sensitivity of the various physical quantities to the one-loop effects. In order to set the stage for our analysis, it is convenient to make a number of general observations.

Radiative corrections are known to play a crucial role in many low-energy weak interactions, such as forbidden processes, muon decay, and the analysis of the universality of the weak interactions**). Recently, considerable attention has been given to their effect on the theoretical prediction of intermediate boson masses [9-11] and the determination of $\sin^2 \theta_W$ [12-13]. A test of the theory beyond the tree level at LEP energies could confirm the validity of the standard model as a fully-fledged quantum field theory, or signal the need for significant modifications. Whatever the final conclusion, this is an important task because our current belief in the gauge theories of electroweak interactions is largely motivated by its renormalizability. On the other hand, unlike 'pure QED','

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**) See, for example, Ref. [8].
where the fundamental physical quantities are known to a very high degree of accuracy, the theoretical predictions in the standard model are affected, to some extent, by a number of uncertainties. This is due to the presence of unknown parameters (such as the Higgs particle mass) and, in some cases, to complications arising from the strong interactions. Moreover, even lowest-order predictions for many processes are not free of ambiguities due to the current uncertainties in the determination of \( \sin \theta_w \). Although there are relationships between observables, such as the \( \mu - m_Z \) interdependence in which \( \sin \theta_w \) is effectively eliminated to zeroth order, and others involving only charged currents, such as the analysis of universality in which it plays no significant role, the fact remains that this parameter appears nearly everywhere in the discussion of neutral-current phenomena. For such processes a judicious definition and an accurate determination of \( \sin \theta_w \) become important. We want to stress that any definition of \( \sin \theta_w \) in terms of observable quantities is theoretically admissible. Some definitions, however, may be particularly convenient. For example, it is clearly advantageous to consider definitions in terms of which the radiative corrections are theoretically clean (free, as far as possible, from uncontrolled parameters) and small in magnitude. A possible approach would be to define \( \sin \theta_w \) from low-energy neutral-current processes, either in the semileptonic or purely leptonic processes

\[
v(\bar{v}) + N \rightarrow v(\bar{v}) + \ldots , \tag{1.1}
\]

\[
v(\bar{v}) + e \rightarrow v(\bar{v}) + \ldots . \tag{1.2}
\]

As discussed in Refs. [12] and [14], there are a number of drawbacks in such definitions. Indeed, the determination of \( \sin \theta_w \) from process (1.2) is at present severely limited by statistics. A precision measurement of process (1.1) and its interpretation is complicated because of strong interaction effects. Moreover, if \( \sin \theta_w \) is defined in terms of the amplitudes (1.1) or (1.2) at \( q^2 = 0 \), theoretical predictions for the higher-order corrections to any physical quantity (or process) at the vector boson scale contain sizeable contributions from various polarization functions involving light quark masses. This is a clear signal of dependance on the dynamics of strong interactions. It is well known that in the case of the photon vacuum polarization this problem can be circumvented by relating the amplitude to experimental data on \( e^+ e^- \rightarrow \text{hadrons} \) via dispersion relations. In the other cases the situation is more complicated, and one has to deal with light quark masses or appeal to other approximations\(^*\). On the

\(^*\) See for example, the discussion in Section III C in the first work quoted in Ref. [12].
other hand, it has been shown in Refs. [8] and [14] that by defining \( \sin \theta_w \) through the intermediate vector boson (IVB) parameters, such as \( W \) and \( Z \) masses and their decay rates, the theoretical predictions of higher-order corrections to physical quantities at the vector boson scale contain the large logarithms associated with light quarks only through the renormalization of the fine structure constant. More precisely, the only way in which the 'large logs' enter into the calculations is through the usual renormalized photon vacuum polarization function evaluated at \(-q^2 = m_Z^2 (q^2 = q^2 - q_0^2)\), i.e.

\[
\bar{\Pi}_{\gamma\gamma}(m_Z^2) = \bar{\Pi}_{\gamma\gamma}(m_Z^2) - \bar{\Pi}_{\gamma\gamma}(0),
\]

(1.3)

where the \( \bar{\Pi} \)'s on the r.h.s. denote the unrenormalized quantities. The reason for this is easy to understand: once \( \sin \theta_w \) is defined through IVB amplitudes (which are evaluated at invariant momenta \(-q^2 = m_W^2, m_Z^2\)) the only parameter defined in terms of a \( q^2 = 0 \) amplitude is the electric charge; as a consequence, 'mass singularities' associated with light quarks appear only through \( \bar{\Pi}_{\gamma\gamma}(m_Z^2) \). As this quantity has been computed quite accurately, one of the main sources of uncertainty in the higher-order effects can be avoided\(^\ast\). We emphasize that this feature is characteristic of any definition of \( \sin \theta_w \) in terms of the IVB parameters. The simplest and most natural choice among such definitions introduced in the first paper of Ref. [11], is

\[
\sin^2 \theta_w = 1 - \frac{m_W^2}{m_Z^2}.
\]

(1.4)

Equation (1.4) is regarded as exact to all orders in perturbation theory and can be easily implemented at the Lagrangian level. In terms of this \( \sin \theta_w \), the theoretical predictions for the vector boson masses are particularly simple [11]

\[
m_W = \frac{\mu}{s_W (1-\Delta \gamma)^{1/2}},
\]

(1.5)

\[
m_Z = \frac{\mu}{s_W c_W (1-\Delta \gamma)^{1/2}},
\]

(1.6)

where

\[
s_W = \sin \theta_w, \quad c_W = \cos \theta_w
\]

\[
\mu = \left( \frac{m_W}{G/2} \right)^{1/2} = 37.281 \text{ GeV},
\]

(1.7)

\(^\ast\) As well as the papers of Refs. [11], [12], and [14], see the recent discussion in Verzegnassi [15].
and \( G = (1.16635 \pm 0.00002) \times 10^{-5} \) GeV\(^{-2}\) is the Fermi constant. The \( O(\alpha) \) correction \( \Delta \alpha \) has been evaluated in considerable detail elsewhere [11, 12, 14]. For \( m_L = 36 \) GeV and \( m_H = m_Z \), one finds \( \Delta \alpha = 0.07 \). The value of \( \Delta \alpha \) is not too sensitive to small variations of \( m_H, m_L \), and \( \sin \theta_w \). For this reason we can take the above estimate as the 'standard' value of \( \Delta \alpha \) for our discussion. Since at LEP the \( Z \) mass will be determined with very high precision \([\Delta m_Z/m_Z = 0(10^{-3})]\), it is convenient to express all the parameters in terms of \( m_Z \) and the accurately known quantities \( \alpha \) and \( G \). From Eq. (1.6) it follows that

\[
\sin^2 \theta_w = \frac{1}{2} \left( 1 - \left[ 1 - \frac{4\mu^2}{m_Z^2(1-\Delta \alpha)^2} \right]^{1/2} \right), \quad (1.8)
\]

which leads, in conjunction with Eq. (1.4), to

\[
m_w^2 = \frac{m_Z^2}{2} \left[ 1 + \left[ 1 - \frac{4\mu^2}{m_Z^2(1-\Delta \alpha)^2} \right]^{1/2} \right]. \quad (1.9)
\]

[In deriving Eqs. (1.8) and (1.9) we have excluded the second solution of Eq. (1.6) as it obviously contradicts neutral-current phenomenology.]

By allowing \( m_H \) and \( m_L \) to vary over a wide range (10 GeV \( \leq m_H \leq 1 \) TeV, 20 GeV \( \leq m_L \leq 60 \) GeV) and defining

\[
c_w^2 = \frac{1}{2} \left( 1 - \left[ 1 - \frac{4\mu^2}{m_Z^2(1-\Delta \alpha)^2} \right]^{1/2} \right), \quad (1.10)
\]

we find

\[
\sin^2 \theta_w = c_w^2 = 3 \times 10^{-3}. \quad (1.11)
\]

This is comparable to the expected uncertainty in \( \sin^2 \theta_w \) due to the experimental resolution of \( m_Z \) at LEP:

\[
|\delta s_w^2|_{\Delta m_Z} = 2\, \frac{c_w^2}{s_w^2} \frac{\Delta m_Z}{m_Z} = 0(10^{-3}). \quad (1.12)
\]

The above parametrization allows us to circumvent the problem of using quantities extracted from experiments with different accuracy, such as \( m_w \) from pp...
experiments. In this way all physical amplitudes for LEP processes will depend
on one parameter*), the Z boson mass \( m_Z \), which can be accurately determined by
comparing the theoretical expressions for observable quantities with the exper-
imental data. In particular, \( \sin^2 \theta_W \) and \( m_W \) can be calculated from Eqs. (1.8)
and (1.9). It will then be convenient to define the radiative corrections to the
various processes by the difference between the lowest order and the radiatively
corrected results evaluated with the same value for \( m_Z \) (the one determined at
the end of the fitting procedure by including radiative corrections).

The plan of the paper is as follows. In Section 2 we describe a convenient
parametrization of the lowest-order amplitude which takes into account the main
effects ('leading logs') due to one-loop corrections. Section 3 lists some re-
levant formulas for lowest-order cross-sections and asymmetries. In Section 4 we
compare our 'improved' lowest-order estimates with the results of complete one-
loop calculations. Our conclusions are presented in Section 5. In the Appendix
we give some explicit formulas for the \( \gamma-Z \) box diagrams and for the whole set
of the one-loop 'weak' corrections.

The calculations reported in this paper are performed in the 't Hooft-
Feynman gauge. Conventions (metric, \( \gamma \)-matrices, and spinors) are as in Ref. [16].

2. PARAMETRIZATION OF THE LOWEST-ORDER AMPLITUDE FOR \( e^+e^- \rightarrow \mu^+\mu^- \)

In order to discuss the most convenient parametrization, it is useful to
express the lowest-order amplitude in terms of unrenormalized coupling cons-
tants. Thus we write

\[
m^{(0)}_{\gamma\gamma}(e^+e^- \rightarrow \mu^+\mu^-) = \frac{e^2}{s} J^\gamma_\Lambda(e)J^\gamma_\Lambda(\mu) - \frac{g^2_0}{c^2_{\text{WO}}} J^\Lambda_\Lambda(e)J^\Lambda_\Lambda(\mu) \frac{m^2_Z}{m^2_R - s}
\]

(2.1)

where \( e_0 \) and \( g_0 \) stand for the bare electric charge and the SU(2) coupling cons-
tant, respectively, \( c^2_{\text{WO}} = m^2_W/m^2_Z \), and \( s \) is the invariant squared momentum
\( (s = \frac{1}{2} E^{cm}_m) \). The currents \( J^\gamma_\Lambda \) and \( J^\Lambda_\Lambda \) are defined as

\[
J^\gamma_\Lambda(e) = \bar{e} \gamma_\Lambda e, \quad J^\gamma_\Lambda(\mu) = \bar{\mu} \gamma_\Lambda \mu,
\]

\[
J^\Lambda_\Lambda(e) = \bar{e} \gamma_\Lambda (V_0 + \lambda_0 \gamma_5) e, \quad J^\Lambda_\Lambda(\mu) = \bar{\mu} \gamma_\Lambda (V_0 + \lambda_0 \gamma_5) \mu.
\]

(2.2)

[In the following we will use the abbreviated expressions
\( J^\gamma_\Lambda(e)J^\gamma_\Lambda(\mu) \)
and
\( J^\Lambda_\Lambda(e)J^\Lambda_\Lambda(\mu) \).]

*) The Z decay width is discussed in Section 2. It is pointed out that, when
this quantity is expressed in terms of \( G \) and \( m_Z \), the residual radiative
corrections are very small.
The vector and axial-vector couplings are given by

\[ V_0 = s_w^2 - \frac{1}{4}, \quad A_0 = -\frac{1}{4}. \]  

(2.3)

In Eq. (2.1), \( m_R^2 \) is a complex renormalized mass

\[ m_R^2 = m_Z^2 - i m_Z \Gamma_Z, \]  

(2.4)

where \( m_Z \) and \( \Gamma_Z \) are the physical mass and the total decay rate of the \( Z \). The use of the above complex mass in the lowest-order amplitude is customary and will be discussed later in this section.

We now proceed to express Eq. (2.1) in terms of renormalized parameters. Thus, we write

\[ e_0^2 = e^2 \left( 1 - \frac{\delta e^2}{e^2} \right), \]  

(2.5)

where \( \delta e^2 \) is the charge renormalization counter-term. Similarly, recalling

\[ c_W^2 = m_W^2/m_Z^2, \quad g_0^2 = e^2/s_w^2 \]  

and writing \( m_W^2 = m_W^2 - \delta m_W^2, \quad m_Z^2 = m_Z^2 - \delta m_Z^2 \), we find to \( O(\alpha) \),

\[ g_{WO}^2 = \frac{g_0^2 c_W^2}{c_W^2} \left[ 1 - \frac{\delta e^2}{e^2} + \left( c_W^2 - 1 \right) \left( \frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2} \right) \right], \]  

(2.6)

where

\[ g = \frac{e}{\sin \theta_W} \]  

(2.7)

and, in accordance with Eq. (1.4), \( c_W \equiv m_W/m_Z \). Following the renormalization method of the first paper of Ref. [11], the counter-terms \( \delta m_W^2, \delta m_Z^2 \), and \( \delta e^2 \) are judiciously chosen in such a way that the renormalized parameters \( m_W, m_Z \) coincide with the physical masses of \( W \) and \( Z \), and \( e \) is identified with the conventionally defined charge of the positron. As explained in the above-mentioned paper, this requires that, to \( O(\alpha) \), \( \delta m_W^2 = -\text{Re} \tilde{\nu}_{WW}(-k^2 = m_W^2), \quad \delta m_Z^2 = -\text{Re} \tilde{\nu}_{ZZ}(-k^2 = m_Z^2) \), where \( \tilde{\nu}_{WW} \) and \( \tilde{\nu}_{ZZ} \) are the coefficients of \( \delta_{\mu\nu} \) in the one-particle irreducible self-energy parts associated with \( W \) and \( Z \), respectively.

* In Ref. [11], \( \tilde{\nu}_{WW} \) and \( \tilde{\nu}_{ZZ} \) are denoted as \(-A_{WW}\) and \(-A_{ZZ}\).
Because the conventional electric charge renormalization involves a subtraction at \( q^2 = 0 \) \( [\delta e^2/e^2 = \bar{\Pi}_{\gamma\gamma}(0)] \), the counter-term \( \delta e^2/e^2 \) contains mass singularities of the light fermions. On the other hand, such terms do not occur in \( \delta m^2_\tau \) and \( \delta m^2_W \) because these quantities involve amplitudes defined at large \( q^2 \) values (i.e. non-exceptional invariant momenta). A glance at Eqs. (2.5) and (2.6) tells us that, if the renormalized parameters \( e^2 \) and \( q^2/c_W^2 \) are regarded as 'zeroth order' couplings, then the \( \delta e^2/e^2 \) counter-term will induce large radiative corrections to the 'zeroth order (Born) amplitude'.

We now outline a method for absorbing the large radiative corrections contained in \( \delta e^2/e^2 \), in a suitable redefinition of the effective lowest-order couplings.

Equations (2.5) and (2.6) have to be used in conjunction with one-loop diagrams in order to obtain finite (i.e. non-ultraviolet divergent) results beyond the tree approximation. As the contribution of \( \delta e^2 \) is usually absorbed in the real part of the renormalized vacuum polarization function \( \text{Re} \bar{\Pi}^{(r)}_{\gamma\gamma}(s) \), it is convenient to introduce this correction explicitly. This can be done by replacing \( e^2 \) by the energy-dependent parameter

\[
\delta^2(s) = \frac{e^2}{1 - \text{Re} \bar{\Pi}^{(r)}_{\gamma\gamma}(s)}
\]

(2.8)

i.e. the usual e.m. running coupling constant at a scale \( k^2 = -s \). On the other hand, using Eqs. (2.6), (2.7), and (2.8) we obtain

\[
\frac{g^2_0}{c_W^2} = \delta^2(s)B(s)[1 - \Delta \tau - \frac{\delta e^2}{e^2} + \left( \frac{c_W^2}{s_W^2} - 1 \right) \left( \frac{\delta m^2_\tau}{m^2_\tau} - \frac{\delta m^2_W}{m^2_W} \right)],
\]

(2.9)

where

\[
B(s) = \frac{1}{s_W^2c_W^2} \frac{e^2}{\delta^2(s)} \frac{1 - \Delta \tau}{1 - \frac{m^2_\tau}{1}} \frac{\mu^2}{\mu^2} = \frac{e^2}{\delta^2(s)}. \]

(2.10)

The second equality in Eq. (2.10) follows immediately from Eq. (1.6). Comparison with Eq. (34b) of the first paper of ref. [11], reveals that the mass singularities in \( \Delta \tau \) cancel those in \( \delta e^2/e^2 \); as a consequence, the expression between square brackets in Eq. (2.9) is free from such singularities. This suggests expressing the zeroth-order amplitude for \( e^-e^- \rightarrow \mu^+\mu^- \) in terms of the scale-dependent quantities \( \delta^2(s) \) and \( B(s) \), while including the \( O(a) \) terms between square brackets in Eq. (2.9) as part of the radiative corrections. In this way
the higher-order corrections are free, for all values of s, from the large contributions associated with the mass singularities induced by electric charge renormalization. Therefore, we conclude that a convenient parametrization of the lowest-order amplitude is given by

$$\mathcal{M}_0 (e^+ e^- \rightarrow \mu^+ \mu^-) = \frac{e^2(s)}{s} \left[ J Y J - \frac{s}{m^2_R} B(s) J_D J^D \right],$$

(2.11)

where it is understood that, in the currents $J^{s^2}_{\text{WO}}$, $s^2$ is replaced by the renormalized expression of Eq. (1.8). We note that $\delta^2(s)$ factors out in Eq. (2.11); as a consequence, and as shown explicitly in Section 3, $\delta^2(s)$ cancels in observables involving ratios of cross-sections, such as the forward-backward and left-right asymmetries. We will see in Section 4 that the formulae for the asymmetries derived on the basis of Eq. (2.11) represent a very good approximation to the complete one-loop calculation of electroweak effects when $s \ll m^2_Z$. It is also interesting to discuss $B(s)$ in the framework of a more general class of $SU(2)_L \times U(1)$ theories in which $m^2_W / C W^2 Z^2 = \phi$, where $\phi$ is a new fundamental parameter that is different from unity at the tree level. As is well known, this is the case if there are Higgs scalars transforming according to a class of 'non-standard' representations. In this case, since $\phi$ is known phenomenologically to be very close to unity, it seems reasonable to assume that the physics responsible for $\phi \neq 1$ represents a small perturbation and, as a consequence, the corrections to $\mu$ decay defined by $\Delta \tau$ are only slightly affected. If that is the case, Eq. (1.5) remains essentially unaltered, but in Eq. (1.6) $m^2_Z$ should be replaced by $J_0 m^2_Z$. Therefore, in this more general scenario Eq. (2.12) is still true with $B(s)$ replaced by

$$\left. B(s) \right|_{\phi \neq 1} = \frac{\delta m^2_Z}{\mu^2} \frac{e^2}{e^2(s)}.$$

(2.12)

Before proceeding to the discussion of the cross-sections, we turn our attention to the $Z$ propagator in order to justify the use of $m^2_R$ in Eq. (2.1) and elucidate other aspects of the analysis.

Let us define $m_0$ to be the bare $Z$ mass parameter appearing in the Lagrangian. Then, by including self-energy effects, one gets the full propagator

$$\Delta_{\mu \nu}(k) = \frac{\delta_{\mu \nu}}{k^2 + m_0^2 - \Pi(k^2)},$$

(2.13)
where we have neglected \( k_\mu k_\nu \) terms which would give contributions proportional to external lepton masses. Setting \( k^2 = -s \), the scalar part of the propagators becomes

\[
\Delta(s) = \frac{1}{-s + m_0^2 - \tilde{\Pi}(-s)}. \tag{2.14}
\]

Let us define \( \tilde{s} = m_0^2 \) as the complex valued squared momentum at which the denominator vanishes, i.e.

\[
\tilde{s} = m_0^2 - \tilde{\Pi}(-\tilde{s}) \tag{2.15}
\]

Introducing the physical quantities \( m_Z \) and \( \Gamma_Z \) through the relations

\[
m_Z^2 = m_0^2 - \text{Re} \tilde{\Pi}(-m_R^2) \tag{2.16}
\]

\[
m_Z \Gamma_Z = \text{Im} \tilde{\Pi}(-m_R^2) \tag{2.17}
\]

we obtain the identity

\[
\Delta(s) = (m_R^2 - s)^{-1} \left[ 1 - \tilde{\Pi}'(-m_R^2) - \left( \frac{m_R^2 - s}{m_Z^2} \right) Q(s, m_R^2) \right]^{-1}, \tag{2.18}
\]

where

\[
\tilde{\Pi}'(-m_R^2) = \frac{d\tilde{\Pi}}{dk^2} \bigg|_{k^2 = -m_R^2} \]

and

\[
Q(s, m_R^2) = \frac{m_Z^2}{(m_R^2 - s)^2} \left[ \tilde{\Pi}(-s) - \tilde{\Pi}(-m_R^2) - (m_R^2 - s) \tilde{\Pi}'(-m_R^2) \right] \tag{2.19}
\]

The quantities \( \tilde{\Pi}'(-m_R^2) \) and \( Q(s, m_R^2) \) in Eq. (2.18) are \( O(\alpha/\pi s_W^2) \) and may be treated as part of the higher-order corrections. Using this fact and noticing that \( m_R^2 - m_Z^2 = O(\alpha/\pi s_W^2) m_Z^2 \), it is clear that in Eq. (2.18) we may replace

\[
\tilde{\Pi}'(-m_R^2) \rightarrow \tilde{\Pi}'(-m_Z^2), \quad (m_R^2 - s)Q(s, m_R^2) \rightarrow (m_Z^2 - s)Q(s, m_Z^2), \quad \text{with an error } O[(\alpha/\pi s_W^2)^2].
\]
with respect to the leading contribution. The unrenormalized $Z$ propagator at the pole $(s = m_Z^2)$ has then the form

$$\Delta(s = m_Z^2) = \frac{1 + \bar{n}'(-m_Z^2)}{-1 - m_Z^2 \Gamma_Z},$$  \hspace{1cm} (2.20)

where we have neglected terms $O(s/m_W^2)$ relative to the leading term.

As noted by Wetzel [3], and as is clear from Eq. (2.20), a full first-order calculation around the resonance requires the inclusion of the one-loop corrections to the $Z$ width $\Gamma_Z$. This can be done by using the results of Ref. [14], where all the one-loop corrections to the leptonic width of the vector bosons were presented, and Ref. [17] where the hadronic widths are discussed. As shown in these papers, the main radiative corrections to the decay rates can be absorbed into the vector boson mass by a suitable parametrization of the lowest-order result. To illustrate this point, we consider for simplicity the decay rate $\Gamma(Z \to \nu \bar{\nu})$. To zeroth order we have

$$\Gamma_0(Z \to \nu \bar{\nu}) = \frac{\gamma}{2 s W W},$$  \hspace{1cm} (2.21)

where $\gamma = (\alpha/12)\mu$, and $\mu$ is defined in Eq. (1.7). Computation of the one-loop corrections (employing the definition $s_W^2 = 1 - m_W^2/m_Z^2$) gives [14]

$$\Gamma(Z \to \nu \bar{\nu}) = \Gamma_0(Z \to \nu \bar{\nu}) \left[ 1 + \frac{3}{2} \Delta r + \epsilon(\nu \bar{\nu}) \right],$$  \hspace{1cm} (2.22)

where $\Delta r$ is the radiative correction discussed in Section 1, and $\epsilon(\nu \bar{\nu})$ represents the remaining weak correction. Recalling Eq. (1.6) we see that Eqs. (2.21) and (2.22) can be recast in the form

$$\Gamma(Z \to \nu \bar{\nu}) = \frac{m_Z^3}{12 \pi} \frac{G}{\sqrt{s}} \left[ 1 + \epsilon(\nu \bar{\nu}) \right].$$  \hspace{1cm} (2.23)

In Ref. [10] it was shown that for a wide range of $\sin^2 \theta_W$ values, $|\epsilon(\nu \bar{\nu})| \leq 0.1-0.2\%$ and $|\epsilon(\mu^+ \mu^-)| \leq 0.1-0.2\%$. These corrections are smaller than the expected experimental resolution of the $Z$ width at LEP. For this reason, in order to simplify the calculations, one may employ the lowest-order results for $\Gamma$ expressed in terms of $G$ and $m_Z$ (i.e. neglecting the $\epsilon$ terms). The $\epsilon$ corrections, however, may become sizeable in the case of a heavy doublet with large mass splitting (very large $m_t$). In Table 1 we show some results for the $t$ corrections for several values of the top, Higgs, and $Z$ masses.
Table 1

The quantities \( \eta(\nu\nu) \equiv \varepsilon(\nu\nu) \times 10^3 \) and \( \eta(\mu^+\mu^-) \equiv \varepsilon(\mu^+\mu^-) \times 10^3 \) for various values of the top, Higgs, and Z masses (GeV)

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<th>( m_t = 300 )</th>
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<td>( \eta(\mu^+\mu^-) )</td>
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3. **LOWEST-ORDER CROSS-SECTION AND ASYMMETRIES FOR e^+e^- \rightarrow \mu^+\mu^-**

To complete our discussion we give the expression of the 'Born' differential cross-section, by employing the parametrization introduced in Section 2, in the approximation of neglecting the external lepton masses. For unpolarized particles we get

\[
\frac{d\sigma_B (\theta)}{d\Omega} (s) = \frac{s^2 (s)}{4s} \left[ (1 + z^2) F_1 (s, m_Z^2) + 2z F_2 (s, m_Z^2) \right],
\]

(3.1)
where $\theta$ is the c.m.s. scattering angle between the initial $e^+$ and the outgoing $\mu^+$, $z \equiv \cos \theta$, and $\alpha^2(s) = e^2(s)/4\pi$.

The functions $F_i(s, m_Z^2)$ ($i = 1, 2$) can be suitably expressed as

$$F_1(s, m_Z^2) = 1 - \frac{2s}{q(s)} B(s) V^2 \cos \delta_R + \frac{s^2}{q^2(s)} B^2(s) (V^2 + A^2)^2$$ (3.2)

$$F_2(s, m_Z^2) = -\frac{2s}{q(s)} B(s) A^2 \cos \delta_R + \frac{s^2}{q^2(s)} B^2(s) 4V^2 A^2$$ (3.3)

where (we recapitulate all the notation for the convenience of the reader)

$s_w^2 = \sin^2 \theta_w$ is defined through Eq. (1.8),

$$V = s_w^2 - \frac{1}{4}, \quad A = -\frac{1}{4},$$ (3.4)

$$B(s) = \left(\frac{m_Z}{37.281 \text{ GeV}}\right)^2 \frac{e^2}{\bar{e}_w^2(s)},$$ (3.5)

and $q(s)$ and $\delta_R(s)$ are defined through the relation

$$\frac{1}{M_R^2 - s} = \frac{1}{q(s)} e^{i\delta_R(s)}.$$ (3.6)

Thus

$$\tan \delta_R(s) = \frac{m_Z \Gamma_Z}{m_Z^2 - s} \quad (0 \leq \delta_R \leq \pi)$$ (3.7)

and

$$q(s) = \left[\left(m_Z^2 - s\right)^2 + m_Z^2 \Gamma_Z^2 \right]^{1/2}.$$ (3.8)

Also the $Z$ width $\Gamma_Z$ is accurately expressed in terms of $m_Z$ as

$$\Gamma_Z \equiv \Gamma(Z + \text{all}) = \frac{m_Z^2 G}{12\pi/2} \left[6 - 12s_w^2 + 24s_w^4 + 3 \left[1 + \frac{m_Z^2}{s_w^2}\right] F\left(s_w, \frac{m_t^2}{m_Z^2}\right)\right],$$ (3.9)

with ($m_t$ = top quark mass)
\[
F\left(\frac{s}{M^2}, \frac{m_t^2}{m_Z^2}\right) = 5 - \frac{28}{3} s_W^2 + \frac{80}{9} s_W^4 + \left(1 - 4 \frac{m_t^2}{m_Z^2}\right)^{1/2} \left[1 - \frac{8}{3} s_W^2 + \frac{32}{9} s_W^4 - \frac{m_t^2}{\mu_Z^2} \left(1 + \frac{16}{3} s_W^2 - \frac{64}{9} s_W^4\right)\right], \tag{3.10}
\]

and $s_w^2$ is defined in Eq. (1.8).

A useful observable is the integrated forward-backward asymmetry, defined as

\[
A_{FB} = \frac{\int \frac{d\sigma}{d\Omega} (\theta) - \int \frac{d\sigma}{d\Omega} (\pi - \theta)}{\int \frac{d\sigma}{d\Omega} (\theta) + \int \frac{d\sigma}{d\Omega} (\pi - \theta)}, \tag{3.11}
\]

and the integration in Eq. (3.11) is carried out over the angle of acceptance of the detector (usually $|z| \lesssim 0.8$). For acceptance over the whole solid angle ($|z| \lesssim 1$) we have

\[
A_{FB}\bigg|_{|z| \lesssim 1} = \frac{3}{4} \frac{F_2}{F_1}. \tag{3.12}
\]

At low energy ($s \ll m_Z^2$) this reduces to the approximate expression

\[
A_{FB}\bigg|_{|z| \lesssim 1} = -\frac{3}{32} B(s) \frac{s}{m_Z^2} \frac{s}{s}. \tag{3.13}
\]

On the other hand, exactly on resonance ($s = m_Z^2$) $\delta_R(m_Z^2) = \pi/2$, $\varphi(m_Z^2) = m_Z \Gamma_Z/2$, and

\[
A_{FB}\bigg|_{|z| \lesssim 1} = \frac{3B^2(m_Z^2) V^2 A^2}{\left(\frac{\Gamma_Z}{m_Z}\right)^2 + B^2(m_Z^2)(V^2 + A^2)} = \frac{3V^2 A^2}{(V^2 + A^2)}. \tag{3.14}
\]

Because $\sin^2 \theta_w$ is close to 1/4, $V^2 \ll 0$, and therefore the value of expression (3.14) is very small. On the other hand, as $s$ moves away from $m_Z^2$, $\cos \delta_R$ rapidly increases in magnitude, the first term in Eq. (3.3) becomes important, and the asymmetry sizeable. Moreover, $\cos \delta_R$ changes sign as we cross the resonance and, as a consequence, the asymmetry changes sign very close to $s = m_Z^2$. Also the left-right asymmetry is a useful experimental quantity. A longitudinal polarization for the $e^-$ beam is required in this case. We obtain
\[ A_{LR} = \int \frac{d\sigma_L}{d\Omega} - \int \frac{d\sigma_R}{d\Omega} = \int \frac{d\sigma_R}{d\Omega} + \int \frac{d\sigma_L}{d\Omega} . \] (3.15)

By integrating over the whole solid angle, we find (for a complete polarization of the e\(^-\) beam),

\[ A_{LR}\big|_{\angle 1} = \frac{2VA}{F_1(s, m_Z^2)} \left[ -B(s) \frac{s}{s} \cos \delta_R + B^2(s) \frac{s^2}{s^2} (V^2 + A^2) \right] \] (3.16)

Exactly on resonance

\[ A_{LR}\big|_{\angle 1} = \frac{2B^2(m_Z^2)}{\left( \frac{s}{m_Z^2} + B^2(s)(V^2 + A^2) \right)^2} = \frac{2VA}{V^2 + A^2} . \] (3.17)

4. \textbf{EFFECT OF THE HIGHER-ORDER ELECTROWEAK CORRECTIONS ON } \( e^+ e^- \rightarrow u^+ u^- \)

We will now compare the simple formulae obtained in Section 3 with complete one-loop calculations of electroweak corrections to the forward-backward asymmetry \( A_{FB} \) and the left-right asymmetry \( A_{LR} \). The comparison is useful because it shows to what extent the observables are sensitive to interesting radiative corrections not included in the zeroth-order expressions of Section 3. We will carry out the comparison with Ref. [7]. For simplicity we will employ the fixed value \( m_Z = 94 \text{ GeV} \) (extension to different masses is straightforward) and we will use the value of \( \sin^2 \theta_W \) obtained from Eq. (1.10), which corresponds to \( \Delta r = 0.07 \) (i.e. \( m_L = 36 \text{ GeV}, \ m_H = m_Z \)). For \( m_Z = 94 \text{ GeV} \), this leads to \( \sin^2 \theta_W = 0.2156 \).

The first comparison with Ref. [7] involves the theoretical prediction of the forward-backward asymmetry at relatively low energies, namely \( s = (34.5)^2 \text{ GeV}^2 \). It is important to point out that in Ref. [7] 'pure QED' corrections (i.e. \( \gamma \) vertex corrections, \( \gamma \gamma \) and \( \gamma Z \) box diagrams, and bremsstrahlung) are not included, but vacuum polarization effects are retained together with other electroweak contributions.

To evaluate \( B(s) \) we need the quantity

\[ \frac{\alpha^2}{\bar{\alpha}^2(s)} = 1 - \text{Re} \Pi^{(r)}(s). \] (4.1)
For $s$ in the range of interest we find from Ref. [14]

$$\text{Re} \, \Pi_{\gamma \gamma}^{(r)}(s) = \text{Re} \, \Pi_{\gamma \gamma}^{(r)} [(93 \text{ GeV})^2] + \frac{20}{9} \frac{a}{s} \ln \left[ \frac{s}{(93 \text{ GeV})^2} \right]$$

(4.2)

and

$$\text{Re} \, \Pi_{\gamma \gamma}^{(r)} [(93 \text{ GeV})^2] = (6 \pm 0.04 \pm 0.05) \times 10^{-2} ,$$

(4.3)

where the first error reflects uncertainty from the dispersive integral in evaluating the light-quark contributions, and the second error is the uncertainty due to the top-quark mass ($20 \text{ GeV} \leq m_t \leq 60 \text{ GeV}$). Inserting Eqs. (4.2) and (4.3) into Eqs. (4.1) and (3.5) we find $\epsilon^2 / \epsilon^2 (s) = 0.95$ and $B(s) = 6.040$ at $s = (34.5 \text{ GeV})^2$. Using these values in Eq. (3.13) [or the more exact formulae of Eqs. (3.2), (3.3), and (3.12)] leads to the theoretical prediction

$$\Lambda_{FB}^{(0)} [s = (34.5 \text{ GeV})^2] \bigg|_{|z| \leq 1} = -0.088 \quad (m_Z = 94 \text{ GeV}) .$$

(4.4)

The superscript indicates that we have used the 'zeroth-order' formulae of Section 3. Comparison of Eq. (4.4) with the complete results of Ref. [7] shows very good agreement for a large range of values of $m_H$ and $m_t$. This can be traced to the fact that, according to the argument of Section 2, the electroweak corrections are dominated by vacuum polarization effects, and these are included both in the complete calculation of Ref. [7] and the simple 'zeroth-order' formulae of Section 3.

Next we consider the case $s = m_Z^2$. In this situation $\Lambda_{FB}^{(0)}$ is proportional to $(s^2_w - 1/4)$. Inserting $s^2_w = 0.2156$ into Eq. (3.14) leads to

$$\Lambda_{FB}^{(0)} \bigg|_{s = m_Z^2} = 0.055 \quad \left( \begin{array}{c} m_Z = 94 \text{ GeV} \\ s^2_w = 0.2156 \end{array} \right) .$$

(4.5)

From a comparison with the results of these complete calculations it can be seen that for certain values of $m_H$ and $m_t$, Eq. (4.5) shows deviations (typically $\pm 8\%$) at $m_Z = 94 \text{ GeV}$. Allowing $s^2_w$ to depend on $m_H$ and $m_t$, at fixed $m_Z$, according to the tables shown in the contribution of G. Altarelli, and inserting the corresponding values in the 'zeroth-order' expression of Eq. (3.14), improves the agreement for small $m_H$ but worsens the comparison for large $m_H$. The fact that the electroweak effects not included in the zeroth-order formulae of Section 3 become more important for $\Lambda_{FB}$ at $s = m_Z^2$ can be traced to the fact that the zeroth-order contribution is suppressed by the very small factor $\nu^2$ =
\( (s_w^2 - 1/4) \), whilst the radiative corrections contain contributions of \( O(\alpha/(s_w^2)) \) not inhibited by \( v^2 \).

In the case of the left-right asymmetry at \( s = m_Z^2 \), our zeroth-order estimate (Eq. 3.17) is

\[
A_{LR}^{(0)} \mid s = m_Z^2 = 0.27 \left( m_Z = 94 \text{ GeV} \right) \left( s_w^2 = 0.2156 \right). \tag{4.6}
\]

Comparison with the complete results of Ref. [7] shows that the deviations of Eq. (4.6) are smaller in this case (\( \xi \pm 4\% \)). This can be readily understood since the zeroth-order contribution is only suppressed by a factor \( V \) whilst the radiative corrections contain again contributions of \( O(\alpha/s_w^2) \).

5. CONCLUSIONS

If the quantum correction \( \Delta r \) turns out to be anywhere near the theoretical prediction \( \Delta r = 0.07 \) of the standard model, it is clear that it should be very visible in both the forward-backward asymmetry \( A_{FB} \) and the left-right asymmetry \( A_{LR} \) in the vicinity of the \( s_0^0 \) pole. For example, if \( \Delta r \) were zero or ignored, then for \( m_Z = 94 \) GeV the value extracted from Eq. (1.8) would be \( \sin^2 \theta_w = 0.1955 \), instead of \( \sin^2 \theta_w = 0.2156 \). At \( s = m_Z^2 \) the predicted values would be

\[
A_{FB}^{(0)} \mid s = m_Z^2 = 0.13 \left( m_Z = 94 \text{ GeV} \right) \left( \Delta r = 0 \right) \tag{5.1}
\]

\[
A_{LR}^{(0)} \mid s = m_Z^2 = 0.42 \left( m_Z = 94 \text{ GeV} \right) \left( \Delta r = 0 \right) \tag{5.2}
\]

to be compared with Eqs. (4.5) and (4.6), respectively, or the more detailed values in the tables. Equations (5.1) and (5.2) are approximately 2.4 and 1.6 times as large, respectively, as the radiatively corrected predictions! It is also clear that the simple formulae of Section 3 incorporate the leading effects of the corrections.

On the other hand, if one wants to probe detailed aspects of the corrections, such as the dependence on \( m_H \), one must measure \( A_{FB} \) (\( A_{LR} \)) on resonance to better than \( \xi \pm 1\% \) (3.5\%). In turn, according to Ref. [7], such measurements would lead to determinations of \( \sin^2 \theta_w \) to better than 1.5\%. For such accurate comparisons of theory and experiment it would be necessary to take into account the complete one-loop corrections.
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A) γ-Z BOX DIAGRAMS IN e⁺e⁻ → μ⁺μ⁻

Purely e.m. corrections are of primary importance and may sizeably change the naive (tree order) expectations for our process. They are discussed in the contribution of R. Kleiss. We will limit ourselves to a brief description of the γ-Z box diagrams. Graphs of this type have usually been analysed in the 'soft photon' limit, i.e. by extracting that part which factorizes into the lowest-order amplitude. To our knowledge they were first computed exactly by Brown, Decker and Paschos in Ref. [6]. We have redone the calculation and we agree with their findings. We have obtained the following results:

\[
\frac{e^2}{s^2 c_w^2} J_z J_z \times
\]

\[
\times \frac{2}{w} \left[ \frac{1}{m^2_R - s} \left[ f(t) + g(s) \right] - \frac{1}{s} p(s) \right]
\]

\[- \frac{g^2}{s^2 c_w^2} \left[ (V + \lambda)^2 J_L^2 J_L + (V - \lambda)^2 J_R^2 J_R \right] f_2(s, t, u) \cdot
\]

\[
\times \frac{\alpha}{w} \left[ \frac{1}{m^2_R - s} \left[ f(u) + g(s) \right] - \frac{1}{s} p(s) \right]
\]

\[+ \frac{g^2}{s^2 c_w^2} (V + \lambda)^2 (J_L^R J_L^R + J_R J_L^R) f_2(s, t, u) \cdot
\]

where (λ is the fictitious photon mass introduced to regularize the infrared divergence)
\[ J_{L, R} = \sqrt{s} \lambda (1 \pm \gamma_5) u \quad (e, \mu) \],

\[ f(t) = 2 \ln \left( -\frac{t}{s} \right) \left[ \ln \left( \frac{M_R^2 - s}{m_Z^2} \right) + \ln \left( \frac{f s}{\lambda} \right) \right] + \frac{1}{2} \ln^2 \left( -\frac{t}{s} \right) + L_{12} \left( 1 + \frac{t}{m_Z^2} \right), \]

\[ g(s) = (L_e + L_\mu) \left[ \ln \left( \frac{M_R^2 - s}{m_Z^2} \right) + \ln \left( \frac{f s}{\lambda} \right) \right] - \frac{1}{4} (L_e^2 + L_\mu^2) - \frac{\pi^2}{3}, \]

\[ P(s) = \frac{s^2}{3} - 2L_{12} \left( 1 - \frac{s}{m_Z^2} \right) - (L_e + L_\mu) \ln \frac{M_R^2 - s}{m_Z^2}, \]

and

\[ L_e = \ln \frac{s}{m_e^2}, \quad L_\mu = \ln \frac{s}{m_\mu^2}, \quad L_{12}(x) = -\int_0^x \frac{dt}{t} \ln (1 - t). \]

Also the function \( f_2(s,t,u) \) has been introduced in Ref. [6] and is defined as

\[ f_2(s,t,u) = \frac{1}{u^2} (u - t - m_Z^2) \beta(s,t) + \frac{1}{t} \gamma(s,t) \]

with

\[ \beta(s,t) = \ln \left( -\frac{t}{s} \right) \ln \left( \frac{M_R^2 - s}{m_Z^2} \right) + L_{12} \left( 1 + \frac{t}{m_Z^2} \right) - L_{12} \left( 1 - \frac{s}{m_Z^2} \right), \]

\[ \gamma(s,t) = \left[ \frac{m_Z^2}{s} - 1 \right] \ln \left( \frac{M_R^2 - s}{m_Z^2} \right) + \ln \left( -\frac{t}{m_Z^2} \right). \]

By combining direct and crossed box diagrams we obtain the final result:

\[ M_{\text{box}}^{(\gamma-Z)} = M\text{res}_{\gamma} \left( f(u) - f(t) \right) - \frac{\alpha^2}{s w^2} [((V + A)^2 J_{L, L} + (V - A)^2 J_{R, R}^* J_{R, R}^*) f_2(s,t,u) \]

\[ - (V^2 - A^2)(J_{L, R}^* J_{R, L} + J_{R, L}^* J_{L, R}) f_2(s,u,t) \],

which is indeed the combination reported in Ref. [6].
B) **PURELY WEAK CORRECTIONS**

It is convenient to separate the weak corrections into two parts:

1) self-energy and vertex corrections;
2) Z-Z and W-W box diagrams.

1) The whole set of corrections of class (1) can be expressed by defining
vector and axial-vector couplings, which take into account all possible effects.
We obtain

\[
m^{1\text{-loop}}(1\text{-boson}) = \frac{\alpha^2(s)}{s} \left[ J_{\gamma} J_{\gamma} - \frac{s}{M^2_R - s} B(s) J_Z J_Z \right]
\]

where

\[
J_{\gamma} = \bar{v}_f \gamma_\lambda (A_{\gamma} + B_{\gamma} \gamma_5) u_f
\]

\[
J_Z = \bar{v}_f \gamma_\lambda (A_Z + B_Z \gamma_5) u_f
\]

(f = e, \mu)

with

\[
A_{\gamma} = 1 + \frac{\alpha}{4\pi} F_{\gamma}(s),
\]

\[
B_{\gamma} = \frac{2}{4\pi} G_{\gamma}(s),
\]

\[
A_{Z} = V \left[ 1 + \frac{\alpha}{4\pi} F_{Z}(s) \right],
\]

\[
B_{Z} = A \left[ 1 + \frac{\alpha}{4\pi} G_{Z}(s) \right].
\]

The functions \( F_{\gamma}, G_{\gamma}, F_{Z}, \) and \( G_{Z} \) can be expressed as

\[
F_{\gamma}(s) = -\frac{V^2 + A^2}{s^2 c^2 w^2 w} \left[ V \left( \frac{s}{m_w^2} \right) - \frac{1}{2} \right] - \frac{1}{4s^2} \left[ U \left( \frac{s}{m_w^2} \right) - \frac{1}{2} \right] - \frac{1}{2} \left[ \left( \frac{4m_w^2}{s} + 3 \right) I \left( \frac{s}{m_w^2} \right) + \frac{2}{3} \right],
\]

\[
G_{\gamma}(s) = -\frac{2vA}{s^2 c^2 w w} \left[ V \left( \frac{s}{m_w^2} \right) - \frac{1}{2} \right] - \frac{1}{4s^2} \left[ U \left( \frac{s}{m_w^2} \right) - \frac{1}{2} \right],
\]
\[ F_Z(s) = \frac{(v^2 + 3a^2)}{8s^2c_w} \left[ v\left(\frac{s}{m_z}\right) - \frac{1}{2}\right] \]

\[ + \frac{1}{4vs_w^2} \left[ c_w^2 \left[ U\left(\frac{s}{m_w}\right) - \frac{1}{2} \right] - \frac{1}{2} \left[ V\left(\frac{s}{m_w}\right) - \frac{1}{2}\right] \right] \]

\[ + \frac{1}{s} \left[ 3c_w^2 + \frac{1}{6} + \frac{4m_w^2}{s} \left( c_w^2 + \frac{1}{3} \right) \right] I\left(\frac{s}{m_w}\right) - \frac{1}{9} \]

\[ + \frac{1}{2s_w^2} N_Z \left[ 4 \left( 1 - s_w^2 \right) \left( \ln \frac{s}{m_z} - i\pi \right) + 'top 1' \right] \]

\[ - \frac{c_w^2}{v(c_w^2 - s_w^2)} \left[ a_w^0 + \frac{1}{c_w^2} H(m_z) + R_1 + \frac{2}{3} s_w^2 + 28s_w \right] , \]

\[ G_Z(s) = -\frac{1}{s_w^2} (A^2 + 3v^2) \left[ V\left(\frac{s}{m_w}\right) - \frac{1}{2}\right] \]

\[ + \frac{1}{4As_w^2} \left[ c_w^2 \left[ U\left(\frac{s}{m_w}\right) - \frac{1}{2} \right] - \frac{1}{2} \left[ V\left(\frac{s}{m_w}\right) - \frac{1}{2}\right] \right] + \frac{1}{2s_w^2} N_Z , \]

where

\[ N_Z = n_w^0 + \Pi_0 + \frac{1}{c_w^2} H(m_z) + \frac{1}{c_w^2} H(m_z) + R_1 \]

\[ + \frac{1}{m_z^2 - s} [ f_{ZZ}(s) - f_{ZZ}(m_z^2) - (s - m_z^2) f_{ZZ}(m_z^2) ] \]

\[ - \frac{4}{c_w^2} \left( 1 - 2s_w^2 + \frac{8}{3} s_w^4 \right) \left( \ln \frac{s}{m_z} - \frac{1}{2} \right) + 'top 2' . \]
The elementary functions appearing in the above expressions are defined as

\[
V \left( \frac{s}{m^2} \right) = \left( 1 + \frac{m^2}{s} \right)^2 \left[ 2\text{Li}_2 \left( \frac{m^2}{s} \right) - \ln^2 \left( \frac{s + m^2}{m^2} \right) - \frac{\pi^2}{3} \right] \\
+ \left( 3 + \frac{2m^2}{s} \right) \ln \frac{m^2}{s} + 2 \left( 2 + \frac{m^2}{s} \right) + \text{i} \left[ 3 + \frac{2m^2}{s} - 2 \left( 1 + \frac{m^2}{s} \right) \ln \frac{m^2 + s}{m^2} \right],
\]

\[
U \left( \frac{s}{m^2} \right) = \frac{4m^2}{s} \left( 1 + \frac{m^2}{2s} \right) \left[ \cos^{-1} \left( 1 - \frac{s}{2m^2} \right) \right]^2 - 2 \left( 1 - \frac{m^2}{s} \right)
- 2 \left( 1 + \frac{m^2}{s} \right) R(s) \tan^{-1} \left( \frac{1}{R(s)} \right),
\]

with \( R(s) = \left( \frac{4m^2}{s} - 1 \right)^{1/2} \) and \( 0 \leq \cos^{-1} \leq \pi \),

\[
I \left( \frac{s}{m^2} \right) = 2 \left[ R(s) \tan^{-1} \left( \frac{1}{R(s)} \right) \right],
\]

\[
f_{zz}(s) = \left[ \frac{s^2}{2} \left( 4c^2 - \frac{8}{3} c^2 - \frac{5}{3} \right) + s \left( 3c^2 + \frac{1}{3} - \frac{1}{12 c^2} \right) \right] I \left( \frac{s}{m^2} \right)
+ \frac{1}{12c^2} \left( \frac{2m^2}{z} - 10 - s - \frac{2m^2}{z} \right) I_H(s, m_H, m_Z),
\]

\( (m_H = \text{Higgs particle mass}) \) and,

\[
I_H(s, m_H, m_Z) = -1 + \frac{a}{1 - a} \ln a + \frac{\Delta}{2s} \ln a + \text{Re} \left( \frac{r}{2s} \ln \frac{m_H^2 + m_Z^2 - s - r}{m_H^2 + m_Z^2 - s + r} \right),
\]

with \( a = m_H^2/m_Z^2 \), \( \Delta = m_H^2 + s - m_Z^2 \), \( r = \sqrt{\Delta^2 - 4m_H^2} \).
\[ W^0_W = \frac{13}{12c_w^2} \ln c_w^2 + \frac{22}{9} - \frac{34}{3} c_w^2 - 8c_w^4 + \frac{17}{36c_w^2} + 2(4c_w^2 - 1)^{1/2} \tan^{-1} \left[ \frac{1}{(4c_w^2 - 1)^{1/2}} \right] \left( \frac{17}{3} c_w^2 + 4c_w^4 - \frac{4}{3} \right) - \frac{1}{12c_w^2} \]

\[ W_0' = \frac{20}{9} + \frac{c_w^2}{3} - 4c_w^2 - \frac{5}{36c_w^2} - \frac{1}{12c_w^2} \ln c_w^2 + (4c_w^2 - 1)^{1/2} \tan^{-1} \left[ \frac{1}{(4c_w^2 - 1)^{1/2}} \right] \times \frac{6c_w^2 + 1}{6c_w^2 - 1} - \frac{2}{3} + \frac{1}{4c_w^2 - 1} \left[ - \frac{16}{3} c_w^2 + \frac{68}{3} c_w^4 + 16c_w^6 - \frac{1}{3} \right] \]

\[ H(m_Z) = \frac{31}{18} + \frac{1}{12} a^2 - \frac{1}{2} a - \frac{1}{24} a^3 - 6a^2 + 18a \ln a + \frac{1}{24} (a^3 - 4a^2 + 12a) \times \left[ 8(a - 4) E_H \ln \frac{1 + E_H}{1 - E_H} - 28(4 - a) E_H' \tan^{-1} \left( E_H' \right) \right] \]

with \( a = m_H^2/m_Z^2 \), \( E_H = \sqrt{1 - \frac{4}{a}} \), \( E_H' = \sqrt{\frac{4}{a} - 1} \),

\[ h(m_Z) = \frac{31}{36} - \frac{1}{2} a + \frac{1}{6} a^2 - \frac{1}{24} (2a^3 - 9a^2 + 18a - 12) \ln a \]

\[ + \frac{1}{24} (a^3 - 7a^2 + 24a - 36) \left[ 8(a - 4) \frac{1}{E_H} \ln \frac{1 + E_H}{1 - E_H} - 28(4 - a) \frac{1}{E_H} \tan^{-1} \left( E_H' \right) \right] \]

\[ R_1 = - \frac{7}{4} \left( 1 + \frac{1}{2c_w^2} \right) - \frac{1}{4c_w^2} (4 - 7c_w^2) \ln c_w^2 + \frac{1}{6} A - \frac{3}{4} \frac{A}{A - 1} \ln \left( \frac{A}{1} \right) \]

and \( A = m_H^2/m_w^2 \).
\( \delta M \) is defined through the relation

\[ \Delta r = \Pi_{YY}(m_Z^2) - \Pi_{YY}(0) + 2\epsilon_M, \]

and \( \epsilon_M \equiv (a/4s_w^2)\delta M. \)

The expression for \( \epsilon_M \) can be found in Eqs. (40) and (62) of Ref. [14].

\[ \text{top} 1' = \frac{2}{3} \left( 1 - \frac{8}{3} s_w^2 \right) + (s) \]

\[ + \frac{1}{c_w^2 - s_w^2} \left[ \frac{3 m_t^2}{2 m_Z^2} \left[ \frac{1}{2} - \bar{I}(m_Z, m_t) \right] - 4(V_t^2 + A_t^2) t(m_Z) \right], \]

where \( m_t = \text{top-quark mass} \)

\[ t(s) = \left( 1 + \frac{2m_t^2}{s} \right) \bar{I}(s, m_t) - \ln \frac{s}{m_t^2} + 2 \]

and

\[ \bar{I}(s, m_t) = -2 + \eta \ln \frac{\eta + 1}{\eta - 1}, \]

\[ \eta = \sqrt{\frac{4m_t^2}{s}}, \]

also \( V_t = \frac{1}{4} - \frac{2}{3} s_w^2; \quad A_t = \frac{1}{4}, \)

\[ \text{top} 2' = \frac{1}{c_w^2} \left\{ 4 \left( V_t^2 + A_t^2 \right) \left[ \frac{m_t^2 t(m_Z) - st(s)}{m_Z^2 - s} - t(m_Z) \right] \right. \]

\[ + \frac{3 m_t^2}{2 m_Z^2} \left[ \frac{1}{2} - \bar{I}(m_Z, m_t) + \frac{m_t^2}{m_Z^2 - s} \left[ \bar{I}(s, m_t) - \bar{I}(m_Z, m_t) \right] \right] \right\} . \]
2) W-W and Z-Z box diagrams:

\[
E_{W} \equiv \frac{\alpha^2}{2s_{w}c_{w}} \left[ \left[ (V + A) \right]^4 J_{RR} + (V - A) \right]^4 J_{RR} \times \\
I_{ZZ}(s, t) - (V^2 - A^2) \left( J_{R} J_{L} + J_{L} J_{R} \right) I_{ZZ}(s, u) \right] \\
= \frac{a^2}{8s_{w}} J_{L} J_{L} I_{1}(s, t, \frac{m_{W}^2}{s}) \\
= \frac{a^2}{8s_{w}} J_{L} J_{L} I_{1}(s, t, \frac{m_{W}^2}{s})
\]

Also

\[I_{ZZ}(s, t) \equiv I_{1}(s, t, m_{Z}^2) - I_{2}(s, t, m_{Z}^2),\]

where \((q^2 = -t, -u, \quad q^2 > 0)\),

\[I_{1}(s, q^2, m^2) = - \frac{q^2 (m^2 + s - q^2)^2 + s (m^2 - q^2)^2 - \frac{1}{2} s^2 q^2}{(s - q^2)^2 m^4} f(s, q^2, m^2) \]

\[+ \frac{(2m^2 - s)s + 2q^2 (s - q^2)}{(s - q^2)^2 s} \left[ \cos^{-1} \left( 1 - \frac{s}{2m^2} \right) \right]^2 \]

\[- \frac{2(m^2 - q^2) + s}{(s - q^2)^2} \left[ \frac{x^2}{6} - L_{12} \left( 1 - \frac{q^2}{m^2} \right) \right] \]

\[- \frac{1}{(s - q^2)} \left[ \ln \frac{m^2}{q^2} + 2R(s) \tan^{-1} \left[ \frac{1}{R(s)} \right] \right],\]

\[I_{2}(s, q^2, m^2) = - \frac{q^2}{m^4} f(s, q^2, m^2) - \frac{2}{s} \left[ \cos^{-1} \left( 1 - \frac{s}{2m^2} \right) \right]^2,\]
\[ f(s, \alpha^2, m^2) = \frac{1}{2\xi \cos \varphi} \int_{r_1}^{r_2} \ln \left( \frac{1 - 2t \cos \varphi + t^2}{t} \right) dt, \]

and

\[ R(s) = \sqrt{\frac{4m^2}{s}} - 1 \]

\[ \xi = \frac{\sqrt{q^2 s}}{2m^2}, \quad \cos \varphi = \sqrt{1 + \xi^2 \left( 1 - \frac{4m^2}{s} \right)} \]

\[ r_1 = \cos \varphi - \xi, \quad r_2 = \cos \varphi + \xi, \]

for \( s < 4m^2 \).

Some special limits of the functions \( I_1 \) and \( I_2 \) can be derived:

\[ q^2 \to 0 \quad I_1(s, 0, m^2) = \left( \frac{2m^2 - s}{s^2} \right) \left[ \cos^{-1} \left( 1 - \frac{s}{2m^2} \right) \right]^2 - \frac{4}{s} R(s) \tan^{-1} \frac{1}{R(s)} + \frac{2}{s} \]

\[ I_2(s, 0, m^2) = -\frac{2}{s} \left[ \cos^{-1} \left( 1 - \frac{s}{2m^2} \right) \right]^2, \]

\[ q^2 \to s \quad I_1(s, s, m^2) = \frac{3}{2} \ln \left( \frac{m^2}{s^2} \right) - \frac{1}{25} - \frac{2}{2s} \left[ \cos^{-1} \left( 1 - \frac{s}{2m^2} \right) \right]^2 \]

\[ + f(s, s, m^2) \left[ -\frac{s}{m^2} - \frac{1}{2s} \right] \]

\[ + \frac{\left( \frac{s^2}{m^2} + \frac{1}{2} - \frac{3}{4} \frac{s^2}{m^4} \right)}{2s \left( 1 - \frac{s}{2m^2} \right)^2} \left[ f(s, s) + \frac{1}{1 - \frac{s}{m^2}} \right] + 2R(s) \tan^{-1} \frac{1}{R(s)} \]

\[ I_2(s, s, m^2) = -\frac{s}{m^2} f(s, s, m^2) - \frac{2}{s} \left[ \cos^{-1} \left( 1 - \frac{s}{2m^2} \right) \right]^2 \]
\[ f(s, s, m^2) = \frac{m^2}{s \left( 1 - \frac{s}{2m^2} \right)} \left[ \cos^{-1} \left( 1 - \frac{s}{2m^2} \right)^2 + L_{12} \left( 1 - \frac{s}{m^2} \right) - \frac{x^2}{6} \right] \]

\[ q^2 \to 0, \quad s \to 0 \]

\[ I_1(0, 0, m^2) = -\frac{1}{2m^2} \]

\[ I_2(0, 0, m^2) = -\frac{2}{m^2} \]
RADIATIVE CORRECTIONS IN $SU_2 \times U_1$:
LEP/SLC

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ABSTRACT

We show the sensitivity of various experimental measurements to one-loop radiative corrections in $SU_2 \times U_1$. Models considered are the standard GSW model as well as extensions of it which include extra quarks and leptons, SUSY and certain technicolor models. The observation of longitudinal polarization is a great help in seeing these effects in asymmetries in $e^+e^- \rightarrow \mu^+\mu^-, \tau^+\tau^-$ on $Z^0$ resonance.

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1. Introduction

One of the central questions of high-energy physics is that of determining what theory underlies the standard weak-interaction model of Glashow, Salam, and Weinberg (GSW) at distances much shorter than those we currently explore. The standard model has had dramatic success in predicting the features of the weak neutral current and in locating the masses of the $W$ and $Z$ bosons. But it is a theory which leaves undetermined a very large number of fundamental parameters, including a dimensionful parameter, the mass of the Higgs boson. If these parameters are truly determined by theory and not just put in ad hoc, then we must find a still more fundamental theory which reduces to the standard model at ordinary energies. Where do we look for signs that the standard model requires correction in this way? In many specific schemes which have been explored, the first sign of the presence of a new level of physics beyond the standard model is the appearance of novel particles associated with excitations in the new sector. Because of this, the search for novel particles has been a major preoccupation of physicists working at the highest-energy $e^+e^-$ and $p\bar{p}$ colliders. In the late 1980's, SLC, LEP, and the HERA and Tevatron colliders will explore for new states in the mass region up to about 100 GeV. The further direct exploration will need to wait for the large hadron-hadron colliders planned for the 1990's.

One might well hope to evade the requirement of attaining increasingly higher center-of-mass energies by searching for indirect effects of the new sector. The most sensitive such searches, however, have required that the new states make themselves visible through couplings which change quark or lepton flavor. Such indirect searches have had far less power when the new states couple to ordinary matter in a way that does not depend on flavor, as, for example, if they couple to ordinary matter only via the weak gauge bosons.

This situation will change, however, when the $Z^0$ resonance is produced in $e^+e^-$ collisions at SLC and LEP, and when $W$ bosons are produced in quantity at the
Tevatron and at LEP2. The $W$ and $Z$ couple directly to all particles, familiar or novel, which have weak interactions. Even those particles which are too heavy to be pair-produced at the $Z^0$ will affect the properties of these resonances through their virtual effects in loop diagrams. These loop effects are, in general, quite small, correcting the mass of the $Z^0$ by amounts of relative size $\alpha/\pi \sim 10^{-3}$. But the $Z$ and $W$ are elementary, weakly-interacting objects whose properties can be computed precisely by Feynman diagrams; they are also prominent resonances, with certain specific properties which allow precision experiments. We judge that the available technology, both experimental and theoretical, is sufficient that such tiny effects can be unambiguously observed.\(^1\)

There is one conceptual problem in isolating these loop effects which we should now discuss. Since these effects are typically of order 0.1% in size, their identification requires determining the parameters of the standard model to an accuracy of 0.1% or better. The standard model contains three parameters to which the properties of the $Z^0$ are directly sensitive: the two gauge coupling constants $g$ and $g'$ and the Higgs vacuum expectation value $v$ (or, alternatively, the fine structure constant $\alpha$, the $W$ mass, and the $Z^0$ mass). These are of special importance because they enter the tree-level expressions for leptonic processes. To define these parameters, we will recast them as the combinations:

$$\alpha, \ M_Z, \ \text{and} \ G_\mu.$$ (1.1)

$M_Z$ is conventionally defined as the position of the pole in the $Z$ propagator; this mass can be straightforwardly extracted from the shape of the $Z^0$ resonance. $\alpha$ is given by the electron charge from Thomson scattering measured at $q^2 = 0$. $G_\mu$ is conventionally defined from the muon lifetime $\tau_\mu$ by extracting specific purely electromagnetic radiative corrections:

$$\tau_\mu^{-1} = \frac{G_\mu^2 m_\mu^5}{192\pi^3} \left[ 1 + \frac{\alpha}{2\pi} \left( \frac{25}{4} - \pi^2 \right) \left( 1 + \frac{2\alpha}{3\pi} \ln \frac{m_\mu}{m_e} \right) \right]$$

$$\times \left[ 1 + \frac{3}{5} \frac{m_\mu^2}{M_W^2} \left( 1 - \frac{8 m_e^2}{m_\mu^2} \right) \right].$$ (1.2)
A recent CERN experiment by G. Bardin et. al.\textsuperscript{2} gives

$$G_\mu = (1.16637 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2}. \quad (1.3)$$

$G_\mu$ and $\alpha$ are the best known electroweak constants of Nature. LEP and SLC will soon measure $M_Z$ to 4 significant figures, an accuracy adequate for our purposes.

To completely specify the GSW model, one must also provide values for the Higgs boson self-coupling $\lambda$ (or, alternatively, the Higgs boson mass $m_H$), the fermion masses $m_f$, and the quark mixing angles $\theta_i$. The expressions we will eventually derive will depend weakly on $m_H$ and on $m_t$, the top quark mass. For these parameters, we will simply choose standard values—100 GeV for $m_H$ and 30 GeV for $m_t$—and use these, except where we state otherwise, in all of our calculations. We will, of course, display separately the dependence of our results on $m_H$ and $m_t$.

To test the standard model, and to find new contributions which go beyond it, one needs an additional experiment which can provide the same level of accuracy. One such experiment, which is difficult but conceptually quite familiar, is the accurate determination of the $W$ boson mass. In this paper, we would like to emphasize a second set of experiments at LEP/SLC; the various asymmetries in $e^+e^- \rightarrow \mu^+\mu^-\tau^+\tau^-$. We will show that these asymmetries, measured on $Z^0$ resonance where statistics are expected to be good, will test the GSW model at the one-loop level and possibly reveal the existence of particles beyond GSW. We will further show that the ability to observe longitudinal polarization, either in initial-state electron beams or in final state $\tau^-$ polarization, is especially useful for probing radiative corrections.

An auxiliary quantity which will enter our analysis is $\sin^2 \theta_w$. It is not a free parameter but rather only a bookkeeping device to be defined in terms of the set (1.1) and $m_H, m_t$. Sirlin has introduced the convention of defining $\sin^2 \theta_w$ in
terms of the measured $W$ and $Z$ boson masses. This definition is a very sensible one for $SU(2) \times U(1)$ (but less clear for other gauge groups) because it is in principle unambiguous. It is also a very convenient definition for the following reason: One part of the 1-loop correction to any weak process for $q^2 \sim -M_Z^2$ arises from the renormalization of $\alpha$ from $q^2 = 0$ to $q^2 = -M_Z^2$ due to QED vacuum polarization diagrams like those in Fig. 1. This correction is universal ($g$ and $g'$ are renormalized the same way by the large logs from photon vacuum polarization fermion loops) and is present in any theory which unifies weak and electromagnetic interactions. Marciano has pointed out this correction makes an unusually large contribution to the $W$ boson mass. It is also the largest correction to the lepton pair production polarization asymmetry. (Observation of the shift in the longitudinal polarization asymmetry or some other weak process due these QED corrections would indicate that $\alpha_2 = g^2/4\pi$ is renormalized the same way by the large logs from the diagrams in Fig. 1 as is $\alpha$ and that therefore some sort of unification of weak and electromagnetic interactions is correct.) Including the QED renormalization effect in the definition of $\sin^2 \theta_w$, then, greatly reduces the size of all standard model weak radiative corrections. We would prefer, however, to define $\sin^2 \theta_w$ so that it can be determined simply from the measurement of $\alpha$, $G_\mu$, and $M_Z$. We therefore chose to define $\theta_w$ by the formula:

$$\sin 2\theta_w \equiv \left( \frac{4\pi \alpha}{\sqrt{2} G_\mu M_Z^2 \cdot (1 - \Delta r_0)} \right)^{1/2}, \tag{1.4}$$

where

$$\Delta r_0 \equiv 0.06. \tag{1.5}$$

This factor corrects the zeroth-order Born formulae to include the effect of the renormalization of $\alpha$; our precise choice is made to establish a convention. A more precise evaluation of the QED renormalization factor for $q^2 \gg m_f^2$ reduces to
\[ \Delta r_{\text{QED}}^{\text{free fields}} = \frac{\alpha}{3\pi} \sum_{\text{fermions}} Q_f^2 \kappa_c \left( \ln \frac{-q^2}{m_f^2} - \frac{5}{3} \right) \]  

for light quarks and leptons when strong interactions are neglected. Here \( \kappa_c = 3(1) \) is the number of colors for quarks (leptons) and \( Q_f \) is the fermion electric charge; \( Q_e = -1 \) for electrons. To compute \( \Delta r_{\text{QED}} \) properly, one includes the effects of strong interactions; hadronization of the light quarks, \( u, d, c, s \) is taken into account via a dispersion relation in \( e^+ e^- \rightarrow \text{hadrons} \) while the \( t \) and \( b \) quarks and \( e, \mu \text{ and } \tau \) are treated as free particles. An accurate evaluation of this correction, for \( q^2 = -(94 \text{ GeV})^2 \), and \( m_T = 30 \text{ GeV} \), gives

\[ \Delta r_{\text{QED}} = 0.0597 \pm 0.0013 \]  

We have displayed an uncertainty due to strong interaction effects and note that it makes its way into one loop radiative corrections even in purely leptonic processes via the renormalization of \( \alpha \). Marciano and Sirlin have shown that placing the factor \( (1 - \Delta r_{\text{QED}}) \) in analogy with putting \( (1 - \Delta r_0) \) in the denominator of (1.4) sums all of the leading QED infrared logarithms. This definition of \( \sin^2 \theta_w \) agrees with that of Sirlin at tree level but differs by \( \mathcal{O}(\alpha) \) corrections.

We should warn the reader that the analysis presented here ignores QED radiative corrections associated with radiation of real or virtual photons from external legs, since these corrections depend on details of the particular experimental arrangement. They have been adequately considered by others. Thus, the graphs of Fig. 2 and all permutations are specifically excluded from our analysis of all four-lepton processes in either \( t \) or \( s \) channels. We do, however, include the QED vacuum polarization graphs of Fig. 1, as we have discussed.

Our analysis will proceed as follows. In Section 2 we will outline a formalism for the calculation of all one-loop radiative correction effects in four-lepton processes and will derive there effective matrix elements for charged and neutral current processes which include all one-loop corrections in \( SU_2 \times U_1 \) broken primarily
by Higgs doublets. It will be convenient then to divide one-loop corrections into two groups: oblique corrections affecting only gauge-boson vacuum polarization amplitudes and direct corrections involving 1PI vertex, fermion self-energy and box corrections. The first class, oblique, includes all of those corrections which do not involve the external particles. Examples are vector-ghost graphs, extra quarks and leptons, squarks, technicolor, and small effects due to v.e.v.'s of scalars transforming in $SU_2$ representations different from doublets. The second class include all those corrections involving the external particles; examples are many supersymmetric particles—winos, neutralinos, the first two generations of sleptons—and some extended technicolor models which are expected to give masses to the light leptons. For oblique couplings all one-loop radiative corrections can be absorbed into four functions; this is a great help in classifying the effects of new particles. In Section 3 we will define some physical quantities: the initial state longitudinal polarization, forward-backward, transverse polarization and final-state $\tau^-$ polarization asymmetries in $e^+e^- \to \mu^+\mu^-\tau^+\tau^-$, the ratio of Bhabha scattering to $\mu$ pair production and the $Z^0$ width (all measurable at LEP1/SLC) the $W^\pm$ mass and width (measurable at LEP2 and the Tevatron) and various ratios of neutrino scattering on electrons at low $q^2$ (to be measured by the CHARM II collaboration). We give the one-loop GSW prediction for these quantities and show the sensitivity to high Higgs' and top-quark masses. In Section 4, we give the response of the various asymmetries in $e^+e^- \to \mu^+\mu^-$ and $\tau^+\tau^-$, Bhabha scattering and $W^\pm$ mass to one-loop effects due to new physics; notably extra generations of quarks and leptons, SUSY and Technicolor. In Section 5 we give some conclusions.

Throughout this paper we will use the Euclidean metric, so that on $Z^0$ resonance $q^2 = q^2_0 - q^2 = -M_Z^2$. All work on one-loop radiative corrections in the standard GSW model is from the work of Lynn and Stuart. All work on the contributions to radiative corrections from beyond the standard model is from the work of Lynn and Lynn and Peskin. We apologize for any references omitted.
2. General Scheme for $SU_2 \times U_1$ Radiative Corrections

To study weak radiative corrections as a function of the kinematic variables associated with a given reaction, it is useful to construct an effective 4-fermion vertex for neutral current processes which includes all of the possible 1-loop subgraphs. We will approach this object in stages. Let us first note that, if the external particles in a process are $e, \mu, \tau$, then at $Z^0$ energies we may ignore the masses of these particles. That in turn implies that helicity is conserved at each gauge boson vertex. Thus, if we define the polarization of one initial- and one final-state fermion, the process contains only one invariant amplitude. For example, the cross-section for the reaction $e^+e^- \rightarrow \mu^+\mu^-$ takes the form:

$$
\frac{d\sigma}{dt} \ (e^- (P)e^+ \rightarrow \mu^- (P') \mu^+) = \frac{4\pi \alpha^2}{s} \cdot k_{PP'}^2 \cdot |M_{PP'}(-s)|^2,
$$

(2.1)

where $P$ and $P'$ denote longitudinal polarizations $L$ or $R$. $k_{PP'}^2$ is a kinematic factor from the Dirac algebra, equal to $(u/s)^2$ for $L \rightarrow L$ and $R \rightarrow R$ and to $(t/s)^2$ for $L \rightarrow R$ and vice versa. $M_{PP'}$ is the invariant amplitude which contains all the nontrivial information about the coupling; it is defined in such a way that $M_{PP'}$ equals 1, independently of $P$ and $P'$, for the simple s-channel photon exchange diagram of lowest-order QED for electrons. In the GSW theory, at leading order, but generalizing to arbitrary fermions in the initial and final states, $M$ is given by:

$$
M_{PP'}(q^2) = \frac{Q}{q^2} Q' \frac{-s}{q^2 + M_Z^2 - i\text{Im} \Pi_{ZZ}^{1\text{-loop}}(q^2)}
$$

\begin{equation}
\left( I^3 - Q \sin^2 \theta_w \right) \left( I^3' - Q' \sin^2 \theta_w \right) \\
\cos \theta_w \sin \theta_w \cos \theta_w \sin \theta_w
\end{equation}

(2.2)
where we have inserted the tree level $Z^0$ width (imaginary part of the one-loop $Z^0$ self energy) so that this expression remains finite on resonance. In the case in which only light quarks and leptons can be produced at $s = M_Z^2$ we have$^{12}$

$$
Im \Pi_{ZZ}^{1-\text{loop}}(-M_Z^2) \equiv \Gamma_2^0 M_Z
$$

$$
= \frac{\alpha M_Z^2}{3 \sin^2 \theta_W \cos^2 \theta_W} \sum_{\text{fermions}} \left[ \left( \frac{\Gamma_1^0}{2} - Q \sin^2 \theta_W \right)^2 \left( 1 + 2 \frac{m_f^2}{M_Z^2} \right) \right.
$$

$$
+ \left( \frac{\Gamma_2^0}{2} \right)^2 \left( 1 - 4 \frac{m_f^2}{M_Z^2} \right) \left( 1 - 4 \frac{m_f^2}{M_Z^2} \right)^{1/2} \cdot C_{QCD}.
$$

(2.3)

with fermion masses $2m_f < M_Z$ and left-handed isospin component $I_L^0$. The last factor gives the QCD corrections to the lowest order width; $C_{QCD}$ is $3 \cdot (1 + \alpha_{\text{strong}}(-M_Z^2)/\pi)$ for quarks and 1 for leptons.

It is straightforward to add to the above expression for $\mathcal{M}$ the effects for all possible 1-loop subdiagrams. At the same time, one must correct the factors of $\alpha$ and $G_\mu$ and $M_Z$ in the tree level expression from their bare to their physical values. For simplicity, let us first carry out the analysis only for oblique corrections. Oblique corrections are all of those corrections which affect only vector particle vacuum polarization amplitudes. These explicitly include the standard QED corrections of Fig. 1. A typical 1PI vector self energy, a formally infinite object containing no counterterms, is defined in Fig. 3. The calculation of $\Pi_{ij}$, $i,j = Z^0, W^\pm$, A (photon) or $i,j = SU_2$ currents 1, 2, 3 and electromagnetic current $Q$ is clearly just a matter of counting the representations of $SU_2 \times U_1$ particles in the theory and being careful about mass diagonalization and Clebsch-Gordon coefficients.

Including only these oblique corrections, $\mathcal{M}$ takes the form (with the abbreviation $\sin^2 \theta_W = s_W^2$)
\[ M = Q \frac{1}{1 - \Delta_\alpha(q^2)} \left( \frac{-s}{q^2} \right)^2 Q' \left( I_3 - Q(s_\beta^2 + \Delta_p(q^2) - i s_\beta c_\beta \text{Im} \Pi_{Z\alpha}^1(q^2)) \right. \]
\[
\left. \times \left( \frac{-s}{(q^2 + M_Z^2)(1 - \Delta_p(q^2) - 0.06) - \text{Im} \Pi_{Z\alpha}^{1\text{loop}}(q^2)(1 - C_\Gamma \Delta_p(q^2))} \right) \right) \]
\[
\times \left( I_3' - Q'(s_\beta^2 + \Delta_p(q^2) - i s_\beta c_\beta \text{Im} \Pi_{Z\alpha}^{1'}(q^2)) \right) \]
\]

where

\[ \Delta_p(q^2) = \Delta_p(-M_Z^2) \]
\[ - 4\pi \alpha \text{Re} \left( \frac{\Pi_{33}(-M_Z^2) - \sin^2 \theta_w \Pi_{33Q}(-M_Z^2)}{M_Z^2} + \frac{\Pi_{33}(q^2) - \sin^2 \theta_w \Pi_{33Q}(q^2)}{q^2} \right) \]

(2.5)

\[ \Delta_\rho(q^2) = \Delta_\rho(0) \]
\[ + \text{Re} \left( \frac{\Pi_{ZZ}(-M_Z^2) - \Pi_{ZZ}(0)}{M_Z^2} + \frac{\Pi_{ZZ}(q^2) - \Pi_{ZZ}(-M_Z^2)}{q^2 + M_Z^2} \right) \]

(2.6)

\[ \Delta_\alpha(q^2) = \frac{4\pi \alpha}{q^2} \left( \Pi_{QQ}(q^2) - q^2 \Pi_{QQ}'(0) \right) \]

(2.7)

and \( \Delta_\rho(0), \Delta_p(-M_Z^2) \) are given by

\[ \Delta_\rho(0) = \frac{4\pi \alpha}{M_Z^2 \cos^3 \theta_w \sin^2 \theta_w} (\Pi_{33}(0) - \Pi_{11}(0)) \]

(2.8)

\[ \Delta_p(-M_Z^2) = \frac{\sin^2 \theta_w \cos^2 \theta_w}{\cos 2\theta_w} \cdot \text{Re} \left( \Delta_\alpha(-M_Z^2) - \Delta_\rho(0) - 0.06 \right. \]
\[ - \frac{4\pi \alpha}{\sin^2 \theta_w \cos^2 \theta_w} \cdot \frac{1}{M_Z^2} \left( \Pi_{33}(-M_Z^2) - \Pi_{33}(0) - \Pi_{33Q}(-M_Z^2) \right) \]·

(2.9)

This expression makes obvious the constraint on \( M \) which follows from the renormalizability of the electroweak interactions; it must be free of one-loop divergences. One can see that the various vacuum polarization amplitudes in
(2.4) are assembled into combinations whose divergences cancel explicitly. We have replaced the vacuum polarization amplitudes of vector bosons \( W, Z \) and \( A(\text{photon}) \) with vacuum polarization amplitudes of the weak isospin and electromagnetic currents. Denoting the weak isospin currents by 1, 2, 3 and the electromagnetic current by \( Q \), we have

\[
\Pi_{AA} = e^2 \Pi_{QQ}, \quad \Pi_{ZA} = \frac{e^2}{\cos \theta_w \sin \theta_w} (\Pi_{3Q} - \sin^2 \theta_w \Pi_{QQ}),
\]

\[
\Pi_{WW} = \frac{e^2}{\sin^2 \theta_w} \Pi_{11}, \quad \Pi_{ZZ} = \frac{e^2}{\sin^2 \theta_w \cos^2 \theta_w} (\Pi_{33} - 2 \sin^2 \theta_w \Pi_{3Q} + \sin^4 \theta_w \Pi_{QQ}).
\]

The combination \( \Delta_\sigma(q^2) \) is simply the properly subtracted photon vacuum polarization. The combination \( \Delta_\rho(q^2) \) is also independently observable, giving at \( q^2 = 0 \) the one-loop oblique particle correction to the \( \rho \) parameter\(^{13} \) which measures the relative strength of charged and neutral weak currents:

\[
\rho - 1 = \Delta_\rho(0). \tag{2.11}
\]

The quantity \( \Delta_\rho(q^2) \) is also free of ultraviolet divergences: \( \Pi_{3Q} \) vanishes at \( q^2 = 0 \). It may have a divergent slope at that point, but the relation \( Q = I^3 + \frac{Y}{2} \) implies that \( \Pi_{33} \) has an identical (and, thus, cancelling) divergence.

It is easy to see that only three independent combinations of vacuum polarization amplitudes can appear in the most general neutral current matrix element for oblique corrections due to any representations of \( SU_2 \times U_1 \); there are only three neutral vector self energies: \( Z - Z, Z - A \) and \( A - A \). The fourth possible vector self energy, the \( W - W \), will result in a fourth (and last) independent combination which will of course enter into the effective matrix element governing charged-current processes at the one loop-level. It is also easy to see why these particular combinations arise: since we have used \( \alpha, G_\mu \) and \( M_Z \) as renormalized input data, the shift in these quantities from their bare to renormalized values
\[ \frac{\delta_b \alpha}{\alpha} = -\Pi_{AA}'(q^2 = 0) \]  

\[ \frac{\delta_b G_\mu}{G_\mu} = -\frac{\Pi_{WW}(q^2 = 0)}{M_\mu^2} \]  

\[ \frac{\delta_b M_Z^2}{M_Z^2} = \text{Re} \frac{\Pi_{ZZ}(q^2 = -M_Z^2)}{M_Z^2} \]  

will enter our expressions. Also, the quantities \( \Pi_{AA}(q^2) \), \( \Pi_{ZZ}(q^2) \) and \( \Pi_{ZA}(q^2) \) enter neutral current processes directly. There are also factors of \( \Pi_{ZA}(0) \) (when this does not automatically vanish) coming from the Ward identity governing the \( SU_2 \) vector boson wavefunction renormalization but we will drop these throughout this paper. Note the inclusion in (2.4) of the appropriate \( O(\alpha^2) \) contributions to the imaginary part of the \( Z^0 \) inverse propagator. This is necessary in order that we may examine the corrections to cross sections to the 0.1 percent level on the \( Z^0 \) resonance; it is not necessary off resonance or for the discussion of asymmetries because it cancels out in ratios of cross sections at this level of accuracy. We will discuss the constant \( C_\Gamma \) and why \( \Delta_p(q^2) \) alone occurs in the shift in the imaginary part of the \( Z^0 \) inverse propagator later.

Equation (2.4) is the main result of this section. It makes explicit (at least for the case of oblique corrections) the reduction of the variety of weak radiative corrections to a few (for oblique corrections there are four) basic quantities which determine the experimentally accessible parameters. In particular, it should be noted that for \( e^+e^- \rightarrow f\bar{f} \) with \( f \neq e, \nu_e \) on the \( Z^0 \) resonance where only the \( Z \) propagator term is important, all of the various possible asymmetries and cross sections in fermion pair production are determined entirely by the relative coefficients of \( I^3 \) and \( Q \) in the fermion-fermion-\( Z^0 \) vertex. Thus, all of the asymmetries in \( e^+e^- \rightarrow \mu^+\mu^- \) measure the same quantity, \( \Delta_p(-M_Z^2) \) and corrections to, say, the charge asymmetry are proportional to those of the polarization asymmetry with the constant of proportionality a function only of \( \sin^2 \theta_W \).
We now turn to the direct coupling corrections: 1PI vertex, fermion self energy parts and box diagrams. The box diagrams form a gauge invariant set and so are finite to one-loop. Combinations of 1PI vertex parts and external fermion line self-energies are also finite as we will explain now.

The Ward identities of $SU_2 \times U_1$ govern the relationship between the vector boson wavefunction renormalizations $Z_W$ (for weak isospin) and $Z_B$ (weak hypercharge) and their respective bare couplings $g^0, g^0'$ and the renormalized finite couplings $g, g'$. In analogy with QED we have ($\sin \theta_W \equiv s_\theta$)

\[
Z_B^{1/2} g^0' = g'
\]

\[
Z_W^{1/2} g^0 = g \left(1 + \frac{\Pi_{ZA}(0)}{c_\theta s_\theta M_Z^2} \right) + O(s_\theta^5).
\]

(2.15)

(2.16)

Since exactly the combination $Z_B^{1/2} g^0'$ and $Z_W^{1/2} g^0$ occur in the bare Lagrangian, the fermion wavefunction renormalization must also properly subtract the fermion-vector boson vertices. Note that we will drop all factors of $\Pi_{ZA}(0)/c_\theta s_\theta M_Z^2$ which is a non-zero constant for internal gauge boson lines. Consequently, if we divide the 1PI fermion self-energy and fermion-boson vertex parts into left-handed and right-handed parts as in Figs. 4 and 5 with $\gamma = \frac{1}{2}(1 \pm \gamma_5)$ the left- and right-handed spin projection operators, the combinations

\[
\Gamma_{\pm \pm} = \tilde{\Gamma}_{\pm \pm} + \left[A_{\pm}^e + 2m_e^2 \frac{\partial}{\partial p^0} (C - A_{\pm}^e) \right]_{p^0 = -m_e^2}
\]

(2.17)

will be finite. All other fermion-vector boson couplings are treated in a completely analogous way.

We close this section with a discussion of the effective matrix element to one loop for charged current scattering and its relation to the shift in the W mass. At tree level the W couples only left-handedly and it is easy to see that, neglecting the masses of external particles, any 4-fermion charged current process may be written in terms of the effective vertex

\[
\mathcal{M}^{CC} = \frac{1}{2\sin \theta_W} \cdot \frac{s}{q^2 + \cos^2 \theta_W M_Z^2 - iM_W^{1\text{-loop}}(q^2)} \cdot \frac{1}{\sin \theta_W}
\]

(2.18)
We have inserted the tree level $W$ width so that this expression remains finite on $W$ resonance. In the case in which only light quarks and leptons can be produced there we have\textsuperscript{12}

\[
\text{Im} \Pi_{WW}^{1-\text{loop}} (-c_w^2 M_Z^2) \equiv \Gamma^0_W M_Z \cos \theta_w
\]

\[
= \sum_{\text{fermions}} \frac{\alpha M_Z^2 c_w^2}{12\delta_s^2} |U_{ff'}|^2 \left[ 1 - \frac{1}{2} (\delta_f + \delta_{f'}) + \frac{1}{2} (\delta_f - \delta_{f'})^2 \right] \times \left[ 1 - 2(\delta_f + \delta_{f'}) + (\delta_f - \delta_{f'})^2 \right]^{1/2} \cdot C_{QCD}
\]

(2.19)

where $\delta_f = m_f^2/c_w^2 M_Z^2$, $m_f + m_{f'} < M_W$ and $U_{ff'}$ is a Kobayashi-Maskawa quark mixing matrix element. Using the methods of this section it is easy to show that the 1-loop oblique corrections to (2.18) are

\[
M^{CC}(q^2) = \frac{1}{2\sin^2 \theta_w} (-s)
\]

\[
\times \left[ (1 - 0.06) \left[ (q^2 + \cos^2 \theta_w M_Z^2)(1 - \Delta_W(0)) + \cos^2 \theta_w M_Z^2 \cdot \Delta_W(q^2) \right] - i \text{Im} \Pi_{WW}^{1-\text{loop}}(q^2) \right]^{-1}
\]

(2.20)

where $\Delta_W(q^2)$ is obviously finite:

\[
\Delta_W(q^2) = \Delta_W(-M_W^2) + \text{Re} \left( \frac{\Pi_{WW}(-M_W^2) - \Pi_{WW}(q^2)}{M_W^2} \right)
\]

(2.21)

with

\[
\Delta_W(-M_W^2) = -\frac{1}{\sin^2 \theta_w} \Delta_\rho(-M_Z^2) + \text{Re} \Delta_\omega(-M_Z^2) - 0.06 - \frac{4\pi\alpha}{\sin^2 \theta_w \cos^2 \theta_w} \times \frac{\text{Re} \left( \Pi_{11}(-M_W^2) - \Pi_{11}(0) - \cos^2 \theta_w \Pi_{3Q}(-M_Z^2) \right)}{M_Z^2}.
\]

(2.22)

This is the fourth and last function needed to discuss the effects of oblique corrections; it corresponds to the fourth possible vector self energy correction, the $W-W$. It is clear from (2.20) that there are no oblique corrections to charged current processes at $q^2 = 0$ because we have used such a process, muon decay,
to define one of the input parameters in our renormalization scheme. The position of the pole of the effective charged current vertex gives the $W$ mass and we recover the result for oblique corrections

$$M_W^2 = M_Z^2 c_W^2 \left( 1 + \Delta_W(-M_W^2) \right). \quad (2.23)$$

The formula for the $W^{\pm}$ mass shift was first written for GSW by Sirlin\textsuperscript{3,41} and later by others.\textsuperscript{14} Again, it is simple to include direct corrections of vertices, fermion self-energies and boxes in charge current processes. We have, in analogy with (2.17), the finite quantity

$$\Gamma^e_{\mu W} = \tilde{\Gamma}^e_{\mu W} + \frac{1}{2} \left[ A_e^+ + 2m_e^2 \frac{\partial}{\partial p^2} (C^e - A_e^+) \right]_{p^2 = -m_e^2} + \frac{1}{2} A_e^+(0) \quad (2.24)$$

where the 1PI charged vertex part is defined in Fig. 6. The effective charged current matrix element is defined so that it requires no radiative corrections, either oblique or direct, at $q^2 = 0$ since we use muon decay as an input parameter. Care has to be taken to transmit the direct corrections of muon decay into the neutral current and charged current matrix elements $M_{PP}$ and $M^{CC}$. We call the reader’s attention to the one-loop box diagrams contributing to muon decay depicted in Fig. 7 and the definition of the form factor $V^{\mu \nu \mu \nu}_+$. These are $UV$ finite and will contribute to both the shift in the $W^{\pm}$ mass and to the various asymmetries in $e^+e^- \rightarrow \mu^+\mu^-$, $\tau^+\tau^-$. Also, we have to be very careful to include those QED corrections which are traditional in the definition of $G_\mu$ from $\tau^{-1}$ in Eq. (1.2) throughout.

Thus, all one-loop corrections to neutral and charged current processes in $SU_2 \times U_1$ can be boiled down to the calculation of the following combinations of 1PI parts

\textsuperscript{41} In GSW $\Delta_W(-M_W^2)$ is related to Sirlin’s $\Delta r$ for the $W^{\pm}$ mass shift by

$$\Delta_W(-M_W^2) = \frac{-s_W^2}{\cos 2\beta_w} (\Delta r_{\text{vac, pol}} - 0.06)$$
1. oblique corrections:

\[ \Delta_W, \Delta_\rho, \Delta_\alpha, \Delta_\rho \] are combinations of vacuum polarization amplitudes.

2. direct corrections:

(a) box diagrams are UV finite

(b) vertex parts \[ \Gamma^{fZ}_\pm, \Gamma^{fA}_\pm, \Gamma^{fW}_+ \] are combinations of 1PI vertex parts and self energies of fermions \( f, f' \).

These are inserted into effective matrix elements for neutral \( \mathcal{M}_{PP} \) and charged \( \mathcal{M}^{CC} \) current processes. All cross section to one loop are just functions of \( \mathcal{M}^{CC} \) and \( \mathcal{M}_{PP} \), apart from kinematics.

In the next section we will define some physically measurable quantities, mostly on \( Z^0 \) resonance \( q^2 = -M_Z^2 \), and display the GSW one-loop corrections to them. In Section 4 we will give the response of these quantities to new physics, mostly obliquely coupled; e.g. extra quarks and leptons, SUSY, technicolor, etc.

3. Measurables: Response to One-loop in GSW

In the last section we saw that all one-loop corrections in \( SU_2 \times U_1 \) could be incorporated into effective matrix elements for neutral current \( \mathcal{M}_{PP} \) and charged-current \( \mathcal{M}^{CC} \) four-fermion processes if the masses of external fermions were small. In this section we will define a number of physical measurables and give the response of these to one-loop corrections in the standard GSW model with three generations of quarks and leptons. All of the results of this section are from the work of Lynn and Stuart\(^{5,11}\) whose computer program calculates all four-fermion processes to one-loop in GSW excluding the graphs of Fig. 2.

Let us start with the various asymmetries in \( e^+e^- \rightarrow \mu^+\mu^- \) or \( \tau^+\tau^- \). These are obviously given by various combinations of the cross sections (2.1). We first discuss the calculation of the initial state longitudinal polarization asymmetry in lepton pair production. This quantity is defined as:

\[
A_{LR} = \frac{\sigma(e^-(L)e^+ \rightarrow \ell^-\ell^+) - \sigma(e^-(R)e^+ \rightarrow \ell^-\ell^+)}{\sigma(e^-(L)e^+ \rightarrow \ell^-\ell^+) + \sigma(e^-(R)e^+ \rightarrow \ell^-\ell^+)}.
\] (3.1)
We begin by examining the longitudinal polarization asymmetry, \( A_{LR} \), because it will be shown that of the available asymmetries in \( e^+e^- \rightarrow \mu^+\mu^- \) on \( Z^0 \) resonance it is the most sensitive to radiative corrections. The shifts in the forward-backward asymmetry, \( A_{FB} \), and transverse polarization asymmetry \( A_L \) due to higher order corrections can be expressed in terms of the shift in \( A_{LR} \). Hence no new information is available from them. It should be borne in mind that initial state longitudinal polarization will not be available in the early stages of LEP. However the observed sensitivity is presented as a strong argument for its eventual inclusion. Also, a careful measurement of the \( \tau^- \) final state longitudinal polarization asymmetry (defined below) is equally sensitive to the effects of radiative corrections and should be a higher priority at LEP.

Unless otherwise stated we will assume, because we are primarily interested in LEP/SLC physics, that the energy of the reaction \( e^+e^- \rightarrow \mu^+\mu^- \), \( \tau^+\tau^- \) is tuned precisely to the \( Z^0 \) resonance; then the \( Z \) propagator is purely imaginary, so that not only does the diagram with a \( Z^0 \) in the \( s \)-channel dominate, but (modulo the imaginary parts of 1PI vertex and self-energy parts) there are also no interference terms with the photon exchange diagram. The contribution of the photon exchange diagram to the cross section is \( \sim 10^{-2} \) times that of the \( Z^0 \) exchange diagram and so we will concentrate primarily on corrections to the \( Z^0 \) pole part.

At leading order in perturbation theory, \( A_{LR} \) on resonance is given precisely by the asymmetry in the couplings of \( e(L) \) and \( e(R) \) to the \( Z^0 \):

\[
A_{LR}\Big|_0 = \frac{\left( \frac{1}{2} - \sin^2 \theta_w \right)^2 - (\sin^2 \theta_w)^2}{\left( \frac{1}{2} - \sin^2 \theta_w \right)^2 + (\sin^2 \theta_w)^2},
\]

\[
= \frac{-2\pi \theta}{1 + v_\theta^2},
\]

(3.2)

where \( v_\theta = (4\sin^2 \theta_w - 1) \). Let us now define:

\[
\delta A_{LR} = A_{LR} - A_{LR}\big|_0,
\]

(3.3)
where $A_{LR}|_0$ is to be evaluated using (1.4). $\delta A_{LR}$ is directly measurable, in the sense that it may be computed directly from the physical quantities $\alpha$, $G_\mu$, $M_Z$, and $A_{LR}$. We have argued above that the indicated difference can eventually be measured to a few tenths of a percent by precision experiments\(^1\) at the $Z^0$. Our main focus, then, will be on assessing the size of the difference $\delta A_{LR}$ to be expected in a variety of models of physics.

To clarify the situation, let us for the moment consider only oblique corrections (gauge boson vacuum polarization amplitudes) and, of these, only those entering the $Z^0$ exchange part of $M_{PP'}$. Then we find that\(^6,11,15\)

$$\delta A_{LR}^{\text{oblique}} = -8 \cdot \frac{1 - v_\beta^2}{(1 + v_\beta^2)^2} \cdot \Delta p(-M_Z^2)$$

$$= 64 \cdot \frac{\sin^2 \theta_w}{(1 + v_\beta^2)^2} \cdot 4\pi \frac{\text{Re}}{M_Z^4} \left[ -\Pi_{11}(0) + \Pi_{33}(-M_Z^2) - \Pi_{3Q}(-M_Z^2) \right]$$

$$+ \cos^2 \theta_w \sin^2 \theta_w \left[ \Pi_{QQ}(-M_Z^2) + M_Z^2 \Pi_{QQ}'(0) + 0.06 \frac{M_Z^2}{4\pi \alpha} \right].$$

(3.4)

Thus, $A_{LR}$ is only sensitive on $Z^0$ resonance to the quantity $\Delta p(-M_Z^2)$ for oblique corrections. In adding direct corrections, we must be careful about two sources. Those from muon decay used to define our input parameter $G_\mu$ will just be added on to $\Delta p(-M_Z^2)$ since they effectively only change the value of $\sin^2 \theta_w$ when included.\(^3\) Direct correction to the electron-$Z^0$ vertex will appear, since this is a left-right asymmetry, in the combination $\Gamma^+ - \Gamma^-$. We have excluded the QED detector dependent graphs\(^9\) of Fig. 2, the neutral current boxes do not have the correct $Z^0$ pole structure to contribute heavily and $\Gamma_Z$ drops out in the ratio of cross sections. Thus, including only the $Z^0$ pole terms we have\(^{15}\)
\[ \delta A_{LR} = -8 \frac{1 - v^2}{1 + v^2} \Delta \rho(-M_Z^2) - \frac{64 s^2 c^2}{(1 + v^2)^2} \]

\[ \times \left\{ \Gamma_{\nu}^{\mu \nu \varphi}(0) + \Gamma_{\mu}^{\nu \varphi \nu}(0) + \left( \frac{1 - 2 s^2}{c^2} \right)^2 \right\} \]

\[ \times \left( \Gamma_{\mu}^{\mu \nu \varphi}(-M_Z^2) - \Gamma_{\nu}^{\mu \varphi \nu}(-M_Z^2) \right) \]

\[ + V_{\mu}^{\nu \mu \nu \varphi}(0) \right\} \] (3.5)

where the \( \Gamma \)'s are effective vertex parts depicted in Figs. 4, 5 and 6 and Eqs. (2.17) and (2.24) and \( V_{\mu}^{\nu \mu \nu \varphi}(0) \) are the box diagrams from muon decay depicted in Fig. 7 evaluated with zero external momenta.\(^2\) All of the one-loop GSW corrections to \( A_{LR} \) except those of Fig. 2 have been calculated by Lynn and Stuart (and later by Hollik\(^1\)), including corrections to photon exchange, boxes and a careful treatment of the various imaginary parts of one-loop vertex and self-energies. The results are given in Table I and, indeed, the above corrections are dominant. Notice the dramatic dependence of \( A_{LR} \) on the precise values of \( M_Z \) relative to \( G_\mu^{-\frac{1}{2}} \) as well as a large Higgs and top quark masses. In order to make a sensible (to us at least) statement about the size of the GSW weak corrections due to internal gauge bosons, etc., which are not taken into account by the QED renormalization of \( \alpha \) from \( q^2 = 0 \) to \( -M_Z^2 \) we display the quantity

\[ \delta A_{LR}^{G_{SW}}(m_t = 30, m_H = 100) = A_{LR}(m_t = 30, m_H = 100) - A_{LR}|_0 \]

\[-\left\{ \begin{array}{c}
0.0257 \\
0.0242 \\
0.0237
\end{array} \right\} \quad \text{for} \quad M_Z = \left\{ \begin{array}{c}
90 \\
94 \\
98
\end{array} \right\} \text{GeV}. \]

\[ \] (3.6)

Here, all masses are in GeV. Thus, GSW weak corrections are measurable in the initial state longitudinal polarization asymmetry. Further, we see the variation with large top quark mass and Higgs' mass

\[ ^2 \text{In (3.5) we have not displayed the QED corrections traditional in the definition (1.2) of } G_\mu. \]

\[ ^1 \text{They are, of course, included in our numerical evaluations throughout this section.} \]
\[ \delta A_{LR}^{GSW} (m_t = 180, m_H = 100, M_Z = 94) = -0.0242 + 0.0294 \]  
(3.7)

\[ \delta A_{LR}^{GSW} (m_t = 30, m_H = 1000, M_Z = 94) = -0.0242 - 0.009 . \]  
(3.8)

The next quantity to be examined is the forward-backward asymmetry, which for 2-particle final states is the same as the charge asymmetry:

\[ A_{FB} = \frac{\int d\phi \left[ \int_0^1 - \int_{-1}^0 \right] d \cos \theta \frac{d \sigma}{d \eta} (e^+ e^- \to f \bar{f})}{\int d\phi \left[ \int_0^1 + \int_{-1}^0 \right] d \cos \theta \frac{d \sigma}{d \eta} (e^+ e^- \to f \bar{f})} . \]  
(3.9)

For a fermion of charge \( Q \) and left-handed isospin \( I_3 \), the zeroth-order formula for this asymmetry on \( Z^0 \) resonance is:

\[ A_{FB}^f \big|_0 = \frac{3}{4} \left( \frac{-2v_\theta}{1 + v_\theta^2} \right) \left( \frac{2I_L^3 a_f}{a_f^2 + I_3^2} \right) , \]  
(3.10)

where

\[ a_f = I_L^3 - 2Q \sin^2 \theta_w \]  
(3.11)

with left-handed weak isospin component \( I_L^3 \) for the fermion. The forward-backward asymmetry in GSW including all one-loop corrections except those of Fig. 2 is given in Table II from the work of Lynn and Stuart. It has also been calculated in GSW by many others.\(^{16}\) We note that it is dramatically dependent on the precise value of \( M_Z \). This is easy to understand since at tree level \( A_{FB} \approx 3v_\frac{\theta}{2} \) on \( Z^0 \) resonance and small shifts in \( s_\frac{\theta}{2} \) away from \( 1/4 \) give large shifts in \( v_\theta = 4 \sin^2 \theta_w - 1 \). If we define the shifts from the calculated value in analogy with (3.6)

\[ \delta A_{FB}^{GSW} = A_{FB}^{GSW} - A_{FB} \big|_0 \]  
(3.12)

then the GSW weak corrections are

\[ \delta A_{FB}^{GSW} (m_H = 30, m_H = 100) = - \begin{cases} 0.0008 & \text{for } M_Z = \begin{cases} 90 \\ 94 \end{cases} \\ 0.0076 & \text{for } M_Z = \begin{cases} 94 \\ 98 \end{cases} \\ 0.0123 & \text{for } M_Z = \begin{cases} 98 \\ 100 \end{cases} \end{cases} \]  
(3.13)
and the response to heavy top quarks and Higgs' are

\[ \delta A_{FB}^{GSW} (m_t = 180, m_H = 100, M_Z = 94) = -0.0076 + 0.0075 \]  \hspace{1cm} (3.14) 

\[ \delta A_{FB}^{GSW} (m_t = 30, m_H = 1000, M_Z = 94) = -0.0076 - 0.0038 \]  \hspace{1cm} (3.15) 

The charge asymmetry is less sensitive to the effects of radiative corrections than is the left-right longitudinal polarization asymmetry. This is easily understood from (2.4) as follows. On \( Z^0 \) resonance the shift due to oblique corrections in the charge asymmetry or any other asymmetry or cross section in \( e^+e^- \rightarrow f\bar{f} \), with \( f \neq e, \nu_e \) a light fermion depends only on \( I_3, Q \) and the particular combination \( \Delta_\rho (M_Z^2) \); this is essentially only a statement about the dominance of the \( Z^0 \) exchange graph for neutral current processes on resonance. Using the methods of the last section, one may easily show that the shifts in this asymmetry is for final state fermions \( f \)

\[ \delta A_{FB}^f \approx \frac{3 I_3^2}{2 (I_3^2)^2 + a_f^2} \cdot \left( a_f + \frac{v_\theta Q}{2} \frac{1 + v_\theta^2 a_f^2 - \frac{(I_3^2)^2}{2}}{1 - v_\theta^2 a_f^2 + (I_3^2)^2} \right) \cdot \delta A_{LR} \]  \hspace{1cm} (3.16) 

in the case in which the \( Z^0 \) pole terms only are included. Since \( a_\mu = v_\theta/2 \) both terms on the right-hand side are suppressed by \( v_\theta \) for the charge asymmetry in \( e^+e^- \rightarrow \mu^+\mu^- \). In fact, since at tree level on \( Z^0 \) resonance

\[ A_{FB}^\mu \big|_0 = \frac{3}{4} \left( A_{LR} \big|_0 \right)^2 \]  \hspace{1cm} (3.17) 

we have

\[ \delta A_{FB}^\mu \approx \frac{3}{2} A_{LR} \big|_0 \cdot \delta A_{LR} = -\frac{3 v_\theta}{1 + v_\theta^2} \cdot \delta A_{LR} \]  \hspace{1cm} (3.18) 

including both oblique and direct corrections if \( \mu - e \) universality is assumed.

This suppression (especially for small \( M_Z \)) by a factor of \( v_\theta \) of the contribution of new heavy oblique particles is true for any asymmetry formed from \( e^+e^- \rightarrow \mu^+\mu^- \), \( \tau^+\tau^- \) on \( Z^0 \) resonance in which the longitudinal polarization of
incoming or outgoing particles is unobserved. Thus the capacity for longitudinal polarization measurement in leptonic processes on $Z^0$ resonance is crucial for the observation of small effects due to radiative corrections in GSW.

Anticipating the possibility of obtaining transverse polarization on both the $e^+(P^+)$ and $e^-(P^-)$ beams at LEP, we may define the transverse or azimuthal ($\phi$) asymmetry $A_\perp$ on $Z^0$ resonance

$$ A_\perp = \frac{4}{P_\perp P_-} \frac{\int d\Omega \cos 2\phi \frac{d\sigma}{dt}(e^+e^- \to \mu^+\mu^-)}{\int d\Omega \frac{d\sigma}{dt}(e^+e^- \to \mu^+\mu^-)}. \quad (3.19) $$

The GSW one-loop calculations of Lynn and Stuart for this quantity excluding the graphs of Fig. 2 are given in Table III. From the lowest order $Z^0$ resonance formula

$$ A_\perp|_0 = \frac{v^2_e - 1}{v^2_e + 1} \quad (3.20) $$

we may display the shifts due to the one-loop weak corrections in analogy with (3.6)

$$ \delta A_\perp^{GSW}(m_t = 30, m_H = 100) = \begin{cases} 0.0103 \\ 0.0043 \\ 0.0007 \end{cases} \text{ for } M_Z = \begin{cases} 90 \\ 94 \\ 98 \end{cases} \text{ GeV} \quad (3.21) $$

As well as large top and Higgs' masses

$$ \delta A_\perp^{GSW}(m_t = 180, m_H = 100, M_Z = 94) = 0.0043 + 0.0036 \quad (3.22) $$

$$ \delta A_\perp^{GSW}(m_t = 30, m_H = 1000, M_Z = 94) = 0.0043 - 0.0026 \quad (3.23) $$

Note that the response of $A_\perp$ to radiative corrections is also a lot smaller than that of $A_{LR}$. In fact we may easily show from examination of the tree-level formula and the methods of the previous section that
\[ \delta A_\perp \simeq \frac{-2v_\theta}{1 - v_\theta^2} \delta A_{LR} \simeq \frac{2}{3} \delta A_{FB}^\mu \] (3.24)

when only \( Z^0 \) pole terms are taken into account for both direct and oblique corrections.

We now indicate why this suppression by \( v_\theta = 4s_\theta^2 - 1 \) occurs for radiative corrections on \( Z^0 \) pole for any asymmetry in \( e^+e^- \rightarrow \mu^+\mu^- , \tau^+\tau^- \) in which the longitudinal polarization is not observed. Such an asymmetry, at tree level, must be only a function of the quantity

\[ \left( \frac{\ell^+ \text{ vector coupling to } Z^0}{\ell^+ \text{ axial vector couplings to } Z^0} \right)^2 = v_\theta^2 . \] (3.25)

From (2.4) and (3.5) we can see that the effect of radiative corrections on \( Z^0 \) pole exchange diagrams is to displace

\[ s_\theta^2 \rightarrow s_\theta^2 + \Delta_p(M_Z^2) + \frac{s_\theta^2 c_\theta^2}{1 - 2s_\theta^2} \]

\[ \times \left\{ \Gamma_+^{\nu\nu}(\omega) + \Gamma_+^{\nu\nu}(\omega) + \left( \frac{1 - 2s_\theta^2}{c_\theta} \right)^2 \left( \Gamma_\perp^{\nu\nu}(M_Z^2) - \Gamma_\perp^{\nu\nu}(-M_Z^2) \right) + V_{\perp+}^{\nu\nu}(\omega) \right\} \]

(3.26)

if \( \mu - e - \tau \) universality is obeyed. Thus, the shift in any asymmetry \( A \) without longitudinal polarization in \( e^+e^- \rightarrow \mu^+\mu^- , \tau^+\tau^- \) is

\[ \delta A = (\text{const})v_\theta \delta v_\theta = (\text{const}')v_\theta \delta A_{LR} . \] (3.27)

This suppression by a factor \( v_\theta \) is especially disastrous for small \( M_Z \) and makes the observation of small effects due to radiative corrections extremely difficult in asymmetries formed without the observation of longitudinal polarization. Asymmetries with longitudinal polarization on \( Z^0 \) resonance escape this argument because they are, at tree level, functions only of \( v_\theta \) rather than \( v_\theta^2 \). Thus, the shifts due to radiative corrections in asymmetries with the observation of longitudinal polarization \( \sim \delta v_\theta \) and so avoid the suppression factor.
One asymmetry whose response to radiative corrections is therefore not suppressed by the factor \(v_\theta\) is the \(\tau^-\) polarization asymmetry on \(Z^0\) resonance.

\[
A_{\tau-pol} = \frac{\sigma(e^+e^- \to \tau^+\tau^-L) - \sigma(e^+e^- \to \tau^+\tau^-R)}{\sigma(e^+e^- \to \tau^+\tau^-L) - \sigma(e^+e^- \to \tau^+\tau^-R)}.
\]  

(3.28)

On \(Z^0\) resonance in \(SU_2 \times U_1\) (leaving aside the question of hadronization of the \(\tau^-\) decay products and the graphs of Fig. 2) we have

\[
A_{\tau-pol} = A_{LR}
\]  

(3.29)

so that this too can be read off from Table II. The \(\tau^-\) polarization asymmetry would in principle provide a good test of the one-loop radiative corrections and should be a high priority at LEP/SLC.

For completeness, we display \(A_{FB}\) and \(A_{LR}\) as functions of the center-of-mass energies \(\sqrt{s}\) in Figs. 8 and 9. The dashed lines include no radiative corrections, not even the 0.06 QED correction inserted in our Born terms \(s^2_\theta\) in Eq. (1.4). The solid lines include all GSW corrections except those of Fig. 2 for \(m_t = 30\) and \(m_H = 100\). The dotted line in Fig. 9 includes the 0.06 from the "trivial" QED correction in the definition of \(\sin^2 \theta_w\) given in Eq. (1.4) and shows that the bulk of the shift is indeed due to this. Remember though that the observation of this shift would indicated that some sort of unification picture of weak and electromagnetic interactions is correct but it would not indicate which particular gauge group is demanded.

Let us now make use of our complete expression for \(\mathcal{M}\) to analyze Bhabha scattering. In terms of the functions introduced in (2.4), the Bhabha scattering cross-section, including 1-loop corrections, is given by

\[
\frac{d\sigma}{dt}(e^- (L)e^+ \to e^+e^-) = \frac{4\pi\alpha^2}{s} \left\{ |\mathcal{M}_{LL}(-s) + \mathcal{M}_{LL}(-t)|^2 k_{LL}^2 + |\mathcal{M}_{LR}(-s)|^2 k_{LR}^2 + |\mathcal{M}_{LR}(-t)|^2 \right\}
\]  

(3.30)
\[
\frac{d\sigma}{dt}(e^{-}(R)e^{+} \rightarrow e^{+}e^{-}) = \frac{4\pi\alpha^{2}}{s}\left\{|M_{RR}(-s) + M_{RR}(-t)|^{2}k_{RR}^{2} + |M_{RL}(-s)|^{2}k_{RL}^{2} + |M_{RL}(-t)|^{2}\right\}
\]

(3.31)

At the $Z^{0}$ resonance, the cross-section is dominated over the whole range of $t$ by the contributions of the $Z$ in the $s$-channel and the photon in the $t$-channel. The relative size of these two contributions is a strong function of $t$: The photon term dominates near zero angle $\theta$ (the angle between incoming and outgoing electrons) the $Z$ at $90^{\circ}$. The two terms are roughly equal for $\cos \theta \simeq 0.8$. If, then, we consider the polarization asymmetry in Bhabha scattering near $90^{\circ}$, we are measuring exactly the same correction that we have found already in the $\mu$ and $\tau$ polarization or charge asymmetries. We should investigate whether Bhabha scattering can also give new, independent information on weak radiative corrections.

New information about one-loop radiative corrections might come from the first term in (2.4) in the $t$-channel in forward or near forward directions. However, because we used $\alpha(q^{2} = 0)$ as an input parameter, the renormalization will engineer itself so that radiative corrections will disappear at $t = 0$, the far forward direction, in the photon exchange graph. Our only hope, then, to see effects of radiative corrections in Bhabha scattering not already contained in $e^{+}e^{-} \rightarrow \mu^{+}\mu^{-}$ is in the endcap region $\cos \theta \sim 0.8$ when $\sqrt{-t} \sim 30$ GeV. Thus, one might hope to observe new effects in Bhabha scattering in the near-forward direction. GSW radiative corrections to Bhabha scattering have been calculated by many authors. There is one subtlety in the measurement however because Bhabha scattering is used in the forward direction to calibrate the luminosity in the first place. The luminosity (which usually suffers a rather large systematic error $\sim 5\%$ for our purposes) is only necessary for the measurement of an absolute cross section so that we should study a ratio of cross sections in order to see small effects clearly. Let us define the quantity:
\[ X = \left( \frac{\frac{d\sigma}{dt}(e^+e^- \rightarrow e^+e^-)}{\frac{d\sigma}{dt}(e^+e^- \rightarrow \mu^+\mu^-)} - 1 \right)_{s=M_Z^2} \cdot (1 - \cos \theta)^2. \] (3.32)

This and all other four lepton processes have been calculated to one-loop in GSW by Lynn and Stuart excluding the graphs of Fig. 2. Their results are displayed as a function of \( \cos \theta \) in Table IV. The large shift in \( X \) as the top quark mass changes from 30 to 180 GeV is primarily due to the diminishment of the \( Z^0 \) width (\( X \) is proportional to \( \Gamma_Z^2 \)) as the top becomes too heavy to produce on \( Z^0 \) resonance. Radiative corrections to \( X \) in the near-forward and forward direction are small because radiative corrections to the \( Z^0 \) width tend to be small as we shall see below.

Off \( Z^0 \) resonance we may hope to exploit the \( Z - A \) interference terms in the endcap region in Bhabha scattering or by studying \( \mu \) pair production to get information about the new quantity \( \Delta_\rho \). This new information would also be in the \textit{shape} of the cross sections in \( e^+e^- \rightarrow \mu^+\mu^- \) with or without polarization as one scans across the \( Z^0 \) resonance. Let us examine the corrections to the \( Z^0 \) width \( \Gamma_Z \). To relative \( O(\alpha) \) the definition of the width is subtle because it is extracted experimentally by studying the shape of the resonance; we define it by writing the one-loop \( Z^0 \) propagator near the \( Z^0 \) pole as

\[ G_Z \rightarrow \frac{Z_Z}{q^2 + M_Z^2 - i\Gamma_Z M_Z - i\epsilon} \] (3.33)

so that including oblique corrections

\[ \Gamma_Z = \Gamma_Z^0 \cdot \left( 1 - C_\Gamma \Delta_\rho (-M_Z^2) + \Delta_\rho (-M_Z^2) + 0.06 \right). \] (3.34)

Note that, as expected, the \( Z^0 \) width involves the first derivative of the \( Z^0 \) self energy on resonance through the parameter \( \Delta_\rho \). Oblique corrections to the imaginary part of the \( Z^0 \) inverse propagator are due to the shifts in the coupling
constants and the appropriate part of the $Z - A$ mixing\textsuperscript{13}; the same sources of shift as in the polarization asymmetry. Therefore the $Z^0$ width is corrected by the same function $\Delta_\rho(-M_Z^2)$ as is the polarization asymmetry. The constant $C_\Gamma$ is easily calculated from these considerations:

$$C_\Gamma = \frac{\partial}{\partial s_\phi^2} \ln (c_\phi^2 s_\phi^2 \Gamma_\phi^0).$$ (3.35)

In the case in which only light fermions can be created on $Z^0$ resonance,

$$C_\Gamma =$$

$$\sum_{\text{fermions}} \frac{Q(I_L^2 - 2Q \sin^2 \theta_w) \left(1 + \frac{2m_f^2}{M_Z^2}\right) \left(1 - \frac{4m_f^2}{M_Z^2}\right)^{1/2}}{\sum_{\text{fermions}} \left[\left(I_L^2 - Q \sin^2 \theta_w\right)^2 \left(1 + \frac{2m_f^2}{M_Z^2}\right) + \left(I_L^2\right)^2 \left(1 - \frac{4m_f^2}{M_Z^2}\right)\right] \left(1 - \frac{4m_f^2}{M_Z^2}\right)^{1/2}} C_{\text{QCD}}.$$ (3.36)

where $C_{\text{QCD}} = 1$ for leptons and $3 \cdot \left(1 + \frac{\Delta_\rho(-M_Z^2)}{\tau}\right)$ for quarks. For 3 light $m_f^2 \ll M_Z^2$ generations of quarks and leptons, $C_\Gamma \to 1$ as $s_\phi^2 \to 1/4$. Also $\Delta_\rho(-M_Z^2)$ is small in GSW so that including oblique corrections

$$\Gamma_Z \simeq \Gamma_Z^0 \left(1 + \frac{1}{8} \delta A_{\text{oblique}} + 0.06\right).$$ (3.37)

Thus shifts in the $Z^0$ width from radiative corrections tend to be small. The $Z^0$ width has been calculated by Lynn and Stuart including all GSW corrections to one loop, with the proviso that fermion masses are small in 1PI one-loop vertex parts only (helicity conserving vertices). In particular, the top quark mass and associated Higgs' exchange graphs have been neglected within one-loop 1PI fermion self-energies and vertex parts but have been included in vector boson self-energies and in (2.3). This may be a bad idea for heavy $t$ quarks and a more complete calculation should be done. Also, all strong interaction corrections are

\textsuperscript{13} In the case of direct-coupling theories we need to include the imaginary part of the 2-loop $Z^0$ self-energy.
neglected. These results, displayed in Table V, show that $\Gamma_Z$ is a strong function of the precise $Z^0$ mass $\Gamma_Z \sim G_\mu M_Z^2$ and of whether the top quark is light enough to be produced on resonance. All GSW effects in $\Gamma_Z$, especially the creation of $b$ and $t$ quarks (which involves Higgs' exchange in internal loops because for large top mass the Yukawa couplings are large) and toponium (if recent CERN reports on the $t$ quark mass are vindicated) must be clearly understood before this quantity is used to count neutrino species. We note that this requires knowledge of the strong interactions.

Next, let us examine the GSW radiative corrections to the precise $W^\pm$ mass.\textsuperscript{14} This was of course first calculated to one-loop by Sirlin\textsuperscript{314} and later by Consoli \textit{et. al.}.\textsuperscript{14} An independent check was then done by Lynn and Stuart whose results appear in Table VI. We note the strong dependence on $Z$, top and Higgs' masses.

To see the effects of weak GSW corrections clearly we form the quantity

$$\delta M_W = M_W - \epsilon_8 M_Z$$

$$\delta M_W^{GSW} (m_t = 30, m_H = 100) = -\begin{pmatrix} 170 \\ 180 \\ 170 \end{pmatrix} \text{MeV for } M_Z = \begin{pmatrix} 90 \\ 94 \\ 98 \end{pmatrix} \text{GeV} .$$

(3.39)

To see the effects of heavy top and Higgs' masses we display the shifts

$$\delta M_W^{GSW} (m_t = 180, m_H = 100, M_Z = 94) = (-180 + 780) \text{MeV }$$

(3.40)

$$\delta M_W^{GSW} (m_t = 30, m_H = 1000, M_Z = 94) = (-180 - 160) \text{MeV} .$$

(3.41)

\textsuperscript{14} Besides the QED corrections traditional in the definition (1.2) for $G_\mu$, Sirlin's formula for the GSW shift reads in our notation

$$\Delta r - 0.06 = \left[ \Gamma_{\nu^+\nu^-} (0) + \Gamma_{\mu^+\mu^-} (0) + V_{\nu^\tau\mu^\tau} (0) - \frac{1 - 2s_w^2}{s_w^2} \Delta_W (-M_Z^2) \right]_{GSW}$$

+ QED corrections .
A precise determination of the $W^\pm$ mass must be a high priority at LEP2 which will have enough energy to produce $W$'s in pairs via $e^+e^- \rightarrow W^+W^-$.  

In analogy to the $Z^0$ width (3.33) we define the $W^\pm$ width by writing the propagator near the $W^\pm$ pole as

$$G_W \rightarrow \frac{Z_W}{q^2 + M_W^2 - i\Gamma_W M_W - i\epsilon} \quad (3.42)$$

so that including only oblique corrections

$$\Gamma_W = \Gamma_W^0 \left[ 1 + 0.06 + \frac{1}{2} \Delta W(-M_W^2) + \Delta W(0) + q^2 \left. \frac{\partial}{\partial q^2} \Delta W(q^2) \right|_{q^2 = -M_Z^2} \right] \quad (3.43)$$

We do not display the one-loop GSW results for $\Gamma_W$ here\textsuperscript{11,14} but note only that it also is a strong function ($\Gamma_W \sim G_\mu M_W^3$) of $M_Z$.

Finally, we should compare the information from high energy LEP/SLC experiments on radiative corrections to that from low energy data, say, $\nu e$ scattering\textsuperscript{18} from the CHARM II collaboration. It is easy to write down the cross sections—all in the $t$ channel—for the three processes $\nu e \rightarrow \nu e$, $\nu e \rightarrow \nu e$ and $\nu e \rightarrow \nu e$ in terms of the effective matrix elements $M_{PP}$ and $M_{CC}$. Again, because we worry about the neutrino beam luminosity, we will form only ratios of cross sections:

$$R_{\nu\overline{\nu}} = \frac{\sigma(\nu e \rightarrow \nu e)}{\sigma(\nu e \rightarrow \nu e)} = \frac{|M_{\nu e\ell\ell}(-t)|^2 + \frac{1}{3} |M_{\nu e\ell\ell}(-t)|^2}{\frac{1}{3} |M_{\nu e\ell\ell}(-t)|^2 + |M_{\nu e\ell\ell}(-t)|^2} \quad (3.44)$$

$$R_{NC,CC} = \frac{\sigma(\nu e \rightarrow \nu e)}{\sigma(\nu e \rightarrow \nu e)} = \frac{|M_{\nu e\ell\ell}(-t)|^2 + \frac{1}{3} |M_{\nu e\ell\ell}(-t)|^2}{|M_{CC}(-t)|^2 \cdot \left(1 - \frac{m_e^2}{2m_e^2} \right)^2} \quad (3.45)$$

In (3.44) and (3.45) we have assumed $t$ small enough so as to neglect the $\cos \theta$ dependence in the $Z$, $W$ propagators. We have also neglected a slight $\cos \theta$ dependence in the box and vertex diagrams and have written $R_{NC,CC}$ in the target electron rest frame. The largest part of the effects of radiative corrections can be read off from (2.4) by considering only the oblique corrections.
\[ R_{\nu\bar{\nu}} \simeq \left( 1 - \frac{\bar{\nu}_\theta}{1 + \bar{\nu}_\theta} \right) \left/ \left( 1 + \frac{\bar{\nu}_\theta}{1 + \bar{\nu}_\theta} \right) \right. \] (3.46)

\[ R_{NC;CC} \simeq \frac{1 - \bar{\nu}_\theta + \bar{\nu}_\theta^2}{12} \left( 1 - \Delta_p(-t) \right)^{-2} \left( 1 - \frac{m_{\mu}^2}{2m_e E_\nu} \right)^{-2} \] (3.47)

\[ \bar{\nu}_\theta = 4\left( s_p^2 + \Delta_p(-t) \right) - 1 \] (3.48)

so that \( R_{\nu\bar{\nu}}(t = 0) \) depends on the combination \( \Delta_p(0) \) while \( R_{NC;CC}(t = 0) \) depends on \( \Delta_p(0) \) as well as \( \Delta_p(t = 0) \). This confirms that these quantities measure radiative corrections which are different than those measured in experiments on \( Z^0 \) and \( W^\pm \) resonance which access primarily \( \Delta_p(-M_Z^2) \) and \( \Delta_W(-M_W^2) \). A complete calculation of all GSW radiative corrections (except the QED diagrams of Fig. 2) was done by Lynn and Stuart with no assumptions about \( \cos \theta \) dependence. Their results are given in Tables VII and VIII. Note again the dependence on the precise value of \( M_Z, m_t, m_H \). Thus \( R_{\nu\bar{\nu}} \) and \( R_{NC;CC} \) can be used to eliminate a large fraction of the parameter space of the Tables. Then the various asymmetries in \( e^+e^- \rightarrow \mu^+\mu^- \), \( \tau^+\tau^- \) and the \( W^\pm \) mass can be used to give a clear test of GSW at the one-loop level.

We have now examined the results of one-loop radiative corrections due to the GSW theory. In the next section we will examine the response of various experimentally measurable quantities to corrections due to representations of new particles in \( SU_2 \times U_1 \) beyond GSW.

4. New Physics

In this section we examine the corrections to the various asymmetries in \( e^+e^- \rightarrow \mu^+\mu^- \) and \( \tau^+\tau^- \) on \( Z^0 \) resonance and to the \( W^\pm \) mass from new physics; that physics designed to "explain" the constant parameters in GSW. We will examine extra generations of quarks and leptons, SUSY and technicolor (not extended) contributions to one-loop. All of the results of this section are taken from the work of Lynn\(^{15}\) and Lynn and Peskin.\(^6\)
For the asymmetries, we have shown that this can be reduced to the calculation of only $\delta A_{LR}$ on $Z^0$ resonance with the others related by

$$\delta A_{FB}^\mu \simeq \frac{-3v_t}{1 + v_t^2} \delta A_{LR}$$

$$\simeq \begin{bmatrix} 0.12 \\ 0.44 \\ 0.68 \end{bmatrix} \delta A_{LR} \quad \text{for } M_Z = \begin{bmatrix} 90 \\ 94 \\ 98 \end{bmatrix} \text{ GeV} \quad (4.1)$$

$$\delta A_\perp \simeq \frac{-2v_t}{1 - v_t^2} \delta A_{LR}$$

$$\simeq \begin{bmatrix} 0.08 \\ 0.31 \\ 0.45 \end{bmatrix} \delta A_{LR} \quad \text{for } M_Z = \begin{bmatrix} 90 \\ 94 \\ 98 \end{bmatrix} \text{ GeV} \quad (4.2)$$

$$\delta A_{rad} = \delta A_{LR} \quad (4.3)$$

Note the numerical suppression of the longitudinally unpolarized asymmetries relative to those with longitudinal polarization, especially for low $Z^0$ masses. If $M_Z$ turned out to be small ($\sim 90 \text{ GeV}$), we would have little hope of seeing the effects of one-loop radiative corrections in $Z^0$ resonance lepton asymmetries in which the longitudinal polarization of an initial or final state charged lepton were unobserved. Thus the capacity for observation of longitudinal polarization is crucial for the observations of small effects on $Z^0$ resonance which could betray the existence of new particles.

The astute reader will notice that these relations are not quite satisfied in the Tables I and II. This is due to the inclusion of certain two-loop effects in the numerical evaluations of $A_{LR}$ and $A_{FB}$. We thus estimate that two-loop effects contribute to $A_{FB}$ and $A_{LR}$ at the level of about $\pm 0.001$ and $\pm 0.002$ respectively. These should be understood (with a lot of further work) before comparison with LEP/SLC data is to be made. Further, we note that the effects of hard and soft bremmstrahlung may have to be understood to $O(\alpha^4)$ in the
cross sections as well although most of these effects are expected to cancel in $A_{LR}$ and $A_{\perp}$ (but not in $A_{FB}$) as shown by Bohm and Hollik.\(^9\)

If the standard model is modified in the region of a few hundred GeV, we must add to the contributions of GSW the contributions from new physics. To facilitate this analysis, let us divide models of this new physics into two classes: those with direct and oblique couplings. The second class contains those models with no vertices linking the new particles present in the model with the three light families of quarks and leptons. The new particles then influence leptonic processes only indirectly, by means of their influence on the electroweak gauge bosons. Models with additional heavy quarks and leptons, and models, such as technicolor, which mainly modify the Higgs sector, have only these oblique couplings. Supersymmetric models, which postulate bosonic partners of the light fermions, and models with right-handed leptonic currents are directly coupled. However, even in these models, the partners of any quarks and new heavy leptons have only oblique couplings. As we have seen in the previous two sections, oblique theories are much easier to analyze, because they contribute to the basic process $e^+e^- \rightarrow \mu^+\mu^-$ only through vacuum polarization diagrams. Thus we have only to calculate

$$\delta A_{LR}^{\text{oblique}} = -64 \frac{\cos^2 \theta_W \sin^4 \theta_W}{(1 + v^2)^2} \Re \left[ \frac{\cos 2\theta_W}{\cos \theta_W \sin \theta_W} \frac{\Pi_{Z\Lambda}(-M_Z^2)}{M_Z^2} - \Pi_{AA'}(0) \right]$$

(4.4)

for the various new representations of particles. We will now present a number of such calculations. We will find that new particles in the few hundred GeV mass range give contributions to $\delta A_{LR}$ which are typically of order 0.01. This is a small correction on an absolute scale but, as we have already noted, quite sizeable compared with the expected accuracy\(^1\) of experiments at the $Z^0$. Let us now calculate these contributions for heavy fermions and bosons with definite $SU(2) \times U(1)$ quantum numbers, and also for the pseudo-Goldstone bosons of technicolor models.
Let us first compute the shift $\delta A_{LR}$ associated with a pair of heavy fermions which couple to the weak interactions as a conventional left-handed doublet. Let the masses of the two members of a weak doublet to be $m_T, m_B$. If the fermions have color, we must include a factor $N_c$ for the color multiplicity.

Some typical values of the contribution to the longitudinal polarization asymmetry from quark and lepton doublets are shown in Fig. 10 and 11. Note that, for a doublet with large isospin splitting, $\delta A_{LR}$ increase proportional to $(m_T^2 - m_B^2)/M_W^2$. For $m_T >> M_Z, m_B$, we find:

$$\delta A_{LR} \rightarrow \dfrac{4 \sin^2 \theta_W}{(1 + v_d^2)^2} \dfrac{\alpha}{\pi} N_c \left[ \dfrac{m_T^2}{M_W^2} \right]. \quad (4.5)$$

A similar effect has been noted some time ago in the analogous calculation for the $\rho$ parameter and has been used there to put a bound of a few hundred GeV on the isospin splittings of quark doublets. Experiments on the polarization asymmetry will clearly place a much stronger bound. A more interesting aspect of our result is that, even in the case of exact isospin degeneracy, a single additional generation of fermions produces an observable effect which is almost independent of mass. For $m_T = m_B >> M_Z$, one finds that certain of the $\Delta_i$ tend to zero:

$$\Delta_\alpha(q^2) \rightarrow 0 \quad (4.6)$$

$$\Delta_\rho(0) \rightarrow 0 \quad (4.7)$$

so that there is decoupling of a heavy degenerate doublet for the $\rho$ parameter or in the renormalization of $\alpha$. However, in contrast to the $\rho$ parameter, we find

$$\delta A_{LR} \rightarrow -\dfrac{8 \sin^2 \theta_W}{(1 + v_d^2)^2} \dfrac{\alpha}{3\pi} N_c$$

$$\approx \begin{cases} -0.0040 & \text{for one quark doublet} \\ -0.0013 & \text{for one lepton doublet} \end{cases} \quad (4.8)$$
Numerically, this result is already correct to better than 5% for $m_T = m_B > 100$ GeV. It is a bit surprising at first sight that isospin-degenerate heavy fermions do not decouple. It has been known for some time, though, that heavy fermions need not decouple from low-energy processes in models with axial-current couplings; the contributions of heavy fermions to the axion mass and to the mass of the $W^\pm$ provide other examples of this phenomenon. This result that $\delta A_{LR}$ is independent of the fermion mass presumably breaks down when the fermions become strongly coupled to the Higgs sector; this requires $m_T \approx 500$ GeV. Nevertheless, this result, depicted graphically in Fig. 12 shows that, in principle, sufficiently accurate determinations of the longitudinal polarization asymmetries, and to a lesser extent the charge and transverse polarization asymmetries, are capable of "counting" heavy generations of quarks and leptons with non-zero axial vector couplings to gauge particles.

The corresponding calculations for a weak doublet of scalars are also quite straightforward. As examples, we display in Figs. 13 and 14 the oblique contributions to $\delta A_{LR}$ from the scalars which are the supersymmetric partners of, respectively, a quark and a lepton doublet. For simplicity we examine the case in which "left" and "right" squarks (and sleptons) $\tilde{q}_L$ and $\tilde{q}_R$, the SUSY partners of left and right handed quarks $q_L$ and $q_R$ (and leptons), are good mass eigenstates. The more general case with left-right squark mixing has also been examined and has been included in the supergravity-motivated SUSY models discussed later. Note that the divergence which we saw above in the case of large isospin splittings for fermions is still present for bosons; for $m_T \gg m_B$, $M_Z$ Eq. (4.5) is replaced by:

$$\delta A_{LR} \rightarrow \frac{4 \sin^2 \theta_W}{\alpha} \frac{N_c}{\pi} \left[ \frac{m_T^2}{M_Z^2} \right].$$

(4.9)

However, the contribution of the bosons to $\delta A_{LR}$ does tend to zero in the limit of large but isospin-degenerate boson masses. In this case, the Appelquist-Carazzone decoupling theorem does apply.
These considerations apply only to those bosons which form multiplets under the weak-interaction symmetry. This will not generally be true for bosons which arise from the Higgs sector. In that case, the new bosons will have definite electric charge and perhaps also definite quantum numbers under a custodial isospin symmetry, but they should not have definite $SU(2) \times U(1)$ quantum numbers if they are formed in the symmetry-breaking process. In one class of models, however, a fairly general analysis can still be made: In technicolor models, in which new bosons appear as pseudo-Goldstone bosons associated with the symmetry-breaking, one can use an effective-Lagrangian description of the interactions of these Goldstone bosons to compute their influence on weak-interaction processes.\textsuperscript{23} The various vacuum polarization amplitudes depend on the value of an explicit cutoff, $\Lambda_T$, which should be taken to be the mass scale of the new hadrons of the technicolor theory. Since the Goldstone bosons are described only by an effective Lagrangian, valid for energies well below $\Lambda_T$, it is unreasonable to expect that their contribution will be cutoff-independent and unambiguous. The effective theory of technicolor bosons is not then renormalizable from the point of view of $SU_2 \times U_1$ (although of course the gauge theory of technifermions is) and this will induce large radiative corrections $\sim \ln(\Lambda_T^2/M_Z^2)$. Further, there are usually a huge number of such Goldstone bosons and their mass matrix breaks global $SU_2$ quite badly since some are very heavy while others are constrained to be the longitudinal components of the $W^\pm$ and $Z^0$. Thus, technicolor (and most composite) models give very large radiative corrections to low energy processes and will affect $A_{LR}$ and $M_W$ dramatically.

The contribution to $\delta A_{LR}$ from pseudo-Goldstone bosons is easily assembled from (4.4). In Fig. 15, we display this contribution for two typical multiplets, a set of states with the quantum numbers of $LQ$ (and their antiparticles) and a set of color-octet states with the quantum numbers of $\bar{Q}Q$, where, in each case, \( L \) and \( Q \) represent, respectively, a lepton and a quark doublet. The particles within each multiplet are taken to have almost exactly equal masses; that is the
expected situation. We note that the corrections are huge for technicolor and should be observable at SLC/LEP.

We next move on to direct corrections from SUSY. As shown in the previous section, these also can be interpreted as effective additions to $\sin^2 \theta_w$ on $Z^0$ resonance and so Eqs. (4.1) and (4.2) still apply for $A_{FB}$ and $A_{\perp}$ but we must now use (3.5) (dropping of course the 0.06 in $\Delta_p$ since this was included in the GSW prediction) for $\delta A_{LR}$. We show in Figs. 16 and 17 the shifts $\delta A_{LR}$ due to a generic class of SUSY theories (including both oblique and direct-corrections) in which the SUSY breaking scale is motivated by supergravity (SUGRA). These models give mass spectra which depend on the gravitino mass $m_{3/2}$. In particular, as $m_{3/2}$ gets large the squarks in an isospin doublet become degenerate and heavy and thus decouple (as in Figs. 13 and 14) from the rest of the model at the one-loop level. In some models (the renormalization group or "no-scale" models), a very large top quark mass is used to break the internal symmetry $SU_2 \times U_1 \to U_1^{\text{QED}}$ after inclusion of radiative corrections and this effect can be seen in the figures. The dependence on the specific Majorana SUSY-breaking "gaugino" masses $M_2$ (for $SU_2$) and $M_1$ (for $U_1$) is slight.

We now go on to discuss radiative corrections in Bhabha scattering. Unless the new particles are light enough to be produced on $Z^0$ resonance there will be no interference between the $Z^0$ and photon exchange diagrams in $\mu$-pair production or Bhabha scattering. Then from (2.4) we can see that on resonance the only new information in Bhabha scattering not suppressed kinematically is in the quantity $\Delta_\alpha$ in the $t$-channel photon exchange for $\cos \theta \gg 0$ with $\theta$ the angle between incoming and outgoing electrons. Let us study the response of the ratio $X$ defined in (3.32) to new physics at one loop. Neglecting direct corrections, $s$-channel photon exchange, $t$-channel $Z^0$ exchange and interference terms between $s$-channel $Z^0$ and $t$-channel photon exchange we have on $Z^0$ resonance
\[
X = (8(1 + \cos \theta)^2 + 32) \left[ \frac{\Gamma_2^0 s_\theta^2 c_\theta^2 (1 - C_\Gamma \Delta_p (-M_Z^2))}{M_Z (1 - \Delta_\alpha (-t))} \right]^2 \\
\times \left[ \left( s_\theta^2 + \Delta_p (-M_Z^2) \right)^4 + \left( -\frac{1}{2} + s_\theta^2 + \Delta_p (-M_Z^2) \right)^4 \left( 1 + \cos \theta \right)^2 \right]
\]

\[\text{(4.10)}\]

Note however that \(\Delta_\alpha (\cos \theta = 1) = 0\) so that forward Bhabha scattering again contains information only about \(\Delta_p (-M_Z^2)\). Our only hope then is in the endcap region \(\cos \theta \sim 0.7\) to 0.9 where \(\sqrt{-t} \sim 40\) to 20 GeV. We would comment though that for many oblique corrections \(\Delta_\alpha\) is disappointingly small in this region. For example in theories with extra quarks and leptons whose mass matrices break badly a global \(SU(2)\) isospin symmetry, the \(\mu\) pair longitudinal polarization asymmetry blows up quadratically with large splitting within the doublet. In \(\Delta_\alpha\) evaluated in the endcap region such a doublet would decouple completely and there would be no contribution if the lightest member had mass much larger than 40 GeV.

We now discuss the shifts in the precise value of the \(W^\pm\) mass due to new physics,\(^6,14,15\) we have only to examine the quantity

\[
\delta M_W = \frac{c_\theta M_Z}{2} \left( \frac{-s_\theta^2}{1 - 2s_\theta^2} \right) \text{Re} \left\{ \frac{\Pi_{WW} (0)}{M_W^2} - \Pi'_{AA} (0) \right\} \\
- \frac{c_\theta^2}{s_\theta^2} \frac{\Pi_{ZZ} (-M_Z^2)}{M_Z^2} + \frac{1 - s_\theta^2}{s_\theta^2} \frac{\Pi_{WW} (-M_Z^2)}{M_W^2} \\
+ \Gamma_+^{\mu W} (0) + \Gamma_+^{\mu W} (0) + V_{\mu e}^{\nu \mu e \nu} (0)
\]

\[\text{(4.11)}\]

for the various new representations of particles where \(\Gamma_+^{\mu W}, \Gamma_+^{\mu W}\) and \(V_{\mu e}^{\nu \mu e \nu}\) came from muon decay (see Figs. 4,5,6,7 and Eqs. (2.17) and (2.24)). Some interesting results are
(1) for an extra doublet with large isospin breaking in the $m_T \gg m_B$ mass matrix

\[
\frac{\delta M_W}{M_W} \to \frac{\alpha N_c}{\pi} \frac{1}{32} \frac{m_T^2}{s^2_b(1-2s^2_b)} \frac{m^2_T}{M_B^2}
\]  

(4.12)

(2) for an extra quark or lepton doublet with large degenerate mass

\[
\delta M_W \to -\frac{\alpha N_c}{24\pi(1-2s^2_b)} \cdot M_W \approx -\left\{ \begin{array}{ll} -14 \text{ MeV} & \text{for leptons} \\ -42 \text{ MeV} & \text{for quarks} \end{array} \right. \]  

(4.13)

The shifts $\delta M_W$ are plotted against those of $\delta A_{LR}$ for an extra quark and lepton doublet in Figs. 18 and 19.

(3) the shift due to a squark or slepton doublet with large isospin splitting in the mass matrix is $m_T \gg m_B$

\[
\frac{\delta M_W}{M_W} \to \frac{\alpha N_c}{\pi} \frac{1}{32} \frac{m^2_T}{s^2_b(1-2s^2_b)} \frac{m^2_T}{M_B^2}
\]  

(4.14)

(4) the shift due to a heavy degenerate squark or slepton doublet is

\[
\delta M_W \to 0
\]  

(4.15)

(5) Technicolor models can change the value of the $W^\pm$ mass considerably. The shift $\delta M_W$ is plotted against $\delta A_{LR}$ in Fig. 20.

(6) We give the complete shifts $\delta M_W$ from two classes or SUGRA models including all oblique and direct couplings; boxes, vertices and fermion and vector boson self-energies in Figs. 21 and 22. These are from the work of Lynn\(^{15}\) and Lynn and Peskin.\(^{6}\)

Finally, we would like to add a comment on anomalous vacuum expectation values (v.e.v.'s). All of the previous work in this paper was based on the assumption that the symmetry breaking $SU_l \times U_1 \to U_1^{\text{QED}}$ was done by Higgs' doublets. Let us imagine that some representation of scalars which is not a doublet (say,
a Higgs' triplet) acquired a small v.e.v.: There would be two effects of such a representation on various experimentally measurable quantities;

a) the new representation of scalars would enter into one-loop diagrams such as in Figs. 3,4,5,6,7 and contribute to radiative corrections as previously discussed.

b) The Higgs' v.e.v.'s would affect tree level formulae. We will now discuss this latter contribution. The $M_W^2 - M_Z^2$ mass relation would be changed to

$$\frac{M_W^2}{M_Z^2} = \rho \frac{g^2}{g^2 + g'^2}$$

$$\rho = 1 + \delta \rho$$

$$\delta \rho = 2 \frac{\sum_{\text{scalars}} \lambda_i^2 (r_i^2 + I_i - \frac{3}{4} Y_i^2)}{\sum_{\text{scalars}} \lambda_i^2 Y_i^2}$$

where $\lambda_i, I_i, Y_i$ are the v.e.v., weak isospin and hypercharge of the $i^{th}$ scalar representation; for neutral particles $I_{3i} = -\frac{Y_i}{2}$. If we assume $\delta \rho \ll 1$ and throw away terms $O(\delta \rho)^2$ and $O(\alpha \delta \rho)$ in keeping with the experimental value $\rho \approx 1$, the formulae (2.4) and (2.20) will still be correct provided we make the displacements.\(^{25}\)

$$\Delta_\rho \rightarrow \Delta_\rho + \delta \rho$$

$$\Delta_\rho \rightarrow \Delta_\rho - \frac{s_\theta^2 c_\theta^2}{1 - 2 s_\theta^2} \delta \rho$$

$$\Delta_\alpha \rightarrow \Delta_\alpha$$

$$\Delta_W \rightarrow \Delta_W + \frac{c_\theta^2}{1 - 2 s_\theta^2} \delta \rho.$$  

This would cause the shifts in the physical quantities listed in Table IX. These have been evaluated for $M_Z = 94$ GeV and $\delta \rho = 0.01$ in order to see the generic magnitude of the effect. Note that, contrary to popular belief, $R_{NC;CC}$ is not the best place to look for the effects of anomalous v.e.v.'s; $M_W$ and $A_{LR}$ are.
5. Conclusions

Radiative corrections can affect the values of the various asymmetries in $e^+e^- \rightarrow \mu^+\mu^-$, $\tau^+\tau^-$ as well as the $W^\pm$ mass in measurable ways. Our most important conclusion is that all radiative corrections to four-lepton processes can be reduced to a few functions. The fact that precision probes of the weak interactions involve only a few specific quantities is a great aid in evaluating and comparing experiments on weak radiative corrections. Especially exciting is the possibility of seeing the effects of new particles from beyond the standard model, even if they are too heavy to be produced directly at LEP/SLC, via radiative corrections. We present a summary list of generic values of shifts to various physical quantities due to radiative corrections from various sources in Table X. Also present there is an estimate of the theoretical uncertainty due to strong interactions and light hadrons from the QED vacuum polarization diagrams of Fig. 1 entering as in Eq. (1.7). Lepton asymmetries on $Z^0$ resonance are more sensitive to the effects of radiative corrections with the observation of longitudinal polarization than without.
REFERENCES


4. W. J. Marciano in Cornell Workshop, Ref. 1; ibid, Ref. 8.


20. M. E. Peskin (unpublished)
TABLE CAPTIONS

I. Initial state longitudinal polarization asymmetry $A_{LR}$ on $Z^0$ resonance in $e^+e^- \rightarrow \mu^+\mu^-$ for various $M_Z$, $m_H$, $m_t$ to one-loop in GSW. All masses are in GeV.

II. Forward-backward or charge asymmetry $A_{FB}$ on $Z^0$ resonance in $e^+e^- \rightarrow \mu^+\mu^-$ for various $M_Z$, $m_H$, $m_t$ to one-loop in GSW. All masses are in GeV.

III. Transverse polarization asymmetry $A_{\perp}$ on $Z^0$ resonance for $e^+e^- \rightarrow \mu^+\mu^-$ for various $M_Z$, $m_H$, $m_t$ to one-loop in GSW. All masses are in GeV.

IV. The ratio of Bhabha scattering to $\mu$ pair production on $Z^0$ resonance

$$X = \left[ \frac{d\sigma}{dt} (e^+e^- \rightarrow e^+e^-) \right]_{t=M_Z^2} \left[ \frac{d\sigma}{dt} (e^+e^- \rightarrow \mu^+\mu^-) \right]^{-1} (1 - \cos \theta)^2$$

for various $\cos \theta$, $M_Z$, $m_H$, $m_t$ to one-loop in GSW. All masses are in GeV.

V. The $Z^0$ decay width $\Gamma_Z$ to one-loop in GSW for various $M_Z$, $m_t$, $m_H$. Fermion masses have been neglected in 1PI vertex and fermion self energy parts but included in 1PI vector boson self-energies. Strong interactions have been neglected. All masses are in GeV.

VI. The $W^\pm$ mass $M_W$ to one-loop in GSW for various $M_Z$, $m_t$, $m_H$. All masses are in GeV.

VII. $R_{\nu\nu}$ ratio of neutral current muon neutrino to muon antineutrino-electron scattering to leptons for incoming neutrino energy $E_\nu = 70$ in electron rest frame for various $M_Z$, $m_H$, $m_t$. All masses are in GeV.

VIII. $R_{NC;CC}$ ratio of neutral to charged current muon neutrino electron scattering to leptons for incoming neutrino energy $E_\nu = 70$ in electron rest frame for various $M_Z$, $m_H$, $m_t$. All masses are in GeV.

IX. Shifts of various quantities due to v.e.v.'s of scalars not in doublet representations.

X. Responses at one-loop of various asymmetries on $Z^0$ resonance and the $W^\pm$ mass to new one-loop physics. Numbers are generic, calculated using $M_Z = 94$ GeV.
### Table I

Initial state longitudinal polarization asymmetry $A_{LR}$ on $Z^0$ resonance in $e^+e^- \to \mu^+\mu^-$ for various $M_Z$, $m_H$, $m_t$ to one-loop in GSW. All masses are in GeV.

$$A_{LR}(q^2 = -M_Z^2)$$

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Table II

Forward-backward or charge asymmetry $A_{FB}$ on $Z^0$ resonance in $e^+e^- \rightarrow \mu^+\mu^-$ for various $M_Z$, $m_H$, $m_t$ to one-loop in GSW. All masses are in GeV.

$$A_{FB}(q^2 = -M_Z^2)$$

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</table>

| $m_t = 60$ |          |             |              |
| 90    | .0044      | .0037       | .0025        |
| 92    | .0256      | .0237       | .0208        |
| 94    | .0599      | .0572       | .0533        |
| 96    | .1014      | .0982       | .0936        |
| 98    | .1460      | .1425       | .1375        |

| $m_t = 90$ |          |             |              |
| 90    | .0047      | .0041       | .0033        |
| 92    | .0269      | .0251       | .0226        |
| 94    | .0619      | .0593       | .0558        |
| 96    | .1039      | .1008       | .0965        |
| 98    | .1487      | .1454       | .1407        |

| $m_t = 130$ |          |             |              |
| 90    | .0046      | .0040       | .0036        |
| 92    | .0282      | .0265       | .0245        |
| 94    | .0643      | .0618       | .0587        |
| 96    | .1069      | .1039       | .1001        |
| 98    | .1522      | .1489       | .1447        |

| $m_t = 180$ |          |             |              |
| 90    | .0032      | .0028       | .0030        |
| 92    | .0292      | .0276       | .0263        |
| 94    | .0668      | .0645       | .0620        |
| 96    | .1105      | .1077       | .1045        |
| 98    | .1564      | .1533       | .1498        |
Table III

Transverse polarization asymmetry $A_{\perp}$ on $Z^0$ resonance for $e^+e^- \rightarrow \mu^+\mu^-$ for various $M_Z$, $m_H$, $m_t$ to one-loop in GSW. All masses are in GeV.

\[ A_{\perp} (q^2 = -M_Z^2) \]

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Table IV

The ratio of Bhabha scattering to $\mu$ pair production on $Z^0$ resonance for various $\cos \theta$, $M_Z$, $m_t$, $m_H$ to one loop in GSW. All masses are in GeV. $X = \left[ \frac{\sigma(e^+e^- \rightarrow e^+e^-)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \right]_{s=M_Z^2} (1-\cos \theta)^2$

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| $m_t = 1000$ |
|-------------------------|-------------------------|
| $\cos \theta$ | $M_Z = 90$ | $M_Z = 94$ | $M_Z = 98$ |
| 1.00 | 0.0516 | 0.0507 | 0.0464 |
| 0.99 | 0.0537 | 0.0528 | 0.0484 |
| 0.90 | 0.0549 | 0.0541 | 0.0498 |
| 0.80 | 0.0552 | 0.0546 | 0.0505 |
| 0.70 | 0.0552 | 0.0549 | 0.0510 |
| 0.60 | 0.0550 | 0.0550 | 0.0515 |
| 0.50 | 0.0546 | 0.0548 | 0.0518 |
| 0.40 | 0.0538 | 0.0544 | 0.0520 |
| 0.30 | 0.0526 | 0.0537 | 0.0520 |
| 0.20 | 0.0510 | 0.0526 | 0.0517 |
| 0.10 | 0.0490 | 0.0510 | 0.0511 |
| 0.0 | 0.0466 | 0.0490 | 0.0501 |
| -0.10 | 0.0439 | 0.0465 | 0.0490 |
| -0.20 | 0.0409 | 0.0438 | 0.0463 |
| -0.30 | 0.0379 | 0.0409 | 0.0441 |
| -0.40 | 0.0349 | 0.0378 | 0.0414 |
| -0.50 | 0.0319 | 0.0348 | 0.0384 |
| -0.60 | 0.0291 | 0.0318 | 0.0354 |
| -0.70 | 0.0265 | 0.0289 | 0.0324 |
| -0.80 | 0.0240 | 0.0262 | 0.0295 |
| -0.90 | 0.0217 | 0.0237 | 0.0267 |
| -0.99 | 0.0197 | 0.0215 | 0.0242 |
| -1.00 | 0.0195 | 0.0213 | 0.0240 |
Table V

The $Z^0$ decay width $\Gamma_Z$ to one-loop in GSW for various $M_Z$, $m_t$, $m_H$. Fermion masses have been neglected in 1PI vertex and fermion self energy parts but included in 1PI vector boson self-energies. Strong interactions have been neglected. All masses are in GeV.

$\Gamma_Z$

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### Table VI

The $W^\pm$ mass $M_W$ to one-loop in GSW for various $M_Z, m_t, m_H$. All masses are in GeV.

$M_W$

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</tr>
<tr>
<td>92</td>
<td>80.86</td>
<td>80.78</td>
<td>80.62</td>
</tr>
<tr>
<td>$m_t = 60$</td>
<td>94</td>
<td>83.30</td>
<td>83.22</td>
</tr>
<tr>
<td>96</td>
<td>85.69</td>
<td>85.62</td>
<td>85.46</td>
</tr>
<tr>
<td>98</td>
<td>88.05</td>
<td>87.98</td>
<td>87.82</td>
</tr>
<tr>
<td>90</td>
<td>78.55</td>
<td>78.47</td>
<td>78.30</td>
</tr>
<tr>
<td>92</td>
<td>81.06</td>
<td>80.98</td>
<td>80.81</td>
</tr>
<tr>
<td>$m_t = 90$</td>
<td>94</td>
<td>83.50</td>
<td>83.43</td>
</tr>
<tr>
<td>96</td>
<td>85.90</td>
<td>85.82</td>
<td>85.67</td>
</tr>
<tr>
<td>98</td>
<td>88.26</td>
<td>88.18</td>
<td>88.03</td>
</tr>
<tr>
<td>90</td>
<td>78.79</td>
<td>78.70</td>
<td>78.53</td>
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<td>92</td>
<td>81.30</td>
<td>81.22</td>
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</tr>
<tr>
<td>$m_t = 130$</td>
<td>94</td>
<td>83.75</td>
<td>83.67</td>
</tr>
<tr>
<td>96</td>
<td>86.16</td>
<td>86.08</td>
<td>85.92</td>
</tr>
<tr>
<td>98</td>
<td>88.53</td>
<td>88.45</td>
<td>88.29</td>
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<tr>
<td>90</td>
<td>79.14</td>
<td>79.06</td>
<td>78.88</td>
</tr>
<tr>
<td>92</td>
<td>81.66</td>
<td>81.57</td>
<td>81.40</td>
</tr>
<tr>
<td>$m_t = 180$</td>
<td>94</td>
<td>84.12</td>
<td>84.04</td>
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<td>96</td>
<td>86.54</td>
<td>86.46</td>
<td>86.29</td>
</tr>
<tr>
<td>98</td>
<td>88.92</td>
<td>88.84</td>
<td>88.67</td>
</tr>
</tbody>
</table>
Table VII

$R_{\nu \bar{\nu}}$ ratio of neutral current muon neutrino to muon antineutrino-electron scattering to leptons for incoming neutrino energy $E_\nu = 70$ in electron rest frame to one-loop in GSW for various $M_Z, m_H, m_t$. All masses are in GeV.

$$R_{\nu \bar{\nu}} = \frac{\sigma(\nu\mu e \to \nu\mu e)}{\sigma(\bar{\nu}\mu e \to \bar{\nu}\mu e)}$$

<table>
<thead>
<tr>
<th>$M_Z$</th>
<th>$m_H = 100$</th>
<th>$m_H = 1000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_t = 30$</td>
<td>90</td>
<td>1.0325</td>
</tr>
<tr>
<td></td>
<td>94</td>
<td>1.2862</td>
</tr>
<tr>
<td></td>
<td>98</td>
<td>1.5224</td>
</tr>
<tr>
<td>$m_t = 180$</td>
<td>90</td>
<td>1.1598</td>
</tr>
<tr>
<td></td>
<td>94</td>
<td>1.4352</td>
</tr>
<tr>
<td></td>
<td>98</td>
<td>1.6841</td>
</tr>
</tbody>
</table>

Table VIII

$R_{NC,CC}$ ratio of neutral to charged current muon neutrino electron scattering to leptons for incoming neutrino energy $E_\nu = 70$ in electron rest frame to one-loop in GSW for various $M_Z, m_H, m_t$. All masses are in GeV.

$$R_{NC,CC} = \frac{\sigma(\nu\mu e \to \nu\mu e)}{\sigma(\nu\mu e \to \nu_e\mu)}$$

<table>
<thead>
<tr>
<th>$M_Z$</th>
<th>$m_H = 100$</th>
<th>$m_H = 1000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_t = 30$</td>
<td>90</td>
<td>0.1158</td>
</tr>
<tr>
<td></td>
<td>94</td>
<td>0.1295</td>
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<td></td>
<td>98</td>
<td>0.1424</td>
</tr>
<tr>
<td>$m_t = 180$</td>
<td>90</td>
<td>0.1204</td>
</tr>
<tr>
<td></td>
<td>94</td>
<td>0.1344</td>
</tr>
<tr>
<td></td>
<td>98</td>
<td>0.1471</td>
</tr>
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Table IX
Shifts in various quantities due to v.e.v.'s of scalars not in doublet representations

<table>
<thead>
<tr>
<th>Shift</th>
<th>Formula for Shift</th>
<th>Shift Evaluated</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta A_{LR} = \delta A_{rpol} )</td>
<td>( \frac{64s^2_{\mu}c^2_{\mu}}{(1 + v^2_{\rho})^2} \delta \rho )</td>
<td>.021</td>
<td>initial or final state longitudinal polarisation asymmetry ( e^+e^- \rightarrow \mu^+\mu^- ), ( \tau^+\tau^- ) on Z(^0) resonance</td>
</tr>
<tr>
<td>( \delta A_{FB} )</td>
<td>( \frac{192s_{\mu}s_{\rho}c^2_{\rho}}{(1 + v^2_{\rho})^3} \delta \rho )</td>
<td>.0094</td>
<td>charge asymmetry ( e^+e^- \rightarrow \mu^+\mu^- ) on Z(^0) resonance</td>
</tr>
<tr>
<td>( \delta A_{\perp} )</td>
<td>( \frac{-16s_{\mu}s_{\rho}c^2_{\rho}}{(1 + v^2_{\rho})^3(1 - 2s^2_{\rho})} \delta \rho )</td>
<td>.0066</td>
<td>transverse polarisation asymmetry ( e^+e^- \rightarrow \mu^+\mu^- ) on Z(^0) resonance</td>
</tr>
<tr>
<td>( \delta M_W )</td>
<td>( \frac{c^2_{\rho}}{2(1 - 2s^2_{\rho})} M_W \delta \rho )</td>
<td>570 MeV</td>
<td>( W^\pm ) mass</td>
</tr>
<tr>
<td>( \delta X(\cos \theta = 1)/X(\cos \theta = 1) )</td>
<td>( \frac{C_F}{2} + \frac{(s^2_{\rho})^3 + (-\frac{1}{2} + s^2_{\rho})^3}{(s^2_{\rho})^3 + (-\frac{1}{2} + s^2_{\rho})^3} \frac{4s^2_{\mu}c^2_{\mu}}{1 - 2s^2_{\rho}} \delta \rho )</td>
<td>-.013</td>
<td>ratio Bhabha to ( e^+e^- \rightarrow \mu^+\mu^- ) on Z(^0) resonance Eq. (4.10) at ( \cos \theta = 1 )</td>
</tr>
<tr>
<td>( \delta \Gamma_Z )</td>
<td>( \frac{C_F}{1 - 2s^2_{\rho}} + 1 ) ( \Gamma_Z \delta \rho )</td>
<td>38 MeV</td>
<td>Z(^0) width</td>
</tr>
<tr>
<td>( \delta \Gamma_W )</td>
<td>( \frac{3}{2} \frac{c^2_{\rho}}{1 - 2s^2_{\rho}} \Gamma_W \delta \rho )</td>
<td>53 MeV</td>
<td>( W^\pm ) width</td>
</tr>
<tr>
<td>( \delta R_{nu} )</td>
<td>( \frac{64s^2_{\mu}c^2_{\mu}}{(1 + v^2_{\rho} + v^2_{\rho})^2} R_{nu} \delta \rho )</td>
<td>.033</td>
<td>ratio of neutral ( \nu_{\mu}\bar{e} ) to ( \bar{\nu}<em>{\mu}e ) scattering ( E</em>{\nu} = 70 ) GeV</td>
</tr>
<tr>
<td>( \delta R_{NC:CC} )</td>
<td>( 2 + \frac{4(1 - 2v_{\rho})s^2_{\mu}c^2_{\mu}}{(1 - v^2_{\rho} + v^2_{\rho})(1 - 2s^2_{\rho})} ) ( R_{NC:CC} \delta \rho )</td>
<td>.0043</td>
<td>ratio of neutral to charge ( \nu_{\mu}\bar{e} ) scattering ( E_{\nu} = 70 ) GeV</td>
</tr>
</tbody>
</table>
Table X

Responses at one loop of various asymmetries on $Z^0$ resonance and the $W^\pm$ mass to new one-loop physics.

Numbers are generic, calculated using $M_Z = 94$ GeV

<table>
<thead>
<tr>
<th>One-Loop Physics</th>
<th>$\delta A_{LR} = \delta A_{rpol}$</th>
<th>$\delta A_{FB}$</th>
<th>$\delta A_\perp$</th>
<th>$\delta M_W$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSW Weak</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_t = 30$</td>
<td>$-0.03$</td>
<td>$-0.01$</td>
<td>$0.005$</td>
<td>$-180$</td>
</tr>
<tr>
<td>$m_H = 100$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy Top Quark</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_t \approx 180$ GeV</td>
<td>$0.03$</td>
<td>$0.0075$</td>
<td>$0.004$</td>
<td>$780$</td>
</tr>
<tr>
<td>Heavy Higgs $\sim 1$ TeV</td>
<td>$-0.01$</td>
<td>$-0.0045$</td>
<td>$-0.003$</td>
<td>$-160$</td>
</tr>
<tr>
<td>Heavy Quark Pair</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Large I Splitting</td>
<td>$0.02$</td>
<td>$0.01$</td>
<td>$0.007$</td>
<td>$300$</td>
</tr>
<tr>
<td>b) Degenerate</td>
<td>$-0.004$</td>
<td>$-0.002$</td>
<td>$-0.001$</td>
<td>$-42$</td>
</tr>
<tr>
<td>Heavy Lepton Pair</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Large I Splitting $m_\nu = 0$</td>
<td>$0.012$</td>
<td>$0.006$</td>
<td>$0.004$</td>
<td>$300$</td>
</tr>
<tr>
<td>b) Degenerate</td>
<td>$-0.0013$</td>
<td>$-0.0006$</td>
<td>$-0.0004$</td>
<td>$-14$</td>
</tr>
<tr>
<td>Heavy Squark Pair</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Large I Splitting</td>
<td>$0.02$</td>
<td>$0.01$</td>
<td>$0.007$</td>
<td>$300$</td>
</tr>
<tr>
<td>b) Degenerate</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Heavy Slepton Pair</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Large I Splitting</td>
<td>$0.012$</td>
<td>$0.006$</td>
<td>$0.004$</td>
<td>$300$</td>
</tr>
<tr>
<td>b) Degenerate</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Winos</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) $m_{3/2} \ll 100$ GeV</td>
<td>$0.005$</td>
<td>$0.0025$</td>
<td>$0.001$</td>
<td>$100$</td>
</tr>
<tr>
<td>b) $m_{3/2} \gg 100$ GeV</td>
<td>$&lt;0.001$</td>
<td>$&lt;0.001$</td>
<td>$&lt;0.001$</td>
<td>$&lt;10$</td>
</tr>
<tr>
<td>Technicolor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$SU_6 \times SU_8$</td>
<td>$-0.04$</td>
<td>$-0.018$</td>
<td>$-0.012$</td>
<td>$-500$</td>
</tr>
<tr>
<td>$O_{16}$</td>
<td>$-0.07$</td>
<td>$-0.032$</td>
<td>$-0.021$</td>
<td>$-500$</td>
</tr>
<tr>
<td>Strong Interaction Uncertainty</td>
<td>$\pm 0.0033$</td>
<td>$\pm 0.0016$</td>
<td>$\pm 0.001$</td>
<td>$\pm 25$ MeV</td>
</tr>
</tbody>
</table>
FIGURE CAPTIONS

1. QED vacuum polarization graphs.

2. QED detector dependent graphs specifically excluded from 4-fermion processes considered in this paper. To the we must add all permutations of the photon lines. See Ref. 9.

3. Unrenormalized 1PI vector boson self-energy. i, j refer to the particles Z, A (photon), W± or to the SU2 isospin 1, 2, 3 and electromagnetic Q currents. Solid lines with arrows are fermions, dashed lines scalars, wavy lines vector particles, and dotted lines with arrows are ghosts.

4. Unrenormalized 1PI fermion self energy.

5. Unrenormalized 1PI fermion Z0 vertex part.

6. Unrenormalized 1PI fermion W± vertex part. Graphs are similar to those in Fig. 5.

7. 1PI box diagrams contributing to muon decay and the form factor Vμνμν.".

8. Forward-backward asymmetry AFβ as a function of √s. The dashed lines include no radiative corrections, not even from QED diagrams of Fig. 1, and the solid lines include all one-loop GSW corrections except those of Fig. 2 for m_t = 30 and m_H = 100. All masses are in GeV.

9. Longitudinal polarization asymmetry ALR = A_{ρ→} as a function of √s. The dashed lines and solid lines are as in Fig. 7. The dotted lines include only the QED graphs of Fig. 1 contribution to the renormalization of α.

10. Contribution of an extra quark doublet to δALR for various isospin splitting ratios m_B/m_T as a function of m_T. All masses are in GeV.

11. Contributions of an extra lepton doublet to δALR for various isospin splitting ratios m_μ/m_τ as a function of m_τ. All masses are in GeV.
12. Contributions of heavy degenerate generations of quarks and leptons to $\delta A_{LR}$ as a function of the fermion mass. All masses are in GeV.

13. Contribution of a squark doublet to $\delta A_{LR}$ for various isospin splitting ratios $m_{\tilde{B}}/m_{\tilde{T}}$ as a function of $m_{\tilde{T}}$. We ignore mixing between "left" and "right" squarks and take their masses degenerate. All masses in GeV.

14. Contribution of a slepton doublet to $\delta A_{LR}$ for various isospin splitting ratios $m_{\tilde{\tau}^c}/m_{\tilde{\ell}^-}$ as a function of $m_{\tilde{\ell}^-}$. We ignore mixings between "left" and "right" sleptons and take their masses degenerate. All masses are in GeV.

15. Contributions of various technicolor models to $\delta A_{LR}$ as a function of a mass scale which occurs in such models. The contribution from a GSW Higgs is displayed for reference relative to a "standard" value $m_H = 100$. All masses are in GeV.

16. SUSY shifts $\delta A_{LR}$ in models in which the gaugino masses are related to the SUSY breaking gravitino mass $m_{3/2}$ from SUGRA. All masses are in GeV.

17. SUSY shifts $\delta A_{LR}$ in models in which the gaugino masses are independent of $m_{3/2}$. In some models they are due to radiative corrections and are small. All masses are in GeV.

18. Shift $\delta M_W$ versus shift $\delta A_{LR}$ for an extra quark doublet as a function of $m_T$. We include the shift due to heavy GSW Higgs for reference. All masses are in GeV.

19. Shift $\delta M_W$ versus shift $\delta A_L$ for an extra lepton doublet as a function of $m_{\tilde{\ell}^-}$. We include the shift due to the GSW Higgs for reference relative to a "standard" value $m_H = 100$. All masses are in GeV.
20. Shift $\delta M_W$ versus $\delta A_{LR}$ for two technicolor models as a function of a scale which enters into such models. We plot also the shift due to the GSW Higgs' for reference relative to a "standard" value $m_H = 100$. All masses are in GeV.

21. SUSY shifts $\delta M_W$ in models in which the gaugino masses are related to the SUSY breaking gravitino mass $m_{3/2}$ from SUGRA. All masses are in GeV.

22. SUSY shifts $\delta M_W$ in models in which the gaugino masses are independent of $m_{3/2}$. In some models they are due to radiative corrections and are small. All masses are in GeV.
Fig. 1

\[ a_i \rightarrow \mu^a_j = \varepsilon_{\mu \nu} \Pi_{ij} - q_\mu q_\nu \Pi_{ij} \]

\[ \sum \text{fermions} + \sum \text{vectors} + \sum \text{scalars} \]

\[ \sum \text{vectors} + \sum \text{vectors} \]

\[ \sum \text{scalars} + \sum \text{scalars} \]

\[ \sum \text{ghosts} \]

Fig. 2

\[ \sum \text{fermions} \]

\[ \sum \text{fermions} \]

\[ \sum \text{scalars} \]

\[ \sum \text{ghosts} \]

Fig. 3

\[ d, s, b \]

\[ u, c, t \]

\[ A, Z^\pm, W^\pm \]
Fig. 5

\[ e \rightarrow e = \frac{i e}{\sqrt{2}} \frac{1^L Q z^2}{s_W c_W} \gamma_\lambda \tilde{f}^{\text{eeZ}} + \frac{i e}{\sqrt{2}} \frac{1^R Q z^2}{s_W c_W} \gamma_\lambda \tilde{f}^{\text{eeZ}} \]

\[ e \rightarrow W \gamma_\lambda \tilde{f}^{\text{eeW}} \]

Fig. 6

\[ \sum \text{ vectors fermion} + \sum \text{ vectors } \neq A \text{ fermion} + \sum \text{ vectors scalars fermion} + \sum \text{ vectors scalars fermion} \]

Fig. 7

\[ \nu_\mu \rightarrow e = \frac{-4G_F}{\sqrt{2}} \left( \bar{\nu}_\mu \gamma_\chi \gamma_\mu \nu_\chi \gamma_\mu \nu_\chi \right) V^{\nu_\mu}_{\nu_\mu} e^e \]

\[ = \sum \text{ vectors } \neq A \text{ fermions} + \sum \text{ scalars fermions} \]

\[ + \sum \text{ vectors } \neq A \text{ scalars fermions} \]

Fig. 7
Fig. 21

Fig. 22
HARD BREMSSTRAHLUNG IN $e^+e^- + \mu^+\mu^-$: MATCHING THEORY AND EXPERIMENT

R. Kleiss
CERN, Geneva, Switzerland

ABSTRACT

We discuss the problem of matching the theoretical knowledge of the radiative corrections to the cross-section for $e^+e^- + \mu^+\mu^-$ with the experimental conditions. We argue that the optimal way to do this is by using Monte Carlo event generators. The importance of a careful treatment of hard-photon bremsstrahlung effects is stressed.

1. INTRODUCTION

This study concerns itself with high-precision comparisons between the data on $e^+e^- + \mu^+\mu^-$ to be expected from LEP, and the one-loop radiatively corrected predictions of possible models for electroweak interactions. The virtual corrections to this cross-section around the $Z^0$ resonance are treated in detail elsewhere in this publication [1]. There are, however, two other pieces of information that are essential to an experimentally verifiable theoretical prediction. In the first place, virtual corrections augmented by soft-photon bremsstrahlung are not enough: hard-bremsstrahlung effects have to be taken into account, and therefore the cross-section for $e^+e^- + \mu^+\mu^-\gamma$ is relevant. In the second place there must be a recipe for obtaining from the theoretical input a prediction which includes all features of the experimental detection and analysis. The procedure which in our view is optimal is the use of an event generator whose predictions can be made arbitrarily precise while taking into account all possible experimental constraints [2]. After having treated these points, we will sketch the structure of an event generator and present some numerical results.

Throughout this note we shall not consider polarization effects. Some of these are discussed by Böhm and Hollik [3].

2. THE SOFT-PHOTON CROSS-SECTION

In the LEP precision measurements on the muon \textsuperscript{*)} pair production process

$$e^+(p_+) \ e^-(p_-) + \mu^+(q_+) \ \mu^-(q_-) , \quad (1)$$

\textsuperscript{*)} All results presented in this note are such that they can easily be generalized to pair production of other light fermions (except $e^+e^-$). For quarks, one should of course include a factor of 3 for colour. For $e^+e^-$ production, see Ref. [4].
the object of actual physical interest is

$$\frac{d\sigma}{dQ} = f_0(s, E_0', \cos \theta),$$  \hspace{1cm} (2)

which is the (standard model) prediction for process (1) including all one-loop virtual corrections. Here $s$ is the c.m. energy squared, and $\theta$ is the angle between $p_+^*$ and $q_-^*$ (or $p_-^*$ and $q_+^*$). In order to cancel the infrared divergences, Eq. (2) also includes a contribution from soft-photon emission. Photons are called soft if their energy is less than a certain value $E_0$. For infinitesimally small $E_0'$, the soft-photon cross-section factorizes exactly into the Born term expression and a so-called infrared factor.

The function $f_0$ is the subject of other studies, both in this publication [1] and elsewhere [3]. Into it enter all the subtleties connected with a choice of renormalization scheme and (therefore) the relations between the parameters of the theory. Here we disregard all this and treat $f_0$ as just some given function which contains a number of (from our viewpoint) arbitrary parameters:

- $m_Z$: the mass of the $Z^0$ boson;
- $\Gamma_Z$: the total decay width of the $Z^0$ boson;
- $m_{\mu}$: the $\mu$ mass;
- $e^+$: the $e^+$ electric charge;
- $g_V (g_{V}^\mu)$: the $e^+ e^- Z (\mu^+ \mu^- Z)$ vector coupling constant;
- $g_A (g_A^\mu)$: the $e^+ e^- Z (\mu^+ \mu^- Z)$ axial-vector coupling constant.

These parameters also enter into the hard-bremsstrahlung cross-section, the only model-dependence residing in their numerical values. In the following we shall work with $f_0$ as if it were, say, just a table of values for some $\theta$-values, at fixed $s$ and $E_0$. For the sake of reference we give in appendix B the corrections to the Born terms that arise from virtual photon loops.

Before we finish this section one remark is in order. As will become clear later on, $E_0$ is bounded from below by the condition that $f_0$ must be positive: therefore $E_0$ cannot be infinitesimal\(^{1}\), and the soft-photon factorization is not exact if the lowest-order cross-section fluctuates very much. This implies that we have to augment $f_0$ by a term which takes into account the fact that around the $Z^0$ resonance the cross-section fluctuates appreciably over

\(^{1}\) In practice $E_0$ is limited to about $10^{-3} E_0$, where $E_0$ is the beam energy. Therefore around the $Z^0$ peak $E_0$ will typically be of the order of 0.1-0.5 GeV. The resonant cross-section ($\Gamma_{\text{tot}} \approx 3$ GeV) can vary appreciably over this energy interval.
an energy interval of size $E_0$. This so-called tail effect has been derived by several authors [2,5]; it reads

$$\frac{d\sigma_{\text{tail}}}{dQ} = \frac{1}{128\pi^2 s} \left[ A_1 (1 - \cos \theta)^2 + A_0 (1 + \cos \theta)^2 \right],$$

(3)

$$A_\pm = 2 \text{e}^{2e'} (g_V g_Y' \pm g_A g_A') A^T \text{Re} (\chi) + \left[ (q_V^2 + q_Y'^2) (q_A^2 + q_A'^2) - 4 g_V g_A g_Y g_Y' A_R \right] |\chi|^2,$$

$$\chi = s/(s - m_2^2 + i m_2 \Gamma_2),$$

$$A^T = (\beta_1 + \beta_2) \left[ \left( \frac{\zeta^2 + \gamma^2}{\zeta} - 1 \right) \psi - \frac{\gamma}{\zeta} \phi \right],$$

$$A^R = \beta_1 \left[ (\zeta^2 + \gamma^2 - 1) \psi + \frac{(\zeta^2 + \gamma^2)(\zeta - 2) + \zeta \phi}{\gamma} \right] + 2 \beta_2 \left[ (\zeta - 1) \psi - \gamma \phi \right],$$

$$\beta_1 = \frac{e^2}{2 \alpha^2} \left( \ln \frac{s}{m_e^2} - 1 \right), \quad \beta_2 = \frac{ee'}{\pi^2} \ln \tan \frac{\theta}{2},$$

$$\psi = \frac{1}{2} \ln \left[ \frac{(E_0/E_D - \zeta)^2 + \gamma^2}{\zeta^2 + \gamma^2} \right],$$

$$\phi = \arctan \left( \frac{E_0/E_D - \zeta}{\gamma} \right) - \arctan \left( - \frac{\zeta}{\gamma} \right),$$

$$\zeta = 1 - \frac{m_2^2}{s}, \quad \gamma = \frac{m_2 \Gamma_2}{s}.$$

In the following, we denote by 'soft-photon cross-section' the sum of the two contributions (2) and (3):

$$\frac{d\sigma_{\text{soft}}}{dQ} = f_0(s, E_0, \cos \theta) + \frac{d\sigma_{\text{tail}}}{dQ}.$$  (4)

We now come to the second contribution to the event rate, namely that from hard-photon bremsstrahlung.
3. **THE HARD-PHOTON CROSS-SECTION**

The multidifferential cross-section for

\[ e^+(p_+)e^-(p_-) \rightarrow \mu^+(q_+)\mu^-(q_-)\gamma(k) \]  

(5)

is most easily derived with the helicity amplitude techniques introduced by the CALKUL collaboration [6] and the spinor product formalism of Refs. [7] and [8]. A derivation of the results of this section is described in detail in Ref. [9]. For any two light-like momenta \( k_1^\mu \) and \( k_2^\mu \), the spinor product is defined by

\[ s(k_1^\mu, k_2^\mu) = (k_1^Y +ik_1^Z) \left( \frac{k_2^0 - k_2^X}{k_1^0 - k_1^X} \right)^{1/2} - (k_2^Y +ik_2^Z) \left( \frac{k_1^0 - k_1^X}{k_2^0 - k_2^X} \right)^{1/2}, \]  

(6)

which implies \( |s(k_1^\mu, k_2^\mu)|^2 = 2(k_1^0 k_2^0) \). Using these relativistically invariant, antisymmetric products, we construct two radiation factors:

\[ V_p = \frac{-s(p_+, p_-)^* e'}{s(k,p_+)s(p_-, k)} \quad \text{and} \quad V_q = \frac{-s(p_+, p_-)^* e'}{s(k,q_+)s(q_-, k)} \]  

(7)

There occur four different combinations of photon and \( g^\mu \) propagators:

\[ Z_1(x) = i8 \left( \frac{ee'}{x} + \frac{G_1^0 K_Z}{x - m_Z^2 + i\pi Z Z} \right), \]

\[ G_1 = (q_\gamma - q_A^\gamma)(q_\gamma - q_A^\gamma), \quad G_2 = (q_\gamma - q_A^\gamma)(q_\gamma + q_A^\gamma), \]

\[ G_3 = (q_\gamma + q_A^\gamma)(q_\gamma - q_A^\gamma), \quad G_4 = (q_\gamma + q_A^\gamma)(q_\gamma + q_A^\gamma). \]  

(8)

For massless electrons and muons there are eight non-zero helicity amplitudes:

\[ M_1 = s(p_+, q_-)^2 \left[ Z_1(s^*)V_p + Z_1(s)V_q \right], \]

\[ M_2 = -s(p_+, q_-)^2 \left[ Z_2(s^*)V_p + Z_2(s) V_q \right] \],

\[ M_3 = -s(q_-, p_-)^2 \left[ Z_3(s^*)V_p + Z_3(s) V_q \right] \],

\[ M_4 = s(p_-, q_-)^2 \left[ Z_4(s^*) V_p + Z_4(s) V_q \right] \],

(9)

(cont.)
\[ M_5 = -s(p_-q_+)^2 [z_1(s')v_p^* + z_1(s)v_q^*], \]  
\[ M_6 = +s(p_-q_+)^2 [z_2(s')v_p^* + z_2(s)v_q^*], \]  
\[ M_7 = +s(q_+p_-)^2 [z_3(s')v_p^* + z_3(s)v_q^*], \]  
\[ M_8 = -s(q_+p_-)^2 [z_4(s')v_p^* + z_4(s)v_q^*], \]  
(9)

where \( s = (p_+ + p_-)^2 \) and \( s' = (q_+ + q_-)^2 \).

When the photon is emitted parallel to an electron or muon, the effect of a finite \( m_e \) or \( m_\mu \) becomes noticeable, leading to a mass effect factor

\[
W_m = \left[ \frac{1 - \frac{m_e^2}{p_+ \cdot k}}{p_+ \cdot p_- + (p_+ \cdot k)^2} \right] \left[ \frac{1 - \frac{m_e^2}{p_- \cdot k}}{q_+ \cdot q_- + (q_+ \cdot k)^2} \right] \left[ \frac{1 - \frac{m_\mu^2}{q_+ \cdot k}}{q_+ \cdot q_- + (q_+ \cdot k)^2} \right].
\]

(10)

Unless the photon is collinear with an \( e \) or \( \mu \), \( W_m \) equals 1.

The hard-photon cross-section is given by

\[
\alpha_{\text{hard}}^5 = \frac{(2\pi)^{-5}}{2s} \left( \frac{1}{4} \sum_{i=1}^{4} |M_i|^2 W_m \right) \times [\delta^4(p_+ - p_- - q_+ - q_- - k) d^4q_+ d^4q_- d^4k \Delta(q_+^2 - m_\mu^2) \Delta(q_-^2 - m_\mu^2) \Delta(k^2)].
\]

(11)

In principle, Eq. (11) contains all information about the hard-photon effects. Any experimentally accessible prediction is obtained by integrating Eqs. (4) and (11) over a suitably defined phase space. Our procedure to this end is described in the next section.

4. EVENT GENERATORS

To this order of perturbation theory, Eqs. (4) and (9) contain all experimentally accessible information about our subject process. In order to justify the method which we advocate as a means of arriving from these

\* See also the fourth reference in [6], where the cross-section is given without referring to spinor products.
expressions to a definite prediction for an actual experimental situation, we make the following observations.

i) The bremsstrahlung cross-section fluctuates over many orders of magnitude in those small regions of phase space where the photon is collinear with a charged particle. In such cases it is important to choose phase-space variables that describe these peakings well. There is, however, no set of variables describing all peakings simultaneously in any simple way.

ii) The 'soft photon' cross-section (4) is a smooth function of its one variable, cos θ, but its analytical form is extremely involved. Indeed, several authors prefer to obtain the values of the contributing diagrams in a numerical form only.

iii) The phase-space restrictions corresponding to any realistic experimental set-up are typically very complicated functions of the variables even in idealized cases. Moreover, they tend to change with time as the experiment evolves.

iv) For precision tests such as these, it is unacceptable that the one-loop prediction (the subject of the tests) should only be known approximately. Therefore, any approximation in Eqs. (4) or (9), or in the phase-space cuts, should be avoided.

From these considerations it is clear that our approach must be numerical: an analytic integration would on the one hand be wellnigh impossible to perform, and on the other hand be useless for all experiments except one. Moreover, detailed detector data such as acceptances are only available in numerical form. However, ordinary numerical integration using, for instance, the VEGAS program [10], is also unattractive because of the non-existence of a 'good' set of variables. Another problem is the fact that it is typically quite difficult and time-consuming to fold a reasonably complete and realistic detector simulation in with the integration.

In order to combine realism and flexibility, it is most advantageous to construct a computer simulation of the (radiatively corrected) pair production process: an event generator.

We define an event generator as a program package which, possibly after an initialization step, gives an event upon every call to it. An event is a completely specified final state, i.e. a set of particle momenta: in our case \( q^\mu, q'^\mu, \) and \( k^\mu \). The events should come out uncorrelated but obey a probability distribution as given by the model. Finally, for a generated set of events, it must be known to what cross-section the sample corresponds (possibly only after the sample has been generated).

By keeping or rejecting events according to one's experimental cuts, a number for the total accepted cross-section can easily be obtained, whilst
making one- or more-dimensional plots of any variable, no matter how complicated its definition, is trivial. An additional advantage is that it is also possible to get an idea of the fluctuations that can be expected\(^*\), which is especially important if the cuts are tight so that only a small number of events is expected. Finally, a sample of generated ('Monte Carlo') events can be stored to be used over and over again for different analyses.

5. OUTLINE OF AN EVENT GENERATOR

Having argued that we need an event generator, we shall now sketch its structure: a detailed write-up is presented in Ref. [2]. Technically speaking, we use the Monte Carlo procedure called importance sampling. A description is given in Appendix A: at this point we will just mention that there are two steps to this method. First we generate events according to an approximate distribution; then we modify the distribution of the events thus obtained by discarding a number of them, so that the surviving events satisfy the exact distribution. The quality of this procedure depends on our ability to find an approximant that 'looks like' the exact cross-section, and at the same time can be generated quickly. In our case we need one approximant for Eq. (4) and one for Eqs. (9), which we will denote by \( h_{\text{soft}} \) and \( h_{\text{hard}} \) respectively\(^**\): \( h_{\text{soft}} \) is defined on the two-dimensional phase space with \( k'^0 = 0 \), and \( h_{\text{hard}} \) on the complete five-dimensional hard-photon phase space.

The program structure is sketched in Fig. 1. There are three main steps: initialization, event generation, and evaluation.

1) Initialization. We integrate \( h_{\text{soft}} \) and \( h_{\text{hard}} \) over the respective phase spaces, resulting in the two approximate cross-sections, \( A_{\text{soft}} \) and \( A_{\text{hard}} \). Here the values of the model parameters (\( m_z, \Gamma_z \), etc.) have to be supplied, as well as \( E_b', E_p \), and possibly some phase-space cuts.

2) Generation. Using \( A_{\text{soft}} \) and \( A_{\text{hard}} \) as relative weights we first decide whether the event to be generated will have \( k'^0 > E_0 \) ('hard' event) or \( k'^0 < E_0 \) ('soft' event, in which case we set \( k'^0 = 0 \)). In either case the phase-space variables are generated by mappings (see Appendix A), so that they are distributed according to \( h_{\text{hard}} \) or \( h_{\text{soft}} \). For details about such mappings, we refer the reader to Refs. [2, 12].

For a generated event we calculate its weight, defined as the ratio of the exact expression [Eqs. (4) or (9)] over the corresponding approximate one. If the approximations are 'good' to within a reasonable factor (i.e. if they

\(^*\) This implies that we have some control over the random number source, which is usually the case. For an excellent review on the subject of random numbers, see Ref. [11]

\(^**\) Actually we use a superposition of several approximants for Eqs. (9). In the following we disregard this technicality.
Fig. 1 General outline of an event generator as discussed in the text.

have the same peaking behaviour, etc.) this weight will not fluctuate very much over phase space.

Finally, we apply a rejection algorithm (see Appendix A) in which we compare the event weight with a random number. If the event is rejected we must start all over again; if it is accepted the event can be submitted for storage, analysis, etc.

Two remarks are in order here. First, the exact expressions (4) and (9) enter only in the calculation of weights: this allows for easy modification.
Secondly, although internally the program uses approximants and event weights, the resulting events satisfy the exact cross-section, and no weight is assigned to them.

3) **Evaluation.** By bookkeeping in the generation stage (B) we can obtain the exact cross-section for a generated event sample. Let \( N \) be the number of generated events before rejection, \( W_1 \) the sum of their weights, and \( W_2 \) the sum of their weights squared. The Monte Carlo result for the exact cross-section is then [11]

\[
\sigma_{\text{hard}} = A_{\text{hard}} \left( \frac{W_1}{N} \right)
\]

and the estimated error is

\[
\Delta \sigma_{\text{hard}} = A_{\text{hard}} \left( \frac{W_2 - W_1^2}{N} \right)^{1/2} / N.
\]

Of course, similar formulae hold for the soft events.

One final remark is in order regarding the choice of \( E_0 \) (see also Refs. [2] and [12]. Our results should not depend on the value of \( E_0 \), it being an artificial parameter of the calculation. However, if we want to interpret Eq. (4) as a probability function it must be positive, and this puts a lower limit on \( E_0 \). On the other hand, to justify the soft-photon approximation in Eq. (4), and the fact that \( k^\mu = 0 \) in the soft-photon region, the value of \( E_0 \) cannot be too large, either. Typically, \( E_0 \) is of the order of 1% of the beam energy \( E_b \).

6) **RESULTS**

We now return to physics. As stated above, it is impossible to present predictions that are valid for all experiments, since so much depends on details of the detector and the analysis. Instead, we have assumed a set of reasonable cuts and present our results as illustrations, qualitatively similar to what is to be expected for an actual experiment. Our requirements for accepting a Monte Carlo event are that

i) the muons must have energies of at least 10 GeV;
ii) they must be separated from the beam directions by at least 5°;
iii) they must be back-to-back to within an **collinearity angle** \( \zeta \).

Experimentally, \( \zeta = 10° \) is typical; we have let \( \zeta \) vary between 1° and 180°.

We shall discuss results for two sets of weak model parameters:

\[
M_Z = 90 \text{ GeV}, \quad \Gamma_Z = 2.258 \text{ GeV}, \quad \sin^2 \theta_W = 0.243,
\]

and

\[
M_Z = 94 \text{ GeV}, \quad \Gamma_Z = 2.667 \text{ GeV}, \quad \sin^2 \theta_W = 0.216.
\]
The interrelation between the parameter values is discussed in Refs. [1] and [3]. The coupling constants are such that \( \alpha = 1/137 \), and

\[
q_A = q_A^1 = e/4 \sin \theta_w \cos \theta_w, \quad q_V = q_V^1 = q_A^1 (1 - 4 \sin^2 \theta_w).
\] (14)

By running the event generator at a number of values of \( \sqrt{s} \) around \( m_Z \)\(^*\) we obtain Fig. 2 to Fig. 9. For the function \( f_\theta \) of Eq. (4) we used the results of Lynn and Stuart [3], augmented by those of Ref. [5]. We concentrate on \( \sigma \), the total event rate, and \( A \), the forward-backward asymmetry.

Figure 2 shows \( \sigma \) as a function of \( \sqrt{s} \) for \( m_Z = 90 \) GeV and \( \zeta = 10^\circ \). The solid line is the Born term expectation, the dashed line the Monte Carlo result (the error is about equal to the width of the line). Below and on the resonance the correction is large and negative, whilst above the resonance it grows rapidly. These features are easily understood as arising from 'initial-state radiation' [5, 13]. The correction is completely due to well-understood, model-independent QED corrections; the interesting, non-trivial effects of the one-loop 'weak' diagrams are invisible at this scale. Note that the peak of the resonance curve is shifted by about 300 MeV. Figure 3 gives the same curves for \( m_Z = 94 \) GeV.

---

*) At each value of \( \sqrt{s} \) about 100,000 events were generated.
Fig. 4 The forward-backward asymmetry $A$ as a function of energy, for $m_q = 90$ GeV. Solid line: lowest-order prediction. Dashed line: fully first-order corrected prediction for the experimental cuts described in the text.

Fig. 5 A close-up of Fig. 4 of the region around $\sqrt{s} = m_q$, for $m_q = 90$ GeV. Solid line: lowest-order prediction. Dashed line: fully corrected result. Dotted line: the prediction including only the 'photonic' radiative corrections discussed in the text.

Fig. 6 The forward-backward asymmetry $A$ as a function of energy, for $m_q = 94$ GeV. Solid line: lowest-order prediction. Dashed line: fully first-order corrected prediction for the experimental cuts described in the text.

Fig. 7 A close-up of Fig. 4 of the region around $\sqrt{s} = m_q$, for $m_q = 94$ GeV. Solid line: lowest-order prediction. Dashed line: fully corrected result. Dotted line: the prediction including only 'photonic' radiative corrections discussed in the text.
Fig. 8 The correction $\delta \sigma$ to the total cross-section as a function of the acollinearity angle $\zeta$, for different values of $\sqrt{s}$.

Fig. 9 The shift $\Delta A$ in the forward-backward asymmetry cone to radiative corrections, as a function of the acollinearity angle $\zeta$, for different values of $\sqrt{s}$.

In Fig. 4 we give the asymmetry $A$ for the same events as in Fig. 2. The corrected asymmetry is always lower than the Born result. This feature, as well as the large suppression at higher energies, can be qualitatively explained from initial-state radiation. Figure 5, which is a close-up of Fig. 4, shows some effect of the 'weak' diagrams: the dashed line includes all one-loop corrections, and the dotted line only the QED corrections. Again, Figs. 6 and 7 show the same result for $m_Z = 94$ GeV.

In order to give an idea of the influence of the cuts, we plot the $\zeta$ dependence of the percentage correction to $\sigma$ under, on, and above the $Z^0$ resonance at 90 GeV (Fig. 8). As expected, for $\zeta > 0$ more and more hard-photon events are cut out, and the correction decreases logarithmically with $\zeta$. Since the hard-photon contribution is most important above the resonance, this effect is largest for those energies. It is also seen that the correction is essentially independent of $\zeta$ if $\zeta$ is above $10^\circ$, indicating that this is also a theoretically attractive value. Similar considerations hold for Fig. 9, which depicts the shift in $A$ as a function of $\zeta$. Here the correction grows as $\zeta$ decreases because the hard-bremsstrahlung contribution is negative.

REFERENCES

[1] See, for instance the contributions to this report by M. Consoli; M. Greco; B.W. Lynn and R.G. Stuart; W. Wetzel (and references quoted therein).
    A more complete treatment including the $Z^0$ resonance is in preparation.
[12] Descriptions of this kind of event generator can be found in F.A. Berends,
APPENDIX A

We quickly review three elementary event generation techniques [2, 12]. The problem is formulated as follows. Given a probability density \( f(x) \) on \((x_0, x_1)\), generate numbers \( x \) at random in \((x_0, x_1)\) such that their density is given by \( f(x) \).

1) **Rejection algorithm**: Pick a pair of numbers \((x, y)\), \( x \) lying in \((x_0, x_1)\) and \( y \) in \([0, (x_0, x_1) f(x)]\). If \( f(x) < y \), reject the \( x \) value and try again. This hit-and-miss method always works (also in more dimensions) but may become inefficient if \( f(x) \) fluctuates badly.

2) **Mapping algorithm**: Suppose that the integral \( F(x) \) of \( f(x) \) is known:
\[
f(x) = \frac{dF(x)}{dx}, \text{ and that } F^{-1}(x) \text{ is known and computable.}
\]
Pick a number \( y \) in \([F(x_0), F(x_1)]\) and compute \( x = F^{-1}(y) \). Then the \( x \)'s are distributed correctly. This method is very efficient, but of course is only applicable to a limited class of functions, and restricted to one dimension.

3) **Importance sampling algorithm**: Find a function \( g(x) \) to which we can apply (2), and generate an \( x \) value. Then calculate \( w(x) = f(x)/g(x) \) and apply (1) to the distribution \( w(x) \).
This works if we can find a \( g(x) \) which, on the one hand, allows mapping, and on the other hand follows \( f(x) \) closely enough so that \( w(x) \) is reasonably smooth. In particular, some upper limit on \( (x_0, x_1) w(x) \) should be known, and \( w(x) \) must be positive.
APPENDIX B

Here we give those corrections, entering in $f_0$ of Eq. (2), that can be ascribed to virtual photon loops. The lowest-order cross-section for process (1) consists of a QED part (superscript $Q$), a resonant $Z^0$ part (superscript $Z$) and an interference part (superscript $I$), given by

\[
\frac{d\sigma}{dQ}^Q = \frac{g^2}{4\pi} (1+c^2), \quad c = \cos \theta (p^*, q^*),
\]  

\[
\frac{d\sigma}{dQ}^Z = \frac{g^2}{4\pi} |x|^2 \left[ (C^2 + c^2) (1+c^2) + 8C_v^2 c^2 \right],
\]

\[
\frac{d\sigma}{dQ}^I = \frac{g^2}{4\pi} (Re \chi) \left[ 2C^2_v (1+c^2) + 4C^2_A c \right],
\]

where $\chi$ is given in Eq. (3), and $C_v = Q_v/(-e)$, $C_A = Q_A/(-e)$. We consider the effects due to vertex corrections, fermion self-energies, box graphs with either two photons or one photon and one $Z^0$, and photon self-energy graphs. We also discuss soft bremsstrahlung. Together these make up the corrections that have been included in Ref. [5]. The generalization of the formulae below to the case of light quarks can also be found there.

The vertex and fermion self-energy corrections lead to an angle-independent correction factor,

\[
\delta_{\nu e} = \delta_{\nu e} (m^2_e) + \delta_{\nu \mu} (m^2_\mu),
\]

with

\[
\delta_{\nu e} (m^2_e) = \frac{2g}{\pi} \left[ -1 + \frac{1}{3} \ln \left( \frac{s}{m^2_e} \right) - \frac{3}{4} \ln \left( \frac{s}{m^2_e} \right) - \frac{1}{4} \ln \left( \frac{s}{m^2_e} \right) + \left( 1 - \ln \left( \frac{s}{m^2_e} \right) \right) \ln \frac{m_e}{\lambda} \right],
\]

where $\lambda$ is a small fictitious photon mass used to regularize the infrared divergence. The photon self-energy (vacuum polarization) contribution reads:

\[
\delta_{\nu e} = -2 \operatorname{Re} \Pi = \delta_{\nu e} (m^2_e) + \delta_{\nu \mu} (m^2_\mu) + \delta_{\nu \tau} (m^2_\tau) + \delta_{\nu \mu} (m^2_\mu) + \delta_{\text{had}},
\]

the various contributions originating from lepton loops and quark loops (or equivalently from hadrons). The leptonic terms are given by

\[
\delta_{\nu e} (m^2_e) = -2 \operatorname{Re} \Pi (m^2_e) = \frac{2g}{\pi} \left( -\frac{5}{9} + \frac{1}{3} \ln \frac{s}{m^2_e} \right),
\]

\[
\delta_{\nu \mu} (m^2_\mu) = \frac{2g}{\pi} \left( -\frac{5}{9} + \frac{1}{3} \ln \frac{s}{m^2_\mu} \right),
\]

\[
\delta_{\nu \tau} (m^2_\tau) = \frac{2g}{\pi} \left( -\frac{5}{9} + \frac{1}{3} \ln \frac{s}{m^2_\tau} \right),
\]

\[
\delta_{\text{had}} = \frac{2g}{\pi} \left( -\frac{5}{9} + \frac{1}{3} \ln \frac{s}{m^2_{\text{had}}} \right),
\]

where $s = p^2$ is the squared momentum and $m_{\text{had}}$ is the mass of the hadron.
whereas, for the hadronic vacuum polarization, the numerical evaluation of a
dispersion integral over the total hadronic cross-section \( \sigma_{\text{had}}(s) \) is used:

\[
\tilde{\sigma}_{\text{had}} = \frac{s}{2\pi^2} \int \frac{\sigma_{\text{had}}(s')}{s - s'} \, ds'.
\]  

(B.5)

The imaginary part of the vacuum polarization above the threshold for the
production of the known three charged leptons and five quarks is

\[
\text{Im} \, \Pi = \frac{1}{3} \alpha \sum_i Q_i^2 = \frac{20}{9} \alpha,
\]  

(B.6)

where the \( Q_i \) denote the lepton or quark charges over which the summation runs.

For a more detailed discussion of the vacuum polarization and the way in
which it ought to be taken into account, we refer the reader to the contributions
to this publication mentioned in Ref. [1].

The effect of the box diagrams with two photons is

\[
\frac{d\sigma^{\gamma\gamma}}{d\Omega^\gamma} = \frac{d\sigma^Q}{d\Omega^\gamma} \delta^{\gamma\gamma} + \frac{d\sigma^I}{d\Omega^\gamma} \delta^{I\gamma},
\]  

(B.7)

with

\[
\delta^{\gamma\gamma} = \frac{2a}{w} \left[ -4 \ln \tan \frac{1}{2} \theta \ln \frac{2E}{\lambda} - \frac{2}{1 + c^2} \left[ c \left( \ln^2 \sin \frac{1}{2} \theta + \ln^2 \cos \frac{1}{2} \theta \right) \right.ight.
\]

\[ \left. - \cos^2 \frac{1}{2} \theta \ln \sin \frac{1}{2} \theta + \sin^2 \frac{1}{2} \theta \ln \cos \frac{1}{2} \theta \right] \right],
\]  

(B.8)

and

\[
\delta^{I\gamma} = \frac{a}{2w} \left[ V_1 - \frac{m_Z^2}{s - m_Z^2} 2wV_2 + \left( A_1 - \frac{m_Z^2}{s - m_Z^2} 2wA_2 \right) \frac{c^2(1+c^2) + 2C_A^2 c}{C_V(1+c^2) + 2C_A^2 c} \right],
\]  

with

\[
V_1 = -8 \ln \tan \frac{1}{2} \theta \ln \frac{2E}{\lambda} - c \left( \ln^2 \sin \frac{1}{2} \theta + \ln^2 \cos \frac{1}{2} \theta \right)
\]

\[
\frac{\cos^4 \frac{1}{2} \theta}{\sin^4 \frac{1}{2} \theta} + \frac{\ln \sin \frac{1}{2} \theta \ln \cos \frac{1}{2} \theta}{\cos^2 \frac{1}{2} \theta \sin^2 \frac{1}{2} \theta},
\]  

(B.10a)
\[ V_2 = 2 \ln \tan \frac{1}{2} \theta - \frac{1}{2} c \left( \frac{\ln \sin \frac{1}{2} \theta}{\cos^4 \frac{1}{2} \theta} + \frac{\ln \cos \frac{1}{2} \theta}{\sin^4 \frac{1}{2} \theta} \right) - \frac{c}{1-c^2}, \]  
(B.10b)

\[ A_1 = -c \left( \frac{\ln^2 \sin \frac{1}{2} \theta}{\cos^4 \frac{1}{2} \theta} - \frac{\ln^2 \cos \frac{1}{2} \theta}{\sin^4 \frac{1}{2} \theta} \right) + \frac{\ln \sin \frac{1}{2} \theta + \ln \cos \frac{1}{2} \theta}{\cos^2 \frac{1}{2} \theta + \sin^2 \frac{1}{2} \theta}, \]  
(B.10c)

\[ A_2 = -\frac{1}{2} c \left( \frac{\ln \sin \frac{1}{2} \theta}{\cos^4 \frac{1}{2} \theta} - \frac{\ln \cos \frac{1}{2} \theta}{\sin^4 \frac{1}{2} \theta} \right) + \frac{1}{1-c^2}. \]  
(B.10d)

The contribution of the \( \gamma-Z \) box diagrams can best be taken into account as one part which is proportional to the resonant \( Z^0 \) propagator, and another part describing the rest of the contribution.

The resonating part is given by

\[ \frac{d\sigma_{\gamma Z, \text{res}}}{d\Omega} = \frac{d\sigma_{\gamma Z}}{d\Omega} \delta_{\gamma Z} + \frac{d\sigma_{Z}}{d\Omega} \delta_{\gamma Z}, \]  
(B.11a)

\[ \delta_{\gamma Z} = \frac{2a}{v} \left[ -2 \ln \tan \frac{1}{2} \theta \ln \frac{|Z(s)|^2}{m^2_{Z^2}} + 2 \ln^2 \cos \frac{1}{2} \theta - 2 \ln^2 \sin \frac{1}{2} \theta \right. \]

\[ + \frac{\ln^2 \sin \frac{1}{2} \theta}{\cos^4 \frac{1}{2} \theta} - \frac{\ln^2 \cos \frac{1}{2} \theta}{\sin^4 \frac{1}{2} \theta} \left( \frac{1 + \frac{1}{u^2}}{m^2_{Z^2}} \right), \]  
(B.11b)

\[ \left( \text{where } u = -s \cos^2 \frac{1}{2} \theta, \ t = -s \sin^2 \frac{1}{2} \theta, \ Z(s) = s - m^2_{Z^2} + \text{im}_{Z^2} \right) \]

\[ \delta_{\gamma Z}^I = \frac{1}{2} \delta_{\gamma Z} + \frac{4a}{s-m^2_{Z^2}} \left( \ln \tan \frac{1}{2} \theta \right) \delta_{\gamma Z}, \]  
(B.11c)

where \( \text{Li}_2 \) denotes the dilogarithm and \( \delta_{\gamma Z}^I \) is the phase shift of the \( Z^0 \) resonance:

\[ \delta_{\gamma Z} = \arctan \frac{\text{Re}_{Z^2}}{\text{Im}_{Z^2}}, \]  
(B.12)

where the arctan is defined in the interval \([0, \pi]\), such that \( \delta_{\gamma Z} \to 0 \) and \( \pi \) when \( s \to 0 \) and \( \infty \), respectively.
The non-resonating part is given by the expression

\[
\frac{d\sigma_Z^{\text{non-res}}}{dQ} = -\frac{\alpha^2}{8\pi} \text{Re}(N),
\]

\[
N = (1+\cos \theta)^2 \left\{ (C_{V+A}^2)^2 [1+X^2(s)(C_{V-A}^2)] + (C_{V-C}^2)^2 [1+X^2(s)(C_{V-C}^2)] \right\} f_N(s,t,u)
- 2(1-\cos \theta)^2 (C_{V-C}^2) [1+X(s)(C_{V-C}^2)] f_{N}(s,u,t),
\]

\[
f_N(s,t,u) = \frac{(u-t-m_Z^2)}{u^2} \left[ \ln \frac{m_Z^2 - s + i m_Z r_Z}{m^2_Z} \ln \left( -\frac{t}{s} \right) + \text{Li}_2 \left( 1 + \frac{t}{m_Z^2} \right) - \text{Li}_2 \left( 1 - \frac{s}{m_Z^2} \right) \right]
\]

\[
+ \frac{1}{u} \left[ \frac{m_Z^2}{s} - 1 \right] \ln \left( 1 - \frac{s}{m_Z^2} \right) + \ln \left( -\frac{t}{m_Z^2} \right). \tag{B.13b}
\]

This expression was first derived by Brown et al. [3] and later confirmed by Consoli and Sirlin [1]. The exact form of \(d\sigma_Z/dQ\) is given by Consoli and Sirlin [1]. Finally, we have to consider the soft-bremsstrahlung contribution that factorizes the lowest-order cross-section:

\[
\delta_s = 2 \frac{\alpha}{\pi} \left[ -\frac{1}{6} \ln^2 \frac{s}{m_e^2} + \frac{1}{2} \ln \frac{s}{m_e^2} - \frac{1}{4} \ln \frac{s}{m_e^2} - 1 \right] \ln \frac{2E}{\lambda} + (m_e + m_\mu) + 4 \ln \tan \frac{1}{2} \theta \ln \frac{2E}{\lambda} + 2 \ln^2 \left( \sin \frac{1}{2} \theta \right)
- 2 \ln^2 \left( \cos \frac{1}{2} \theta \right) - \text{Li}_2 \left( \sin^2 \frac{1}{2} \theta \right) + \text{Li}_2 \left( \cos^2 \frac{1}{2} \theta \right). \tag{B.14}
\]

Upon including all these corrections, the result for \(f_6\), as far as this note is concerned, is

\[
f_6(s,E_0^2,c) = \frac{d\sigma}{dQ} \left( 1 + \delta_{VC} + \delta_{VP} + \delta_s + \delta_{YY} \right)
+ \frac{d\sigma}{dQ} \left( 1 + \delta_{VC} + \frac{\delta_{VP}}{2} + \frac{m_e m_\mu}{s^2 m_Z^2} \text{Im} W + \delta_{YY} \right) + \delta_{YY} + \delta_s
+ \frac{d\sigma}{dQ} \frac{d\sigma_{non-res}}{dQ}, \tag{B.15}
\]
which can be included in Eq. (4) to obtain the fully QED-corrected expression for $d_0^{\text{soft}}$. If we add to this the effect of the $g^0$ self-energy diagrams, the bulk of the virtual corrections is accounted for. However, since we are discussing precision experiments, all other diagrams have to be included as well in the final expression for $f_0$. This is discussed in the various other contributions to this publication.
CHARGE ASYMMETRY OF $\mu^+\mu^-$

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The measurement of the charge asymmetry of the $\mu^+\mu^-$ state on top of the $Z^0$ has long been considered as a fundamental test of the standard model. To contribute useful information, however, it should provide a value of $\sin^2 \theta_W$ with a relative error approaching 1%. Here we show that this is indeed possible.

The asymmetry is given by the general formula,

$$\Lambda_{\text{ch}} = \frac{3}{2} \frac{A^2 B_1 + 2V^2 A^2 B_2}{1 + 2V^2 B_1 + (V^2 + A^2) B_2},$$

where

$$B_1 = \frac{g m^2 s (s-m^2)}{(s-m^2)^2 + m^2 r^2} \quad \text{and} \quad B_2 = \frac{g m^4 s^2}{(s-m^2)^2 + m^2 r^2}.$$

On top of the $Z^0$ resonance ($s = m^2$), $B_1 = 0$ and $B_2 = R = (g m^2 / r)^2$, where $g = 4.5 \times 10^{-5}$. In the standard model, $A = 1$ and $V = 1 - 4 \sin^2 \theta_W$. We assume that $V_\mu = V_\tau = V$. The total cross-section is

$$\sigma_{\text{tot}} = \frac{4 \pi a^2}{3 s} \left[ 1 + 2V^2 B_1 + (V^2 + A^2) B_2 \right].$$

We assume that a first scan of the $Z^0$ has given its mass and width with the accuracy quoted previously, as well as $\sigma_{\text{tot}}$. We come back on top of the resonance and accumulate data during $N$ days. Figure 1, giving $\Delta V/V$ across the

![Statistical error $\Delta V/V$](image)

Fig. 1 Statistical error $\Delta V/V$

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resonance versus the value it takes on top of it, for a given amount of time,
shows that we should sit on top of the $\zeta^0$.

With our assumption regarding the luminosity, we register $-10^3 \mu^+\mu^-$ pairs
per day in the full solid angle. The expected integrated asymmetry is, on top of
the $\zeta^0$,

$$\Lambda_{\text{ch}} = 3\sqrt{2}.$$  

Figure 2 shows this as a function of $\sin^2 \theta_w$, which in our case ($\sin^2 \theta_w = 0.23$)
amounts to $-2\%$. For our choice of $\theta_w$, the relation giving the error on $\sin^2 \theta_w$
can be written:

$$\frac{d\sin^2 \theta_w}{\sin^2 \theta_w} = \left(\frac{1 - 4 \sin^2 \theta_w}{4 \sin^2 \theta_w}\right) \frac{dV}{V} = 0.09 \frac{dV}{V}.$$  

Differentiating the formulae for $\sigma_{\text{tot}}$ and $\Lambda_{\text{ch}}$, we get in full generality,

$$\frac{dV}{V} = \frac{1}{2} \frac{d\Lambda_{\text{ch}}}{\Lambda_{\text{ch}}} + \frac{1}{4} \frac{d\sigma_{\text{tot}}}{\sigma_{\text{tot}}} + \frac{1}{2} \frac{d\Gamma}{\Gamma} - \frac{1}{4} \frac{m}{\Gamma} \frac{1}{\sqrt{\sigma_{\text{tot}}}} \frac{d(s-m^2)}{s}.$$  

For $\sin^2 \theta_w = 0.23$ the last coefficient is numerically equal to $-100$. We note
that, as expected, the absolute value of $s$ does not matter here: what counts is
the uncertainty on the distance of the measurement from the $Z$ pole. We adopt
$\Delta E/E < 2 \times 10^{-4}$ for the uncertainty on the reproducibility and stability of the
LEP energy. The corresponding relative error on $\sin^2 \theta_w$ is $< 0.4\%$. Errors due to
$\Delta \sigma$ and $\Delta \Gamma$ are much less important.

The main uncertainty will be due to the statistical and the systematic
error on $\Lambda_{\text{ch}}$. A total of 200 days of exposure will bring the statistical error
to what is shown in Fig. 2, i.e. $0.5\%$ for $s^2 = 0.22$. One must demonstrate that
the systematic error can reach a similar level.

![Graph showing $\Lambda_{\text{ch}}$ and $\frac{d\sigma}{s^2}$ as functions of $s^2$.](image)
This implies that
\[
\frac{\Delta A_{ch}}{A_{ch}}_{\text{syst}} = -11\% \quad \text{or} \quad \Delta A_{ch} = 0.22\% .
\]

It has been shown that a careful treatment of the radiative corrections can in principle bring the corresponding uncertainty to the 0.2% level: there is no fundamental problem, but much work and good co-operation between experiment and theory are needed.

Table 1 gives the published values of the systematic error on $A_{ch}$ of the PETRA and PEP experiments [1-4]. Also indicated are the main sources of uncertainty according to the authors. We note that several $e^+e^-$ experiments, absent from the table, have not fully pushed their study of systematics (because of their large statistical errors), and confine themselves to safe upper limits.

Several sources of uncertainty present at PETRA and PEP will have disappeared at LEP:

i) Since $R_{\mu\mu} > 180$ on the $Z^0$, the $\gamma\gamma$ and cosmic-ray backgrounds will be negligible;

ii) since in their respective domains of energy the momentum resolution of LEP detectors will be better than that of PETRA-PEP detectors, charge mis-identification will no longer occur;

iii) confusion between $Z^0 \rightarrow e^+e^-$ and $Z^0 \rightarrow \mu^+\mu^-$ seems to be totally excluded in LEP detectors;

Table 1

The systematic error on $A_{ch}$ in four PETRA and PEP experiments, and the main contributions to these errors

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Syst. error (%)</th>
<th>$s$ (GeV)</th>
<th>Source</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAC</td>
<td>0.3</td>
<td>29</td>
<td>Mainly Bhabha</td>
<td>1</td>
</tr>
<tr>
<td>TASSO</td>
<td>0.5</td>
<td>34</td>
<td>Radiative corrections</td>
<td>2</td>
</tr>
<tr>
<td>HRS</td>
<td>0.5</td>
<td>29</td>
<td>Bhabha + possible experimental bias</td>
<td>3</td>
</tr>
<tr>
<td>JADE</td>
<td>$\pm 0.9$</td>
<td>34</td>
<td>Error in charge determination is dominant</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.5$</td>
<td>42</td>
<td>- - -</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>$\pm 1.5$</td>
<td></td>
<td>- - -</td>
<td>4</td>
</tr>
</tbody>
</table>
iv) the contamination from $\tau^+\tau^-$, which is also dependent on the momentum resolution, will be very small: furthermore, $\tau^+\tau^-$ pairs have in principle the same asymmetry.

We are therefore left with systematic errors due to the detector itself. At a given $(\theta, \phi)$ the detector may, for various reasons, have a slightly different acceptance or reconstruction efficiency for positive and negative tracks. In principle such an effect could be eliminated by running for half of the time with opposite polarity of the solenoid. However, part of the effect would remain—for instance, a possible effect depending on $E \times B$. Furthermore, the procedure may be cumbersome. But we can argue that the sophistication of LEP detectors, in particular of their trigger system, and the large counting rates available on top of the $Z^0$, would make it possible to measure such a bias (if it exists) with a sufficient accuracy.

For the $\mu$ trigger and detection, let us mention two separate functions:

i) the reconstruction of the $\mu$ in the tracking detectors;

ii) its signature in the calorimeters and $\mu$ identifier.

To check a possible bias in (i), we can use $e^+e^-$ events triggered in a way which does not involve the tracking devices, for instance by requiring large energy depositions in the e.m. calorimeters. For (ii), we can do the same using the $\mu^+\mu^-$ events themselves if, at the trigger level, identification is required on a single leg.

These are the usual procedures in the presently running $e^+e^-$ detectors, and we feel that such trigger modes will be quite possible at LEP.

If $f^+$ and $f^-$ are the efficiencies (integrated over solid angle and time) for positive and negative tracks, let us define

$$\epsilon = \frac{1}{2} (f^+ - f^-) \text{ (small)},$$

and

$$\bar{\epsilon} = \frac{1}{2} (f^+ + f^-) \leq 1.$$  

It is easily found that $\Delta A_{\text{ch}} = A_{\text{ch}}^{\text{meas}} - A_{\text{ch}}^{\text{true}} = \epsilon / \bar{\epsilon} \leq \epsilon$. To reach $\Delta A_{\text{ch}} = 0.2\%$, it would therefore be necessary to measure a possible difference of acceptance of 0.4%.

Let us suppose that we register $10^5 e^+e^-$ as described above. Assuming $\bar{f} = 0.97$ (typical for existing detectors) we will miss the reconstruction of $\approx 3000$ tracks of each sign. A difference of 10% on these numbers is quite noticeable with enough statistical significance.

We do not see that it will be much of a problem to reach the level quoted. Therefore we feel that a long running period, which gives a small statistical error and thereby a small systematic one on $A_{\text{ch}}$, is fully justified: 100 days
would allow us to approach the 1% limit on $\Delta \sin^2 \theta / \sin^2 \theta$, and 200 days would allow us to get below it.

This discussion could be repeated for the case where one beam is longitudinally polarized with polarization $P_e \ (-60\%)$.

The formula giving the asymmetry (see G. Altarelli’s paper [5]) is now linear in $V$: $A_{\text{ch pol}} = 1.4 V P_e$. The asymmetry is substantially larger than without polarization ($-4$ times larger on top of the $Z^0$ for $\sin^2 \theta_W = 0.23$). Therefore less time is needed to reach the same relative statistical accuracy.

Since here

$$\frac{\Delta V}{V} = \frac{\Delta A_{\text{ch}}}{A_{\text{ch}}} - \frac{\Delta P_e}{P_e},$$

the same error $\Delta A_{\text{ch}}$ gives a smaller error on $\Delta V/V$ than in the previous case (half the amount for our choice). However, $\Delta P_e/P_e$ should also be kept below the 10% level and possibly reach 5%.

Finally, we recall that longitudinal polarization makes it possible to perform a different type of asymmetry measurement, which consists in measuring the difference in counting rate when the polarization is flipped. Any kind of final state can be used, with no need to distinguish between charges. The statistical and systematic errors (from the detector side) are much smaller, but knowledge of the polarization, of the possible secondary effects related to its manipulation, and of the normalization has to be good.

* * *

REFERENCES

[5] G. Altarelli, Precision tests of the electroweak theory at the $Z^0$, this report.
In this note we recall how to detect τ pairs at LEP (e⁺e⁻ → τ⁺τ⁻), and show that the τ helicity in the above process can be measured with an accuracy corresponding to an error of 0.002 on sin² θ_W. At this level, effects might show up, which signify new physics beyond the standard model. The derivation below is not new [1] nor is it approximation-free. For instance, the radiative corrections have not been taken into account. We nevertheless feel it is a realistic enough exercise which shows that, with LEP, precise tests of the standard model can be performed.

Tau pairs have long been detected at e⁺e⁻ colliders. The τ's are short-lived particles (their flight path at LEP will be several millimetres) which decay into low-charged-multiplicity final states. Table 1 lists their principal decays [2]. The branching ratios are quite consistent amongst the various experiments, although theory has some trouble reproducing inclusive branching ratios into one and three charged prongs (+ neutrals) [3].

In the detector, τ pairs appear as low-charged-multiplicity events. Since the τ leptons are quite energetic, their decay products are narrowly collimated around their parent directions. The topology of the events is thus: two almost

<table>
<thead>
<tr>
<th>τ decay mode</th>
<th>Branching ratio a) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>μνν</td>
<td>18.5 ± 1.1</td>
</tr>
<tr>
<td>eνν</td>
<td>16.5 ± 0.9</td>
</tr>
<tr>
<td>νν</td>
<td>10.3 ± 1.2</td>
</tr>
<tr>
<td>γν</td>
<td>22.1 ± 2.4</td>
</tr>
<tr>
<td>Kν</td>
<td>1.3 ± 0.5</td>
</tr>
<tr>
<td>1 charged (+ neutrals)</td>
<td>83.1 ± 2.5</td>
</tr>
<tr>
<td>3 charged (+ neutrals)</td>
<td>17.0 ± 1.3</td>
</tr>
</tbody>
</table>

a) from Ref. [2].
back-to-back jets, each containing one or three charged particles. Provided a two-track trigger can be efficiently implemented, more than 90% of the $\tau$ pairs should be detectable as 1-vs.-1, 1-vs.-3, or 3-vs.-3 events. We have discarded the events where both $\tau$'s decay into electrons or muons, since $\mu$ pairs or Bhabha events with initial-state radiation would represent a substantial contamination. The background is very small (it is already negligible at PEP and PETRA). Experimental cuts (e.g. for lepton tagging) will somewhat reduce the statistics, but not overwhelmingly so.

Now if a $\tau$ is longitudinally polarized, with value $P_L(\tau)$, the angular distributions of its decay products present asymmetries with respect to the spin direction in the rest frame. These asymmetries are reflected in the lab. frame energy spectra of the decay products. In other words, the lab. energy spectra of the $\tau$ decay products can be used to analyse the $\tau$ polarization.

For instance, the $\tau$ spectrum from the $\tau \rightarrow \pi \nu$ decays is given by

$$\frac{dN(\tau^\ast)}{dx} = 1 - P_L(\tau^\ast) (2x - 1),$$

where $x = E_\nu/E_{\text{beam}}$. Figure 1 shows the dependence of the $\tau$ polarization on $\sin^2 \theta_w$, whilst Fig. 2 gives a feeling of how the pion spectrum (1) depends on $\sin^2 \theta_w$.

The same behaviour is observed for the $\phi \nu$ final state, although with a reduced sensitivity due to the fact that, the $\phi$ being a spin-1 particle, there are more ways of sharing the $\tau$ spin among the final particles:

$$\frac{dN(\tau^\ast)}{dx} = 1 - 0.46 P_L(\tau^\ast) (2x - 1).$$

Fig. 1 Dependence of $\tau$ polarization on $\sin^2 \theta_w$

Fig. 2 Pion spectrum
The distortion of the shape of the lepton spectrum in the purely leptonic $\tau$ decays can also be used (Fig. 3):

$$\frac{dN(\tau^\prime)}{dx_1} = a(x_1) - P_L(\tau^\prime) b(x_1),$$

with

$$a(x_1) = \frac{5}{3} - 3x_1^2 + \frac{4}{3}x_1^3; \quad b(x_1) = \frac{1}{3} - 3x_1^2 + \frac{8}{3}x_1^3.$$

From an experimental point of view we have not only to consider the analysing power of a given decay mode but also to worry about statistics (the branching ratio) and systematics (mainly the background which remains owing to the difficulty in telling one decay mode from another).

In Table 2 we compare the potentials of the most promising decay modes. We also give the precision that can be expected on $\sin^2 \theta_W$ from these modes, for an exposure of 100 pb$^{-1}$.

**Table 2**

<table>
<thead>
<tr>
<th>Mode</th>
<th>No. of decays (x1000)</th>
<th>Visible decays (x1000)</th>
<th>Background</th>
<th>$\Delta \sin^2 \theta_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu^+\mu^-$</td>
<td>30</td>
<td>20</td>
<td>None</td>
<td>$6 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\mu^+e^-$</td>
<td>33</td>
<td>?</td>
<td>Easy to remove $\pi^0, \eta^\prime, K^0$</td>
<td>$&gt; 6 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td>13</td>
<td>5</td>
<td>$\omega \nu$</td>
<td>$2 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\nu$</td>
<td></td>
<td></td>
<td>Difficult to remove</td>
<td>$&gt; 10^{-2}$</td>
</tr>
</tbody>
</table>
average LEP detector. The detector used has $\Delta p_T/p_T^2 = 0.24$, a polar angle acceptance between 20° and 160°, the ability to tag leptons above 3 GeV/c, and an electromagnetic calorimeter providing an e/\mu separation of better than 500:1.

The best results are obtained for the $\nu\nu$ mode, the dominant factor being the analysing power. As an example, Fig. 4 shows the results of a simulation of the ALEPH detector, for an exposure of 10 pb$^{-1}$. The error on $\sin^2 \theta_W$, $\Delta \sin^2 \theta_W = 0.005$, is dominated by statistics. The main systematic error comes from those $\tau \to \phi \nu$ decays where the $\phi^0$ from the $\phi$ decay escapes detection. We estimate the corresponding level of systematic uncertainty to $\Delta \sin^2 \theta_W = 0.002$. Hence an overall precision of $\Delta \sin^2 \theta_W = 0.001$ can be obtained with a 100 pb$^{-1}$ exposure at LEP.

As concerns the decay $\tau_L \to K^0 S (\phi L) \nu$ mentioned in the literature [1], it is not a problem for the LEP detectors, with their hadronic calorimeters. Although less precise, the results from the leptonic modes yield valuable cross-checks for beating down the systematic errors. The $\phi \nu$ decay mode is quite an experimental challenge. It has the highest branching ratio, but the analysing power is poor and there are many backgrounds which are difficult to remove.

We have explained how it will be possible with LEP and unpolarized beams to determine the electroweak mixing parameter $\sin^2 \theta_W$ to a precision enabling non-trivial tests of the standard model: this is by measuring the $\tau$ polarization in the reaction $e^+ e^- \to \tau^+ \tau^-$ at the $Z^0$, using the $\tau \to \nu \nu$ decay mode. The experimental error is limited more by statistics than by systematic uncertainties. The systematics can be cross-checked by comparing the different decay modes of the $\tau$'s.
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   G. Goggi, ibid., p. 483.


Known results for e.m. radiative effects at LEP/SLC energies are reviewed. The relevance of higher-order terms for precision measurements of the $Z^0$ mass and width is discussed.

Precision tests of the electroweak theory at LEP/SLC energies require an accuracy at the level of 1% and therefore demand a correct treatment of e.m. radiative corrections [1]. Indeed, first-order corrections, for example, reduce the $Z$ peak cross-section by more than 50%, or shift the zero in the $\mu$ forward-backward asymmetry by about $(\pm 360)$ MeV, for an energy resolution of $(10^{-1}-10^{-2})$. It is therefore important that high-order corrections are properly taken into account if the $Z$ mass and width have to be measured to an accuracy of the order of 50 MeV.

Previous studies of these effects have been presented over the past few years [2, 3]. In this note we briefly update the theoretical results for the reaction $e^+e^- \rightarrow \mu^+\mu^-$. A more complete discussion of this, and also of other processes of immediate experimental interest, will be presented elsewhere [4].

Our considerations apply to a typical experiment in which the following requirements are satisfied:

i) The muons should be back-to-back within a certain acollinearity angle $\theta$ of a few degrees ($\theta \lesssim 5^\circ$). The energy resolution $\Delta \omega$ depends upon $\theta$ [see Eq. (1)].

ii) An electromagnetic calorimeter of finite and small angular resolution $\delta$ is centred along the muon direction. In principle it does not discriminate between a charged particle and the accompanying collinear photons.

Then, using (i) and (ii), one would be sure that all but a fraction $\Lambda = \Delta \omega/E$ of the beam energy ($s = 2E$) is taken by the muons and the accompanying hard photons. For small $\Lambda$ and $\delta$, fully analytic expressions can be used, neglecting hard-photon effects of order $[(\alpha/s)\Lambda, (\alpha/s)\delta]$. On the contrary, all double logarithmic terms of the form

\[ \frac{\alpha}{\pi} \ln \left( \frac{s}{\Delta m_e^2} \right) \ln (\Lambda, \Gamma/m), \quad \frac{\alpha}{\pi} \ln \delta^2 \ln \Lambda, \]
where $m$ and $\Gamma$ are the mass and the width of the Z, or simple logs such as

$$\frac{\alpha}{\pi} \ln \left( \frac{S-\frac{\alpha}{\pi}}{e} \right), \quad \frac{\alpha}{\pi} \ln (\Delta, \Gamma/m, \delta^2)$$

can be resummed to all orders, using known results on the exponentiation of the infrared and mass singularities.

For a given acollinearity angle $J$, the maximum energy $k_{\text{max}}$ taken by undetected soft photons, which defines the energy resolution $\Delta \omega$, is given by

$$k_{\text{max}} = \Delta \omega = \frac{f_s}{1 + \cos J} \left[ (1 - \cos J) + 2 \left( (1 - \cos J) \left( \frac{m^2}{\mu} \right) \left( 1 + \cos J \right) \right)^{1/2} \right].$$  \quad (1)

Then for $J = 1^\circ$, $3^\circ$, and $5^\circ$, one obtains $\Delta = \Delta \omega/E = (1.7)\%$, $(5.1)\%$ and $(8.3)\%$, respectively.

We define, as usual,

$$\beta_{e,\mu} = \frac{2\alpha}{\pi} \left[ \ln \left( \frac{S-\frac{\alpha}{\pi}}{e} \right) - 1 \right] \quad \text{and} \quad \beta_{\text{int}} = \frac{4\alpha}{\pi} \ln (\operatorname{tg} \theta/2),$$

where $\theta$ is the c.m. scattering angle. The first-order radiative corrections to the Born cross-sections, which include vertex, vacuum polarization, and box diagrams, and soft-photon bremsstrahlung give, using the notation of Ref. [2],

$$\frac{d\sigma}{d\Omega} = \left. \left( \frac{d\sigma_{\text{QED}}}{d\Omega} \right) \right|_0 \left[ 1 + (\beta_{e,\mu} + 2\beta_{\text{int}}) \ln \Delta + c^{\text{QED}}_F \right]$$

$$+ \sum_{i=V, A, VA} \left. \left( \frac{d\sigma_{\text{int},i}}{d\Omega} \right) \right|_0 \left[ 1 + (\beta_{e,\mu} + \beta_{\text{int}}) \ln \Delta \right.$$  

$$\left. + \frac{1}{\cos \delta_R} \operatorname{Re} \left[ e^{i\delta_R} (\beta_{e,\mu} \ln \bar{E} + \beta_{\text{int}} \ln \bar{I}) + c^{\text{int},i}_F \right] \right]$$

$$+ \left. \left( \frac{d\sigma_{\text{res}}}{d\Omega} \right) \right|_0 \left[ 1 + \beta_{\mu} \ln \Delta + \beta_{e,\mu} \ln |\bar{E}| + 2\beta_{\text{int}} \ln |\bar{I}| \right.$$  

$$\left. - \beta_{e,\mu} \delta(s, \Delta \omega) \cot \delta_R + c^{\text{res}}_F \right],$$  \quad (2)

where

$$(m^2 - i\Gamma - s)^{-1} = e^{i\delta_R} \sin \delta_R (m^2)^{-1},$$
\[ E = \frac{\Delta}{1 + \Delta \frac{s}{\sin \delta_R}}, \quad I = \frac{\Delta}{\Delta + \frac{m_R^2}{s} e^{-i\delta_R / \sin \delta_R}} \]  

(3)

And

\[ C_{F,\text{QED}}^0 = \frac{3}{4} (\beta_e + \beta_\mu) + \hat{C}_{\text{tot}} \theta_{\text{VP}} + \frac{2a}{w} \left( \frac{x^2}{3} - \frac{1}{2} \right) + x_Y, \]

\[ C_{F,\text{int},i}^0 = \frac{3}{4} (\beta_e + \beta_\mu) + \frac{1}{2} \hat{C}_{\text{tot}} \theta_{\text{VP}} + \frac{2a}{w} \left( \frac{x^2}{3} - \frac{1}{2} \right) + x_i^i, \quad (i = V, A, VA), \]

\[ C_{F,\text{res}}^0 = \frac{3}{4} (\beta_e + \beta_\mu) + \frac{2a}{w} \left( \frac{x^2}{3} - \frac{1}{2} \right) + f(Y_V, Y_A). \]  

(4)

For the sake of simplicity, in the above equation we have not reported the explicit form of \( \delta(s, \Delta \omega), x_i^i \), and \( f(Y_V, Y_A) \); they can be found in Ref. [2]. Furthermore, \( \delta_{\text{tot}}^{\text{VP}} \) refers to the full vacuum polarization correction, which includes all leptons and hadrons.

In calorimetric-type measurements, where collinear hard radiation \( (k > \Delta \omega) \) from the final particles is detected within a small cone of half opening angle \( \delta \), one has to add the following correction factor [5-7] to each term in the r.h.s. of Eq. (2):

\[ \delta_{\text{coll}} = \delta_0^0 \frac{4a}{w} \left[ \ln \left( \frac{E}{\Delta \omega} - \frac{3}{4} \right) \ln \left( \frac{E_0}{m_\mu} \right) - \frac{1}{2} \ln \left( \frac{E}{\Delta \omega} \right) + \frac{1}{2} \left( \frac{9}{4} - \frac{x^2}{3} \right) \right]. \]  

(5)

Then in agreement with the Kinoshita-Lee-Nauenberg theorem on the mass singularities [8], the \( m_\mu \)-dependence disappears after adding Eq. (5) to Eq. (2) and the overall correction factor to the Born cross-sections can be simply obtained from Eq. (2) by the substitution

\[ \beta_\mu \left( \ln \Delta + \frac{3}{4} \right) + \frac{2a}{w} \left[ \ln \frac{4}{\delta^2} \left( \ln \Delta + \frac{3}{4} \right) + \left( \frac{3}{2} - \frac{x^2}{3} \right) \right]. \]  

(6)

From the known results on the exponentiation of soft and collinear divergences [9], one then obtains the final result,

\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma_{\text{QED}}}{d\Omega} \right)_0 \left[ \frac{\theta^{E} + 2\theta_{\text{int},i}^{\mu} + \beta_0}{\Delta} \right] + \sum_{i=V,A,VA} \left( \frac{d\sigma_{\text{int},i}}{d\Omega} \right)_0 \]

\[ \times \left[ \frac{\beta_0 + \beta_{\text{int},i}}{\cos \delta_R} \left( \frac{1}{e^{i\delta_R / \sin \delta_R}} - \frac{e^{-i\delta_R}}{\hat{C}_F^0} \right) \right] \]

\[ + \left( \frac{d\sigma_{\text{res}}}{d\Omega} \right)_0 \left[ \theta^{E} \theta^{E} \left[ \frac{2\beta_0}{\Delta} \right] - \beta_0 \delta(s, \Delta \omega) \cot \delta_R + \hat{C}_F^0 \right]. \]  

(7)
Fig. 1  a) Cross-section integrated on the scattering angle $\theta$, for $\Delta = 10^{-1}$ and $\delta \leq 1'$. The electroweak parameters are $m = 92$ GeV, $\Gamma = 2.9$ GeV, and $\sin^2 \theta_w = 0.23$. Dashed curve: Born cross-section; dot-dashed curve: first-order correction; full curve: all-orders correction.  
b) Same as (a) for $\Delta = 10^{-2}$.

Fig. 2 Integrated forward-backward asymmetry for $\sqrt{s} - m$. The notation is the same as that of Fig. 1a.
where

\[ \beta_6 = \frac{2a}{w} \ln \left( \frac{4}{\delta^2} \right) \quad \text{and} \quad C_F^{(j)} = C_F^{(j)} (\beta_\mu + \beta_6) + \frac{2a}{w} \left( \frac{3}{2} - \frac{s^2}{3} \right). \]

The contribution of the muon loop to the vacuum polarization factor \( \delta_{\text{vp}}^{\text{tot}} \) is kept unchanged.

The effect of higher-order terms on the total cross-section is shown in Figs. 1a, b for \( \delta = 1 \) and \( \Delta = 0.1 \) and 0.01, respectively, and compared with the Born terms and first-order corrections. Both the size and the position of the peak are clearly affected. Reasonable changes in \( \delta \) do not appreciably alter the result.

In Fig. 2 the forward-backward asymmetry is also shown for the same choice of the parameters. Notice the shift of the zero, which strongly depends on the value of \( \Delta \). Clearly, a precise determination of the value of \( m \) is quite sensitive to the energy resolution of the experiments.

To conclude: precise measurements of the mass and the width of the neutral weak boson with LEP/SLC experiments depend crucially on a correct treatment of e.m. radiative effects. To achieve this, higher-order terms have to be compulsorily taken into account.

REFERENCES

[9] See, for example, Ref. [6] and references therein.
The large number of $Z^0$'s that will be produced at LEP opens up the possibility of detecting other interesting decays besides $Z^0 \rightarrow f\bar{f}$. Here we collect together a number of results obtained by several authors about these rare decays. All the processes are studied in the framework of the standard model, with one Higgs doublet and three generations of leptons and quarks. Where possible, the numerical calculations have been redone with $\sin^2 \theta_W = 0.23$ and $m_Z = 92$ GeV.

$Z^0 \rightarrow H^0 \mu^+\mu^-$

The decay proceeds mainly via the diagram of Fig. 1; the rate has been evaluated in Ref. [1]. The result for the differential rate is

$$\frac{1}{\Gamma(Z^0 \rightarrow \mu^+\mu^-)} \frac{d\Gamma(Z^0 \rightarrow H^0\mu^+\mu^-)}{dx} = \frac{\alpha}{4\pi \sin^2 \theta_W \cos^2 \theta_W} \times$$

$$\left[ 1 - x + \frac{x^2}{12} + \frac{2}{3} \frac{m_H^2}{m_Z^2} \left( x^2 - 4 \frac{m_H^2}{m_Z^2} \right)^{1/2} \right] \times$$

$$\left( x - \frac{m_H^2}{m_Z^2} \right)^{1/2},$$

(1)

where $x = 2p_H/m_Z$, and the kinematical limits are

$$\frac{2m_H}{m_Z} < x < 1 + \frac{m_H^2}{m_Z^2}.$$
Table 1
Number of events of $Z^0 \rightarrow H^0 \mu^+ \mu^-$ for $10^6 Z^0$

<table>
<thead>
<tr>
<th>Mass of Higgs (GeV)</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of events</td>
<td>62.2</td>
<td>38.0</td>
<td>24.1</td>
<td>15.4</td>
<td>9.8</td>
<td>6.2</td>
<td>3.8</td>
<td>2.3</td>
<td>1.3</td>
<td>0.7</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Fig. 2 Relative rate of $Z^0 \rightarrow H^0 \mu^+ \mu^-$ for different values of the Higgs mass. The dashed line is obtained with $\sin^2 \theta_W = 0.22$ and $m_Z = 93.8$ GeV.

In Fig. 2 we plot the integrated rate relative to $\Gamma(Z^0 \rightarrow \mu^+ \mu^-)$ for different values of $m_H$. The dashed curve is obtained with $\sin^2 \theta_W = 0.22$ and $m_Z = 93.8$.

Table 1 gives the number of events per $10^6 Z^0$. These results seem to indicate the possibility of detecting the Higgs boson in $Z^0$ decay if $m_H \lesssim 50$ GeV.

$Z^0 \rightarrow H^0 \gamma$

This two-body decay occurs through the one-loop coupling of Fig. 3. We should remember that the similar decay $Z^0 \rightarrow \gamma \gamma$ is forbidden because two photons cannot have a total angular momentum equal to 1. The rate has been evaluated in Eq. (2), and the result is

$$\frac{\Gamma(Z^0 \rightarrow H^0 \gamma)}{\Gamma(Z^0 \rightarrow \mu^+ \mu^-)} = \frac{\alpha^2}{\pi^2 \sin^2 \theta_W} \left( \frac{E_Y}{m_Z} \right)^3 \frac{A^2}{1 + (1 - 4 \sin^2 \theta_W)^2} = 2.33 \times 10^{-5} \left( \frac{E_Y}{m_Z} \right)^3 A^2.$$

}\ (2)
Fig. 3 The fermion and W loops in the decay $Z^0 \rightarrow H^0 \gamma$

The contribution to $A$ of the fermion loop is given by

$$A_F = \frac{-2 Q_F (T^3_L - 2 Q_F \sin^2 \theta_W)}{\cos \theta_W} \int_0^1 dx \int_0^{1-x} dy \frac{(4xy-1)m_F^2}{m_F^2 - y(1-y) + xy(m_Z^2 - m_H^2)},$$

(3)

where $Q_F$ is the charge and $T^3_L$ is the third component of the isospin of the fermion. If the fermion is light, the contribution is proportional to $m_F^2/m_W^2$ and can be ignored. If $m_F \rightarrow \infty$, $A_F$ approaches the value

$$A_F \rightarrow \frac{2}{3} Q_F \frac{(T^3_L - 2 Q_F \sin^2 \theta_W)}{\cos \theta_W}.$$

So $A_F$ (heavy lepton) $\approx 0.03$, $A_F$ (heavy up) $\approx 0.10$, $A_F$ (heavy down) $\approx 0.09$.

The contribution of the W loop is

$$A_w = -4 \cos \theta_W m_w^2 \int_0^1 dx \int_0^{1-x} dy \left[ (3 - \tan^2 \theta_W) + xy \left[ 1 + \frac{m_H^2}{2m_w^2} \right] \tan^2 \theta_W ight. \left. - \left( \frac{m_H^2}{2m_w^2} \right)^2 \right] \times \left[ m_Z^2 - y(1-y)m_Z^2 + xy(m_Z^2 - m_H^2) \right]^{-1}.$$

(4)

Expanding in powers of $m_H^2/m_w^2$,

$$A_w = - \left[ 4.5 + 0.25 \frac{m_H^2}{m_w^2} \right].$$

The two amplitudes have different signs, but the W contribution is much larger than the fermion-loop term even for a heavy-fermion generation. Then, neglecting the fermion contribution, the total rate is

$$\frac{\Gamma(Z^0 \rightarrow H\gamma)}{\Gamma(Z^0 \rightarrow \mu^+\mu^-)} = 5.9 \times 10^{-5} \left( 1 - \frac{m_H^2}{m_Z^2} \right)^3 \left( 1 + 0.11 \frac{m_H^2}{m_Z^2} \right).$$
Fig. 4 Relative rate of $Z^0 \rightarrow H^0 \gamma$ for different values of the Higgs mass

Table 2

<table>
<thead>
<tr>
<th>Mass of Higgs (GeV)</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of events</td>
<td>1.7</td>
<td>1.6</td>
<td>1.5</td>
<td>1.4</td>
<td>1.3</td>
<td>1.1</td>
<td>1.0</td>
<td>0.8</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The numerical results are shown in Fig. 4, where for comparison the rate $\Gamma(Z \rightarrow H^0 \mu^+ \mu^-)$ is also plotted. Table 2 gives the number of events per $10^6 Z^0$.

$Z^0 \rightarrow W^+ l^- \nu, \quad Z^0 \rightarrow W^- q_{-1/3} \bar{q}_{2/3}$

The possibility of detecting $W$ vector bosons in $Z^0$ decay has been considered in Ref. [3]. The diagrams involved are shown in Fig. 5. The total rate, setting the masses of the fermions equal to zero, is

$$\frac{\Gamma(Z^0 \rightarrow W^+ l^- \nu)}{\Gamma(Z^0 \rightarrow \mu^+ \mu^-)} = \frac{\alpha \cot^2 \theta_W}{8\pi(1-4s^2+8s^4)} \left[ \frac{2-4s^2+4s^4}{c^2} H_1 + \frac{c^2}{c^2} H_2 + \frac{4s^2-2}{c^2} H_3 - 4c^2 H_4 \right].$$ (5)
In Eq. (5), \( s^2 = \sin^2 \theta_w \), \( c^2 = \cos^2 \theta_w \), and

\[
H_1 = \int \frac{1 + \cos^2 \theta_w}{2 \cos \theta_w} \ h_1(x) \ dx ,
\]

where

\[
h_1(x) = 2 \ x \ \log \frac{x + \sqrt{x^2 - 4c^2}}{x - \sqrt{x^2 - 4c^2}} + \sqrt{x^2 - 4c^2}\left( \frac{1}{c^2} + \frac{c^2}{x^2} - \frac{x}{c^2} + \frac{x^2}{12c^2} - \frac{7}{3} \right),
\]

\[
h_2(x) = \frac{4(x^2 - 4c^2)}{(1-x^2)^2} \left( \frac{1}{c^2} + \frac{c^2}{x^2} - \frac{x}{c^2} + \frac{x^2}{12c^2} + \frac{8}{3} \right),
\]

\[
h_3(x) = \frac{4(1+c^2-x)(1+c^2)}{x} \ \log \frac{x + \sqrt{x^2 - 4c^2}}{x - \sqrt{x^2 - 4c^2}}
- \sqrt{x^2 - 4c^2}\left( \frac{1}{c^2} + \frac{c^2}{x^2} - \frac{x}{c^2} + \frac{x^2}{12c^2} + \frac{5}{3} \right),
\]

\[
h_4(x) = \left[ \frac{4(1+c^2-x)(2+c^2)}{1-x} - 4 \right] \log \frac{x + \sqrt{x^2 - 4c^2}}{x - \sqrt{x^2 - 4c^2}}
- \frac{2x}{1-x} \frac{x^2 - 4c^2}{(1/c^2 + c^2 - x - x/c^2 + x^2/12c^2 + 8/3).}
\]

The numerical result is

\[
\Gamma(Z^0 \rightarrow W^+ \nu) = 2.4 \times 10^{-7}.
\]

\[
\Gamma(Z^0 \rightarrow \mu^+ \mu^-) = \Gamma(Z^0 \rightarrow e^+ e^-).
\]
A similar formula also holds for $\Gamma(z^0 + W^+ q_{-1/3} \bar{q}_{2/3})$:

$$\frac{\Gamma(z^0 + W^+ q_{-1/3} \bar{q}_{2/3})}{\Gamma(z^0 + \mu^+ \mu^-)} = \frac{3\alpha \cot^2 \theta_W |K|}{8\pi(1-4s^2+8s^4)} \left( \frac{2 - 4s^2 + (20/9)s^4}{c^2} H_1 + c^2 H_2 + \right.$$ 

$$+ \frac{4s^2 - 2 - (16/9)s^4}{c^2} H_3 - 4c^2 H_4 \right) .$$

(7)

Adding up the two rates (ignoring the $b,t$ doublet), we obtain

$$\frac{\Gamma(z^0 + W + \text{anything})}{\Gamma(z^0 + \mu^+ \mu^-)} \leq 2.1 \times 10^{-6} .$$

$z^0 \rightarrow 2S+1 \ell J + \gamma, \ z^0 \rightarrow 2S+1 \ell J + H$

This process, the radiative or Higgs decay of the $z^0$ in a quark-antiquark system, has been considered in Ref. [4]. Using the notation

$$R_{\gamma, H}(2S+1 \ell J) = \frac{\Gamma(z^0 \rightarrow 2S+1 \ell J + \gamma)}{\Gamma(z^0 \rightarrow \mu^+ \mu^-)} ,$$

we only report the results for $R_{\gamma}(3S_1)$ and $R_{H}(1S_0)$, which are the most prominent modes:

$$R_{\gamma}(3S_1) = C_\gamma A \frac{m^2}{m_Z^2} \left( 1 - \frac{m^4}{m_Z^4} \right) ,$$

$$R_{H}(1S_0) = C_H A \left[ \left( 1 - \frac{m^2}{m_Z^2} \right)^2 - \frac{m_H^2}{m_Z^2} \left( 2 + 2 \frac{m^2}{m_Z^2} - \frac{m_H^2}{m_Z^2} \right) \right]^{3/2} .$$

(8)

Here $m$ is the mass of the onia, $m_H$ is the Higgs mass, and

$$A = \frac{1}{1 + (1 - 4 \sin^2 \theta_W)^2} ,$$

$$C_\gamma = \frac{24\alpha \ell^2 R_0^2(0)}{F_0^2} m^{-3} ,$$

$$C_H = \frac{3G_F R_0^2(0)}{2/2\pi \ m} .$$
In order to estimate $R_0(0)$, the value of the wave function at the origin of the onia, one can use the formula

$$\Gamma(3S_1 \rightarrow e^+e^-) = 4\alpha^2 Q^2 F_0^2(0) m^{-2}$$

and the experimental regularity in all known vector mesons, $\Gamma e^+e^-/Q^2 \lesssim 10^{-5}$ GeV. One obtains the estimate

$$R_0^2(0) = \frac{m^2}{4\alpha^2} \frac{\Gamma e^+e^-}{Q^2} \approx 0.05 m^2 \text{ (GeV)^3}.$$  

So

$$C_\gamma = \frac{6.2 \times 10^{-3}}{m \text{ (GeV)}} Q^2 \quad \text{and} \quad C_H = 1.8 \times 10^{-7} m \text{ (GeV)}.$$  

Other estimates based on specific potential models can give an answer about an order of magnitude larger or smaller. Considering an onium with $Q_F = 2/3$ and $m = 60$ GeV, the results are

$$R_\gamma(3S_1) \approx 2.1 \times 10^{-5}.$$  

In Fig. 6 we plot $R_\gamma(^1S_0)$ for different values of $m_H$.

Fig. 6 Relative rate of $Z^0 + ^1S_0 + H^0$ for different values of the Higgs mass. The quark in the onia has a charge $Q_F = 2/3$ and a mass of 30 GeV.

$Z^0 \rightarrow l\bar{l}qq$

Four-body $Z^0$ decay is considered in Ref. [5]. The evaluation of this rate may be useful for estimating the background of other processes, such as $Z^0 \rightarrow H^0 l^+l^-$. The authors [5] do not report final formulae but give many figures...
Fig. 7 Diagrams involved in the decay $Z^0 \rightarrow q\bar{q}l\bar{l}$

Table 3
Numerical Results

<table>
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<tr>
<th>q</th>
<th>l</th>
<th>e</th>
<th>μ</th>
<th>τ</th>
</tr>
</thead>
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<tr>
<td>u</td>
<td>$7.5 \times 10^{-4}$</td>
<td>$1.5 \times 10^{-4}$</td>
<td>$8.1 \times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.6 $\times 10^{-4}$)</td>
<td>(1.3 $\times 10^{-4}$)</td>
<td>(7.1 $\times 10^{-5}$)</td>
<td></td>
</tr>
<tr>
<td>d, s</td>
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</tr>
<tr>
<td></td>
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<td>(2.7 $\times 10^{-5}$)</td>
<td></td>
</tr>
<tr>
<td>c</td>
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<td>$2.8 \times 10^{-5}$</td>
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</tr>
<tr>
<td></td>
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<td>(9 $\times 10^{-5}$)</td>
<td>(2.6 $\times 10^{-5}$)</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>$1.2 \times 10^{-4}$</td>
<td>$2.5 \times 10^{-5}$</td>
<td>$3.9 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7 $\times 10^{-5}$)</td>
<td>(2.2 $\times 10^{-5}$)</td>
<td>(3.7 $\times 10^{-6}$)</td>
<td></td>
</tr>
<tr>
<td>t</td>
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</tr>
<tr>
<td>$m_t = 30$ GeV</td>
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<td>(7.1 $\times 10^{-6}$)</td>
<td>(8.6 $\times 10^{-7}$)</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>$1.2 \times 10^{-6}$</td>
<td>$1.2 \times 10^{-7}$</td>
<td>$2.7 \times 10^{-9}$</td>
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</tr>
<tr>
<td>$m_t = 42$ GeV</td>
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<td>(1.1 $\times 10^{-7}$)</td>
<td>(2.7 $\times 10^{-9}$)</td>
<td></td>
</tr>
</tbody>
</table>

for the distributions of the dilepton and diquark masses squared and other quantities. The diagrams involved are shown in Fig. 7.

In Table 3 the numerical results are reported for the total rate relative to $f(Z^0 + \mu^+\mu^-)$, for different combinations of leptons and quarks. The values of the relevant parameters are $\sin^2 \theta_W = 0.22$ and $m_Z = 95$ GeV. The numbers in parentheses are the relative widths corresponding to cuts specified by cutting the lepton opening angle at $> 1$ mrad and imposing an energy greater than 1 GeV.
$Z^0 \to gg\gamma, \ Z^0 \to ggg$

The rate of this decay has been calculated in Ref. [6]. The calculation is quite complicated, so we report the numerical results given by the authors, obtained with $m_t = 20$ GeV, $\sin^2 \theta_W = 0.23$, $\alpha_s = 0.17$.

The decays occur through the diagrams of Fig. 8. The authors consider only the vector couplings of the $Z^0$ in the loop. For the decay $Z^0 \to gg\gamma$ this is all, but in the case where $Z^0 \to ggg$ there is also a contribution from the axial couplings, proportional to the mass of the fermion in the loop. In any case the two contributions add incoherently, so that a lower bound is obtained.

![Fig. 8 Diagrams involved in the decays $Z^0 \to gg\gamma$ and $Z^0 \to ggg$](image)

The numerical results are given by

$$\frac{\Gamma(Z^0 \to gg\gamma)}{\Gamma(Z^0 \to \mu^+\mu^-)} = 1.4 \times 10^{-5}.$$  

(If the mass of the top is set equal to zero, the rate is a factor of two larger.)

$$\frac{\Gamma(Z^0 \to ggg)}{\Gamma(Z^0 \to \mu^+\mu^-)} > 6.1 \times 10^{-5}.$$  

In this case if $m_t = 0$, the result is a factor of three smaller.

In conclusion, we see that all these decays have very small branching ratios and that, with the possible exception of the Higgs search, it appears difficult to observe them.

I wish to thank G. Altarelli for useful discussions, and the Theory Division of CERN for its hospitality.
REFERENCES

NEUTRINO COUNTING BY MEASURING $\sigma(e^+e^- \rightarrow \gamma \nu\bar{\nu})$ AT LEP

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One of the most important experiments to be carried out at LEP is the measurement of $Z^0$ decay into weakly coupled objects with mass lower than $m_Z/2$. In more conventional terms, this experiment looks for neutrinos beyond the three generations which are already known.

One method to measure the partial decay width of the $Z^0$ into neutrinos (or neutrino-like objects), $\Gamma_{\nu\bar{\nu}}$, is to determine the cross-section for radiative $Z^0$ production with subsequent $Z^0$ decay into neutrinos:

$$e^+e^- \rightarrow Z^0 \gamma \rightarrow \nu\bar{\nu}.$$ 

The signature of the process is a single photon in the final state, recoiling against a system which has the mass of the $Z^0$. The experimental challenge is to trigger upon, and to identify, a clean sample of such events where a single photon of a few GeV only is observable. LEP must be operated at a centre-of-mass energy somewhat above the $Z^0$ resonance. The minimal machine energy is defined by the smallest photon energy which the detector can trigger upon.

The dominant background is radiative Bhabha scattering:

$$e^+e^- \rightarrow e^+e^-\gamma,$$

where the final-state electron pair remains within the beam tube or near to it, and thus escapes detection. Other backgrounds appear negligible provided the detector is capable of recognizing the absence of accompanying charged or neutral particles. All LEP detectors are expected to have no problem with that by virtue of their 4\pi calorimetry and tracking devices.

This study is based on earlier work by Barbiellini et al. [1]. The cross-section of $e^+e^- \rightarrow \gamma \nu\bar{\nu}$ is in the electroweak theory given by [2]:

$$\frac{d^2\sigma}{dx dy} = \frac{G^2_s}{\pi} \frac{\cos(1-x) \left( [1 - (x/2)]^2 + (x^2 y^2 / 4) \right)}{6 x (1 - y^2)} \times \left[ 2 + \frac{(N_c/4)(v_e^2 + a_e^2) + (v_e + a_e) \left( 1 - [s(1 - x)/m_Z^2] \right)}{(1 - [s(1 - x)/m_Z^2]) + (\Gamma_{\text{tot}}/m_Z^2)} \right],$$

(1)
where \( x = 2E_\gamma/s \) and \( y = \cos \theta_\gamma \), with \( \theta_\gamma \) denoting the photon emission angle with respect to the incident-beam direction. \( N_\nu \) denotes the number of neutrinos with mass less than \( m_\nu/2 \) (\( N_\nu = 3 \) according to the present dogma), and \( v_e, a_e \) denote the electron vector and axial-vector coupling constants: \( v_e = -1 + 4 \sin^2 \theta_W \), \( a_e = -1 \). In all calculations, we assume \( \sin^2 \theta_W = 0.23 \) and \( m_e = 92 \text{ GeV} \).

To evaluate the precision of this experiment, we have assumed that the standard scan is made across the \( Z^0 \) resonance: 13 equidistant steps of 2 GeV, ranging from \( s = 80 \text{ GeV} \) to \( s = 104 \text{ GeV} \). Of course, only centre-of-mass energies above the \( Z^0 \) resonance are useful for our purpose. We assume, further, a constant luminosity \( L = 10^{31} \text{ cm}^2 \text{ s}^{-1} \), and two days (100%) running at each point.

Figure 1 shows the cross-section integrated over \( \gamma \) (20° < \( \theta_\gamma < 160° \)), as a function of the photon energy, for various values of the centre-of-mass energies.

![Photon Energy Spectrum](image)

Fig. 1 Cross-section of \( e^+e^- \rightarrow \bar{\nu}\nu \) as a function of \( E_F \), integrated over all photon emission angles in the range 20° < \( \theta_\gamma < 160° \), for various centre-of-mass energies.
Fig. 2 Differential cross-section of $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ for $\sqrt{s} = 104$ GeV, as a function of the photon emission angle $\theta_\gamma$, for $8.4 < E_\gamma < 13.6$ GeV. Also shown is the differential cross-section of radiative Bhabha scattering, with the same cuts as applied for the photon, and with electrons vetoed in the range $6^\circ < \theta_e < 174^\circ$.

of the scan. The figure shows at low energies the typical soft-photon divergence of radiative processes, and at higher photon energy a peak corresponding to the $Z^0$ resonance. Figure 2 shows the photon emission angle $\theta_\gamma$, for $\sqrt{s} = 104$ GeV and for photon energies in the range $8.4 < E_\gamma < 13.6$ GeV. The reaction peaks sharply at small photon emission angles with respect to the beam direction.

Table 1 summarizes the results of the proposed experiment. It gives, for all centre-of-mass energies above the $Z^0$ pole, the cross-sections and event numbers for a range of the photon energy corresponding to a recoil mass within $m_\gamma^2 = \pm 2.6$ GeV. The regions of $\theta_\gamma$ integration are $15^\circ < \theta_\gamma < 165^\circ$ and $25^\circ < \theta_\gamma < 155^\circ$.

To study the radiative Bhabha-scattering background, $e^+e^- \rightarrow e^+e^-\gamma$, we have used the respective Monte Carlo generation program of Berends and Kleiss [3]. This program included, however, only single-photon exchange, but no $Z^0$ exchange, leading to a slight underestimation of the cross-section of the background. The Monte Carlo program gave an integrated cross-section for the same range of photon energies as chosen above to signal the reaction $e^+e^- \rightarrow \nu\bar{\nu}\gamma$. For technical reasons, owing to a divergence at electron emission angles $\theta_e = 0^\circ$ or $180^\circ$, the integration was performed in the interval $2.5^\circ < \theta_e < 177.5^\circ$ only, with no correction for the remainder. It was assumed, further, that electrons are detected
### Table 1
Cross-sections and event numbers for the process $e^+e^- \to \nu\nu\gamma$, and cross-sections for the background reaction $e^+e^- \to e^+e^-\gamma$, for two regions of integration of the photon emission angle $\theta$. The event numbers refer to the running conditions of the scan experiment (see the text).

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>$E_\gamma$ (most probable value)</th>
<th>$\sigma$ (nb)</th>
<th>No. of events</th>
<th>$\sigma$ background (nb)</th>
<th>$\sigma$ (nb)</th>
<th>No. of events</th>
<th>$\sigma$ background (nb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>94</td>
<td>0</td>
<td>0.42</td>
<td>729</td>
<td>0.014</td>
<td>0.31</td>
<td>542</td>
<td></td>
</tr>
<tr>
<td>96</td>
<td>3.5</td>
<td>0.17</td>
<td>285</td>
<td>0.016</td>
<td>0.12</td>
<td>211</td>
<td>0.0008</td>
</tr>
<tr>
<td>98</td>
<td>5.5</td>
<td>0.088</td>
<td>152</td>
<td>0.006</td>
<td>0.065</td>
<td>112</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>7.4</td>
<td>0.063</td>
<td>109</td>
<td>0.004</td>
<td>0.047</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>102</td>
<td>9.2</td>
<td>0.051</td>
<td>88</td>
<td>0.002</td>
<td>0.038</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>104</td>
<td>11.0</td>
<td>0.041</td>
<td>71</td>
<td>0.001</td>
<td>0.030</td>
<td>52</td>
<td></td>
</tr>
</tbody>
</table>

From $\theta_e = 6^\circ$ to $174^\circ$, but are invisible below. Events were retained only if the photon had an emission angle and energy within the acceptance cuts of the signal, and the electrons were invisible.

The resulting cross-sections for the radiative Bhabha background are given in Table 1. Although these values have large statistical errors (due to limitation of computer time) they tend to fall significantly below the cross-section for $e^+e^- \to \nu\nu\gamma$.

Figure 2 shows the angular distribution of radiative Bhabha scattering with the same cuts on the photon energy and emission angle as for the signal in question. Electrons are vetoed in the interval $6^\circ < \theta_e < 174^\circ$. The isolation of the signal seems not to be difficult.

For $\sqrt{s} = 96$ GeV and above, the photon energy is $\gtrsim 3$ GeV. Hence, the efficient triggering on single-photon events (without being swamped by background) should pose no insurmountable problem. Taking, for example, the angular interval $25^\circ < \theta_\gamma < 155^\circ$ and centre-of-mass energies of 96 GeV and above, one obtains altogether 522 events, largely free of background.
If a fourth neutrino generation exists, the cross-section will be enhanced by 30%. Such an increase would, in the proposed $Z^0$ scan experiment, show up with a $>5\sigma$ statistical significance.

We conclude that a realistic scan around the $Z^0$ resonance would provide a highly significant measurement of the number of neutrino generations with mass less than $m^*_\nu/2$.

* * *

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TOPONIUM PHYSICS AT LEP

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1. **INTRODUCTION**

Since the discovery of the J/ψ [1] more than a decade ago, heavy quarkonia have played a central role in particle physics. The ψ and Y families [1, 2] of the (cc) and (bb) bound states have become the 'hydrogen atoms' of strong interactions: their spectroscopies provide direct evidence of the quark structure of hadrons, their decay patterns support the quark-gluon picture of perturbative quantum chromodynamics (QCD), and from some of their properties one can even obtain a quantitative determination of the QCD scale parameter Λ.

The standard gauge theory of strong and electroweak interactions predicts the existence of the t-quark, a sixth heavy quark, which should complete the third quark-lepton family. So far the search for the corresponding (t̅t) spectroscopy at PETRA and PEP has not been successful, and the present lower bound on the mass of the (t̅t) ground state has reached 46 GeV [3]. Present UA1 results [4] indicate a mass range from 60 GeV to 100 GeV for toponium states which will be accessible in the near future at SLC and LEP.

Toponium physics at e⁺e⁻ colliders has been extensively discussed in the literature. Earlier work can be found in Refs. [5] to [11]; interesting recent developments have been reviewed in Refs. [12] to [14]. Contrary to the ψ and Y spectroscopies, in the (t̅t) system the weak interactions will play a major role. Production and annihilation decays of toponium states are strongly affected by the Z boson, and furthermore the β-decay of the t-quark becomes increasingly important for t-quark masses above 30 GeV. On the experimental side, compared to quarkonium physics at SPEAR, DORIS, and CESR, the main challenge is the large spread in beam energy which decreases the height of the resonance peaks. As we will see, however, this problem is less severe than one may naively expect, and toponium physics at LEP energies appears to be very promising.

In this paper we study in detail to what extent toponium physics will be experimentally feasible at LEP. In Section 2 we recall the theoretical expectations for the (t̅t) system and give a collection of formulae on which calculations in the following sections are based. Section 3 deals with the scan for the toponium ground state and the spectroscopy of (t̅t) S-states (0) and P-states (χₜ). In Section 4 we examine the feasibility of measuring angular and polarization asymmetries in the vicinity of (t̅t) resonances, and in Section 5 we discuss the prospects for finding Higgs particles in toponium decays. Section 6 is devoted to supersymmetric decay modes of (t̅t) resonances, and in Section 7 we describe how the toponium physics program could be extended if the energy spread of the machine could be decreased. The summary and conclusions of our study are presented in Section 8. In the appendix radiative corrections to some of the formulae of Section 2 are listed.
2. QUARKONIUM PHYSICS AT LEP ENERGIES

The starting point of quantitative quarkonium spectroscopy is the choice of a quark-antiquark potential. The properties of the \( \phi \) and \( \Upsilon \) families are well described by 'QCD-like' potential models which incorporate the QCD predictions of a Coulomb singularity at short distances and linear confinement at long distances. The simplest example is the Cornell model [15] which is simply the sum of a linear term and a Coulomb term with constant \( \alpha_s \). More sophisticated versions include the dominant one-loop radiative corrections [16, 17] to the short-distance part which lead to a running coupling \( \alpha_s(r) \). A particularly simple and successful potential of this kind was suggested by Richardson [16]:

\[
V(r) = -\frac{4}{3} \frac{16\pi^2}{b_0} \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{q^2 \ln \left[ 1 + \left( q^2 / \Lambda^2 \right) \right]},
\]

(2.1)

where

\[
x = |\mathbf{x}|, \quad q = |\mathbf{q}|, \quad b_0 = 11 - \frac{2}{3} n_f,
\]

and \( n_f \) is the number of effective flavours. In order to relate the short-distance behaviour of the potential to a well-defined QCD scale parameter, e.g. \( \Lambda_{\overline{MS}} \), one has to incorporate the complete next-to-leading-order QCD corrections [18]:

\[
V(r) = -\frac{4}{3} \frac{1}{r} \left( \frac{4\pi}{b_0 \ln \left( \frac{\Lambda_{\overline{MS}}^2}{r^2} \right)} \right) \left[ 1 - \frac{b_1}{b_0^2} \ln \left( \frac{\Lambda_{\overline{MS}}^2}{r^2} \right) \right] \ln \frac{1}{\ln \left( \frac{\Lambda_{\overline{MS}}^2}{r^2} \right)} + C \ln \left( \frac{\Lambda_{\overline{MS}}^2}{r^2} \right) + O \left( \ln^2 \left( \frac{\Lambda_{\overline{MS}}^2}{r^2} \right) \right),
\]

(2.2)

\[
b_1 = 102 - \frac{38}{3} n_f, \quad C = \frac{1}{360} \left( 31 - \frac{10}{3} n_f \right) + 2\gamma_E,
\]

where \( \gamma_E = 0.5772... \) is Euler's constant. Contrary to the \( \phi \) and \( \Upsilon \) spectroscopies, the toponium system will probe the \( (Q\bar{Q}) \) potential at distances where it is expected to be described by the perturbative QCD expression (2.2).

Within the theoretical uncertainties, potential models with no resemblance to QCD, such as the logarithmic potential [19] or Martin's power potential [20],

\[
V(r) = A + B r^{0.1},
\]

(2.3)
work equally well as QCD-like models. The reason is that in the region of r probed by the \( \Phi \) and \( \Upsilon \) spectroscopies, all potentials agree numerically (see Fig. 2.1). The determination of the \((Q\bar{Q})\) potential at distances \( r < 0.1 \text{ fm} \) requires heavier quarkonia; the determination of the behaviour at distances beyond 1 \( \text{fm} \) may be possible by means of resonances above threshold. Indications in favour of QCD-like models with running coupling \( \alpha_s \) are their successful predictions of the centre-of-gravity mass of the \( \Upsilon \) 1P-states [16] and the \( \Upsilon \) leptonic width [18, 21], but unequivocal evidence of a Coulomb-like short-distance behaviour can only come from the \( (t\bar{t}) \) system.

The number of various toponium \( S \)-states below the threshold for pair production of \( T \) \((t\bar{q})\) mesons has been predicted as [23]

\[
n_0 = 2(\frac{m_t}{m_c})^{1/2} \quad . \tag{2.4}
\]

The position of the continuum threshold can be estimated, based on the charm and bottom spectroscopy and quark-mass differences which are independent of the normalization of the \((Q\bar{Q})\) potential [24]:
$$2m_T = m_W + 2(m_t - m_c) + 120 \text{ MeV} \quad , \quad (2.5a)$$

$$2m_T = m_Y + 2(m_t - m_b) + 225 \text{ MeV} \quad . \quad (2.5b)$$

From Eq. (2.4) we expect ten $S$-states below threshold for a $t$-quark with 40 GeV mass. Owing to its confining part the $(q\bar{q})$ potential is steeper than the Coulomb potential. Therefore the total number of narrow resonances $n_t$ is bounded from below by the corresponding number for the Coulomb potential,

$$n_t \geq 2n_0 + 4 \sum_{l=1}^{n_0-1} (n_0 - l) = 2n_0^2 , \quad (2.6)$$

where energy levels with $l = 0$ ($l > 0$) contribute 2 (4) states with total angular momentum 0 and 1 ($1-1$, 1, 1, 1+1).

For a 40 GeV $t$-quark there will be more than 200 resonances with a total width of less than 100 keV and angular momenta up to at least $l = 9$, a fantastic spectroscopy! Unfortunately, because of the large spread in beam energy, only the lowest few $S$-states and the $1P$ states appear to be experimentally accessible. This is different from the $\phi$ and $\Upsilon$ spectroscopies, where seven of the predicted eight narrow $(c\bar{c})$ states and nine of the predicted thirty narrow $(bb)$ states have been found up to now.

Figure 2.2 shows the size of the lowest $(t\bar{t})$ resonances as a function of $2m_t$. It is obvious that mass predictions of different potential models will

![Image of Figure 2.2: The sizes of the lowest toponium states as functions of $2m_t$.](image)
differ only for the 1S-, 2S-, and 1P-states. Extensive tables of mass predictions can be found in Refs. [12], [17], [21], [24], and [25]. The sensitivity of the first two S-states with respect to the behaviour of the potential $V(r)$ at distances below 0.1 fm can be inferred from Figs. 2.4 and 2.5, where the 1S-2S mass difference $E_2 - E_1$ and the leptonic width of the ground state are plotted for Martin's potential and two QCD-like potentials (see Fig. 2.3) with a short-distance behaviour corresponding to the QCD scale parameters $\Lambda_{\overline{MS}} = 200$ MeV and $\Lambda_{\overline{MS}} = 500$ MeV [see Eq. (2.2)]. The differences are large. For $2m_c > 60$ GeV the predictions for $E_2 - E_1$ differ by more than 100 MeV. Even more sensitive is the leptonic width of the ground state [for the effect of the $Z$ boson, see Eq. (2.14)],

$$\Gamma_0(\theta) = \frac{64\pi e^2_{\text{em}}}{9a_0^2} \left[ 1 - \frac{16}{3\pi} \alpha_s(m_\pi^2) \right] |\phi_0(0)|^2,$$

(2.7)

which varies from the power potential to the QCD model with $\Lambda_{\overline{MS}} = 500$ MeV by more than a factor of 3! Therefore, we expect that the toponium system will determine the $(Q\bar{Q})$ potential down to distances of at least 0.04 fm. This should provide direct evidence of asymptotic freedom, i.e. Coulomb behaviour of the potential at short distances with a running coupling strength $\alpha_s(r)$.

Figures 2.4 and 2.5 for the 1S-2S mass difference and the 1S leptonic width suggest that the $(t\bar{t})$ system may provide an accurate determination of the QCD scale parameter $\Lambda_{\overline{MS}}$ (see Ref. [26]) via the $(Q\bar{Q})$ potential. As emphasized in Ref. [21], the quantitative dependence of these quantities on $\Lambda_{\overline{MS}}$ has to be considered with caution. It is obtained by assuming that the absolute normalization of the QCD potential at short distances is known theoretically, and that also the $c$-, $b$-, and $t$-quark masses--and therefore the absolute normalization of the empirical $(Q\bar{Q})$ potential--are known. Given the normalization of the short-distance QCD potential, the quark masses are very sensitive to $\Lambda_{\overline{MS}}$. It has been estimated [21], for instance, that for a 60 GeV toponium system a measurement of the three-gluon partial width with an accuracy of 20% could determine the quark masses up to $\pm 150$ MeV. We thus hope that after the discovery of toponium the combined analysis of the $(Q\bar{Q})$ potential and $(t\bar{t})$ decays will lead to a quantitative test of QCD.

The 1S, 1P, and 2S $(t\bar{t})$ states will determine the $(Q\bar{Q})$ potential at distances below 0.1 fm. Also important is the test of flavour independence of the potential, which will result from the measurement of masses and leptonic widths of higher radial excitations. The existence of a flavour-independent potential describing the $(c\bar{c}), (b\bar{b})$, and $(t\bar{t})$ spectroscopies would justify a posteriori the validity of potential models [27] for the $\phi$ and $\Upsilon$ systems which has been seriously questioned in connection with the QCD sum-rule approach.
Fig. 2.3 Martin's power law potential and two QCD-like potentials with $\Lambda_{\overline{MS}} = 200$ MeV and $\Lambda_{\overline{MS}} = 500$ MeV at short distances. The mean square radius of the toponium ground state is shown for different values of $2m_t$. From Ref. [21].

Fig. 2.4 1S-2S mass difference as a function of $2m_t$ for the three potentials of Fig. 2.3. From Ref. [21].

Fig. 2.5 The leptonic width of the ground state (without $Z^0$ contribution) as a function of $2m_t$ for the three potentials of Fig. 2.3. From Ref. [21].
The splitting of toponium energy levels caused by spin-dependent forces is a relativistic effect and is therefore expected to be small. The total fine-structure splitting (fs) of the 1P-states,

$$\Delta m_{fs} = m(1P, J=2) - m(1P, J=0),$$

scales from the (cc) to the (bb) system like the square of the quark velocity in the ground state,

$$\frac{\Delta m^b_{fs}}{\Delta m^c_{fs}} = \frac{(v^2/c^2)_b}{(v^2/c^2)_c},$$

where $\Delta m^c_{fs} = 141 \text{ MeV}$, $\Delta m^b_{fs} = 42 \text{ MeV}$, $(v^2/c^2)_c = 0.23$, and $(v^2/c^2)_b = 0.08$. With $(v^2/c^2)_{t, m=40 \text{ GeV}} = 0.01, \ldots, 0.02$, one obtains for the size of the toponium fine structure

$$\Delta m_{fs} \approx (5-10) \text{ MeV},$$

which seems to be impossible to resolve. The hyperfine splitting (hfs) of the toponium ground state is related to the leptons width,

$$\Delta m_{hfs}(\theta) = \frac{8}{9} \frac{a_s}{e^2} \frac{\alpha_{em}}{\alpha_{em}} \Gamma_0(\theta),$$

from which one obtains

$$\Delta m_{hfs}(\theta) \approx (10-30) \text{ MeV}.$$

A more detailed discussion can be found in Refs. [13] and [17].

Quarkonium decays can proceed as annihilation decays (Figs. 2.6a to 2.6e), single-quark decays (Figs. 2.6f and 2.6g), and hadronic and radiative transitions (Fig. 2.7). In the following we will normalize the partial decay width to a final state $F$ relative to the leptonic width (2.7) (without neutral-current effects), i.e. we will consider the ratios

$$R_F = \frac{\Gamma_F}{\Gamma_0}.$$

The QCD radiative corrections, which have been calculated for most quarkonium decays [26, 28] will be neglected because they are unknown for the important
Fig. 2.6 Annihilation decay modes (a – e) and single-quark decay modes (f, g) of toponium

Fig. 2.7 a) Hadronic and b) electromagnetic transitions between (t̅t̅) resonances

single-quark decay (SQD). A thorough discussion of quarkonium decays can be found in Refs. [5], [6], [8], [28], and [29].

The most frequent final state of toponium decays below 110 GeV is a fermion-antifermion pair (Fig. 2.6a). The corresponding ratio \( r \) reads,

\[
\frac{r_{ff}}{e_2^2} = \frac{c_f}{e_t^2} \left[ \frac{1}{e_t^2} + 2 \frac{e_t e_f v_t v_f}{y^2} \text{Re} \chi_2 + \frac{v_t^2 (v_f^2 + 1)}{y^4} \right] |x_2|^2
\]

\[
+ \delta_{f,b} \left[ \frac{v_t (1 - v_f)}{3 y^2} \text{Re} \chi_2 - \frac{e_t e_f}{3 x} \chi_2 + \frac{1}{18 x^2} \chi_2^2 \right],
\]

(2.14)
where

\[
x = 4 \sin^2 \theta_w, \quad y = 4 \sin \theta_w \cos \theta_w,
\]

\[
x_z = \frac{m_w^2}{m_w^2 - m_z^2}, \quad x_w = \frac{m_w^2 + (m^2/8)}{m_w^2 + (m^2/4)},
\]

\[
c_f = \begin{cases} 3, & \text{quarks} \\ 1, & \text{leptons} \end{cases}, \quad \delta_{f, b} = \begin{cases} 1, & f = b \\ 0, & f \neq b \end{cases},
\]

\[
e_u = e_c = e_t = \frac{2}{3}, \quad e_d = e_s = e_b = \frac{1}{3},
\]

\[
e_{\nu_e} = e_{\nu_\mu} = e_{\nu_\tau} = 0, \quad e_e = e_\mu = e_\tau = -1,
\]

\[
v_u = v_c = v_t = 1 - \frac{2}{3} x, \quad v_d = v_s = v_b = -1 + \frac{1}{3} x,
\]

\[
v_{\nu_e} = v_{\nu_\mu} = v_{\nu_\tau} = 1, \quad v_e = v_\mu = v_\tau = -1 + x.
\]

Except for the \(b \bar{b}\) final state, we have neglected in (2.10) the charged-current contributions which are Cabibbo-suppressed. The \(W\) exchange can also give rise to flavour non-diagonal quark-antiquark decay modes (Fig. 2.6b):

\[
\hat{r}_{q_1 q_2} = \frac{1}{\hat{r}_s(\theta)} \left[ \hat{r}(\theta + \bar{q}_1 q_2) + \hat{r}(\theta + \bar{q}_2 q_1) \right]
\]

\[
= \frac{|V_{tq_1} V_{tq_2}|^2}{3x^2 \alpha_s^2} x_w^2.
\]

These branching ratios are, however, very small because the Kobayashi-Maskawa (KM) matrix elements \(V_{tq_i}\) are known to obey the bounds [30] \(V_{ts} < 0.069\), \(V_{td} < 0.024\). For the familiar decay modes into three gluons, two gluons and a photon, and Higgs scalar and a photon (Figs. 26 c, d, e), one has

\[
r_{ggg} = \frac{10 (x^2 - 9)}{3\alpha_s} \frac{\alpha_s^2}{\alpha_{EM}^2} e_t^2,
\]

\[
r_{gg\gamma} = \frac{8(x^2 - 9)}{9\alpha_s} \frac{\alpha_s^2}{\alpha_{EM}},
\]

\[
r_{gg\gamma} = \frac{4(x^2 - 9)}{9\alpha_s} \frac{\alpha_s^2}{\alpha_{EM}}.
\]
\[
I_{H \gamma}^\theta = \frac{G_F m_\theta^2}{4/2\pi \alpha em} \left(1 - \frac{m_\theta^2}{m_H^2}\right) = \frac{1}{2\pi} \frac{m_\theta^2}{m_W^2} \left(1 - \frac{m_\theta^2}{m_{\tilde{H}}^2}\right) .
\] (2.18)

Very important for toponium physics is the \( \beta \)-decay of the t-quark (SQD) (Fig. 2.6f), which is given by \([\Gamma_0(\theta) = 5 \text{ keV}]:\)

\[
I_{SQD} = \frac{1}{\Gamma_0(\theta)} \left[ \Gamma(\theta \to t\tilde{t}W^+) + \Gamma(\theta \to b\tilde{b}W^+) \right]
\]

\[
= \frac{18}{92 \pi^2} \frac{G_F^2 m_t^5}{\Gamma_0(\theta)} f \left( \frac{m_t^2}{m_W^2}, \frac{m_b^2}{m_t^2} \right)
\]

\[
= 8.4 \left( \frac{m_t}{40 \text{ GeV}} \right)^5 f \left( \frac{m_t^2}{m_W^2}, \frac{m_b^2}{m_t^2} \right) .
\] (2.19)

with

\[
f(q, \mu) = 2 \int_0^{(1-\mu)^2} du \frac{1}{(1-uq)^2} \left[ (1-\mu)^2 + u(1+\mu) - 2u^2 \right] \left[ 1 + u^2 + u^2(1+\mu+u) \right]^{1/2}
\]

\[
= \begin{cases} 
(1-\mu)^2(1-8\mu+\mu^2) - 12\mu^2 \ln \mu , & q = 0 \\
2 & (6[q+(1-q) \ln (1-q)] - 3q^2 - q^3), & \mu = 0 
\end{cases}
\]

\[
f(q, \mu) = f(q, 0) f(0, \mu) + O(10^{-2}) ,
\]

\[0.9 < f \left( \frac{m_t^2}{m_W^2}, \frac{m_b^2}{m_t^2} \right) < 1.3 \text{ for } 30 \text{ GeV} < m_t < 55 \text{ GeV} .
\]

A second important 'single-quark decay' is the two-body decay into a \( b \)-quark and a charged Higgs scalar (Fig. 2.6g), which in some models is predicted with a mass smaller than \( m_t - m_b \). The corresponding ratio \( \Gamma \) reads \([m_b \ll m_t, \Gamma_0(\theta) = 5 \text{ keV}]:\)

\[
I_{bH^\pm} = \frac{1}{\Gamma_0(\theta)} \left[ \Gamma(\theta \to t\tilde{t}H^+) + \Gamma(\theta \to b\tilde{b}H^+) \right]
\]

\[
= \frac{G_F}{4/2\pi} \left| V_{tb} \right|^2 \frac{m_t^3}{\Gamma_0(\theta)} \left(1 - \frac{m_{\tilde{H}}^2}{m_t^2}\right)^2
\]

\[
= 0.8 \times 10^4 \left| V_{tb} \right|^2 \left( \frac{m_t}{40 \text{ GeV}} \right)^3 .
\] (2.20)
Unless the KM-type matrix element \( V_{tb} \) is very small, this process will clearly dominate toponium decays! For superheavy t-quarks with \( m_t > m_W + m_d \), where \( d \) is a quark with electric charge \(-1/3\), the two-body decay into \( d \) and a real \( W \) proceeds with the branching ratio \( [6] \):

\[
\Gamma_{dW} = \frac{G_F |V_{td}|^2}{4\pi/2} \Gamma_0 (\theta) \frac{1}{m_t} \left( 1 + 2 \frac{m_W^2}{m_t^2} \right) \left( 1 - \frac{m_W^2}{m_t^2} \right)^2 .
\]  

(Hadronic transitions (Fig. 2.7a), which are very important in the \( \psi \) family and also in the \( \Upsilon \) family, are expected to be much suppressed in the \( (t\bar{t}) \) system. For the mass range \( 30 \text{ GeV} < m_t < 50 \text{ GeV} \), Kuang and Yan have estimated \( [31] \)

\[
\Gamma (\theta' \rightarrow \psi \psi) = 0.5 \text{ keV} .
\]  

Partial widths for electromagnetic dipole transitions (Fig. 2.7b) between S- and P-states are given by

\[
\Gamma_{if} = \frac{4}{3} \alpha^2 \alpha_{em} \kappa^3 (2J_f + 1) \int_0^\infty dr R_i (r) R_f (r) ,
\]  

where \( \kappa, J_f \), and \( R_{if} \) are the photon momentum, the total angular momentum of the final state, and the radial wave functions of the initial and final states, respectively. Rates for the toponium system have been calculated in Ref. \( [25] \).

The most important ingredients in a calculation of decay widths of toponium states are the wave functions at the origin or, equivalently, the leptonic widths \( \Gamma_\ell \). According to Fig. 2.5, a reasonable estimate for the ground state is

\[
\Gamma_\ell (\theta) = 5 \text{ keV} .
\]  

Furthermore, we choose as input

\[
\Gamma_\ell (\theta') = \frac{1}{3} \Gamma_\ell (\theta) , \quad \Gamma (\theta' \rightarrow \gamma \chi_p) = 1.2 \times \left( \frac{40 \text{ GeV}}{m_t} \right)^{1.25} \text{ keV} ,
\]  

\[
\Gamma_\ell = 2.5 \text{ GeV} , \quad \sin^2 \theta_w = 0.217 ;
\]  

\[
\alpha_s (m) = \frac{12\pi}{23 \ln \left( \frac{m}{100 \text{ MeV}} \right)^2} \quad (n_f = 5) .
\]

\[
m_W = 83.2 \text{ GeV} , \quad m_Z = 94.0 \text{ GeV} ,
\]  

\[
m_t = 2.5 \text{ GeV} ,
\]
We emphasize that the input parameters (2.24) and (2.25) represent only an 'educated guess' based on QCD-like potential models. The leptonic widths could be larger or smaller by a factor of 2, and the dipole transition rate is even more model-dependent (see Ref. [25]).

From Eqs. (2.13) to (2.25) we can calculate the various toponium decay widths. Figure 2.8 shows the total and leptonic widths for the energy range of LEP I, i.e. 60 GeV < \( s \) < 110 GeV, and in Fig. 2.9 the partial widths \( \Gamma_{\bar{f}f} \), \( \Gamma_{\gamma g} \), and \( \Gamma_{QCD} \) are compared. The striking feature, which distinguishes toponium physics from \( \psi \) and \( \Upsilon \) physics, is the dominance of the weak interactions over electromagnetic and strong interactions:

\[
\text{BR}_{\text{weak}} (W^+, Z^+) > \text{BR}_{\text{em}} (\gamma^+) > \text{BR}_{\text{strong}} (ggg) .
\]  

Branching ratios into various final states are plotted in Figs. 2.10 and 2.11 for the 1S- and 2S-state as functions of \( m_\tau \). Owing to the interplay between the single-quark decays and the annihilation decays, which are strongly affected by the \( Z \) boson, the branching ratios vary drastically with \( m_\tau \) and lead to decay patterns which are qualitatively different for \( m_\tau = 70, 80, 90, \) and 100 GeV! In the vicinity of the \( Z \) pole, branching ratios into fermion-antifermion final states dominate. The branching ratio into neutrino pairs is remarkably large. It reaches 10% at \( m_\theta = 80 \) GeV, which, however, is not sufficient to allow for a neutrino counting which could compete with the \( Z^0 \) width determination [25].

From Fig. 2.11 we also conclude that the search for P-states in radiative \( \theta' \) decays appears to be feasible only for toponium masses below 90 GeV.

A special region for toponium physics is the mass range

\[
m_\tau - 2\Gamma_Z < m_\theta < m_\tau + 2\Gamma_Z ,
\]  

where the toponium-\( Z \) interference has to be taken into account [32]. One of the most striking features of this interference is that in the case of degenerate masses, \( m_\theta = m_\tau \), toponium manifests itself as a dip in \( R \) rather than an enhancement of the \( Z \) peak, a phenomenon first observed by Kühn and Zerwas [33].

In order to understand this, let us consider the contribution of toponium to the \( e^+e^- \rightarrow \mu^+\mu^- \) cross-section. In the case \( m_\theta << m_\tau \), one has (see Fig. 2.12a) for the amplitude at \( W = m_\theta \):

\[
A(m_\theta) = \frac{1}{m_\theta^2} + \frac{1}{m_\theta^2} \frac{\gamma^2_{\theta}}{m_\theta} \frac{1}{\theta_\theta m_\theta^2} = A_{B,\gamma} + A_\theta ,
\]

which implies that the \( \theta \) term enhances the photon background term:

\[
|A|^2 = A_{B,\gamma}^2 + A_\theta^2 .
\]
Fig. 2.8 Total width and leptonic width of the toponium ground state as functions of $2m_t$

Fig. 2.9 Partial widths for the total fermion-antifermion, three-gluon, and single-quark decay modes of the toponium ground state as functions of $2m_t$
Fig. 2.10 Branching ratios for $\theta = \frac{(t\bar{t})_S}{(t\bar{t})_T}$ decays as functions of $2m_t$.

Fig. 2.11 Branching ratios for $\theta = \frac{(t\bar{t})_S}{(t\bar{t})_T}$ decays as functions of $2m_t$. 

$\phi$
On the other hand, for \( m_\theta = m_z \) (see Fig. 2.12b) one obtains

\[
\lambda(z) = \frac{1}{i m_z^2} \left[ \frac{1}{m_z^2} + \frac{1}{m_z^2} \right] = \frac{1}{i m_z^2} \left( i m_z^2 Z \right) = i A_{Z}, \quad \text{and} \quad A_{Z} = i A_{\theta}. \tag{2.30}
\]

The interference with the imaginary \( Z \) background term produces a dip

\[
|\lambda|^2 - (\lambda_{Z} - \lambda_{\theta})^2 = 0, \tag{2.31}
\]

where the vanishing of the cross-section follows from

\[
\Gamma_{\theta} = \frac{g_{\theta Z}^2}{m_{\theta}^2} \Gamma_2. \tag{2.32}
\]

If the energy resolution of the machine is larger than the width of the toponium resonance, the effect is weakened. For \( \delta W = 50 \text{ MeV} \), for instance, the cross-section at \( W = m_{\theta} \) is decreased by about 10\% (see Fig. 2.13). The complete pattern \([24, 33-36]\) for a toponium system inside the \( Z^0 \) peak is shown in Figs. 2.14 and 2.15 for \( \delta W = 0 \) and \( \delta W = 48 \text{ MeV} \), respectively.

The amplitude for the process \( f_i \bar{f}_j + f_i \bar{f}_j \), where \( f_i \) represents leptons or quarks, can be obtained directly from \( S \)-matrix theory by imposing analyticity, unitarity, and certain smoothness assumptions. For a set of \( N \) toponium resonances with \( \Gamma_m \ll \Gamma_2 \), \( m = 1, \ldots, N \), one finds for the \( T \)-matrix \([36]\),

\[
T_{ij}(W) = \sigma_i \sigma_j t(W), \tag{2.33}
\]

with

\[
t(W) = \frac{1}{2i} \prod_{m=1}^{N} \left[ \frac{W - m_i - m_j + i \Gamma_m / 2}{W - m_i + i \Gamma_m / 2} \right] \left[ \frac{W - m_i - m_j + i \Gamma_m / 2}{W - m_i + i \Gamma_m / 2} \right] - 1,
\]

\[
|\sigma_i |^2 = 1,
\]

from which one can easily understand the structure of Figs. 2.14 and 2.15.
Fig. 2.13 Energy dependence of the resonance excitation cross-section for a toponium mass a) $m_0 = m_z$, b) $m_0 = m_0 - \Gamma_z/2$, and c) $m_0 = m_0 - \Gamma_z$. The dashed line shows the cross-section without interference terms taken into account; $z = (W - m)/\delta W$, $\delta W = 50$ MeV. From Ref. [33].

Fig. 2.14 $R(\mu^+\mu^-)$ for a $t$-quark mass of 47 GeV without average over the beam profile. From Ref. [24].
Fig. 2.15 $R(\mu^+\mu^-)$ for a $t$-quark mass of 47 GeV averaged over the beam profile. From Ref. [24].

A toponium system with $m_\tau^2 - 2\Gamma_\tau^2 < m_\tau < m_\tau + 2\Gamma_\tau^2$ would lead to an interesting interference pattern and would obviously greatly facilitate the determination of the ($t\bar{t}$) resonances. On the other hand, this would complicate the planned precision tests of the standard model on the $Z^0$ and imply the suppression of interesting toponium decay modes.

Finally, let us estimate the event rates for toponium production at LEP.

In Fig. 2.16 we have plotted the contributions to

$$R^0 = \frac{1}{\sigma_{pt}} \sigma(e^+e^- \rightarrow \text{hadrons}),$$

$$\sigma_{pt} = \sigma(e^+e^- \rightarrow \gamma^{\ast} \rightarrow \mu^+\mu^-),$$

from the continuum ($R^{\gamma,Z}$), the toponium ground state ($R^0$), and the single-quark decays of the toponium ground state ($R_{\text{SQD}}^0$). Radiative corrections have been incorporated as [37]

$$R^0 = R^{(0)} \left[ \frac{(2\Delta W)t}{m_{\theta}} + \epsilon \right],$$

$$\epsilon = \frac{2\alpha_{\text{em}}}{v} \left( \frac{r^2}{6} - \frac{17}{36} \right) + \frac{13}{12} t,$$

$$t = 2 \frac{\alpha_{\text{em}}}{v} \left( \ln \frac{m_\theta^2}{m_e^2} - 1 \right),$$
\[ R(0) = \frac{9\pi}{2\alpha_e^2} \frac{\Gamma(\theta)}{2\pi} \frac{r_e e^-}{\delta W} \text{BR}(\theta \to \text{had}) , \]  

(2.35b)

where \( R(0) \) is the uncorrected value of \( R \) [see Eqs. (2.7) and (2.14)], \( m_e \) is the electron mass, and \( \delta W \) is the design energy spread [38],

\[
\delta W \text{ (MeV)} = \frac{4.43 \times 10^{-3} \ W^2 \text{ (GeV}^2)}{[1 - 0.41 \times 10^{-5} \ W^3 \text{ (GeV}^3)]^{1/2}} .
\]  

(2.36)

In the calculation for Fig. 2.16 the invariant mass of fermion-antifermion pairs in fermion-antifermion-photon final states has been restricted to more than 10 GeV *).

---

*) We thank R. Kleiss for supplying the computer program for performing the radiative correction to the continuum cross-section (see Ref. [39]).
The rates of hadronic events from $\Theta$-decays and continuum production are given by

$$N_{\text{had}}^\Theta = \sigma_{pt} R^\Theta L T,$$  \hspace{1cm} (2.37a)

$$N_{\text{had}}^C = \sigma_{pt} R^{Y,Z} L T,$$  \hspace{1cm} (2.37b)

where $L$ and $T$ are the luminosity and the running time. The reference cross-section $\sigma_{pt}$ is given by

$$\sigma_{pt} = \frac{86.8 \text{ nb}}{[W \text{ (GeV)}]^2}. \hspace{1cm} (2.38)$$

As an example, let us consider the case $W = m_\Theta = 80 \text{ GeV}$, where one has (see Ref. [38] and Fig. 2.16)

$$L \equiv L_0 = 1.1 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}, \quad \delta W \equiv \delta W_0 = 32 \text{ MeV},$$

$$\sigma_{pt} \equiv \sigma_0 = 13.6 \text{ pb}; \hspace{1cm} (2.39)$$

$$R^\Theta = 9, \quad R^{Y,Z} = 20.$$  

With the conventions

$$1(\text{TH}) \ 'day' = 24 \text{ h},$$

$$1(\text{EXP}) \ 'year' = 2800 \text{ h} = 10^7 \text{ s}, \hspace{1cm} (2.40)$$

one obtains from Eqs. (2.37) and (2.39) the event numbers listed in Table 2.1. Although the background is more than twice as large as the signal, the event rates are large enough to make toponium physics at LEP possible.

The main theoretical uncertainty in the estimate of the toponium production cross-section is the unknown wave function at the origin (see Fig. 2.5). The number of hadronic events $N_{\text{had}}^\Theta$ could be larger or smaller by a factor of 2 compared to the values given in Table 2.1. The expected event rates in the whole LEP energy range are plotted in Fig. 2.17.

In this section we have discussed the theoretical expectations for toponium physics at LEP. Heavy quark-antiquark bound states in the mass range from 60 to 110 GeV are an ideal testing ground for the standard model: their spectroscopy should provide direct evidence for asymptotic freedom and their decays are governed by weak, electromagnetic, and strong interactions which in the above
Table 2.1

Number of hadronic events for signal ($N^\theta_{had}$) and background ($N^C_{had}$) at $W = m_\theta = 80$ GeV

<table>
<thead>
<tr>
<th>Running time T</th>
<th>$\int L dt$ (pb$^{-1}$)</th>
<th>$N^C_{had}$</th>
<th>$N^\theta_{had}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 'day'</td>
<td>0.95</td>
<td>260</td>
<td>118</td>
</tr>
<tr>
<td>1 'year'</td>
<td>111</td>
<td>30,000</td>
<td>14,000</td>
</tr>
</tbody>
</table>

Fig. 2.17 Expected counting rates per day (24 h) for the decay modes $\theta + \text{SQD}$, $\mu^- \mu^+$, $H^- \gamma$, and $\theta' + x_T + \gamma$ as functions of $2m_T$.
mass range are of comparable strength. In the following sections we will study the experimental feasibility of various aspects of toponium physics in more detail: in Section 3 toponium scan and spectroscopy; in Sections 4 and 5 electroweak physics, i.e. asymmetry measurements and the search for Higgs particles; and in Section 6 the possible effects of supersymmetry on toponium decays. After a discussion of the role of the energy spread in Section 7, we will summarize our results in Section 8. In this paper we have not discussed the question of QCD tests through toponium decays, such as \( \theta \rightarrow 3\gamma \), \( \theta \rightarrow \gamma 2\gamma \), etc. An analysis of these decay modes will be treated by the study group of A. Ali et al. on QCD and \( \gamma \gamma \) scattering [40].

3. **TOPONIUM SEARCH AND SPECTROSCOPY**

The search for the toponium S-states is performed in two steps, as follows:

i) The scanning region is reduced to a range of about 2 GeV through a rough measurement of the top (t) quark mass.

ii) This region is then scanned in steps of \( 5W \) (\( 5W \) denotes the spread in the centre-of-mass energy \( W \)), looking for a signal in the total hadronic cross-section--eventually applying additional topological cuts as discussed below.

The strategy for limiting the scanning region to 2 GeV is very different for toponium above or below the \( Z^0 \) mass. If t-quarks are light enough, a large number of T mesons can be produced at the \( Z^0 \) resonance in \( e^+e^- \) reactions, and the t-quark mass can then be determined. If toponium is above the \( Z^0 \), it will be more efficient to search for the top threshold, as discussed later.

3.1 **Top-quark mass determination**

Three methods are now described which make it possible to estimate the t mass in \( Z^0 \) decays and thus predict the scanning region for the toponium search.

3.1.1 **Threshold effect**

The rate of \( t\bar{t} \) events at \( Z^0 \), when normalized to \( \mu^+\mu^- \), shows a simple threshold dependence:

\[
R_t = \frac{\Gamma(Z \rightarrow t\bar{t})}{\Gamma(Z \rightarrow \mu^+\mu^-)} = \frac{3B \left[ (3-B^2)/2 \left(1 - 8/3 \sin^2 \theta_W \right)^2 + B^2 \right]}{(1 - 4 \sin^2 \theta_W)^2 + 1}, \tag{3.1}
\]

where

\[
B^2 = 1 - \frac{4m_t^2}{m_Z^2}.
\]
Life becomes more complicated when first-order QCD corrections are included [41]:

\[
R_t = \frac{3\beta [A + V (1 - 8/3 \sin^2 \theta_W)^2]}{(1 - 4 \sin^2 \theta_W)^2 + 1},
\]

(3.2)

where

\[
V = \frac{3 - \beta^2}{2} \left[ 1 + \frac{4}{3} \alpha_s \left( \frac{1}{2} - \frac{32}{3} \beta \right) \right],
\]

\[
A = \beta^2 \left[ 1 + \frac{4}{3} \alpha_s \left( \frac{1}{2} - \frac{32}{3} \beta \right) \right],
\]

\[
\alpha_s = \frac{12\pi}{25 \log \left( \frac{m_t}{1-\beta^2/200 \text{ MeV}} \right)}.
\]

Figure 3.1 shows how \( R_t \) depends on \( m_t \) with and without QCD corrections; \( m_t \) can be deduced from \( R_t \) with some uncertainties, both theoretical and experimental.

![Graph](image)

Fig. 3.1 Decay rate of \( Z^0 \) into top quarks normalized to \( \mu^+\mu^- \). The dashed line indicates the rate without QCD corrections. From Ref. [41].
a) **Experiment**

The $t\bar{t}$ sample can be isolated using topological cuts (see discussion below) which usually keep more than $F_1 = 90\%$ of $t\bar{t}$ with less than $F_2 = 10\%$ contamination (mass-dependent concept). With such figures it seems reasonable to assume a 5\% systematic uncertainty on the number of $t\bar{t}$. When $m_t$ increases, the signal-to-background ratio decreases while the topological separation improves. With this coarse estimate, one can thus neglect the mass dependence of the systematic error.

b) **Theory**

If $m_t = 40$ GeV, the QCD correction term amounts to about 40\%. With an error on $\alpha_s$ of order 10\% we can see that the theoretical uncertainty is of the same order as the experimental one.

One might hope that the potentially large higher-order corrections sum up to modify the leading term by only a factor $[1 - \exp (-2\pi \alpha_s/3\beta)]$ similar to the result in QED (see Ref. [42]). However, one cannot rigorously exclude some sizeable effect of higher-order corrections.

To conclude, such an approach should give an error on $m_t$ of $\sim 1.5$ GeV for $m_t \sim 40$ GeV.

### 3.1.2 Semileptonic decays

a) **Rest system distribution**

In units of $m_t/2$, the $\mu$ (e) energy spectrum in the rest system is given by [43]:

$$
\frac{1}{\Gamma} \frac{d\Gamma}{dx} = 12 \frac{x^2 (1 - x - \epsilon^2)^2}{1 - x},
$$

(3.3a)

where

$$
\epsilon = \frac{m_b}{m_t}, \quad x = \frac{2E_e(\mu)}{m_t}.
$$

This formula is valid at the quark level and for $0^+ \rightarrow 0^-$ transitions. For $0^- \rightarrow 1^-$ transitions one gets [43]:

$$
\frac{1}{\Gamma} \frac{d\Gamma}{dx} = 12 \frac{x^2 (1 - x - \epsilon^2)^2}{(1 - x)^2} (1 + 2\epsilon^2 - x).
$$

(3.3b)

At $\epsilon = 0$ all these formulae coincide; $\epsilon^2$ is indeed known to be very small: $\epsilon^2 < 1/20$. 
From Fig. 3.2 we see that this type of spectrum allows a very accurate determination of $m_t$ through the end-point.

b) Laboratory system

At $Z^0$, top particles are not produced at rest. Even if it is assumed that they are elastically produced (no fragmentation effect), their decay products cannot be well separated and thus their direction cannot be determined precisely. Therefore it does not seem possible to define the semileptonic decay distribution in the top rest frame.

Assuming an isotropic angular distribution of the lepton in the rest frame, the energy distribution in the lab. system can be generated. The result is shown in Fig. 3.3 in the elastic case, for $m_t = 40$ GeV. The end-point is difficult to determine. However, it is possible to try a global fit of the data (keeping out the low-energy region to avoid background from secondary leptons).
This fit was tried at $m_t = 30$ and 40 GeV and gave back the right mass to better than 200 MeV (with a sensitivity corresponding to about $10^6 Z^0$).

c) Polarization effects

The standard model tells us that, at $Z^0$, t-quarks are polarized with

$$\langle \lambda_t \rangle = - \frac{2a_t v_t}{a_t^2 + v_t^2} = -0.74,$$  \hspace{1cm} (3.4)

where

$$a_t = 1, \quad v_t = 1 - \frac{8}{3} \sin^2 \theta_W.$$  

This formula is only approximative since it neglects the mass of the t-quark. Using formulae given in Ref. [44], the average longitudinal polarization (at the $Z^0$ pole) can be computed:

$$\langle \lambda_t \rangle = \frac{-2a_t v_t \beta}{3/4 (v_t^2 - a_t^2)/\gamma^2 + (3/4 + \beta^2/4) (v_t^2 + a_t^2)}.$$  \hspace{1cm} (3.5)

For light t-quarks, $\beta \approx 1$, and one recovers the previous formula. For $m_t = 40$ GeV, $\langle \lambda_t \rangle = -0.85$, which means that the polarization is still large and mostly longitudinal.

The averaged transverse polarization is given by

$$\langle P_T \rangle = \frac{\sqrt{\beta^2 (r_Z/m_Z)^2 + 4v_t^2 v_t^2}}{2\gamma (1 + \beta^2/3)(v_t^2 + a_t^2) + (v_t^2 - a_t^2)/\gamma}. $$  \hspace{1cm} (3.6)

For $m_t = 40$ GeV, $\langle P_T \rangle = 0.087$, which confirms our expectation that the polarization is mostly longitudinal. In the following calculations, we only take into account the longitudinal polarization as given by the previous formula.

Vector top mesons $T^*$ will also be polarized, and since the usual cascade $T^* \rightarrow T + \gamma$ is negligible with respect to the weak decay, the process

$$T^* \rightarrow e\nu B$$

will act as an analyser of polarization:

$$\frac{d\sigma}{d\cos \theta} = (1 + P \cos \theta).$$  \hspace{1cm} (3.7)
Fig. 3.4 a) Energy spectrum in the lab. system assuming an effective longitudinal polarization of -0.40 and $m_c = 40$ GeV (dotted curve corresponds to no polarization). b) Fit to the data (15,000 semileptonic decays generated). The dashed line corresponds to the expected background from secondary (charm, beauty) leptonic decays from $t\bar{t}$ events.

Assuming $T^*/T = 3$ and remembering that only two helicity states contribute, we can write [45],

$$ P = \frac{1}{2} \langle \lambda_t \rangle, $$

and we arrive at the lepton distribution shown in Fig. 3.4a for $m_c = 40$ GeV.

A total of 15,000 semileptonic decay events have been generated at $m_c = 40$ GeV using previous formulae for the angular distribution of the leptons and the quark-decay formula (3.3) for the energy spectrum.

The resulting distribution is fitted using the formula (see also [45])

$$ \frac{d\Gamma}{dy} \propto (y - y_{\text{max}})^2 \left[ 2y + y_{\text{max}} + \frac{P}{\beta} (6yy_{\text{max}} - 2y - y_{\text{max}}) \right], $$

where

$$ y = \frac{E_\ell}{E_{\text{beam}}}, $$

$$ y_{\text{max}} = \frac{1 + \beta}{2}, $$

$$ \beta = \sqrt{1 - \frac{4m^2}{m_T^2}}. $$

This formula is valid when
\[ \frac{1 - \beta}{2} < y < \frac{1 + \beta}{2}, \]

and has been derived in the approximation \( \epsilon = 0 \) [see Eq. (3.3)].

Starting from \( m_t = 40 \) GeV in the Monte Carlo generation, the fit gives (Fig. 3.4b)

\[ m_t = 40.3 \pm 0.1 \text{ GeV}. \]

\[ \text{d)} \quad \text{Fragmentation effects} \]

Fragmentation introduces a further complexity into the problem. At present, several models seem to describe the charm and beauty data [46]. Although they all give quasi-elastic events, the deformation which they introduce into the spectra is noticeable at the \( \chi^2 \) level when the fit given in the previous paragraph is performed. The resulting value for \( m_t \) is not changed by more than 500 MeV.

To summarize:
- a precise determination of the top mass from \( Z^0 \) events requires an understanding of the shape of the lepton spectrum in the lab system;
- this shape is affected by fragmentation and polarization effects;
- if these effects are reasonably well understood, we can hope to reach a precision of 500 MeV on \( m_t \) using this method.

3.1.3 Top-mass determination using multijet final states

The non-leptonic weak decays of the top particles produce final states containing three jets for each meson decay with, in addition, a few low energetic particles coming from the spectator quarks. Using a cluster algorithm, for instance the standard subroutine LUCLUS from the Lund Monte Carlo program, we can reconstruct some of these jets. In the following we have considered events containing five or six identified jets, and the top mass will be obtained by considering subsamples of two or three jets. If hadrons are produced in addition to top particles, the mass of the selected jet sample will be incorrect and the proportion of correct combinations will decrease.

\[ \text{a)} \quad \text{Lund generation} \]

The standard Lund Monte Carlo program generates only a \( T \bar{T} \) system. This is due to the very high peaking of the fragmentation function at high \( z \) values.

We have selected, for each event, the jet subsample which has its energy closest to the beam energy, and the mass of these systems is shown in Fig. 3.5 for different choices of the \( t \)-quark mass. The measured mass has been rescaled
by the ratio $E_{\text{beam}}/E_{\text{jet}}$ to correct for particle losses. A simplified simulation of the DELPHI apparatus has been used, which incorporates dead regions, expected accuracies on energy and angle reconstruction, and photon conversions. A minimum energy of 500 MeV is needed to accept a photon, and at least 1 GeV is required for a neutral hadron ($K^0_L$, n).

In one year of running at the $Z^0$ resonance ($10^5$ hadronic events), clear mass peaks can be obtained, allowing a determination of the top mass with a negligible statistical uncertainty. The systematic uncertainty depends mainly
on the relation between the t-quark and the T-meson masses; it can be minimized by the use of a Monte Carlo program and a reasonable theoretical input. An accuracy of better than 200 MeV seems possible up to 40 GeV.

b) Peterson-like generation

If we use a different fragmentation function for the t-quarks, one of the main effects is to recover $t\bar{t}q$ events. Using the same method we observe a peak around the initial t-quark mass; its height is decreased by about a factor of 3 in comparison with the result obtained with the Lund scheme, and its mean value is shifted towards higher masses because of the rescaling procedure which overestimates the energy of the T-meson.

The systematic uncertainty comes mainly from the shape of the t-quark fragmentation function. However, changing this function from Lund to Peterson recipes, we have observed a displacement of the peak of less than 500 MeV. (As already mentioned in the previous section, the semileptonic decays can be used to reduce the uncertainty on the fragmentation function.)

In conclusion, we believe that our method can give $m_t$ to better than 500 MeV. *

3.1.4 Toponium above the $Z^0$ mass

If toponium lies in the region above the $Z^0$ mass, one could in principle run at the highest energy, identify top mesons, and measure their mass just as before. An alternative method consists in making a direct search for open-top threshold in the following way: The rise of $R$ above threshold is very sharp [41]. For simplicity we ignore the threshold suppression completely. The topology of open-top events [48] is strikingly different from that of two-jet (q$q$) and three-jet (q$qq$) events. The cuts to be discussed below for the selection of SQDs of toponium allow 95% of the continuum cross-section to be rejected, whilst nearly 90% of the signal is retained. Similar numbers apply also in the case of open-top production.

At each energy we take an integrated luminosity corresponding to at least ten events of open-top production (if above threshold); this will take 30 hours in the worst case.

Depending on the observation of the open-top production at the current energy, we increase or decrease the centre-of-mass energy by half of the previous energy step. This procedure converges quickly, and the threshold of open-top production can be measured with an error of about 2 GeV in six steps which correspond to a total of about ten days of data-taking.

*) For a related discussion, see also Ref. [47].
3.2 S-states

3.2.1 Scan

We assume that an energy interval of 2 GeV will be scanned in steps of $2\delta W$ (centre-of-mass energy spread), requiring a signal of $n$ standard deviations in the total cross-section. For each step we need an integrated luminosity $LT$,

$$LT = n^2 \frac{\sigma_S + \sigma_B}{\sigma_S^2}, \quad (3.10)$$

where $\sigma_S$ is the 1S toponium effective cross-section at the peak and $\sigma_B$ is the background cross-section.

The total integrated luminosity needed to scan the full 2 GeV interval is

$$LT(\text{total}) = n^2 \frac{\sigma_S + \sigma_B}{\sigma_S^2} \frac{2 \text{ GeV}}{2\delta W} \quad (3.11)$$

($\sigma_S$ is proportional to the inverse of $\delta W$). When $\sigma_S \ll \sigma_B$ (as happens above 80 GeV), the total integrated luminosity is proportional to $\delta W$ and can be reduced by decreasing the energy spread of the beam.

As already stated, the topology of the toponium SQD is very different from the background topology. If this decay mode is important, it may be more convenient to search for a signal in the cross-section of 'spherical' events using topological cuts to reduce continuum events [48].

Figure 3.6a shows a plot of thrust ($T$) [49] versus oblateness ($O$) [50] for SQD events from a toponium state with a mass of 100 GeV. Figure 3.6b shows the same plot for $q\bar{q}$ and $qg$ events at the same energy. (For the continuum events, we used the Lund Monte Carlo; for SQD events we also used the Lund Monte Carlo simulating open $t\bar{t}$ production at threshold.) The SQD events are localized mainly in the region of small thrust and oblateness,

$$O < 2(1-T) - 2h,$$

where $h$ is an adjustable parameter than can be optimized. The efficiency of the cut for various choices of $h$ is displayed in Table 3.1. The optimum is achieved for $h = 0.05$, where we can retain 87% of the signal and reject 95% of the background. The cut is not sensitive to the centre-of-mass energy, and we therefore regard these numbers as being constant in the full energy range that we consider.
Fig. 3.6a Thrust versus oblateness for SQD decay for a toponium with a mass of 100 GeV. The line shows the cut described in the text.

Fig. 3.6b Thrust versus oblateness for q̅q and q̅q̅g events at 100 GeV centre-of-mass energy. The line shows the cut described in the text.
Table 3.1
The percentage of SQD events and background events that pass the cut for various values of \( h \)

<table>
<thead>
<tr>
<th>( h )</th>
<th>0</th>
<th>0.01</th>
<th>0.03</th>
<th>0.05</th>
<th>0.07</th>
<th>0.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>SQD</td>
<td>95</td>
<td>97</td>
<td>98</td>
<td>87</td>
<td>76</td>
<td>64</td>
</tr>
<tr>
<td>Background</td>
<td>79</td>
<td>48</td>
<td>12</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Fig. 3.7a Integrated luminosity needed to achieve a signal of 1 st. dev. in one step of the scanning as function of the toponium mass

Fig. 3.7b Integrated luminosity needed for the full scanning of a 2 GeV mass interval asking for a signal of 3 st. dev. as function of the toponium mass

Figure 3.7a shows the integrated luminosity needed to achieve 1 st. dev. signal in one step of the scanning as a function of the toponium mass. The dotted line indicates the region where the topological cut is applied.

Figure 3.7b shows the total integrated luminosity needed for scanning the 2 GeV region as a function of the toponium mass with a 3 st. dev. signal. We see that up to a toponium mass of 150 GeV the total integrated luminosity needed for the full scan is less than 20 pb\(^{-1}\), which corresponds to 20 days of full luminosity or to 2 real months.
Furthermore, it is evident from this figure that these cuts would be of crucial importance if toponium were to have a mass above 100 GeV. In the region quite above that of the \( z^0 \), one might eliminate the radiative tail of the \( z^0 \) by appropriate cuts on missing energy and momentum, but we have not studied this possibility in detail.

3.2.2 Measurement of \( m_c \) and \( \Gamma_{ee}(\theta) \)

We assume that the shape of the background in the resonance region is known. To measure the toponium mass and \( \Gamma_{ee} \) we can fit a Gaussian plus the known background to the total cross-section:

\[
\sigma(W) = \sigma_{PT} R^0 e^{-\left( m - W \right)^2 / 2\sigma_W^2} + \text{Background} ,
\]

where \( R^0 \) has been defined in Eq. (2.35).

Table 3.2 gives the statistical accuracy on the toponium mass, assuming that we take 0.4 pb\(^{-1}\) at nine different energies in the resonance region. The absolute measurement of the mass has a systematic error of about 100 MeV owing to the systematic error on the energy of the beams, although this could be reduced if the beams were transversely polarized. This error is not relevant for the measurement of the mass difference between the 1S- and 2S-states.

<table>
<thead>
<tr>
<th>Mass (GeV)</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
<th>110</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta m_c ) (MeV)</td>
<td>10</td>
<td>20</td>
<td>15</td>
<td>20</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>( \delta \Gamma_{ee}/\Gamma_{ee} ) (%)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>

If the toponium mass is in the region of 80 GeV, its branching ratio to neutrinos can reach the 10% level. In this case the branching ratio \( \Gamma_{vis}/\Gamma_{tot} \) is considerably different from 1, and the measurement of \( \Gamma_{ee} \) must be corrected to take this effect into account. The error on the correction is, in any case, negligible compared to the statistical error.

If toponium should be found in the immediate neighbourhood of the \( z^0 \), both the line shape and height of the resonance are affected by interference with the continuum, and a more complicated analysis has to be performed (see Section 2).

3.2.3 Search for radial excitations

Owing to the low signal-to-background ratio, the search for the other S-states (apart from the 2S-state) is possible only if toponium is below the \( z^0 \) mass; we thus concentrate on a single case: the toponium mass at 80 GeV.
Figures 3.8a and 3.8b give a possible scenario if the toponium mass is in this region. Figure 3.8a is a plot of $R$, and Fig. 3.8b is a plot of $R$ for the events that pass the topological cut described earlier. As input we used the values for $\Gamma_0$ and binding energies listed in Table 3.3, as calculated [24] for the Richardson potential. The comparison of these figures shows that it is easier to find the resonances using the topological cut.

Once the $1S$-state has been found, we restrict the scanning region for the $2S$-state to about 400 MeV. To scan this region we need five steps of 0.7 pb$^{-1}$ per point to get a 3 st. dev. signal. This figure corresponds to about four days of data-taking at full luminosity. In the resonance region, we then need about nine points, of 0.8 pb$^{-1}$ each, to measure the mass with a statistical error lower than 10 MeV and a relative accuracy on $\Gamma_{ee}$ of 20%. This figure corresponds to about seven days at full luminosity.

![Graph of $R$ as a function of the centre-of-mass energy assuming $m_t = 40$ GeV](image)

**Fig. 3.8a** $R$ as a function of the centre-of-mass energy assuming $m_t = 40$ GeV

![Graph of $R$ for the events that pass the topological cut (see text) as a function of the centre-of-mass energy assuming $m_t = 40$ GeV](image)

**Fig. 3.8b** $R$ for the events that pass the topological cut (see text) as a function of the centre-of-mass energy assuming $m_t = 40$ GeV
Table 3.3
Model parameters as calculated for the Richardson potential \((m_t = 40 \text{ GeV})\)

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Gamma_0) (keV)</td>
<td>7.34</td>
<td>1.97</td>
<td>1.08</td>
<td>0.77</td>
<td>0.61</td>
<td>0.52</td>
<td>0.46</td>
<td>0.42</td>
<td>0.39</td>
<td>0.37</td>
</tr>
<tr>
<td>(E_B) (MeV)</td>
<td>-1.485</td>
<td>-572</td>
<td>-218</td>
<td>4</td>
<td>172</td>
<td>310</td>
<td>431</td>
<td>539</td>
<td>638</td>
<td>731</td>
</tr>
</tbody>
</table>

Assuming that we again scan a region of 400 MeV asking for a 3 st. dev. signal for the 3S-state, it will require about 7 pb\(^{-1}\) for the full scanning.

The scan for the other states is much more difficult, as is evident for the rapidly decreasing resonance enhancement due to the decreasing \(\Gamma_0\) and the decreasing spacing between the levels listed in Table 3.3. However, once the first three states have been found and their masses have been measured, the masses of the other states should be fixed by the potential models with an error smaller than the centre-of-mass energy spread (32 MeV in this region). Assuming that the experimental width of each resonance is defined by the energy spread of the beams, and assuming that one knows the background below each resonance, the mass of any state can be measured, in principle, taking data at only two energies, one below and one above the foreseen position of the resonance at a distance of the order of the centre-of-mass energy spread. As an example for this method, the mass of the 5S-state can be measured with a precision of 10 MeV in about seven days of data-taking, but more than twenty days are required in which to achieve the same precision on the mass of the 7S-state.

All these figures could be substantially improved if the centre-of-mass energy spread could be reduced.

3.3 P-states\(^*)\)

As already mentioned in the Introduction, after the discovery of the 1S- and 2S-states the determination of the P-wave \(x_t\) mass will be important in order to check the behaviour of the quark-antiquark potential at distances below 0.1 fm.

As for lighter quarkonium states \([(cc) \text{ and } (bb)]\), the method used to investigate the \((t\bar{t})\) P-states will consist in detecting the photons emitted by E1 transitions in the reaction:

\[
\theta' \rightarrow \gamma x_t \rightarrow \gamma_2 \theta . \tag{3.13}
\]

\(^*)\) D. Boutigny (LAPP) has contributed significantly to the work presented in this subsection.
Contrary to the Ψ and Υ spectroscopies the branching ratio for the dipole transition $\chi_T^+ \gamma_2 \theta$ is larger than 50% for toponia in the mass range from 60 to 80 GeV [25]. The inclusive photon spectrum of $\theta'$ decays will therefore contain two sharp lines of comparable height, which in principle facilitates the search for the $\chi_T$ states.

Owing to the anticipated large t-quark mass, SQCD channels are open for both S- and P-states and, as can be seen from Fig. 2.11, branching ratios for radiative decays decrease steeply when the top mass $m_t$ increases. In addition, the energy level differences between states are mass-dependent and also model-dependent. For example, if $m_{\theta'} = 80$ GeV, the following energy ranges are expected (Section 2):

$$E_{\gamma_1} = 100-150 \text{ MeV},$$
$$E_{\gamma_2} = 370-850 \text{ MeV},$$
$$\Delta m_{FS}^t = 5-10 \text{ MeV}.$$  

In order to estimate the integrated luminosity necessary to determine the centre of gravity of the P-states, we have simulated the signal produced in reaction (3.13) as well as the associated background. To do this, the $\theta'$ production rates have been computed assuming that $R_{\theta'} = 1/3 R_{\theta}$ with $R_{\theta}$ taken from Fig. 2.16 and using the LEP 13 (3 mA) luminosity and beam-spread conditions [38]. The assumed $\theta$, $\theta'$, and P-states properties are shown in Fig. 3.9 for

<table>
<thead>
<tr>
<th>$\theta'$ = 70 GeV</th>
<th>$\theta'$ = 80 GeV</th>
<th>$\theta'$ = 90 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>2S</td>
<td>2S</td>
<td>2S</td>
</tr>
<tr>
<td>3.5% $E_1 = 100$ MeV</td>
<td>1.7% $E_1 = 100$ MeV</td>
<td>0.3% $E_1 = 100$ MeV</td>
</tr>
<tr>
<td>1P</td>
<td>1P</td>
<td>1P</td>
</tr>
<tr>
<td>70% $E_2 = 700$ MeV</td>
<td>45% $E_2 = 800$ MeV</td>
<td>30% $E_2 = 850$ MeV</td>
</tr>
<tr>
<td>1S $\mu^+$ 7%</td>
<td>1S $\mu^+$ 5%</td>
<td>1S $\mu^+$ 3.6%</td>
</tr>
<tr>
<td>$\mu^-$</td>
<td>$\mu^-$</td>
<td>$\mu^-$</td>
</tr>
</tbody>
</table>

Fig. 3.9 Input parameters for radiative transitions
distinct $\theta'$ masses of 70, 80, and 90 GeV. They have been estimated on the basis of QCD-like potential models [see Eq. (2.25), Ref. 25].

The energy spectrum of the photons emitted in reaction (3.13), including the photons from all $\theta$-decay products, has been computed for the corresponding $\theta'$ mass values. As an example, Fig. 3.10a shows the 0 to 1 GeV photon spectrum obtained at $m_{\theta'} = 70$ GeV for an integrated luminosity of 90 pb$^{-1}$ corresponding to approximately one year of operation.

The associated background photons have also been computed for the following contributions:

- the continuum, with values of $R_{Y, Z}$ as shown in Fig. 2.16, generated according to the Lund model;
- the various hadronic decay products of the $\theta'$ (see Fig. 2.11);
- the hadronic decay products of the $x_L$ states [see Ref. (25)].

A $|\cos \theta_Y| < 0.98$ cut was applied in order to essentially eliminate photons from initial-state radiation.

Figure 3.11 shows, for $m_{\theta'} = 70$ GeV and $L dt = 90$ pb$^{-1}$ (corresponding to $1.8 \times 10^4$ events), the number of background photons per bin 10 MeV.
Fig. 3.11 Energy spectrum of background photons coming from processes other than $\theta' \rightarrow \gamma \gamma \theta$ (+ anything) ($W = m_\theta = 70$ GeV). The plot is based on $1.8 \times 10^6$ Monte Carlo events.

The probability per hadronic event to find a background photon of energy $E_\gamma$ is given by

$$P(E_\gamma) = \frac{dN}{dE_\gamma} \Delta E_\gamma F(E_\gamma),$$

(3.15)

where $dN/dE_\gamma$ can be obtained from Fig. 3.11, multiplying the number of photons per 10 MeV bin by $(N_{evts} \times 10 \text{ MeV})^{-1}$; $\Delta E_\gamma$ is determined by the energy resolution of the detector; $F$ denotes the fraction of photons that remain after $\pi^0$ and $\eta^0$ rejection. The evidence of P-state production can be obtained searching for a signal in the single-photon spectrum or by studying the double-photon cascade. As an illustration, we consider the case of a BGO detector with an energy resolution

$$\sigma_E = 1.65\% \sqrt{E}, \quad \text{for} \ 0.02 \text{ GeV} < E < 2.7 \text{ GeV},$$

$$\sigma_E = 1\% E, \quad \text{for} \ E > 2.7 \text{ GeV},$$

(3.16)
and a spatial resolution $\sigma_{X,Y}$ varying between 1 and 6 mm, depending on $E$. Using specific pattern-recognition algorithms for photon isolation, $n^0$ and $n^1$ reconstruction has been attempted [51]. Figure 3.10b and the dashed line of Fig. 3.11 show the spectra of photons that remain after the assumed reconstruction.

For distinct $\theta'$ mass values and integrated luminosities, the number of detected photons with energy $E_1$ or $E_2$ and the number of events where both photons are detected in coincidence in a $2\sigma_E$ window are given in Table 3.4 for the signal and for the background. The results given in Table 3.4 can probably be slightly improved by reduction of systematic errors on energy resolution and optimization of pattern-recognition algorithms.

In any case, owing to the fast increase of $R^{\gamma_1\gamma_2}/R^{\theta'}$ combined with the decrease of radiative transition rates when the toponium mass increases, this two-photon coincidence method of searching for $\chi_{\ell}$ states seems to be limited to toponium masses lower than 80 GeV.

<table>
<thead>
<tr>
<th>Table 3.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energies of $\gamma_1$ and $\gamma_2$, number of signal and background photons in the inclusive spectrum, and number of events requiring $\gamma_1\gamma_2$ coincidence</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$m_{\theta'}$ (GeV)</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J L dt$ (hours)</td>
<td>1000</td>
<td>2800</td>
</tr>
<tr>
<td>$J L dt$ (pb$^{-1}$)</td>
<td>32</td>
<td>90</td>
</tr>
<tr>
<td>$E_{\gamma_1}$ (MeV)</td>
<td>$100 \pm 10$</td>
<td>$100 \pm 10$</td>
</tr>
<tr>
<td>Signal $S_1$</td>
<td>63</td>
<td>177</td>
</tr>
<tr>
<td>Background $B_1$</td>
<td>1254</td>
<td>3540</td>
</tr>
<tr>
<td>$S_1/B_1$ ($\sigma$)</td>
<td>1.8</td>
<td>3.0</td>
</tr>
<tr>
<td>$E_{\gamma_2}$ (MeV)</td>
<td>$700 \pm 25$</td>
<td>$700 \pm 25$</td>
</tr>
<tr>
<td>Signal $S_2$</td>
<td>53</td>
<td>147</td>
</tr>
<tr>
<td>Background $B_2$</td>
<td>675</td>
<td>1834</td>
</tr>
<tr>
<td>$S_2/B_2$ ($\sigma$)</td>
<td>2.0</td>
<td>3.4</td>
</tr>
<tr>
<td>$\gamma_1\gamma_2$ coincidence</td>
<td>33</td>
<td>93</td>
</tr>
<tr>
<td>Signal $S_{1,2}$</td>
<td>113</td>
<td>315</td>
</tr>
<tr>
<td>Background $B_{1,2}$</td>
<td>3.1</td>
<td>5.2</td>
</tr>
<tr>
<td>$S_{1,2}/B_{1,2}$ ($\sigma$)</td>
<td>33</td>
<td>93</td>
</tr>
</tbody>
</table>
Table 3.5

Counting rates in the exclusive channel \( \theta' + \gamma_1 \gamma_2 l^+l^- \)

<table>
<thead>
<tr>
<th>( m_{\theta'} ) (GeV)</th>
<th>( \int L , dt ) (hours)</th>
<th>( \int L , dt ) (pb(^{-1}))</th>
<th>( \gamma_1 \gamma_2 l^+l^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>1000</td>
<td>32</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>2800</td>
<td>90</td>
<td>40</td>
</tr>
<tr>
<td>75</td>
<td>1000</td>
<td>36</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>2800</td>
<td>100</td>
<td>18</td>
</tr>
<tr>
<td>80</td>
<td>1000</td>
<td>39</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2800</td>
<td>110</td>
<td>9</td>
</tr>
<tr>
<td>85</td>
<td>2800</td>
<td>125</td>
<td>4</td>
</tr>
<tr>
<td>90</td>
<td>2800</td>
<td>136</td>
<td>2.5</td>
</tr>
</tbody>
</table>

As an alternative, we have also considered the possibility of using the exclusive reaction

\[
\theta' + \gamma_1 \gamma_2 \rightarrow l^+l^-, \quad (3.17)
\]

which gives the signal event rates shown in Table 3.5 when summing over the three charged-lepton types: \( e^+e^- \), \( \mu^+\mu^- \), \( \tau^+\tau^- \). In this case the background is due to radiation in initial and final states, which produces photons mainly collinear to the beam or to the final leptons. With an angular cut of 10° and neglecting interferences between initial and final radiation, one finds

\[
\tilde{P}(E_\gamma) = 4.5 \times 10^{-2} \frac{\Delta E_\gamma}{E_\gamma} \quad (3.18)
\]

for the number of photons radiated per event, in a certain energy interval \( \Delta E_\gamma \).

The probability per event to contribute as background to the double-cascade process is given by

\[
\tilde{P}(E_{\gamma_1}, E_{\gamma_2}) = (4.5 \times 10^{-2})^2 \cdot \frac{\Delta E_{\gamma_1}}{E_{\gamma_1}} \cdot \frac{\Delta E_{\gamma_2}}{E_{\gamma_2}} \quad (3.19)
\]

The requirements (as previously proposed in Ref. [52]), that the two leptons have an acollinearity \( \lesssim 10° \) in the \( e^+e^- \) or \( \mu^+\mu^- \) case or that there be a number of charged tracks \( \lesssim 3 \) for each back-to-back \( \tau \) decay and a total measured energy
\[ E_{\text{vis}} > 0.4 \text{ /s}, \] lead us to estimate that the background is smaller than the signal up to \( m_\chi \approx 85-90 \text{ GeV} \), in the case of a BGO detector.

With a gaseous electromagnetic detector, for which we assume that the detection of photons down to 100 MeV is feasible with \( \sigma_\gamma/E = 15\%/E \), we estimate that the background becomes as large as the signal near \( m_\chi \approx 80 \text{ GeV} \).

It should be noted that with such a gaseous detector it will not be possible to determine the 1P-state position with precision, but the event rate could put constraints on the model, through the measurement of the dipole transition rate. With a BGO detector a more precise determination of the \( \chi_t \) mass will be possible.

To summarize: Based on the predictions of the present models, the \( \chi_t \) energy-level determination seems feasible for toponium masses in the low-energy range of LEP I.

Using the reaction

\[ \theta' \rightarrow \gamma_1 \gamma_2 \theta, \]

the inclusive one-photon or semi-inclusive two-photon methods appear to be limited to toponium masses of less than 80 GeV. The exclusive method, with the associated detection of \( \mu^- \mu^+ \), \( e^-e^+ \), or \( \ell^-\ell^+ \), seems to be more promising for pushing this limit up to toponium masses of 85-90 GeV.

It is worth recalling here that the toponium production rates are uncertain within a factor of 2. On the other hand, the knowledge of the \( \theta \) and \( \theta' \) energy levels will constrain the expected counting rates as well as the photon energy ranges to be scanned.

4. ANGULAR AND POLARIZATION ASYMMETRIES

As discussed in Section 2, neutral-current effects and W exchange will play an important role in toponium decays. Indeed, toponium seems to be an ideal place for determining the coupling of the neutral current to top quarks. The measurement of asymmetries on toponium determines the relative strength of the neutral-current coupling \( g^V_t \) and the electromagnetic coupling \( e_t \) quite unambiguously, in contrast with open-top production, which has to cope with uncertainties from QCD corrections and quark-mass effects, and which involves a mixture of vector and axial-vector contributions.

We shall base the following discussion mainly on the theoretical formalism developed in Refs. [29] and [24] and on earlier works referred to in those papers. We shall treat four different asymmetry measurements. These are all (apart from those with \( b\bar{b} \) as the final state) dependent on \( g^V_t/e_t \) or its square, although from an experimental viewpoint they are rather different. The ratio \( g^V_t/e_t \) is of course determined in the standard model by \( \sin^2 \theta_W \). Nevertheless it
is desirable to measure this quantity, which is also necessary for deducing the wave function at the origin from $\Gamma_{ee}$, in a model-independent way. Furthermore, this allows an independent determination of quark charge and isospin.

In the analysis presented below, we have always taken an incoherent sum of the resonance and the $\gamma-Z$ continuum. In the neighbourhood of $Z (\{|m_{\tau}^2 - m_e^2| \lesssim 2\Gamma_{Z}^2\}$ a more involved formalism is required (see Refs. [24] and [33]). Here we are only interested in the experimental accuracy that can be expected and in a comparison of the different methods proposed, and for this the simplified treatment is adequate. To determine $a_t^V/e_t$, one could make use of longitudinal beam polarization, or measure the $\tau$ polarization; or one could directly measure the toponium polarization through semileptonic SQDs; $(a_t^V/e_t)^2$ can also be measured through the difference in the forward-backward asymmetries of lepton pairs on and off resonance. We shall now discuss these possibilities in turn, and list the integrated luminosity in pb$^{-1}$ (= 24 hours at $L = 10^{31}$ cm$^{-2}$ s$^{-1}$) which is required in order to achieve a specified accuracy in the detection of the asymmetry.

4.1 Polarization asymmetry

Longitudinally polarized beams will not be available in the first round of LEP experiments. We nevertheless start with the polarization asymmetry $a_{RL}$, which can be expressed in terms of the coupling constants in a particularly simple way. Other asymmetries which can be measured with unpolarized beams are then conveniently expressed in terms of $a_{RL}$ and may thus also serve to measure this quantity.

If longitudinally polarized beams are available, one expects a difference in the production cross-section for right-handed versus left-handed polarization owing to neutral-current effects [53, 54, 29, 24]. The polarization asymmetry defined by

$$a_{RL} = \frac{a_{R} - a_{L}}{a_{R} + a_{L}} ,$$

is in general different on and off resonance. Ignoring the aforementioned interference between resonance and continuum, and taking the effects of beam energy smearing into account, $a_{RL}$ on top of the resonance is given by

$$\langle a_{RL} \rangle = \frac{a_{RL}^{on} + a_{RL}^{off} \eta}{1 + \eta} ,$$

where $\eta = R_{cont}/R_{\theta}$ parametrizes the 'contamination' of the resonance by the continuum background. The asymmetries from resonance and continuum are given by
\[ \alpha_{\text{on}}^{\text{RL}} = -2 \frac{\text{Re} \left( \lambda_{e}^{*} \lambda_{e}' \right)}{\left| \lambda_{e} \right|^2 + \left| \lambda_{e}' \right|^2}, \quad (4.3) \]

\[ \alpha_{\text{off}}^{\text{RL}} = \frac{\sum_{h_{f},h_{e}} h_{e} \left| F^{C}(h_{f},h_{e}) \right|^2}{\sum_{h_{f},h_{e}} \left| F^{C}(h_{f},h_{e}) \right|^2}, \quad (4.4) \]

where

\[ \lambda_{f} = \frac{e^2}{s} e_{f} e_{t} + \frac{e^2}{y} \frac{v_{f} v_{t}}{s - m_{Z}^2 + i m_{Z} \Gamma_{Z}}, \quad (4.5) \]

\[ \lambda_{f}' = \frac{e^2}{y} \frac{a_{f} v_{t}}{s - m_{Z}^2 + i m_{Z} \Gamma_{Z}}, \quad (4.6) \]

\[ y = 2 \sin 2\theta_{W}, \]

\[ F^{C}(h_{f},h_{e}) = \frac{e^2 e_{f} e_{f} e_{e}}{s} + \frac{e^2}{y} \frac{(v_{f} - h_{f} a_{f})(e_{f} e_{e})}{s - m_{Z}^2 + i m_{Z} \Gamma_{Z}}, \quad (4.7) \]

and

\[ a_{u} = a_{v} = 1, \quad a_{d} = a_{e} = -1; \]

\( v_{f} \) has been defined in Eq. (2.14).

The asymmetries \( \alpha_{\text{on}}^{\text{RL}} \) and \( \alpha_{\text{off}}^{\text{RL}} \) (for \( \mu \) pairs) are shown in Fig. 4.1 as a function of \( m_{\phi} \). (Note that \( \alpha_{\text{on}}^{\text{RL}} \) is independent of the final state, but not \( \alpha_{\text{off}}^{\text{RL}} \).)

\[ \text{Fig. 4.1 The polarization asymmetry} \alpha_{\text{on}}^{\text{RL}} \text{ on top of an S-wave toponium resonance as a function of the toponium mass, compared with the corresponding value of the} \mu'\mu' \text{ continuum. From Ref. [24].} \]
As an illustration, we show in Fig. 4.2 the variation of \( \langle \alpha_{RL} \rangle \) for the \( \mu^- \) pair final state—assuming that toponium is located in the 80 GeV region.

For the realistic case where both the electron and the positron beams are only partially polarized, of degree \( P_- \) and \( P_+ \), the relative difference in the cross-section is given by \( \alpha_{RL} \cdot P_{\text{eff}} \), where

\[
P_{\text{eff}} = \frac{P_- - P_+}{1 - P_- P_+} = \frac{2P_-}{1 + P_-^2}.
\] (4.8)

The statistical error expected for this measurement is given by

\[
\Delta \alpha_{RL}^{\text{on}} = \frac{(1+\eta)}{N} \sqrt{\left( \frac{1}{P_{\text{eff}}} - \langle \alpha_{RL} \rangle^2 \right) + \eta^2 (\langle \alpha_{RL} \rangle - \alpha_{RL}^{\text{off}})^2}.
\] (4.9)

In addition, one has to take into account the systematic error from the uncertainty in the beam polarization:

\[
\Delta \alpha_{RL}^{\text{on \, (sys.)}} = \alpha_{RL} \cdot \frac{\Delta P_{\text{eff}}}{P_{\text{eff}}} = \alpha_{RL} \cdot \frac{(1-P_-^2) \Delta P_-}{P_-}.
\] (4.10)
Table 4.1

Polarization asymmetry $\alpha_{RL}^{on}$ for different values of $m_\theta$, the ratio $\eta$, the required statistical error $\Delta \alpha_{RL}^{on}$, and the continuum and resonance contributions to $R$. The number of events and the integrated luminosity needed to achieve $0.1$ statistical error on $\alpha_{RL}^{on}$ for 100% and 50% beam polarization are listed, together with the systematic error on $\alpha_{RL}^{on}$, assuming $\Delta P_{\text{beam}} = \pm 0.05$

<table>
<thead>
<tr>
<th>$m_\theta$ (GeV)</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>83</th>
<th>90</th>
<th>100</th>
<th>110</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{RL}^{on}$</td>
<td>0.32</td>
<td>0.55</td>
<td>0.92</td>
<td>0.99</td>
<td>0.48</td>
<td>-0.90</td>
<td>-0.97</td>
</tr>
<tr>
<td>$\eta$ (hadron events)</td>
<td>0.4</td>
<td>0.77</td>
<td>2.2</td>
<td>3.3</td>
<td>6.4</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>$\Delta \alpha_{RL}^{on}$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$R_{\text{off}} + R_{\text{res}}$ (hadron events)</td>
<td>19</td>
<td>18</td>
<td>29</td>
<td>44</td>
<td>250</td>
<td>440</td>
<td>140</td>
</tr>
</tbody>
</table>

100% polarization

| $N_{\text{evts}}$ | 187 | 297 | 1349 | 2838 | 6851 | 49951 | 65345 |
| $L$ (pb$^{-1}$)    | 0.2 | 0.4 | 1.7  | 2.5  | 1.2  | 6.5   | 32.5  |

50% polarization

| $N_{\text{evts}}$ | 298 | 474 | 1925 | 3878 | 9931 | 70257 | 90152 |
| $L$ (pb$^{-1}$)    | 0.5 | 1.2 | 3.9  | 5.6  | 2.9  | 14.7  | 71.8  |

Syst. error

| $(\Delta P/P = 0.1)$ | 0.02 | 0.03 | 0.06 | 0.06 | 0.02 | 0.05 | 0.06 |

In Table 4.1 we have listed the number of hadronic events needed to achieve $\Delta \alpha_{RL}^{on} = 0.1$, together with the required integrated luminosity and the systematic error. The dilution of $\alpha_{RL}^{on}$ by the continuum is reduced for some masses (particularly above ~ 100-110 GeV) if we base this measurement on final states from SQDs. We have shown in subsection 3.2 that topological cuts will allow 95% of the hadronic continuum to be rejected, whilst 87% of the SQDs are retained.
Table 4.2

The integrated luminosity required in order to achieve $\Delta a^{on}_{RL} = 0.1$ with longitudinally polarized beams: a) selecting SQD events through topological cuts; b) selecting semileptonic SQD events. The underlined numbers indicate cases where these cuts lead to an improvement.

a) SQD events (87% signal, 5% background)

<table>
<thead>
<tr>
<th>$m_B$ (GeV)</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>83</th>
<th>90</th>
<th>100</th>
<th>110</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>0.13</td>
<td>0.13</td>
<td>0.27</td>
<td>0.47</td>
<td>2.89</td>
<td>5.2</td>
<td>1.7</td>
</tr>
<tr>
<td>$R^{\text{off}} + R^{\text{res}}$</td>
<td>2.39</td>
<td>3.6</td>
<td>4.6</td>
<td>5.3</td>
<td>14.8</td>
<td>26.2</td>
<td>11.1</td>
</tr>
<tr>
<td>$N_{\text{evts}}$</td>
<td>190</td>
<td>169</td>
<td>173</td>
<td>259</td>
<td>2691</td>
<td>6954</td>
<td>1138</td>
</tr>
<tr>
<td>$L$ (pb$^{-1}$)</td>
<td>2.6</td>
<td>2.1</td>
<td>2.2</td>
<td>3.1</td>
<td>13.3</td>
<td>29.9</td>
<td>11.4</td>
</tr>
</tbody>
</table>

b) Semileptonic SQD events (no background!!)

<table>
<thead>
<tr>
<th>$m_B$ (GeV)</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>83</th>
<th>90</th>
<th>100</th>
<th>110</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$R^{\text{res}}$</td>
<td>1.1</td>
<td>1.7</td>
<td>1.9</td>
<td>1.9</td>
<td>2.0</td>
<td>2.2</td>
<td>2.1</td>
</tr>
<tr>
<td>$N_{\text{evts}}$</td>
<td>147</td>
<td>125</td>
<td>71</td>
<td>56</td>
<td>133</td>
<td>75</td>
<td>62</td>
</tr>
<tr>
<td>$L$ (pb$^{-1}$)</td>
<td>4.4</td>
<td>3.3</td>
<td>2.2</td>
<td>1.8</td>
<td>4.9</td>
<td>3.1</td>
<td>2.2</td>
</tr>
</tbody>
</table>

The resulting requirements on the counting rate for a statistical error of 0.1 are listed in Table 4.2. This also contains the numbers if we select the almost background-free semileptonic SQDs.

In the early stages of LEP operation, longitudinally polarized beams will not be available. However, for toponium there are a number of other possibilities for determining $a_{RL}$, which we will now discuss.

4.2 Polarization of $\gamma$ leptons

Assuming identical neutral-current couplings of electrons, muons, and $\gamma$ leptons, the longitudinal polarization of outgoing muons and $\gamma$'s is given by
\( \alpha_{RL} \) as defined in Eqs. (4.2) to (4.4). The polarization of \( \tau \)'s can be observed through the momentum distribution of their decay products*. For example, for the energy distribution of \( \tau \)'s or \( \phi \)'s from \( \tau \) decay [55, 56] one finds

\[
\frac{dN}{dz} = \frac{1}{(1-r_h^2)} \left[ 1 + \alpha_{RL} \frac{C_h}{1-r_h^2} \left[ 2z - (1 + r_h^2) \right] \right], \tag{4.11}
\]

\( r_h^2 < z < 1 \),

where

\[
r_h = \frac{m_{\tau,\phi}}{m_\tau},
\]

\[
C_h = \begin{cases} 
1 & \text{for } h = \tau \\
\frac{m_\tau^2 - 2 m_\tau^2}{m_\tau^2 + 2 m_\phi^2} & \text{for } h = \phi.
\end{cases}
\]

For \( \tau \) pair production in the continuum, this technique is discussed in more detail in the report of Altarelli et al. on precision studies [57]. Considering the small combined branching ratio on toponium, the expected statistical error will be large. Nevertheless, this piece of information could be used to resolve the sign ambiguity inherent in the determination of \( g_\ell^V/e_\ell \) through the forward-backward asymmetry.

### 4.3 Forward-backward asymmetry

Neutral-current effects and \( W \) exchange will also lead to a marked difference of the forward-backward asymmetry on and off resonance. The angular distribution for an arbitrary fermion-antifermion final state is given by

\[
\frac{dN}{d \cos \theta} \propto 1 + \cos^2 \theta + 2 \alpha_{FB} \cos \theta, \tag{4.12}
\]

and a non-vanishing \( \alpha_{FB} \) leads to a forward-backward asymmetry,

\[
\frac{N_F - N_B}{N_F + N_B} = \frac{3}{4} \alpha_{FB}. \tag{4.13}
\]

*) The use of \( \tau \) leptons to study weak effects on quarkonia was first emphasized in Refs. [55, 56].
As before, the measurement of $\alpha_{FB}$ for $W = \nu_\theta$ will be affected by the resonance and the continuum asymmetry. Ignoring interference between the two contributions, which is legitimate for $|\nu_\theta - \nu_e| \gg 2\Gamma_e$,

$$|\alpha_{FB}| = \frac{\alpha_{FB}^{on} + \alpha_{FB}^{off}}{1 + \eta} \eta,$$

(4.14)

where $\eta$ again parametrizes the relative strength of continuum versus resonance. Under the same assumptions as before, $\alpha_{FB}^{on}$ is related to the polarization asymmetry (4.3) in a rather simple way [58]:

$$\alpha_{FB}^{on} = \alpha_{RL}^2,$$

(4.15)

and a similar relation can be derived also for quark-antiquark final states [58]:

$$\alpha_{FB}^{on}(q) = \alpha_{RL}^2 \alpha_{RL}(q),$$

(4.16)

where $\alpha_{RL}(q)$ is defined analogously to $\alpha_{RL} = \alpha_{RL}(e)$ in Eq. (4.3).

The statistical error in the determination of $\alpha_{FB}^{on}$ for a sample of $N$ lepton pairs is given by

$$\Delta \alpha_{FB}^{on} = \frac{1 + \eta}{\sqrt{N}} \sqrt{C - \langle \alpha_{FB} \rangle^2 + \eta^2 (\langle \alpha_{FB} \rangle - \alpha_{FB}^{off})^2},$$

(4.17)

with $C = 16/9$ if we use only the forward-backward asymmetry

$$\frac{N_F - N_B}{N_F + N_B} = \frac{3}{4} \langle \alpha_{FB} \rangle,$$

(4.18)

and $C = 8/5$ if we use the full shape of the distribution and evaluate $\langle \cos \theta \rangle$ using

$$\langle \cos \theta \rangle = \frac{1}{2} \langle \alpha_{FB} \rangle \quad \text{and} \quad \Delta \langle \alpha_{FB} \rangle = \frac{2}{\sqrt{N}} \sqrt{\frac{2}{5} - \left(\frac{\langle \alpha_{FB} \rangle}{2}\right)^2}.$$

(4.19)

The asymmetry off resonance is given by

$$\alpha_{FB}^{off} = \frac{h_{f,e} |F_c(h_f, h_e)|^2}{\sum h_{f,e} |F_c(h_f, h_e)|^2}.$$
Fig. 4.3 The $\mu^-\mu^+$ forward-backward asymmetry $a_{FB}$ on top of an S-wave toponium resonance decaying into $\mu^-\mu^+$, compared with the continuum. From Ref. [24].

Fig. 4.4 $\mu^-\mu^+$ forward-backward asymmetry $\langle a_{FB} \rangle$ averaged over the beam profile for top-quark masses of a) 40 GeV and b) 47 GeV (radiative corrections are not taken into account). From Ref. [24].

The asymmetries $a_{FB}^{on}$ and $a_{FB}^{off}$ for $\mu$ pairs are shown in Fig. 4.3 as a function of $m_{\tau}$. Figure 4.4 shows the variation of $\langle a_{FB} \rangle$ for the $\mu$-pair final state with $m_{t} = 40$ GeV. In Table 4.3 we have listed the number of lepton pairs required to achieve either 10% statistical error on $a_{FB}^{on}$, or alternatively 10% error on $|a_{RL}^{on} = |a_{FB}^{on}$. The estimated integrated luminosity is based on $\mu$ pairs only. The inclusion of some or all of the $\tau$-pair final states would reduce the luminosity requirements accordingly.

Toponium annihilation into $b\bar{b}$ and the resulting asymmetries [29] may give important information on the strength of the $W$ exchange amplitude. The relative importance of this channel is strongly mass-dependent; furthermore, the possibility to tag $B$ mesons will depend on the $B$-meson lifetime. In spite of these uncertainties it is worth while keeping this possibility in mind.

In passing we note that also azimuthal distributions might be different on and off resonance, if transversely polarized beams are available. The relevant formulae are given elsewhere [24].
The integrated luminosity required in order to achieve $\Delta a_{RL}^{on} = 0.1$ and, alternatively, $\Delta |a_{RL}^{on}| = 0.1$, through a measurement of the $\mu$-pair asymmetry. For $m_\mu = 100$ and 110 GeV the upper (lower) values are without (with) radiative corrections applied to the continuum. The other energies are always with radiative corrections.

<table>
<thead>
<tr>
<th>$m_\mu$ (GeV)</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>83</th>
<th>90</th>
<th>100</th>
<th>110</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{on}^{RL}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.30</td>
<td>0.84</td>
<td>0.97</td>
<td>0.23</td>
<td>0.81</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>$\alpha_{FB}^{off}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.51</td>
<td>-0.75</td>
<td>-0.98</td>
<td>-0.90</td>
<td>-0.46</td>
<td>0.55</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>$\eta_{\mu\mu}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.13</td>
<td>1.63</td>
<td>3.3</td>
<td>4.4</td>
<td>7.1</td>
<td>12.3</td>
<td>18.8</td>
<td></td>
</tr>
<tr>
<td>$\langle\alpha_{FB}\rangle$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.22</td>
<td>-0.34</td>
<td>-0.34</td>
<td>-0.55</td>
<td>-0.37</td>
<td>0.57</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>$R^{off} + R^{res}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.37</td>
<td>2.27</td>
<td>2.6</td>
<td>3.2</td>
<td>13.8</td>
<td>11.9</td>
<td>3.12</td>
<td></td>
</tr>
</tbody>
</table>

| $\Delta \alpha_{RL}^{on}$ |
| 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| $\Delta a_{FB}^{on}$ |
| 0.06 | 0.10 | 0.18 | 0.20 | 0.10 | 0.18 | 0.19 |
| $N_{\mu\mu}$ |
| 2085 | 1321 | 1863 | 2694 | 12052 | 7300 | 7500 |
| $L (pb^{-1})$ |
| 36 | 33 | 53 | 67 | 81 | 71 | 337 |

| $\Delta |a_{RL}^{on}|$ |
| 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| $N_{\mu\mu}$ |
| 728 | 1321 | 6037 | 10778 | 12052 | 23652 | 27000 |
| $L (pb^{-1})$ |
| 12 | 33 | 171 | 276 | 81 | 228 | 1217 |

4.4 Polarization measurements through semileptonic single-quark decays

A convenient method of determining the polarization asymmetry is based on the analysis of leptons from SQDs [59]. The polarization asymmetry leads to a longitudinal polarization of toponium even for unpolarized beams:

$$\langle \mathbf{S} \cdot \mathbf{n}_e \rangle = \alpha_{RL}^{on}.$$  \hspace{1cm} (4.21)
The parity-violating SQOs provide a convenient tool for analysing the spin of toponium. Since t and \( \bar{t} \) are in a spin triplet S-wave state, their polarization is also given by Eq. (4.21). The angular distribution of leptons from the decay of a top quark is correlated with the top spin through

\[
dN = (1 + \hat{n} \cdot \hat{S}) \, d\Omega .
\]

(4.22)

The remaining t (or \( \bar{t} \)) quark is converted into a top-meson T or T*, and the T and T* both decay weakly [60]. For leptons from this second step this leads to a partial depolarization by a factor of 1/2. The angular distribution of positive leptons from t decay is finally given by

\[
dN = \left[ 1 + \frac{3}{4} \alpha_{RL} \cos \theta \right] \, d\cos \theta ,
\]

(4.23)

and for negative leptons from \( \bar{t} \) decay the sign is obviously reversed.

Events from semileptonic SQOs can be discriminated against the background through their characteristic topology and their hard isolated lepton. The statistical error on the measurement of \( \alpha_{RL} \) through this method is

\[
\Delta \alpha_{RL} = \frac{1}{N} \sqrt{\frac{C_{SQO}}{\langle \alpha_{RL} \rangle^2} - 1} \approx \frac{1}{3} \frac{1}{N} ,
\]

(4.24)

and \( C_{SQO} = 64/9 \) or 16/3 depending on whether we base the analysis on the forward-backward asymmetry or on \( \langle \cos \theta \rangle \). The requirements for the counting rate and running time to achieve \( \Delta \alpha_{RL} = 0.1 \) are listed in Table 4.4.

**Table 4.4**

The integrated luminosity required in order to achieve \( \Delta \alpha_{RL}^{on} = 0.1 \) through a measurement of lepton asymmetries in semileptonic SQOs

<table>
<thead>
<tr>
<th>( m_B ) (GeV)</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>83</th>
<th>90</th>
<th>100</th>
<th>110</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{RL}^{on} )</td>
<td>0.30</td>
<td>0.55</td>
<td>0.92</td>
<td>1.0</td>
<td>0.48</td>
<td>-0.9</td>
<td>-0.97</td>
</tr>
<tr>
<td>( \Delta \alpha_{RL}^{on} )</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>( 4/9 , R_{SQO} )</td>
<td>1.1</td>
<td>1.7</td>
<td>1.9</td>
<td>1.9</td>
<td>2.0</td>
<td>2.2</td>
<td>2.1</td>
</tr>
<tr>
<td>( N_{evts} )</td>
<td>524</td>
<td>503</td>
<td>448</td>
<td>433</td>
<td>510</td>
<td>452</td>
<td>4394</td>
</tr>
<tr>
<td>( L ) (pb(^{-1}))</td>
<td>19.7</td>
<td>16.7</td>
<td>17.4</td>
<td>18.1</td>
<td>23.8</td>
<td>23.7</td>
<td>29.1</td>
</tr>
</tbody>
</table>
The advantage of this method is twofold: first, the branching ratio for semileptonic SQDs is larger than the $\mu$-pair rate, apart from a narrow mass range very close to $m_\tau$; secondly, there are no continuum events which feed into this channel, so that a smaller sample leads to a relatively smaller statistical error.

4.5 Summary

In Table 4.5 we have compared the requirements on the integrated luminosity in units of pb$^{-1}$ ($\approx$ 24 hours at $L = 10^{31}$ cm$^{-2}$ s$^{-1}$). It is apparently quite possible to achieve $\Delta a_{RL} = 0.1$ through the lepton-pair asymmetry within the first year of toponium physics. Longitudinally polarized beams reduce the time requirement by a sizeable amount, and the systematic error from $\Delta P/P$ would dominate relatively quickly in this case. However, for the method based on semileptonic single-quark decays, the luminosity requirements are also modest.

Table 4.5

Comparison of the various methods in terms of integrated luminosities (pb$^{-1}$) required in order to achieve a statistical error of 0.1 on $a_{RL}$

<table>
<thead>
<tr>
<th>$m_\theta$ (GeV)</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>83</th>
<th>90</th>
<th>100</th>
<th>110</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polarized beams ($P = 0.5$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hadron events</td>
<td>0.5</td>
<td>1.2</td>
<td>3.9</td>
<td>5.6</td>
<td>2.9</td>
<td>14.7</td>
<td>71.8</td>
</tr>
<tr>
<td>SQDs</td>
<td>2.6</td>
<td>2.1</td>
<td>2.2</td>
<td>3.1</td>
<td>13.3</td>
<td>29.9</td>
<td>11.4</td>
</tr>
<tr>
<td>Semileptonic SQDs</td>
<td>4.4</td>
<td>3.3</td>
<td>2.2</td>
<td>1.8</td>
<td>4.9</td>
<td>3.1</td>
<td>2.2</td>
</tr>
<tr>
<td>Unpolarized beams</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FB</td>
<td>36</td>
<td>33</td>
<td>53</td>
<td>67</td>
<td>81</td>
<td>$\frac{71}{139}$</td>
<td>337</td>
</tr>
<tr>
<td>Semileptonic SQDs</td>
<td>19.7</td>
<td>16.7</td>
<td>17.4</td>
<td>18.1</td>
<td>23.8</td>
<td>23.7</td>
<td>29.1</td>
</tr>
</tbody>
</table>

A statistical error below 0.1 is easily attainable in this case, and the main limitations will be set by the systematic error. A measurement of $a_{RL}$ to an accuracy of 0.05 would be sensitive to radiative corrections within the standard model. Depending on the toponium mass, this might lead to an independent determination of $\sin^2 \theta_W$ with an accuracy of up to $4 \times 10^{-3}$ [61].
5. SEARCH FOR HIGGS PARTICLES

Higgs particles are an inescapable ingredient of present gauge theories of the electroweak interactions. They are the vestige of symmetry breaking and they are responsible for mass generation of the known particles. Their experimental investigation is, then, of the highest importance [62, 63].

In the minimal Weinberg-Glashow-Salam model, one neutral Higgs particle \( H^0 \) is needed. In extensions of the standard model (e.g. SUSY, technicolour, etc.) many charged and neutral Higgs-like scalars are expected: \( H_1^0, H_2^0, \ldots; H_1^+, H_2^+, \ldots \).

In both cases, the masses of these particles are almost completely undetermined; on the other hand, the fermion-Higgs couplings \( q_{fH} \) are usually assumed to be proportional to the fermion (quark or lepton) masses. This immediately suggests that toponium is an interesting source of Higgs particles with masses \( m_H < m_\theta \).

5.1 Charged Higgs scalars

The detection of charged Higgs particles with masses less than the top-quark mass \( m_t \) will certainly be easy since, in this case, the heavy quark (antiquark) of the bound state will predominantly decay into an \( H^+ (H^-) \) and a \( b (\bar{b}) \) quark (Fig. 2.6g): The amplitude for this decay is proportional to \( \xi_{Htb} (m_t / v) \), where \( \xi_{Htb} \) represents an unknown (t, b) mixing angle and \( v \) is the Higgs vacuum expectation value. The relative widths for toponium decays into charged Higgs scalars and QCD are connected by the relation [8]

\[
\frac{\Gamma}{\Gamma_{QCD}} \approx \xi_{Htb}^2 \frac{10^2}{[m_t (100 \text{ GeV})]^2}.
\]  

(5.1)

It can be seen from this formula that, with the prejudiced value \( \xi_{Htb} = 1 \), the \( H^\pm \) decay becomes the dominant one, and consequently the decay branching ratios indicated in Fig. 2.10 will be very much suppressed. Even if \( \xi_{Htb} \) is reduced by more than one order of magnitude, the toponium decay into charged Higgs scalars will still be sizeable, the preferred decay modes of a \( H^\pm \) being \( H^+ \rightarrow \bar{b}c \), and then \( H^+ \rightarrow t^+\nu \).

It is worth noticing that the UA1 data [4] interpreted as the manifestation of a \( t \)-quark with \( m_t \approx 40 \text{ GeV} \), would exclude a maximally coupled charged Higgs scalar with \( m_H^+ < m_t \).

5.2 Neutral Higgs scalars

The importance of the heavy vector-meson decay into \( H^0 + \gamma \) (Fig. 2.6e) was first recognized by Wilczek [64]. For toponium--neglecting radiative corrections and QCD corrections [65, 66]--the decay width for this channel can be written [cf. (2.18)],
\[
\Gamma(\theta \rightarrow H^0 \gamma) = \frac{1}{8 \sin^2 \theta} \frac{m_\theta^2}{m_W^2} \left(1 - \frac{m_{H^0}^2}{m_\theta^2}\right) \Gamma_0(\theta).
\]

(5.2)

For \( m_{H^0} \) not too close to \( m_\theta \) and using the parameters given in (2.24) and (2.25), we obtain \( \theta \rightarrow H^0 \gamma \) branching ratios in the 1-3\% range for \( \theta \) masses in the LEP energy domain (cf. Fig. 2.10). This has to be compared with the equivalent bottomonium branching ratio, which is smaller by two orders of magnitude.

This reaction has a clean signature characterized by a monochromatic photon, the Higgs mass being determined by the missing-mass technique.

In order to estimate roughly the sensitivity needed to discover an eventual neutral Higgs scalar particle, we have first estimated (Table 5.1) the production rates of \( \theta \) and of continuum events for three \( \theta \) masses, 70, 90, and 110 GeV, using the \( R^\theta \) and \( R^\gamma,Z \) values shown in Fig. 2.16. The corresponding event numbers obtained for 1000 hours of running time under LEP 13 (3 mA) conditions are given in Table 5.2 for different \( m_{H^0} \) values. One notices from this table that the event rates are quite high, even for large Higgs masses.

**Table 5.1**

<table>
<thead>
<tr>
<th>( W ) (GeV)</th>
<th>70</th>
<th>90</th>
<th>110</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N^{\gamma,Z} )</td>
<td>4 700</td>
<td>114 000</td>
<td>68 000</td>
</tr>
<tr>
<td>( N^\theta )</td>
<td>7 500</td>
<td>26 800</td>
<td>3 600</td>
</tr>
</tbody>
</table>

**Table 5.2**

<table>
<thead>
<tr>
<th>( m_\theta ) (GeV)</th>
<th>10</th>
<th>30</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>85</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_{H^0} ) (GeV)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>([ \int L , dt , (pb^{-1})] )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70 [32]</td>
<td>202</td>
<td>160</td>
<td>102</td>
<td>52</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90 [49]</td>
<td>107</td>
<td>95</td>
<td>75</td>
<td>60</td>
<td>43</td>
<td>22</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>110 [65]</td>
<td>39</td>
<td>36</td>
<td>31</td>
<td>28</td>
<td>25</td>
<td>19</td>
<td>16</td>
<td>13</td>
<td>7</td>
</tr>
</tbody>
</table>
We have then computed the inclusive photon background originating from
- the $\gamma$ and $Z$ contribution generated according to the Lund model;
- the contribution due to competing $\Theta$ decays, weighted with their relative
  branching ratios as indicated in Fig. 2.10.

In each case, a $|\cos \theta_{\gamma}| < 0.98$ cut, which virtually does not affect the
signal, was applied to essentially eliminate photons from initial-state radiation.

5.2.1 Hadronic background

The inclusive spectra originating from the various background reactions are
shown in Figs. 5.1, 5.2, and 5.3 for $m_{\Theta} = 70$, 90, and 110 GeV, respectively.
They have been arbitrarily normalized to $10^5$ Lund ($\gamma$, $Z$) events. The corre-
sponding rates of monochromatic photons from the $\Theta + H^0 \gamma$ reaction are also indi-
cated for various Higgs masses.

From these curves, summing up the various inclusive contributions to obtain
the final background spectrum at each energy, it is possible to estimate the
integrated luminosity needed to detect a $H^0$ particle. As an example, Fig. 5.4
shows the typical integrated luminosities needed to detect a $3\sigma$ effect with a $4\pi$ acceptance BGO detector. With a gaseous detector the energy resolution is about
five times worse, whilst the granularity in position allows $m^0$'s up to 20 GeV to
be identified with $~90\%$ probability. In other words, in the high-mass region,
because of these compensating factors the gaseous detectors will show similar

![Graph](image_url)

Fig. 5.1 Complete inclusive photon spectrum for different Higgs masses at
$W = m_\Theta = 70$ GeV, normalized to $10^5$ continuum Lund events
Fig. 5.2 Complete inclusive photon spectrum for different Higgs masses at $W = m_h = 90$ GeV, normalized to $10^5$ continuum Lund events.

Fig. 5.3 Complete inclusive photon spectrum for different Higgs masses at $W = m_h = 110$ GeV, normalized to $10^5$ continuum Lund events.
signal-to-background ratios, and thus Fig. 5.4 also essentially applies. It can be seen that, without any special requirement on the final-state topology, the detection of inclusive photons will allow Higgs mass values such as \( m_{H^0} \lesssim (0.7-0.8) m_W \) to be reached quite easily.

On the other hand, as shown in Fig. 5.3 the inclusive \( \gamma \) background due to hadron jets can be very severe when \( m_0 \sim m_Z^0 \) for large Higgs masses. In order to improve the signal-to-background ratio one can make use of the final-state topology of the events (H^0 + b\bar{b} \gamma). Since only two- and three-jet topologies can simulate the Higgs channel, the single-quark decay mode (\( \sim t\bar{t} \)) can be efficiently eliminated. The two-jet topologies are characterized by angular collimation of high-energy particles so that a conservative angular cut (e.g. 200 me rad for a 10 GeV particle) should remove more than 90% of the background. Finally, in the three-jet cases, the \( \gamma \) background will also emerge from a cluster of collimated particles. It is thus possible to require an isolation criterion in order to reduce this background. Although no detailed simulation yet exists, our present guess is a reduction of the order of 10. These improvements would allow the limit on \( m_{H^0} \) to be increased by about 5 GeV.

In addition to the inclusive photon peak, a Higgs signal would be characterized by the presence of \( b\bar{b} \) jets. The microvertex detectors can provide this
Fig. 5.5 Uncertainty in the Higgs mass determination (per event) as a function of $m_{H^0}$ for a BGO detector (full line) and a gaseous detector (dashed line).

very useful signature since they have a good tagging efficiency ($\geq 50\%$, to be compared with the semileptonic decay method), with low contamination (typically $25\%$ from $c\bar{c}$ pairs when keeping $75\%$ of $b\bar{b}$ events).

With these various requirements, running for less than one year will allow the mass limit on $H^0$ to be pushed up to typical values of 65, 80, and 85 GeV for $m_\theta = 70$, 90, and 110 GeV, respectively.

The precision on the $H^0$ mass per event is indicated in Fig. 5.5 for two types of electromagnetic detectors. It differs typically by a factor of 3-5, and in both cases the precision improves when the $H^0$ mass increases. Obviously the precision of the final determination of the Higgs mass has to include the statistical and systematic errors.

Figure 5.6a shows, as a function of $m_\theta$, the $H^0$ mass region that can be investigated at LEP with the reaction $\theta \rightarrow H^0 \gamma$. The contour of the domain is fixed by $m_\theta > 44$ GeV, $m_\theta > m_{H^0}$, and the rate limit below which this reaction is meaningless. This rate limit has been determined using the parameters of LEP given in Ref. [38], and the luminosity growth was assumed to be linear in energy for beam energies above 60 GeV. It has been taken arbitrarily as 0.1 event per 'day' ($\leq 10$ events per year). As already mentioned, the two-jet background is unimportant unless $m_\theta - m_{H^0}$ (close to the diagonal of the plot), in which case
Fig. 5.6  a) Accessible range of Higgs masses (abscissa) for given toponium mass (ordinate); in the 'back' regions the Higgs search is more difficult owing to background. b-d) Comparison of Higgs search in $\Theta$ and $Z$ decays. The indicated process has a higher rate.
the limit depends on the details of the topological cuts and on the properties of the detector.

5.2.2 Radiative contaminations

Two difficult regions are indicated as 'back' in Fig. 5.6a. They occur in the following two cases:

a) $m_\theta < m_{Z^0}$

In this case the radiative process $Z^0 \rightarrow q\bar{q}\gamma$ has a very high rate. Owing to the strong dependence of the cross-section on the initial energy, only radiation in the final state is contributing. Asking for a minimal angle $\theta$ between the quark and the photon of momentum $k$, the following radiation probability is obtained:

$$P_R = \frac{2\alpha}{\pi} \log \frac{1 + \cos \theta \Delta k}{1 - \cos \theta \Delta k} \ F \left( \frac{k}{E} \right),$$  \hspace{1cm} (5.3)

where $F(k/E)$ is close to 1 when $k/E$ is not too large (otherwise, QED and QCD corrections should be introduced);

$\alpha = 0.225 \ a_{em}$ \ if \ $\sin^2 \theta_w = 0.215$ ;

$\Delta k/k$ is fixed by the energy resolution of the electromagnetic calorimeter.

Now we can estimate the signal-to-background ratio

$$S \quad B = \frac{R_{H^0\gamma}}{R_{Z^0} \cdot P_R} = \begin{cases} 2/3, \text{ BGO detector} , \\ 1/6, \text{ gaseous detector} . \end{cases} \hspace{1cm} (5.4)$$

Given that the $H^0\gamma$ rate is $\leq 2$ per day, the $3\sigma$ level will be reached in 7 (27) days with a BGO (gaseous) detector. The ability to detect $H^0$ in this region will be very much detector-dependent, the situation becoming very difficult anyway when $m_\theta < m_{H^0}$.

b) $m_\theta > m_{Z^0}$ and $m_{H^0} < m_{Z^0}$

In this case the reaction $e^+e^- \rightarrow H^0\gamma$ will produce a photon which has the same energy as that of the radiative process $e^+e^- \rightarrow Z^0\gamma$.

Requiring a minimum angle $\beta$ of the photon with respect to the beam direction, one obtains the following probability for a photon to fall in the peak region, diluted by the Breit-Wigner of the $Z^0$:

---

*) $\alpha$ is the electromagnetic fine-structure constant weighted by the charges of the various flavours times the branching ratios at $Z^0$. 
\[ P_R = \log \left( \frac{(1 + \cos \theta)/(1 - \cos \theta)}{2 \left( \log \frac{2E}{m_e} - \frac{1}{2} \right)} \right) \frac{\Delta E_Y}{\Gamma} \leq 0.2 \frac{\Delta E_Y}{\Gamma} , \tag{5.5} \]

where \( \Delta E_Y \) is the energy resolution of the detector and \( \Gamma \) is the \( Z^0 \) width. Owing to the very large rate of the radiative \( Z^0 \) events compared with the \( \theta \to H^0 \gamma \) (see Fig. 2.16 and Table 5.2), one finds that \( S/B \leq 1/100 \), and we conclude that the search for \( H^0 \) through toponium seems difficult if \( |m_{H^0} - m_{Z^0}| < 10 \text{ GeV} \).

5.2.3 Comparison with other processes

Finally, in order to evaluate the quality of the toponium decay method, we compare it with other processes that are considered more frequently [62].

a) \( Z^0 + H^0 l^+ l^- \)

This process will probably be the first one to give information on the \( H^0 \) mass, since LEP will certainly start to run for a significant length of time on the \( Z^0 \) pole. The rate limit of 0.1 event per day is reached for \( m_{H^0} = 55 \text{ GeV} \) (straight vertical line in Fig. 5.6b when \( e^+e^- \) and \( \mu^+\mu^- \) modes are added).

Taking into account the LEP machine parameters, one can compare, for the accessible mass domain previously defined, the rate given by toponium and the rate given by \( H^0 l^+ l^- \). The border line given in Fig. 5.6b corresponds to equal rates. From this curve one concludes that, on the average, toponium wins when \( m_{H^0} > 30 \text{ GeV} \).

Remarks
- The main background to \( H^0 l^+ l^- \) comes from

\[ Z^0 \to t \quad \bar{t} \quad \rightarrow l^+ + X \rightarrow l^- + X . \]

This background is concentrated at large Higgs masses, which confirms the advantage of toponium in the search for heavy Higgs scalars.
- A very peculiar situation occurs when \( m_{\theta} < m_{H^0} \) with \( m_{H^0} < 55 \text{ GeV} \), in which case only the \( H^0 l^+ l^- \) process is operative.

b) \( Z^0 + \bar{\nu} \nu H^0 \)

This process has a higher rate than the previous one but is much less constrained and requires specific cuts to eliminate the background. The \( H^0 \) mass can be obtained by combining the information from the central detector and from the electromagnetic and hadronic calorimeters. One expects then a resolution of the order of \( 0.8/\Delta m_{H^0} \) (FWHM), i.e. \( \Delta m_{H^0}/m_{H^0} = 0.15 \) for \( m_{H^0} = 30 \text{ GeV} \).
Here as well the main background originates from $t\bar{t}$ decays, and it is also concentrated at large Higgs masses. The comparison with the $\theta \rightarrow H^0\gamma$ method is made in Fig. 5.6c.

\[ e^+e^- \rightarrow Z^0 + H^0 (Z^0 + \mu^+\mu^-, e^+e^-) \]

It has been shown [62] that this process is efficient as long as the condition $W = m^2_{H^0} + (p_{H^0}^2$ is fulfilled, i.e. for $m_{H^0} < 80$ GeV at maximal LEP II energy. This limit is shown in Fig. 5.6d, where the rate comparison with the $\theta \rightarrow H^0\gamma$ method is also made. In this figure we show the border, on the left-hand side of which the reaction $e^+e^- \rightarrow Z^0 + H^0 (Z^0 + ll^-)$ produces more events. One sees that here also, for $m_\theta < 110$ GeV, the $\theta \rightarrow H^0\gamma$ process will be more efficient for producing large Higgs masses.

5.3 Summary and conclusions

After a careful estimate of signal and background rates, we have found that toponium is an excellent place to search for heavy Higgs-like scalar particles.

Charged Higgs particles with masses less than the mass of the top quark ($m_{H^\pm} < m_t$) will be easy to find if $\cos\theta_{Htb}$, which represents the unknown (tb) mixing angle, is not too small.

Using the $\theta \rightarrow H^0\gamma$ toponium decay channel, it appears possible to discover neutral Higgs scalar particles with masses $m_{H^0} < (0.7-0.9)m_\theta$ (depending on the $\theta$ mass) within a reasonable running time. The missing-mass technique will allow the $H^0$ mass to be determined with a precision of less than one GeV. We have also shown that this method compares favourably with other reactions for 3.5 GeV < $m_{H^0}$ < 85 GeV. In particular, if the toponium mass is in the LEP I energy range, the process $\theta \rightarrow H^0\gamma$ will allow $H^0$ mass values to be reached, which are only achievable in the LEP II range through the reaction $e^+e^- \rightarrow Z^0 H^0$.

6. Supersymmetry and toponium decays

A number of toponium decays have been proposed which could allow some of the particles predicted by supersymmetry to be detected and their masses to be determined. The drastically different scenarios which are summarized in Tables 6.1 and 6.2 depend on the choice of the unknown masses of the $t$-quark and of the various postulated new particles. Various supersymmetric models make specific predictions for relations between different masses. Nevertheless, considering the large variety of models, we will consider the masses as free parameters, solely constrained by the available experimental information.

*) For recent reviews, see Refs. [67], [68] and, especially, [69].
Table 6.1
Toponium decays in supersymmetric models
with $m_\ell > m_\ell^\gamma + m_\gamma^\ell$, $m_\ell^\gamma > m_\gamma$

Assume $m_\ell > m_\ell^\gamma$ and ignore nearly degenerate mass assignments
where allowed modes are drastically phase-space suppressed

---

Top heavier
than lightest SUSY
particle

No

SUSY irrelevant
for toponium

Yes

Only SUSY anni-
hilation decays.
Standard decays of open top!
See Table 6.2.

---

$m_\ell > m_\ell^\gamma + m_\gamma^\ell$ (gaugino)

No

Yes

Supersymm. SQBs are dominant!
Toponium decays are drastically modified! Also, open top decays
into $\tilde{t} +$ gaugino

---

$m_\ell > m_\ell^\gamma + m_\gamma^\ell$

Yes

Extremely large decay rates for
$(\ell \ell) \rightarrow (\ell + \ell + \ell + \ell) \; \text{or c.c.}$
$\Gamma_{\text{tot}} \gtrsim 1 \text{ GeV}$
$\gg m(2S) - m(1S)$!
No individual resonances.

---

$m_\ell > m_\ell^\gamma + m_\gamma^\ell$

No

$(\ell \ell) \rightarrow (\ell + \ell + \ell + \ell) \; \text{or c.c.}$
allowed and dominant!
$\Gamma_{\text{tot}} \sim 100 \text{ MeV}$.
Individual resonances remain distinguishable.

---

| 1) | Denotes cases where the appearance of toponium is drastically changed. |
| 1) | Probably excluded by the observation of semileptonic top decays at the collider. |
Table 6.2
Toponion decays in supersymmetric models

with $m_t < m_{\tilde{g}}$, $m_{\tilde{\chi}^0}$, $m_{\tilde{\chi}^+}$ > $m_{\tilde{\chi}^0}$

Supersymmetric annihilation decays for different mass assignments

$m_t \rightarrow m_{\tilde{t}}$ and $m_{\tilde{t}} > m_{\tilde{\chi}^0}$
- complicated mixture of $+\tilde{t} + \tilde{\overline{t}}$
and $+\tilde{\chi} + \tilde{\overline{\chi}}$

$m_{\tilde{t}} > m_{\tilde{\chi}^0}$ and
$m_{\tilde{t}} < m_{\tilde{\chi}^0}$
- decays into gluino pairs are possible (e.g., $+\tilde{g} + \tilde{g}$)

$m_{\tilde{t}} \ll m_{\tilde{\chi}^0}$
- gluino pair production through $\tilde{t}$ exchange

$m_{\tilde{t}} > m_{\tilde{\chi}^0}$
- $+\tilde{g}$ far heavier than $t$
$(m_{\tilde{g}} = 3m_t)$
Gluino exchange suppressed!
Photino exchange allowed!
Small, but not unreasonable rates for
$(tt) + \tilde{\chi} + \tilde{\overline{\chi}}$ and
$+\tilde{g} + \tilde{\gamma}$

1) Denotes cases where the appearance of toponium is
drastically changed.
2) Denotes cases that are consistent with semileptonic
decays of open top and with currently favoured mass
assignments.
3) Note that this scenario is only allowed for
$m_t < m_{\tilde{\chi}^0} < m_{\tilde{\chi}^0} + m_{\tilde{\chi}^0}$ in order to avoid the supersymmetric
SQU. Since $m_{\tilde{\chi}^0}$ is normally assumed to be small, this
can occur only for a very restricted mass range.
4) This mode of interest, since it gives access to
very heavy gluinos.
5) If kinematically allowed, the supersymmetric analogue
of Wilczek's decay could be of interest.
In toponium decays we have to consider two fundamentally different classes of processes: The first class, which is summarized in Table 6.1, consists of single-quark decays into supersymmetric (SUSY) particles. These decays are only possible if $m_t > m^* + m_{\text{gaugino}}$. If it were kinematically allowed, they would dominate the weak decays of open top by a large factor and completely change the appearance of both toponium and open-top mesons. Since open top is expected to be discovered and studied before the toponium search is started, the dramatic change of toponium properties would be anticipated.

The second class of decay modes, summarized in Table 6.2, are annihilation decays. Depending on the masses of the various SUSY particles, the resulting rates will dominate toponium decays in some of the cases; in other cases they might only appear with tiny branching ratios.

In the following we will first present the theoretical framework together with the relevant formulae. We will then present two more specific case studies, based on detailed Monte Carlo calculations. The first one will be a study of $\theta \rightarrow \tilde{g}\tilde{g}$. It will be demonstrated that this decay mode would be an ideal place for a rather accurate gluino mass determination. The second example will be an analysis of $\theta \rightarrow \tilde{g}\tilde{g}$ for a massive gluino. This will serve to demonstrate that rather high gluino masses could be probed in toponium decays.

6.1 Supersymmetric single-quark decays

The mass assignment $m^*_t < m_t$ has been favoured by a number of currently discussed models. If $m_t > m^*_t + m_{\tilde{g}}$ (or $m^*_{\tilde{g}}$) toponium could drastically alter its appearance or even cease to exist as a well-defined resonance. Although these scenarios are already ruled out if the semileptonic top decays observed by the UA1 Collaboration [4] are taken at their face value, we nevertheless mention them for completeness.

Single-quark decays into a scalar quark and a light gaugino (Fig. 6.1) --if kinematically allowed-- can have an extremely high rate [70]. The rate for the hadronic decay is given by

$$\Gamma[(t\tilde{t}) \rightarrow t\tilde{g} + t\tilde{g}] = 2 \frac{\alpha_s}{3} m_t \left( \frac{2 \beta_{\tilde{g}}}{m_t} \right)^2.$$  \hspace{1cm} (6.1)

If not severely suppressed by phase space, the resulting width exceeds 1 GeV and is thus larger than the toponium level spacing. Obviously no $(t\tilde{t})$ resonances

Fig. 6.1 Diagram for the decay of a bound t-quark into scalar quark plus gluino
exist in this case and one will only observe open-top production with a 'smeared' threshold region.

This situation can be described in the following (semiclassical) space-time picture: the constituents' characteristic time of revolution would be large compared with their lifetime, and the notion of a bound state becomes meaningless.

Even if this hadronic mode is kinematically forbidden the corresponding electromagnetic decay mode into a scalar quark and a photino could still proceed at an appreciable rate:

\[
\Gamma[(t\bar{t}) + \tau\bar{\tau} + t\tau] = 2 \frac{1}{2} \frac{1}{2} e^2 t m_t \left( \frac{2p_T}{m_t} \right)^2.
\]  

(6.2)

It would still dominate, by far, all other channels; however, its rate of \(-100-200\ MeV\) would not be sufficient to wipe out the lowest-lying toponium resonances. A determination of the fundamental potential-model parameters \(E(2S) - E(1S)\), \(\Gamma_{ee}(1S)\), and \(\Gamma_{ee}(2S)\) would be feasible.

However, it must again be stressed that if semileptonic top decays are observed at the \(\bar{p}p\) Collider or on top of the \(z^0\) in \(e^+e^-\) machines, SUSY-induced SQDs are ruled out for toponium.

6.2 Annihilation decays

As shown in Table 6.2, a large variety of annihilation decays is possible, depending on the masses of the various postulated particles. In all these cases the properties of open top would be unaffected and only those of toponium would be changed--eventually in a rather drastic manner. In the following we will not try to be completely general and cover all possibilities, but rather we will discuss a few of the more promising choices.

6.2.1 Gluino production through virtual gluons

Toponium (and \(Y\)) decays as a means of searching for gluinos have already been proposed some time ago [71]. Being electrically neutral, gluinos are hard to produce in \(e^+e^-\) collisions. However, as colour octets they may be pair-produced either in hadronic collisions or in toponium decays through the processes indicated in Fig. 6.2. Evidently the rate for these processes depends

![Fig. 6.2 Lowest-order diagrams for gluino production in toponium decays through virtual gluons](image-url)
on the gluino mass only and is independent of all other SUSY parameters—quite
in contrast with the reactions to be discussed later. Compared with the
production of heavy quarks (charm, bottom) the rates are enhanced by a factor
\[ \frac{1}{2} \sum_{\text{abc}} f_{\lambda_{ij}}^2 / (\lambda_{\lambda^2}^2 / 2) = 3. \]
In Fig. 6.3a we show the rates for \( ^3S_1 \) -states and in
Fig. 6.3b those for P-states as a function of \( m_\chi \) normalized to their dominant
conventional hadronic decay rate

\[ R(^3S_1) = \frac{\Gamma(gg\tilde{g})}{\Gamma(gg)} , \]  \hspace{1cm} (6.3a)

\[ R(^3P_0, ^3P_2) = \frac{\Gamma(gg\tilde{g})}{\Gamma(gg)} , \]  \hspace{1cm} (6.3b)

\[ R(^3P_1) = \frac{\Gamma(gg\tilde{q})}{\Gamma(gq\tilde{q})} , \]  \hspace{1cm} (6.3c)

for fixed \( m_\chi = 80 \text{ GeV} \) and a binding energy of 1.5 GeV for the P-states (\( a_s = 0.15 \)).
[We used the analytic forms given in Ref. [72] for heavy-quark production.] For
light gluinos these ratios can become quite sizeable: up to 20% for S-states and
even up to 60% for P-states. However, as discussed in Ref. [25], the hadronic
branching ratios themselves become relatively small for a heavy toponium
P-state. As a specific example, let us consider \( m_\chi = 80 \text{ GeV} \). To obtain the
overall branching ratio, the ratio \( R(^3S_1) \) has to be multiplied by \( a_s = 0.1 \), and the
P-state ratios \( R(^3P_J) \) by 0.20, 0.03, and 0.07 for \( J = 0, 1, 2 \). Furthermore,
taking the branching ratio for dipole transitions into account, one finds at
best \( (m_\tilde{g} = 5 \text{ GeV}) \),
\[ \text{BR}(t^3S_1 + gggg) = 10^{-2} , \]

\[ \sum J \text{BR} [2^3S_1 + \gamma + 3^3P_J (\rightarrow ggg)] = 10^{-3} . \]

The rates are higher by a factor of \(-3\) for \(3^3S_1\), and by a factor of \(-10\) for P-states, if \(m_\theta = 70 \text{ GeV}\), but the experimental verification still looks quite difficult. For heavier gluinos these branching ratios rapidly become worse. Since we suspect that the detection of light gluinos on top of the \(Z^0\) is more favourable, we have not studied this scenario further.

### 6.2.2 Gluino production through virtual scalar quark exchange

A number of authors [70, 73] have speculated that the scalar partner of top could be the lightest of all scalar quarks with a mass comparable to \(m_t\). This would lead to sizeable branching ratios for toponium decays into gluino pairs through scalar quark exchange (Fig. 6.4). The rate would be proportional to \(a_s^2\) and could thus even dominate in some cases. Since this mode also leads to a rather clean signature (two-body decay into energetic gluinos!) it is quite a promising candidate for gluino searches.

Fig. 6.4 Diagram for gluino-pair production in toponium decays through scalar \(t\)-quark exchange

To understand the relevance of this channel for the various \((t\bar{t})\) states, we first discuss the selection rules for decays into two Majorana fermions. The quantum numbers of a fermion-antifermion system are given by

\[ P = (-)^L, \quad C = (-)^{L+S} . \]

This result holds true also for Majorana particles. From the requirement of antisymmetric wave functions for identical fermions, one derives, in addition,

\[ L \text{ even }, \quad \text{for } S = 0 \]
\[ L \text{ odd }, \quad \text{for } S = 1 . \]

This implies that two Majorana particles can only have the following quantum numbers: \(0^-; \ 0^+; \ 1^+; \ 2^+; \ 2^*; \ ... \). Hence, if charge conjugation is conserved \(3^3S_1(1^-) \rightarrow \bar{g}g\) is forbidden, and \(1^3S_0(0^-) \rightarrow \bar{g}g\) and \(3^3P_J(J^*) \rightarrow \bar{g}g\) are allowed. In
general, however, the scalar partners of right- and left-handed quarks are expected to have different masses; parity and charge conjugation symmetry are violated; and \(3S_1 \rightarrow gg\) is allowed [70, 73, 74] and is usually even dominant. For completeness we list all relevant decay rates:

\[
\Gamma(0^{--} \rightarrow gg) = \left(\frac{2}{3}\right)^2 \alpha_s^2 \left[\frac{R(0)}{m}\right]^2 a_5^2 \left(\frac{m}{M}\right) \beta^3,
\]

\(6.7a\)

\[
\Gamma(1^{--} \rightarrow gg) = \left(\frac{2}{3}\right)^2 \alpha_s^2 \left[\frac{R(0)}{m}\right]^2 a_1^2 \beta^3,
\]

\(6.7b\)

\[
\Gamma(1^{++} \rightarrow gg) = 0,
\]

\(6.7c\)

\[
\Gamma(J^{++} \rightarrow gg) = \left(\frac{2}{3}\right)^2 \alpha_s^2 \left[\frac{R'(0)}{m^2}\right]^2 \left[\frac{b_2^2}{5} \left(\frac{3}{4} + \frac{2m_2^2}{m^2}\right) \beta^3, J = 2 \right],
\]

\(6.7d\)

where

\[
\beta \equiv \left(1 - 4 \frac{m_2^2}{m^2}\right)^{1/2},
\]

\[
a_{1,5} \equiv \frac{1}{2} \left(\frac{m_t^2}{m_t^2 - m_R^2 + m_L^2} + \frac{m_t^2}{m_t^2 - m_L^2 + m_R^2}\right),
\]

\[
b_{1,5} \equiv \frac{1}{2} \left[\frac{m_t^4}{(-m_t^2 - m_R^2 + m_L^2)^2} + \frac{m_t^4}{(-m_t^2 - m_L^2 + m_R^2)^2}\right];
\]

\(m\) denotes the mass of the respective bound states and \(m_R, L\) are the masses of the scalar t-quarks. In the extreme case where \(m_L = m_t\), \(m_R \gg m_L\), and \(\beta = 1\), this mode would dominate all \(3S_1\) decays up to \(m_\beta \approx 110\) GeV (excluding the tiny region \(m_\beta + 5\) GeV), but also for less extreme choices it would still be the dominant channel. Since this holds true also for radial excitations, decays into P-states would be quite suppressed.

Apart from the determination of \(\Gamma_{\text{ee}}\) and the level spacings, none of the standard toponium topics (electroweak interferences and decays, gluon properties, Higgs search, ...) would be accessible. However, toponium would then be an excellent place to study a rather clean sample of events with only two gluinos in...
the final state. In subsection 6.3 we show that the mass of gluinos could be measured in this case with an uncertainty of less than 0.5 GeV.

If parity and charge conjugation were conserved \( m_L = m_R \), gluino decays could be important for P-state decays. In this case, dipole transitions from \( n^3S_1 \) to \( \gamma + 3P_1 \) would proceed with normal strength, and P-states could subsequently decay into gluino pairs [74]. In Fig. 6.5 we therefore compare the corresponding rates for decays of \( 1^{**} \) and \( 2^{**} \) states with their conventional modes. If not suppressed by phase space, \( 3P_1 \) annihilation into \( gg \) would even dominate the \( 3P_1 + 3S_1 + \gamma \) dipole transition, and one of the three two-photon cascades would be absent in this peculiar scenario. However, the energy difference between photons from transitions to the various P-states will only amount to a few MeV, so that even a BGO detector will be unable to observe the lines separately. One might try to tag the events with their combined branching ratio of roughly 1% by requiring missing energy from the gluino decay. However, in this case one has to face a 10% contamination from semileptonic SQDs, in particular from \( t + b + \nu \) (hadrons + \( v \)). These could be removed to some extent if one is able to reject B mesons with the microvertex detector and thus achieve another factor of 10 in rejection rate. Alternatively, one could gain a comparable factor by requiring a photon of the (already measured) energy \( E_\gamma = M(2S) - M(1P) \)

Fig. 6.5 Conventional decay rates of \( 1^{**} \) and \( 2^{**} \) \( \chi_c \) states, compared with their decay rates into two gluinos through scalar quark exchange. From Ref. [74].
with the BGO detector. Even then it would be barely possible to identify $2S + \gamma + 1P(\tilde{g}\tilde{g})$.

It should be noted that instead of gluinos, also two photinos could be produced, and all previous formulae apply after the substitution $\alpha_s^2 \rightarrow \alpha_s^2 e^2$. The branching ratio for $1^3S_1 \rightarrow \gamma\gamma$ is comparable to the one for $e^+e^-$ decays and is thus non-negligible if decays into a gluino pair are kinematically forbidden. Since the decay products are invisible, this would effectively only reduce the visible cross-section by at most 10%, and thus this decay mode would remain unnoticed. Tagging the invisible channel through $2^3S_1 \rightarrow \tilde{W}^+ \tilde{W}^- + 1^3S_1$ or $2^3S_1 \rightarrow \gamma\gamma + 1^3S_1$ is possible, in principle; in practice, however, the combined branching ratios are rather low ($\lesssim 10^{-3}$).

If the photino is relatively light and the masses of top and scalar top are comparable, one may have access to high gluino masses through the mode [75] 

$$\theta \rightarrow \tilde{g}\tilde{g}\gamma$$

(6.8)

mediated by scalar top exchange. For $m_L = m_R = \tilde{m}$, the differential rate is given by

$$\frac{d\Gamma(\tilde{g}\tilde{g}\gamma)}{\Gamma(\tilde{g}\tilde{g}\gamma)} = \frac{dy}{dz} \left( \frac{m^2 - 9z^2(\chi + h)^2}{2n^2} \right)$$

$$\left( (2x-y)^2 h+4h-2+2z-2z^2-2xy)(1-z)+4xyh+2h^2+(2xy-h^2-2z^2)(x^2+y^2) \right) R ,$$

(6.9a)

$$R = \left( \frac{n^2}{64} \right) [2n^2 (1+8x-4y-2x^2)+16h^2 (2x-y)^2-n^2 (n^2+4n^2 h+8h^2)$$

$$+ 16hn^2 (1-x+3x-y-x^2)+4n^2 (16y+12xy+y^2+4x^2 y-21x^2+4x^3)$$

$$+ 32h (6-4y-2xy-7x^2-y^2+3x^2 y+2xy^2+2x^3+y^3)$$

$$+ 16 (12z-14-2x^2+4x^2 y+24xy-13x^2 y-3xy^2+5x^3-4xyz+xyz^2) ] ,$$

(6.9b)

where

$$x = \frac{E^g}{m}, \quad y = \frac{E^\gamma}{m}, \quad z = 2 - x - y .$$

In subsection 6.4 we study this case in more detail. As a representative example, we have chosen for the masses $m_\theta = 80$ GeV, $m_g = 60$ GeV, and $m_\tilde{g} = 0$. Although the branching ratio is rather small, we still find that this decay mode can be separated from the conventional decays but with some effort. It should
be stressed that detection of gluinos on top of the Z would be difficult for the masses chosen in our example.

An amusing borderline case would be the mass assignment \( m_{\tilde{t}} + m_{\tilde{\ell}} > m_{\tilde{\tau}} > m_{\tilde{\ell}} \). In this case \( \theta \rightarrow \tilde{\tau}^+ \tilde{\tau}^- \) mediated by gluino exchange could play the dominant role [70]. For completeness we should mention that also \( \theta \rightarrow \tilde{\chi}^0 + \gamma \) and \( \tilde{\tau}^+ + \tilde{\tau}^- \) have been discussed in the literature [73].

A completely different supersymmetric model has been proposed by Samuel and Wess [76]. In this model, supersymmetry is realized in a non-linear form. In the context of quarkonium physics it would lead to the decays \( \theta \rightarrow \gamma \Lambda \Lambda \) and \( \Lambda_{P1,2} \rightarrow \Lambda \Lambda \), where \( \Lambda \) denotes the weakly interacting Goldstone fermion [77, 69]. Experimental observation of the channel \( 2S \rightarrow \gamma \gamma + 1P(\Lambda \Lambda) \) seems difficult.

6.3 Gluino-mass determination from the decay \( \theta \rightarrow \tilde{g}\tilde{g} \)

6.3.1 Assumptions

As discussed in the previous subsection, toponium annihilation into a pair of gluinos could dominate all other channels if \( m_{\tilde{g}} < m_{\tilde{\tau}} \) and if \( \tilde{R} \neq \tilde{L} \). In this case it would be possible to study a rather clean sample of gluino pairs. We shall use this to demonstrate that a study of this decay would allow the gluino mass to be measured with high accuracy. We base this case study on the following assumptions:

- \( m_{\tilde{g}} = 80 \text{ GeV}, \quad m_{\tilde{\tau}} = 35 \text{ GeV}, \quad m_{\tilde{\ell}} = 0 \),
- \( \text{BR}(\theta \rightarrow \tilde{g}\tilde{g}) \) dominant,
- \( \tilde{g} \rightarrow q\bar{q}\gamma \) generated with the squared matrix element

\[
|A|^2 = (P\cdot P_q)^2 \gamma_{q\gamma} + (P\cdot P_{\bar{q}})^2 \gamma_{q\gamma},
\]

- \( q\bar{q} = u\bar{u}, d\bar{d}, c\bar{c} \), and \( b\bar{b} \) generated with equal probability.

6.3.2 Missing-energy distribution

Figure 6.6a shows the total missing energy due to \( \tilde{\gamma} \) for \( m_{\tilde{g}} = 35 \text{ GeV} \). Figures 6.6b-d show how this distribution varies with \( m_{\tilde{g}} \) (25, 30 and 40 GeV). Such variations suggest that \( m_{\tilde{g}} \) be estimated from the width of the missing-energy distribution.

Figure 6.7 displays the dependence of \( m_{\tilde{g}} \) versus the standard deviation \( \sigma \) for these distributions. With 1000 events, we can in principle measure \( m_{\tilde{g}} \) to better than 500 MeV.
Fig. 6.6 Distribution of the energy carried by the two photinos in the decay \( \tilde{\theta} \rightarrow \tilde{g} \tilde{g}, \tilde{g} + q\bar{q}, q\bar{q} \gamma \) with \( m_{\tilde{g}} = 80 \text{ GeV}, m_{\tilde{\theta}} = 0 \). The energy is normalized to the beam energy. a) \( m_{\tilde{g}} = 35 \text{ GeV} \), b) \( m_{\tilde{g}} = 25 \text{ GeV} \), c) \( m_{\tilde{g}} = 30 \text{ GeV} \), d) \( m_{\tilde{g}} = 40 \text{ GeV} \).
Fig. 6.7 Missing-energy dispersion [standard deviation of the distributions (6.6)] versus $m_g$

Remarks:

i) This figure does not include effects due to possible contaminations coming from energy leaks (apparatus).

ii) The missing-energy distribution (Figs. 6.6b-d) can be modified if one takes into account semileptonic decays from heavy flavours coming from gluino decays. This can be vetoed (except $\tau$'s) by eliminating events with $e$ ($\mu$) in the final state.

6.3.3 Estimating the mass of the gluino

In the following, the gluino fragmentation is assumed to be $\delta(1-z)$.

i) One method has already been described above.

ii) It is expected that the sphericity of the events will also vary with $m_g$.

To study this problem we need a hadronization scheme. Since the $\tilde{g}$ lifetime is long compared with typical hadronization schemes (as for $t\bar{t}$ decays), we assume that $\tilde{g}$ dresses up into colour-neutral hadrons $G$ ($\tilde{g}\tilde{g}$) or ($\tilde{g}\tilde{q}\tilde{q}$) before decaying.

The Lund Monte Carlo was used to produce the final hadrons.

Figure 6.8 shows how the sphericity $S$ varies with $m_g$. Error bars correspond to the accuracy on $\langle S \rangle$ for 1000 events. Again we find that the mass estimate can be better than 1 GeV.
Fig. 6.8 Sphericity $S$ versus $m_{\tilde{g}}$ for $\theta \rightarrow \tilde{g} \tilde{g}$ with $m_\theta = 80$ GeV. The hadronization scheme is described in the text. Error bars correspond to the statistical accuracy on $S$ for 1000 events.

iii) Finally, we describe a method which gives direct evidence for a mass peak related to the gluino mass.

Even when the mass of the $\tilde{g}$ is fixed, a full reconstruction of the event is not possible since one constraint is missing. However, as shown in Ref. [78] for a similar case, a Jacobian peak method can be used to fix the unknown mass.

To see this we use the centre-of-mass system for the two $\tilde{\gamma}$'s. In this system (see Fig. 6.9), hadron jets (four jets) are associated with two sub-systems, called A and B, related to the two gluinos $\tilde{g}_1$ and $\tilde{g}_2$. The energy of the $\tilde{\gamma}$ is known (= missing energy/2 in this system). The angle $\alpha$ between $\tilde{\gamma}_1$ and A is fixed once the mass of $\tilde{g}_1$ is fixed.

What remains arbitrary is the choice of the azimuth of $\tilde{\gamma}_1$. The mass of $\tilde{g}_2$ will clearly depend on the choice of this azimuth $\varphi$. It turns out that $m_{\tilde{g}_2}$ is stationary with respect to $\varphi$ when $\tilde{\gamma}$ belongs to the plane defined by $\hat{p}_A$ and $\hat{p}_B$. 
Fig. 6.9 Kinematics in the missing-energy frame (\(P_X + P_Y = 0\)) for \(g_1 \rightarrow \gamma_A g_2\) with \(g_2 \rightarrow \gamma_B g_3\); \(\alpha\) is the angle between \(P_X\) and \(P_A\), \(\beta\) the angle between \(P_Y\) and \(P_B\); \(\phi\) is the azimuthal angle describing the position of \(P\) on the cone with \(P_X\) axis. The origin for \(\phi\) is defined by the \(P_A, P_B\) plane.

Reference [78] shows that the mass \(m_2\) reconstructed with the two possible \(\phi\) values (0, \(\pi\)) shows a peak reflecting this Jacobian effect.

This method has been tried at the parton level and shows the expected peak (Fig. 6.10a).

After hadronization is introduced, the peak is still present (Fig. 6.10b).

Figures 6.10 have been obtained at \(m_2 = 25\) GeV. If we increase \(m_2\), the peak appears less clearly.

Fig. 6.10 a) Distribution at the parton level of the mass of the \(g_2\) system when \(m_2\) is fixed at its nominal value of 25 GeV; b) Same as (a) introducing hadronization.
Fig. 6.11 Distribution of \( m_\gamma \) imposing the equality between \( m_\gamma \) and \( m_\gamma^* \); \(|m_\gamma - m_\gamma^*| \leq 2\) GeV. A peak appears at 25 GeV, the generated value.

As the gluino mass is the quantity we want to measure, we have considered for \( m_{\tilde{g}} \) all the possibilities between 10 GeV and \( m_\nu/2 \). Figure 6.11 shows the distribution for \( m_{\tilde{g}} \) with the condition that \(|m_{\tilde{g}} - m_{\tilde{g}}^*| \leq 2\) GeV. A peak for \( m_{\tilde{g}}^* \) appears, centred around the generated mass of 25 GeV.

6.3.4 Conclusion
Using three independent methods, we arrive at the conclusion that \( m_\gamma \) can be measured to an accuracy of better than 1 GeV.

Most backgrounds to the \( \tilde{g}\tilde{g} \) channel can be efficiently eliminated.

6.4 Search for heavy gluinos in the decay \( \theta \rightarrow g\tilde{g}\tilde{g} \)

6.4.1 Assumptions

As discussed in subsection 6.2, toponium decays could give access to very heavy gluinos through the decay \( \theta \rightarrow g\tilde{g}\tilde{g} \). We shall substantiate this claim by a detailed analysis based on the following assumptions on the masses:

\[
\begin{align*}
\theta &= 80\ \text{GeV}, \\
m_\gamma &= 60\ \text{GeV}, \\
m_{\tilde{g}} &= 0, \\
m_R &= m_L = m_t .
\end{align*}
\]

We use the distributions from Ref. [75] for the energy distributions [see Eq. (6.9)] and the gluino decay as specified in subsection 6.3. For our choice of masses we derive \( \text{BR}(g\tilde{g}\tilde{g}) = 0.5\% \). Figure 6.12a shows the energy distribution of the gluino for our mass assignment, compared with the case \( m_\gamma = 0 \).
Fig. 6.12 a) Energy distribution of the \( \tilde{\gamma} \) (in \( \tilde{\gamma} \gamma g \)) measured in the rest frame of toponium as obtained from Ref. [75]. The energy is normalized to the top mass \( m_t \). The mass of \( \gamma \) and \( g \) are assumed negligible. The full curve corresponds to \( m_{\tilde{\gamma}} = 1.2m_t \), the dashed curve to \( m_{\tilde{\gamma}} = 1.5m_t \), the dash-dotted curve to \( m_{\tilde{\gamma}} = 2m_t \), where \( \tilde{\gamma} \) stands for the scalar partner of top (right- and left-handed scalar quarks are assumed to have equal mass). b) The energy distribution of \( \gamma \) with the mass assignments: \( m_{\tilde{\gamma}} = m_{\tilde{\gamma}} = 40 \text{ GeV} \), \( m_{\tilde{\gamma}} = 0 \), \( m_{\tilde{\gamma}} = 60 \text{ GeV} \). c) With the same hypotheses as (b), energy distribution of \( \tilde{\gamma} \) from \( g \) decays with the same normalization. d) With hypotheses of (b), total energy carried by the two photinos \( \gamma \) and \( \tilde{\gamma} \) produced in \( \theta \rightarrow \gamma \gamma g \) (\( \gamma \rightarrow q\bar{q} \gamma \)) normalized to the top mass.
6.4.2 Missing-energy distributions

Figure 6.12b shows the contribution from the direct \( \gamma \), Fig. 6.12c from the secondary \( \gamma \). Figure 6.12d gives the total contribution. [Note: All energy distributions are normalized to the beam energy (40 GeV).]

6.4.3 Contaminations

i) Single-quark decays from 0 give the largest missing energy. These decays can be identified since they are necessarily accompanied by at least one charged lepton (except for 0ν). The probability of missing this lepton should be kept at the percent level in order to reach a reasonable signal-to-background ratio. We cannot expect topological cuts to help significantly (four jets instead of three). To improve the situation, one could require a missing energy greater than 20 GeV, the maximum value allowed for semileptonic decay. This would force double semileptonic decay and increase the probability of detecting at least one lepton. [Not operative for 0ν(\( \tau \rightarrow \nu + \) hadrons).] The SQD jets cannot exceed the same limit, which means that at least two energetic hadrons have to be lost in order to simulate more than 20 GeV missing energy.

From Fig. 6.12d we find that 20% of the good events are rejected by this cut.

Figure 6.13a gives the distribution of missing energy versus the energy of the most energetic lepton for two primary (\( t \rightarrow b \nu \)) semileptonic

Fig. 6.13 a) Assuming \( 0 \rightarrow b^+ \nu b^- \bar{\nu} \) through single-quark decay, the energy carried by the two neutrinos normalized to the beam energy is the abscissa, the energy of the most energetic lepton is the ordinate with the same normalization. b) Assuming \( 0 \rightarrow b^+ \nu bqq \) with one of the b decaying semileptonically, the same distribution as in (a) is shown.
decays. Keeping lepton candidates with energy below 3 GeV so as to avoid random vetoes, we reject all these events \( (\leq 10^{-3}) \) contamination.

Figure 6.13b gives the same plot for one primary decay followed by a secondary decay \( (b \rightarrow ce\nu) \). With the 3 GeV cut, we retain 1% of these candidates.

Taking into account branching ratios, we are left with \( 5 \times 10^{-4} \) contamination (normalized to the number of 0 events). We have neglected ternary \( (c \rightarrow se\nu) \) decays.

ii) The background coming from light quarks can be eliminated by cutting on coplanarity (Fig. 6.14). With this cut, 70% of the good events are kept, whilst 93% of the background is rejected.

The missing-energy criterion (25 GeV) gives a \( 10^3 \) rejection factor (assuming that only neutrinos give missing energy, i.e. ignoring effects from the apparatus).

Vetoing against charged leptons above 3 GeV leads us finally to a rejection factor of \( 10^5 \) (total).

iii) The most serious background comes from

\[
t \xrightarrow{(SOD)} \text{tvb} \rightarrow \text{had} + \nu
\]

(6.11)

Fig. 6.14 Coplanarity \( (\vec{P}_g, \vec{P}_q, \vec{P}_q) \) distributions of light (not top) quark decays for 3-jet events \( \vec{q}, \vec{q}q, \vec{q}g \).
Fig. 6.15 Assuming $\Theta + \tau v b + X$ with $\tau \rightarrow$ hadrons $+ \nu$, the distribution of the energy carried by the two neutrinos normalized to the beam energy is shown.

The missing-energy criterion ($> 25$ GeV) keeps 28% of these events, as shown in Fig. 6.15.

Vetoing against leptons gives only an additional factor of 2 (decays from accompanying heavy quarks).

Finally we are left with a signal-to-background ratio of ~0.7 due primarily to this channel.

6.4.4 Losses

i) Our cut on missing energy rejects 20% of the good events.

ii) Ten percent of the gluino decays are lost by vetoes on charged leptons ($\tilde{g} \rightarrow b\tilde{b}\gamma$ or $c\tilde{c}\gamma$).

iii) Coplanarity cuts remove ~30% of the good events.

6.4.5 Conclusions

After applying the cuts discussed above, 40 signal events and 60 background events would be kept for an integrated luminosity of 150 pb$^{-1}$. The background events are mainly from $t \rightarrow \tau v b$. We therefore believe that further analysis based, for example, on B-meson rejection through microvertex detectors or on topological cuts to reject low-multiplicity jets from $\tau \rightarrow$ hadrons $+ \nu$ could reduce this background even more. The identification of $\Theta + g\tilde{g}\gamma$ is thus difficult but feasible.
7. **THE ROLE OF THE ENERGY SPREAD**

All our figures have been based on the LEP 13 (3 mA) luminosity [38]

\[
L = (0.8-2.1) \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1},
\]

and a centre-of-mass energy spread [see Eq. (2.36)],

\[
\delta W \text{ (MeV)} = \frac{0.443 \times 10^{-2} \ W^2 \text{ (GeV)}^2}{[1 - 0.41 \times 10^{-6} \ W^3 \text{ (GeV)}^3]^{1/2}}.
\]

The LEP machine can be operated in different modes [38], maximizing \( L \), \( 1/\delta W \), or \( L/\delta W \) (Fig. 7.1). Depending on the signal-to-background ratio, different measurements can be performed in a more efficient way by choosing the appropriate operating mode.

As an example we can study the total time required for the scan of a 2 GeV interval. From formula (3.11) this time is

\[
T = \frac{1}{L} n^2 \left( \frac{\sigma_s + \sigma_B}{\sigma_s^2} \right) \frac{2 \text{ GeV}}{25W}.
\]
Table 7.1
Comparison of different assumptions on energy spread and luminosity

<table>
<thead>
<tr>
<th>W (MeV)</th>
<th>δW (10⁻³³ cm⁻² s⁻¹)</th>
<th>L (10⁻³³ cm⁻² s⁻¹)</th>
<th>δW/L</th>
<th>δW (MeV)</th>
<th>L (10⁻³³ cm⁻² s⁻¹)</th>
<th>δW/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>23</td>
<td>0.79</td>
<td>29.1</td>
<td>23.3</td>
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<tr>
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<tr>
<td>90</td>
<td>42</td>
<td>1.35</td>
<td>31.1</td>
<td>38.3</td>
<td>1.49</td>
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<tr>
<td>100</td>
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<td>36.2</td>
<td>47.8</td>
<td>1.69</td>
<td>28.3</td>
</tr>
<tr>
<td>110</td>
<td>79</td>
<td>1.83</td>
<td>43.1</td>
<td>59.5</td>
<td>1.91</td>
<td>31.2</td>
</tr>
<tr>
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<td>117</td>
<td>2.05</td>
<td>57.1</td>
<td>74.8</td>
<td>2.10</td>
<td>35.6</td>
</tr>
</tbody>
</table>

Since $\sigma_S \propto 1/\delta W$, we notice that $T$ is proportional to $\delta W/L$ if $\sigma_S \ll \sigma_B$. In the other extreme case when $\sigma_B \ll \sigma_S$, $T$ is simply proportional to $1/L$.

For toponium search above $Z^0$ it will be convenient to maximize $L/\delta W$; in the 70 GeV region it is better to maximize $L$. Table 7.1 shows a comparison between our 'assumed' operating mode and the $L/\delta W$ optimized mode [38].

The optimal mode is different for measurements on the resonance. As an example, we can consider the time required to measure the $\alpha_{on}$ asymmetry with an error $\Delta$. From Eq. (4.9) one finds

$$T = \frac{1}{\Delta^2} \frac{1 + \eta}{\log S} \left[ 1 - \langle \alpha_{RL} \rangle^2 + \eta^2 (\langle \alpha_{RL} \rangle - \alpha_{RL}^{\text{off}})^2 \right],$$  

(7.3)

where $\eta = \sigma_B/\sigma_S$.

Under the condition of severe background ($\eta \gg 1$) $T$ is proportional to $(1/L)(\sigma_B/\sigma_S^2)$. In this case the quantity to optimize is $L/(\delta W)^2$. In the other extreme case ($\eta \ll 1$) it is more appropriate to optimize $L/\delta W$.

It has been suggested in various papers on $e^+e^-$ machine physics [79] that it is possible (at least in principle) to substantially reduce the spread in the energy of $e^+e^-$ collisions. A study is at present being made to see whether such schemes could be implemented at LEP and a factor of 4 reduction in $\delta W$ might be possible [80]. We would like to emphasize that such a reduction would not only lead to substantial reductions in the running time and/or improvements in statistical accuracy but it would also give access to new questions of physics. As
Fig. 7.2 $\sigma(\mu^+\mu^-)/\sigma_{pt}$ in the toponium region, assuming $m_Z = 94$ GeV, $m_t = 47$ GeV and $\delta W = 48$ MeV (a) and 12 MeV (b)

an illustration, we show in Fig. 7.2 the $\mu$-pair cross-section in the toponium region for two choices of energy spread, $\delta W = 48$ MeV and 12 MeV, in the toponium region assuming a t-quark mass of 47 GeV. Evidently, resonance and interference effects are strongly enhanced for $\delta W = 12$ MeV. We expect that the second choice of 12 MeV would allow a rather accurate determination of $\sin^2 \theta_W$ through the polarization and asymmetry measurements discussed in Section 4. In particular one might exploit the fact that the relative importance of the photon and $Z^0$ contributions to the amplitude are different on the top and in the wing of the resonance [24].

Another instructive example is provided by the direct formation of axial-vector resonances through the neutral current [81]. The following estimate for the production of a $^3P_1$-state with a mass of 80 GeV is based on the wave functions calculated for Richardson's potential. One finds
\[ \Gamma(1P \rightarrow e^+e^-) = \frac{3}{4\pi^2} \left[ 1 + (1 - 4 \sin^2 \theta \frac{N}{2}) \right] \frac{e^4 Z^4}{(m^2 - m_e^2)^2 + r^2 m_e^2} R'(0)^2 \approx 0.1 \text{ keV} . \]  

(7.4)

A similar result has previously been obtained by Gatto, based on a scaling law for a power potential [82].

This leads to a small resonance enhancement above the large continuum background from \(q\bar{q}\) production plus a small contribution from the radiative tail of \(1S\):

\[ R_{1P} = 0.2 ; \quad R_{q\bar{q}} = 2.0 ; \quad R_{\text{rad}} = 0.2 . \]  

(7.5)

The \(3P_1\) resonance decays mainly through SQD \([BR(SQD) = 0.5]\) or through dipole transitions into \(1S\) \([BR(\gamma + 1S) = 0.5]\). It is thus favourable to use the topology cuts discussed in subsection 3.2 which select SQDs. These cuts reduce the \(1P\), \(q\bar{q}\), and the \(1S\) event rates by the factors 0.6, 0.05, and 0.4, respectively, which gives

\[ R_{1P}^{\text{cut}} = 0.12 , \quad R_{q\bar{q}}^{\text{cut}} = 0.1 , \quad R_{\text{rad}}^{\text{cut}} = 0.08 . \]  

(7.6)

To establish the resonance as a 2\(\sigma\) effect with the unmodified resolution would require an integrated luminosity of 25 pb\(^{-1}\). If, furthermore, one wants to measure three points on and off the resonance, a total of 75 pb\(^{-1}\), corresponding to more than 80 days (maximal luminosity, 24 hours), would be required. (Selecting semileptonic SQDs would reduce these numbers by a factor of \(\approx 2.\))

However, if the energy spread could be reduced by a factor of 5, the luminosity per point would be reduced to 1.2 pb\(^{-1}\) and a scanning region of 20 MeV (the estimated uncertainty of model predictions) could be covered, with an integrated luminosity of 4 pb\(^{-1}\).

8. SUMMARY AND CONCLUSIONS

In the previous sections we have described the theoretical predictions for quark-antiquark bound states at LEP energies, and we have analysed which aspects of toponium physics appear experimentally feasible given the luminosity and beam spread of LEP. When we began our studies it was clear that toponium was an ideal laboratory for the standard model, which could probe strong and electroweak interactions at a comparable strength. It was not clear, however, whether toponium physics could be realistically carried out at LEP. Indeed, as one sees from Fig. 2.16, a toponium mass around 80 GeV appears at first sight as the precise prediction of a pessimist: the background is about twice as large as the
SUMMARY, $m_\theta < m_z$

SUSY?

$\theta$ discovery

$V(r), r = 0.04 - 0.1 \text{ fm}, \text{ ASYMPTOTIC FREEDOM}$

$1S, 2S$

$\alpha_{eq} (m_\theta = 70 \text{ GeV})$

$m_{H^0} > 0.7 m_\theta$

DISCOVERY OF (NON) STANDARD HIGGS SCALARS?

$1P (m_\theta = 70 \text{ GeV})$

$3S - 5S$

Fig. 8.1 Perspectives for toponium physics at LEP with $m_\theta < m_z$

signal and the event rates are low. The main result of our quantitative investigation is, however, that an interesting toponium physics program can be carried out in the whole mass range up to the $W$-pair threshold.

As discussed in Section 2, a distinction has to be made between three different regions for the toponium mass $m_\theta$. For $m_\theta = m_Z$ the toponium-$Z$ system with all its interferences has to be treated as a whole, i.e. toponium physics and $Z$ physics are the same subject. For the case where $m_\theta < m_Z$, our results are summarized in Fig. 8.1. We expect that $Z$ decays into $t\bar{t}$ pairs will allow a determination of the $t$-quark mass with an accuracy of about 500 MeV which would restrict the size of the scanning interval in the toponium search to approximately 2 GeV. The toponium ground state could then be established as a $3\sigma$ effect within two weeks. A further month would be required to determine the $1S-2S$ mass difference and the $1S$ and $2S$ leptonic widths (Section 3). These measurements will determine the quark-antiquark potential down to a distance of 0.04 fm, which should provide direct evidence of asymptotic freedom. The measurement of the single-quark decay rate will lead to a determination of the top-quark lifetime. The observation of higher radial excitations and $P$-states is more difficult. The determination of the $3S$- to $5S$-states and of the $1P$-states will require about three months each for $m_\theta = 70 \text{ GeV}$ (Section 3).
Heavy-quark states are particularly well suited to the search for Higgs particles. One month of running time on the 1S-state is expected to yield a lower bound on the Higgs mass of \( m_H > 0.7 m_\theta \) (Section 5), or the discovery of the standard Higgs scalar or of a non-standard one. Within a second month the mass bound could be improved by 5 GeV. Two months are also required in order to measure the polarization asymmetry \( c^{(0)}_{RL} \) with an uncertainty of 0.1 (Section 4). An interesting candidate for 'new physics' is supersymmetry. If the mass spectrum of the new SUSY particles is such that a two-body decay into two scalar t-quarks or two gluinos is kinematically allowed, the discovery of toponium will simultaneously provide evidence of supersymmetry owing to the qualitative change in the decay branching ratios. If two-body SUSY decays are forbidden, the search for SUSY particles, such as a heavy gluino, in toponium decays will be difficult although not impossible (Section 6).

In the case \( m_\theta > m_Z \), toponium physics will be more difficult owing to the decrease of the production cross-section and the increase of the energy spread; yet it appears feasible up to about 150 GeV. For \( m_\theta = 130 \text{ GeV} \), for instance, our results are shown in Fig. 8.2. We expect that the scan for the ground state and the determination of the parameters of the 1S- and 2S-states will require two and three months respectively (Section 3). A lower bound on the Higgs mass of 80 GeV can presumably be achieved within five months (Section 5). We emphasize

**SUMMARY, \( m_\theta = 130 \text{ GeV} \)**

![Diagram](attachment:image.png)

**Fig. 8.2** Perspectives for toponium physics at LEP with \( m_\theta = 130 \text{ GeV} \)
that the main goals of toponium physics—the test of asymptotic freedom, the efficient search for Higgs scalars, and the measurement of the electroweak couplings of t-quarks—can still be achieved.

Our discussion in Sections 3 to 6 has been based on the current estimates of luminosity and beam spread at LEP. As illustrated in Section 7, a decrease of the energy spread or the optimization of the luminosity/beam-spread ratio would be of great value for toponium physics.

Toponium physics at LEP energies is complementary to \( z^0 \) physics. The toponium spectroscopy will test the idea of asymptotic freedom, and toponium decays are the best way to search for Higgs scalars. Both goals can be achieved at LEP for toponium masses below and above the \( z^0 \) mass.

Acknowledgements
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Our calculation of branching ratios of toponium decays was based on the tree-level formulae of Section 2. For ratios of the partial widths of annihilation decays, the first-order QCD radiative corrections have been computed. They read as follows:

Three-gluon decay [see (2.16)] [83, 84]:

\[
\mathcal{R}_{ggg} = \frac{10(w^2 - 9)}{81\pi} \frac{\alpha_{\text{MS}}^3 (m_0)}{\alpha_{\text{em}}^2 e_c^2} \left[ 1 + \frac{\alpha_{\text{MS}}^2 (m_0)}{\alpha_{\text{em}}^2} \left[ 2.770(7) \beta_0 - 14.0(5) \right] + \ldots \right]. \tag{A.1}
\]

Two-gluon photon decay [see (2.17)] [83, 84]:

\[
\mathcal{R}_{gg\gamma} = \frac{8(w^2 - 9)}{9\pi} \frac{\alpha_{\text{MS}}^2 (m_0)}{\alpha_{\text{em}}} \left[ 1 + \frac{\alpha_{\text{MS}} (m_0)}{\alpha_{\text{em}}} \left[ 2.770(7) \frac{2}{3} \beta_0 - 11.8(4) \right] + \ldots \right]; \tag{A.2}
\]

\( \alpha_{\text{MS}} (\mu) \) and \( \beta_0 \) depend on the number of light quark flavours:

\[
\beta_0 = 11 - \frac{2}{3} n_f, \tag{A.3}
\]

\[
\alpha_{\text{MS}} (\mu) = \frac{4\pi}{\beta_0 \ln \mu^2 / \Lambda_{\text{MS}}^2} - \frac{4\pi \beta_1}{\beta_0^3 \ln^2 (\mu^2 / \Lambda_{\text{MS}}^2)}, \tag{A.4}
\]

\[
\beta_1 = 102 - \frac{22}{3} n_f. \tag{A.5}
\]

Higgs photon decay [see (2.18)] [65]:

\[
\mathcal{R}_{H\gamma} = \frac{1}{2x} \frac{m_0^2}{m_{W}^2} \left( 1 - \frac{m_{H}^2}{m_{W}^2} \right) \left[ 1 - \frac{4}{3} \frac{\alpha}{\pi} a(\kappa) \right], \quad x = 4 \sin^2 \theta_w, \tag{A.6}
\]

\[
a(\kappa) = -\frac{\kappa}{12(1-\kappa)} \left[ \frac{F(1-2\kappa)}{2(1-\kappa)} + \frac{\kappa - 1}{1 - 2\kappa} + \frac{2\kappa(1-2\kappa)}{(1-\kappa)^2} \right] F(1) - F(-1)]
\]

\[+ \left( 4 + \frac{\kappa}{1-\kappa} \right) \left[ \frac{\kappa}{1-\kappa} \right]^{1/2} \arctan \left( \frac{\kappa}{1-\kappa} \right) \]

\[+ \left[ 2 + \frac{\kappa}{1-\kappa} \right] \ln 2 + \ln (1 - \kappa)],
\]
where

\[ \kappa = \frac{m^2}{H} - \frac{m^2}{m_9} \]

\[ F(x) = \int \frac{dy}{y} \ln (1 + y), \]

\[ f(\kappa) = \int \frac{dy}{y - 1/2} \kappa \ln \frac{1 - 4y (1 - \kappa)}{2y(1 - \kappa)}; \]

the scale of \( \alpha_s \) in (A.6) is only determined through the second-order radiative corrections.

The radiative corrections to the hyperfine splitting \( \Delta m_{\text{hfs}} \) (2.11) is given by (85-87):

\[ \Delta m_{\text{hfs}} (0) = \frac{8}{9} \frac{\alpha_{\text{MS}} (m_t)}{\alpha_{\text{em}}} \left[ 1 + \frac{\alpha_{\text{MS}} (m_t)}{\pi} \left[ -\frac{8}{3} - \frac{3}{4} \ln 2 + \frac{21}{8} \xi + \frac{1}{4} \theta (\frac{5}{3} - \xi) \right] \right], \quad \text{(A.7)} \]

\[ \xi = \frac{\langle \ln (Q^2/m_t^2) \rangle}{\langle 1 \rangle}, \quad \langle 1 \rangle = |\langle 0 | \phi \rangle|^2. \]

The expectation values in \( \xi \) have to be evaluated with the toponium wave functions.
NEW PARTICLES


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1. GENERAL INTRODUCTION AND FORMULAE

In this report we do not try to catalogue cross-sections for all the new particles which have ever been proposed. Instead we have made a selection based on certain theoretical biases which we explain below. Most noticeable is the absence of any discussion of composite models, which were assigned to the High-Energies Working Group [1]. In some cases cross-section formulae are too complicated to be reproduced here. The most interesting ones are collected in the 'Electronic Yellow Book' at addresses given in this report.

The success of the Standard Model has brought to the fore a new set of problems, of which we emphasize three. These are the Unification, Flavour, and Mass problems. The Unification problem is to find a more unified mathematical structure which encompasses the known strong, weak, and electromagnetic interactions, and maybe gravity. Possible ideas include Grand Unified Theories (GUTs), supergravity, and, most recently, the superstring. All these theories predict new interactions at high energies, \( \geq 10^{15} \text{ GeV} \), which might lead to observable baryon decay, magnetic monopoles, or neutrino masses--novel phenomena not accessible to LEP. Different GUTs also make quite precise predictions for some low-energy parameters such as \( \sin^2 \theta_w \). Many of these models can be distinguished by precision measurements of \( \sin^2 \theta_w \) in experiments at the \( Z^0 \) peak, as discussed elsewhere [2]. A favoured approach to the Flavour problem, namely that of understanding the proliferation of quark and lepton species, is to postulate that they are composites made of more elementary constituents. Composite theories may predict new four-fermion contact interactions, form factors for quarks and leptons which are no longer point-like, and excited leptons, quarks, and possibly vector bosons. There is no clear idea about the scale of compositeness, but the agreement of many measurements with the Standard Model tells us that it must exceed 100 GeV. There is no guarantee that the compositeness scale will be low enough to be detectable at LEP, but if so it is most likely to show up at high energies, as discussed by the working group [1] on that area of LEP physics. The Mass problem distinguishes itself from the other problems in that the scale at which it is solved by the appearance of new physics must be below 1 TeV. Moreover, most theoretical attacks on the Mass problem predict some new particles--Higgs bosons, supersymmetric particles or technipions--with masses less than 100 GeV. Therefore, LEP is very likely to provide us with key insights into the Mass problem, so we discuss it in more detail.

Consistency of the Standard Model with non-zero vector boson and fermion masses, at the level of perturbative unitarity and of renormalizability, requires [3] the existence of at least one physical spin-0 boson \( H \) with coupling to other particles \( X \): \( g_{HXX} = m_X \), and a mass \( m_H = 0(m_X) = 0(100) \text{ GeV} \)--the infamous Higgs boson [4]. If the Higgs boson is elementary, technical problems arise because
its mass is a very unstable parameter. When one includes a light Higgs boson in a more unified theory containing large mass scales, the inevitable interactions of the Higgs boson with heavy particles feed down the large mass scale, tending to give the $O(100)$ GeV Higgs boson a much heavier mass [5-7]. This is the so-called hierarchy problem. In the absence of new physics below some high scale $\Lambda$, radiative corrections due to loops of light particles make a contribution to the Higgs boson mass which is proportional to $\Lambda$. This is one aspect of the so-called naturalness problem [5]. Supersymmetry [8] introduces such new physics in the form of supersymmetric partners for all the known particles with spins differing by half a unit—bosons to partner fermions and vice versa. To resolve the naturalness problem by partially cancelling the loop diagrams, or to alleviate the hierarchy problem by making light particles insensitive to the large mass scale, the supersymmetric particles must have masses $\Lambda \lesssim 1$ TeV. In many models [9], at least some of these particles have masses $\lesssim 100$ GeV, and are therefore accessible to LEP. Supersymmetry is useful if the Higgs boson is elementary. An alternative is to make it composite, made of constituents bound together by some new interactions which become strong at an energy scale $\Lambda \sim 1$ TeV. In this technicolour [10] scenario, one expects to encounter many new technimesons and technibaryons with masses $\sim 1$ TeV. Many extended technicolour models which give masses to fermions as well as to gauge bosons contain at least some spin-0 technipions with masses $\lesssim 100$ GeV. Thus we have seen that both favoured approaches to the Mass problem tend to predict some new particles with masses $\lesssim 100$ GeV, whose production cross-sections and properties are discussed in subsequent sections of this article.

For convenience, we quote here some useful formulae which apply to the production of any spin-1/2 fermion-antifermion pair, or pair of charge-conjugate spin-0 bosons, produced via virtual $\gamma$ and $Z^0$ exchange in the s-channel. We have the following leading-order cross-section for $f \bar{f}$ production [11, 12]:

$$
\sigma_f^0(Q, T_{3L}, T_{3R}, \beta, s) = \frac{4\alpha^2}{3\pi} \beta \times \left[ \frac{Q^2}{2} \left[ 3-\beta^2 \right] - 2Q \gamma_c C_V \frac{s(s-m_Z^2)}{(s-m_Z^2)^2 + m_Z^2 r_Z^2} \left[ \frac{3-\beta^2}{2} \right] \right]$$

$$
+ \left( C_V^2 + C_A^2 \right) \frac{s^2}{(s-m_Z^2)^2 + m_Z^2 r_Z^2} \left[ c_V^2 \left[ \frac{3-\beta^2}{2} \right] + C_A^2 \left[ \beta^2 \right] \right] \right) \right)$$

(1.1)

where

$$
C_V = \frac{1 - 4 \sin^2 \theta_W}{4 \sin \theta_W \cos \theta_W}, \quad C_A = \frac{-1}{4 \sin \theta_W \cos \theta_W} \quad (1.2)
$$
are the vector and axial-vector couplings of the electron, and

$$C'_V = \frac{-2(T^L_3 + T^R_3)}{4 \sin \theta_w \cos \theta_w}, \quad C'_A = \frac{2(T^L_3 - T^R_3)}{4 \sin \theta_w \cos \theta_w} \tag{1.3}$$

are those of the produced fermion $f$, whose left- and right-handed components have third components of weak isospin $T^L_3$ and $T^R_3$, respectively. The centre-of-mass energy $E_{CM} = s$ and $\beta = (1 - 4m_f^2/s)^{1/2}$ is the centre-of-mass velocity of the produced fermion. The term $\alpha_A^2$ has a different form from the others because the axial current can only generate fermions in a P-wave, not in an S-wave.

These formulae can [11] be carried over to the s-channel pair production of spin-zero particles such as leptons, quarks, and Higgs bosons by setting

$$\sigma_0^x(Q, T_3, s, s) = \frac{\beta^3}{4} \sigma_0^s(Q, T_3, T_3, 1, s). \tag{1.4}$$

Note that these scalar particles are produced in a P-wave by the vector current alone, as required by Bose symmetry, with $C'_A$ (1.3) vanishing.

There are two useful sets of radiative corrections to these leading-order formulae which we also record here. One is the initial-state electromagnetic radiative corrections [13, 14], which yield

$$\frac{\partial \sigma_0}{\partial x} = \frac{a}{x} \left( \ln \frac{s}{m_e^2} - 1 \right) \frac{1 + (1 - x)^2}{x} \sigma_0 [s(1 - x)], \tag{1.5}$$

where $x = 2E_\gamma/E_{CM} = E_{\gamma}/E_{\text{beam}}$, and $\sigma_0$ is the total leading-order cross-section (1.1) or (1.4). These corrections can be important for any process with large local peaks in the cross-section, as provided, for example, by the $Z^0$ or by toponium.

Also important for strongly interacting final states are the final-state QCD radiative corrections:

$$\sigma_{QCD} = N\sigma_0(Q, T^L_3, T^R_3, s, s) \times F_{QCD} \tag{1.6}$$

where $N$ is a statistical factor accounting for the sum over final-state colours (e.g. $N = 1$ for leptons, 3 for colour-triplet quarks, 6 for sextets, 8 for octets, etc.), and to leading order in $\alpha_s$ the correction factor

$$F_{QCD} = 1 + C_{QCD} f(\beta), \tag{1.7}$$
with

\[ C = \frac{4}{3} \text{ for colour } 3 \]
\[ 10/3 \text{ for colour } \frac{4}{3} \]
\[ 3 \text{ for colour } 1, \text{ etc.} \]  \hspace{1cm} (1.8)

and to a very good approximation (±1%) [15]:

\[ f(\beta) = \frac{v}{2\beta} - \left( \frac{3+\beta}{4} \right) \left( \frac{v}{2} - \frac{3}{4\beta} \right), \]  \hspace{1cm} (1.9)

where \( \beta \) is again the centre-of-mass velocity of the produced strongly interacting particles. As \( \beta \to 1 \), \( f(\beta) \to 3/4v \), so that for colour triplet quarks \( F_{QCD} \to 1 + \alpha_s/v \) to first order in \( \alpha_s \). The \( O(\alpha_s^2) \) effects have also been calculated [16] for this case. They depend on the renormalization scheme used and take the values

\[
F_{QCD} = 1 + \frac{\alpha_s}{v} + \begin{cases} 
(7.35 - 0.442 \, N_f) \left( \frac{\alpha_s}{v} \right)^2 \quad & \text{MS scheme} \\
(1.98 - 0.115 \, N_f) \left( \frac{\alpha_s}{v} \right)^2 \quad & \text{\overline{MS} scheme} \\
(4.64 - 0.739 \, N_f) \left( \frac{\alpha_s}{v} \right)^2 \quad & \text{MOM scheme}
\end{cases}
\]  \hspace{1cm} (1.10)

for colour triplets, where \( N_f \) is the number of light quark flavours. As \( \beta \to 0 \), \( f(\beta) \) (1.9) exhibits a Coulomb singularity,

\[ f(\beta) \to \frac{v}{2\beta}, \]  \hspace{1cm} (1.11)

which reflects the presence of bound states just below threshold, and makes a large correction to the leading-order formula (1.1). All the \( O(\alpha_s/\beta)^n \) terms can be evaluated and summed exactly [15] to give the exponential form

\[ F_{QCD} = \frac{z}{1 - \exp(-z)}, \quad z = C \alpha_s \frac{v}{\beta}. \]  \hspace{1cm} (1.12)

A useful interpolation between the limits \( \beta \to 0 \) (1.12) and \( \beta \to 1 \) (1.7) and (1.9) may be obtained by replacing \( z = 2C \alpha_s \, f(\beta) \) in Eq. (1.12). This expression gives large corrections to (1.1) just above threshold, and very close to threshold even this formula breaks down because of non-perturbative effects.
Formulae (1.1) to (1.12) characterize new particle cross-sections away from direct channel resonances such as toponium. Later in this article we will also present some rates for resonance decays into new particles such as Higgses. However, we will not discuss in detail the possible supersymmetric decay modes of toponium, since these are discussed elsewhere [17].

Unless otherwise noted, the following standard values have been used to estimate cross-sections and rates: \( \sin^2 \theta_W = 0.22 \), \( m_{\mu} = 10.6 \text{ GeV} \), and \( \Gamma_{\mu} = 3 \text{ GeV} \).

* * *

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2. HIGGS BOSONS

2.1 Introduction and organization

One major prediction of the Standard Model, the existence of a neutral scalar Higgs boson, has yet to be experimentally confirmed or excluded. Given the success of the Standard Model, and of gauge theories in general, there is a great deal of interest in devising experiments to uncover the nature of electroweak symmetry breaking. LEP will provide an important laboratory for these experiments. There has already been one report [1] dedicated to the subject of Higgs particles at LEP, as well as many review articles about Higgs particles in general [2]. In this section we complement and update Ref. [1], choosing those processes from the existing literature that seem to be most relevant to LEP experiments.

The section is organized as follows. In subsection 2.2 we give a brief outline of the minimal Higgs mechanism in the Standard Model, and some characteristics of the single scalar $H^0$ it introduces. Subsection 2.3 describes possible $H^0$ decays, and subsection 2.4 exhibits the most likely $H^0$ production processes. In subsection 2.5 we consider the possibility of a more complicated Higgs sector and the additional phenomenology it might introduce. Finally, a short outline of technipion phenomenology for LEP is given in subsection 2.6 with the understanding that, although Extended Technicolor theories are currently in disfavour, a similar composite-scalar theory may rise again between now and when the first beams successfully cross.

Whilst most of this section contains theoretical discussions of cross-sections and rates, we also present the results of some Monte Carlo simulations of Higgs production processes and backgrounds which cast light on the observability of some Higgs production mechanisms.

2.2 Minimal Higgs mechanism in the Standard Model

First, we briefly discuss the Higgs [3] mechanism in the Standard Model [4]. Consider a multiplet of complex scalar fields with hypercharge $Y=1$, transforming as a doublet under weak $SU(2)$:

$$
\Phi = \left( \begin{array}{c} \Phi^+ \\ \Phi^0 \end{array} \right),
$$

(2.1)

with a self-interaction potential

$$
V(\Phi) = \frac{1}{4} \mu^2 |\Phi|^2 + \frac{1}{4} \lambda |\Phi|^4 .
$$

(2.2)

Here $\lambda$ and $\mu^2$ are parameters, and we take $\lambda > 0$. We say nothing about $\mu^2$ for the moment. Since the Higgs sector is part of the Standard Model Lagrangian, its kinetic terms must be made invariant under local $SU(2) \times U(1)$ transformations.
This involves the use of four gauge fields \( A^a_\mu (a = 1, 2, 3) \) and \( B_\mu \) to make a covariant derivative, and the corresponding kinetic terms in the Lagrangian become

\[
L = \left| \left( i \partial_\mu - g \mathbf{A}^a_\mu \mathbf{T}^a - g' \frac{\mathbf{Y}}{2} B_\mu \right) \phi \right|^2 - V(\phi),
\]

(2.3)

where \( T \) and \( Y \) are the generators of \( SU(2)_L \) (weak isospin) and \( U(1) \) (hypercharge), respectively, and \( g \) and \( g' \) are the corresponding gauge group couplings. The gauge bosons \( A^a_\mu \) and \( B_\mu \) are the four gauge fields in the electroweak Lagrangian for quarks and leptons, which are as yet massless.

To see how to generate masses for the gauge bosons, we take a closer look at the Higgs potential (2.1) and use the simplified example of a single real scalar field \( \phi \). Figure 2.1 shows the form of Eq. (2.1) for \( \mu^2 > 0 \) and \( \mu^2 < 0 \). Although the mass term seemingly has the wrong sign for \( \mu^2 < 0 \), it is this case that is (perhaps?) physical and of great interest. With \( \mu^2 < 0 \), the minimum of the potential is not at \( \phi = 0 \), but rather at \( \phi = \nu/2 = -\mu^2/\lambda \). A perturbation on expansion can only be done around a local minimum, so the field must be translated to \( \nu/2 \). Let us write

\[
\phi(x) = \nu/2 + \phi'(x),
\]

(2.4)

where \( \phi'(x) \) is the fluctuation around \( \nu/2 \) (we could equally have chosen \( -\nu/2 \), according to spontaneous \textit{w}him).

\[\text{Fig. 2.1 Forms of the Higgs potential (2.2) for } \mu^2 > 0 \text{ and } \mu^2 < 0.\]

Now,

\[
V(\phi') = \frac{1}{2} \lambda \nu^2 \phi'^2 + \frac{\lambda}{2} \nu \phi'^3 + \frac{1}{4} \lambda \phi'^4 + \text{constant},
\]

(2.5)

and we see that there is a mass term for \( \phi' \) with the correct sign:

\[
m_{\phi'} = \sqrt{\lambda \nu^2} = \sqrt{-2\mu^2}
\]

(2.6)
By introducing a potential whose minimum is not at zero we allow the field to develop a non-zero vacuum expectation value \( v/2 \) and a conventional mass term in the Lagrangian.

This trick is now applied to electroweak interactions. In order to generate masses for the weak bosons but have the photon massless, the field which is allowed to acquire a vacuum expectation value is taken to be electrically neutral. Thus the gauge symmetry of the \( U(1)_{\text{em}} \) subgroup is left intact, ensuring a massless photon. Substituting

\[
\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}
\]

(2.7)

into the Lagrangian (2.3), one finds the following quadratic terms:

\[
\frac{1}{4} v^2 g^2 w^\mu w^-_\mu + \frac{1}{8} v^2 (g^2 + g^2) z^\mu z^\mu + (0) A^\mu A^\mu,
\]

(2.8)

where

\[
w^\pm_\mu = \frac{[A^{(1)}_\mu + iA^{(2)}_\mu]}{\sqrt{2}},
\]

(2.9)

\[
A_\mu = \cos \theta_w B_\mu + \sin \theta_w A^{(3)}_\mu,
\]

\[
z_\mu = -\sin \theta_w B_\mu + \cos \theta_w A^{(3)}_\mu
\]

form the basis in which the mass matrix is diagonal, and \( \tan \theta_w \equiv g'/g \). Some of the terms in the Higgs sector Lagrangian have thus turned into mass terms for the charged and neutral gauge bosons, if we identify

\[
m_{w^\pm} = \frac{1}{2} v g, \quad m_z = \frac{1}{2} v \sqrt{g^2 + g^2}, \quad m_A = 0.
\]

(2.10)

Thus we find massive \( W \) and \( Z \) bosons with the relation \( m_w/m_z = \cos \theta_w \), and a massless photon \( A \).

Of the four real fields in Eq. (2.1), three are massless, unphysical Goldstone modes which disappear in a unitary gauge. These three degrees of freedom have been 'eaten' to become the longitudinal polarization states of the now massive gauge bosons: a massless vector state only has two transverse polarization states, whereas a massive vector boson needs three. This leaves one physical massive scalar field, the famous Higgs boson. Replacing \( \phi \) by the translated field \( H^0 \) which may be written
\[ 
\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H^0(x) \end{pmatrix} 
\] (2.11)

in the rest of the Lagrangian, as was done with Eq. (2.4) to get Eq. (2.5), one finds that \( H^0 \) couples to the other particles simply according to their masses [1, 2]:

for fermions: \( g_{fH} = 2^{1/4} \sqrt{G_F m_f} \),

for vector bosons: \( g_{VH} = 2^{5/4} \sqrt{G_F m_V^2} \). (2.12)

While the masses of the gauge bosons and the Higgs couplings are well determined in the Standard Model, the mass of the Higgs boson is almost completely unconstrained, and depends [Eq. (2.6)] on the value of \( \lambda \). If \( \lambda \) becomes too large, perturbation theory breaks down, implying a limit [5, 6] of

\[ m_H < 1 \text{ TeV}. \] (2.13)

When considering a theoretical lower limit for the Higgs mass, radiative corrections to the Higgs potential become significant. For very small or vanishing \( \mu^2 \) [7] there is a local minimum of the potential with \( v \neq 0 \) for \( \mu = O(\alpha^2) \), in which case

\[ m_H^2 = \frac{3\alpha^2}{8} v^2 \left[ 2 \sec^4 \theta_W - \left( \frac{m_f}{m_W} \right)^4 \right], \] (2.14)

which gives a lower limit \( m_H = O(10) \) GeV if \( m_f \ll m_W \). Lower values of \( m_H \) can be accommodated if there exist massive fermions [8], or if \( \mu^2 \) is small and positive [9]. However, it seems unnatural for the Higgs mass to be less than a few GeV.

Even though its mass is not well determined, the nature of the Higgs couplings provides distinct signatures to look for. We now turn to \( H^0 \) decays.

2.3 Decay modes of the neutral Higgs \( H^0 \)

The \( H^0 \) is expected to have a mass between \( O(10) \) GeV and 1 TeV and, since it couples directly to the masses of other particles, to decay into the heaviest available 'daughters'. This is a distinctive signature for the Higgs boson. The leading-order decays are \( H^0 \rightarrow (\ell^+\ell^-), (qq) \) and \( H^0 \rightarrow (W^+W^-, \gamma\gamma) \) if \( m_{H^0} > 2m_W, Z \). Their decay rates are as follows:
2.3.1 $\mathcal{H}^0 \rightarrow f \bar{f}$

The width into $f \bar{f}$ is given [2] by

$$\Gamma(\mathcal{H}^0 \rightarrow f \bar{f}) = \frac{N_C F_f^2 m_{\mathcal{H}^0}^2}{4 \pi} \beta^3,$$

(2.15)

where $\beta = \sqrt{1 - 4m_f^2/m_{\mathcal{H}^0}^2}$ and $N_C = 1$ and 3 for $f = 1$ and $q$, respectively. As an example, for $f = \mu$,

$$\Gamma(\mathcal{H}^0 \rightarrow \mu^+ \mu^-) = 7.3 \times 10^{-9} m_{\mathcal{H}^0} \left(1 - \frac{4m_{\mu}^2}{m_{\mathcal{H}^0}^2}\right)^{3/2}.$$

(2.16)

Table 2.1 gives $\Gamma(\mathcal{H}^0 \rightarrow q\bar{q})$ for various values of $m_{\mathcal{H}^0}$ and $m_q$, while Fig. 2.2a gives $\Gamma(\mathcal{H}^0 \rightarrow q\bar{q})$ as a function of $m_{\mathcal{H}^0}$ for $m_q = 5$ GeV and $m_q = 40$ GeV.

<table>
<thead>
<tr>
<th>$m_{\mathcal{H}^0}$ (GeV)</th>
<th>20</th>
<th>60</th>
<th>100</th>
<th>140</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_q$ (GeV) = 5</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>30</td>
<td>-</td>
<td>-</td>
<td>91</td>
<td>183</td>
<td>226</td>
</tr>
<tr>
<td>40</td>
<td>-</td>
<td>-</td>
<td>68</td>
<td>244</td>
<td>327</td>
</tr>
<tr>
<td>50</td>
<td>-</td>
<td>-</td>
<td>236</td>
<td>375</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2.2 Decay rates of the neutral Higgs in the minimal Standard Model: (a) into $q\bar{q}$ pairs, (b) into $W^+W^-$ and $Z^0Z^0$. 
2.3.2 \( H^0 \rightarrow W^+ W^- \), \( Z^0 \rightarrow Z^0 \)

The rates for decays into the Standard Model intermediate vector bosons are [6]

\[
\Gamma(H^0 \rightarrow W^+ W^-) = \frac{G_F^2 m_{W^0}^5}{8\pi^2} \left[ \frac{(1-x)^{1/2}}{x} \right] (3x^2 - 4x + 4),
\]

\[
\Gamma(H^0 \rightarrow Z^0 Z^0) = \frac{G_F^2 m_{H^0}^5}{16\pi^2} \left[ \frac{(1-x')^{1/2}}{x'} \right] (3x'^2 - 4x' + 4),
\]

where \( x \equiv 4m_W^2/m_{H^0}^2 \) and \( x' \equiv 4m_Z^2/m_{H^0}^2 \). The decay widths (2.17) and (2.18) are shown in Fig. 2.2b. We see that they are much larger than the corresponding \( H^0 \rightarrow f \bar{f} \) decays unless \( m_f > m_{W,Z} \). Note, however, that copious production of a standard Higgs with mass \( m_H > 2m_W \) or \( 2m_Z \) will be negligible even at LEP II.

Higher-order Higgs decay modes include \( H^0 \rightarrow \gamma \gamma \), \( H^0 \rightarrow gg \), and \( H^0 \rightarrow W^\pm W^\mp f \bar{f}' \), \( Z^0 f \bar{f} \).

2.3.3 \( H^0 \rightarrow \gamma \gamma \)

The decay into two photons has contributions from diagrams like those in Fig. 2.3, giving [2, 10, 11]

\[
\Gamma(H^0 \rightarrow \gamma \gamma) = \frac{\alpha^2}{8\pi^2} \frac{G_F^2 m_{H^0}^3}{|N|^2},
\]

where \( I = I_1 + I_{\text{hadrons}} + I_{W^\pm} + \ldots \), and \( |N| = \mathcal{O}(1) \). Unfortunately, this means that \( \Gamma(H^0 \rightarrow \gamma \gamma) \) is estimated [2] to be at most \( \mathcal{O}(10^{-5}) \) of the two-photon decay rate \( \Gamma(h \rightarrow \gamma \gamma) \) for a hadron \( h \) of comparable mass, and hence that \( H^0 \rightarrow \gamma \gamma \) is not an important decay mode for the Higgs, unless it weighs less than 200 MeV.

[Diagram of some diagrams contributing to \( H^0 \rightarrow \gamma \gamma \) decay]

2.3.4 \( H^0 \rightarrow gg \)

Similarly, the decay \( H^0 \rightarrow gg \) proceeds [11-15] via quark (and possibly other strongly interacting particle) loops, as in Fig. 2.4, giving

\[
\Gamma(H^0 \rightarrow gg) = \frac{\alpha^2}{8\pi^2} \frac{m_{H^0}^3}{9} |N|^2,
\]
where $N$ is the sum of contributions $N_j$ from quarks $j = 1, 2, \ldots$, given by

$$N \equiv 3 \sum_j N_j : N_j = \int_0^1 dx \int_0^{1-x} dy \frac{1 - 4xy}{1 - xy (m^2_{H^0}/m_j^2) - i\varepsilon}$$

$$= [2\lambda_j + \lambda_j(4\lambda_j-1)G(\lambda_j)] , \quad (2.21)$$

where $\lambda_j = m_j^2/m_{H^0}^2$ and

$$G(\lambda_j) = -2\left[\arcsin \left(\frac{1}{2\lambda_j}\right)\right]^2 , \quad \lambda_j > \frac{1}{4} , \quad (2.22)$$

$$G(\lambda_j) = \frac{1}{2} \ln^2 \left(\frac{n^+_j}{n^-_j}\right) - \frac{\pi^2}{2} + i\pi \ln \left(\frac{n^+_j}{n^-_j}\right) , \quad \lambda_j < \frac{1}{4} ,$$

where

$$n^+_j = 1 \pm (1-4\lambda_j)^{1/2} .$$

The form of $N_q$ for a single flavour of quark loop is shown in Fig. 2.5. Note that $N_q$ vanishes for $m_{H^0} \gg m_q$, and tends to a constant $N_q \to 1/3$ for

![Fig. 2.5 The loop factor $N_q$ (2.21) appearing in $H^0 \to q\bar{q}$ decay, as a function of $\lambda_q = m_q^2/m_{H^0}^2$.](image-url)
Fig. 2.6 Reduced rate $\Gamma/\alpha_s^2|N|^2$ for $H^0 \to gg$ decay

$m_{H^0} \ll m_q$. Thus only heavy quarks with $m_q > 0(m_{H^0})$ contribute significantly to the sum for $N$ in Eq. (2.20). The width given by Eq. (2.20) divided by $\alpha_s^2|N|^2$ is shown in Fig. 2.6: depending on the quark masses, $\Gamma(H^0 \to gg)/\Gamma(H^0 \to qq)$ can be $(1-10)\%$. Clearly, if $m_{H^0} > 2m_t$ and the top is the heaviest quark, $\Gamma(H^0 \to \bar{t}t) \gg \Gamma(H^0 \to gg)$.

2.3.5 $H^0 \to W^\pm f\bar{f}'$, $Z^0 f\bar{f}$

Finally, if the Higgs is relatively heavy ($m_{H^0} > m_{W^\pm}$, $m_{Z^0}$), the decays $H^0 \to W^\pm f\bar{f}'$ and $H^0 \to Z^0 f\bar{f}$ are possible [14] through the diagrams in Fig. 2.7, and are particularly interesting if $m_f > m_{H^0}/2$. The rates for negligible fermion masses are given by

$$\Gamma(H^0 \to W^\pm f\bar{f}') = \frac{g^4 m_{H^0}}{3072\pi^3} F(\epsilon),$$

(2.23)

where

$$F(\epsilon) = \frac{3(1 - 8\epsilon^2 + 20\epsilon^4)}{1/2} \arccos \left( \frac{3\epsilon^2 - 1}{2\epsilon^2} \right) - (1-\epsilon^2)\left( \frac{47}{2} \epsilon^2 - \frac{13}{2} + \frac{1}{\epsilon^2} \right) - 3(1 - 6\epsilon^2 + 4\epsilon^4) \ln \epsilon,$$

(2.24)

Fig. 2.7 Diagrams contributing to $H^0 \to W^\pm f\bar{f}'$ and $Z^0 f\bar{f}$ decays
and $\epsilon = m_{W^\pm}/m_{H^0}$. Adding together all $\tilde{t}\tilde{f}'$ modes for $W^\pm$ except $\tilde{t}b/\tilde{t}t$, one finds a total,

$$\Gamma(H^0 \rightarrow W^\pm \tilde{t}\tilde{f}') = \frac{3g^4 m_{H^0}}{512\pi^3} F(\epsilon).$$  \hfill (2.25)

Similarly, for $H^0 \rightarrow Z^0 \tilde{t}\tilde{f}$ (neglecting $H^0 \rightarrow Zt\bar{t}$),

$$\Gamma(H^0 \rightarrow Z^0 \tilde{t}\tilde{f}) = \frac{g^4 m_{H^0}}{2048\pi^3 \cos^4 \theta_W} \left( 7 - \frac{40}{3} \sin^2 \theta_W + \frac{160}{9} \sin^4 \theta_W \right) F(\epsilon'),$$  \hfill (2.26)

where $\epsilon' = m_Z/m_{H^0}$. Figure 2.8 shows the branching ratio $\Gamma(H^0 \rightarrow W^\pm \tilde{t}\tilde{q}q'}/\Gamma(H^0 \rightarrow \tilde{t}\tilde{t})$, assuming $m_t = 40$ GeV, as a function of $m_{H^0}$ in the range of $m_{H^0}$ of interest to LEP.

We now turn to some production mechanisms.

2.4 Higgs production processes

2.4.1 Toponium $V_t \rightarrow H^0 + \gamma$

This process takes advantage of the fact that the $H^0$ couples directly to the heavy constituent quarks. Because of its distinctive monochromatic photon signature, this process is a promising way to hunt for the Higgs boson if the mass of $H^0$ is less than the toponium mass. The dominant diagram for toponium $\rightarrow H^0 + \gamma$ is that in Fig. 2.9, which yields the leading-order Wijnsdijk formula [11, 16] for vector toponium $V_t$ decay relative to the decay through $\gamma'$ into $\mu^+\mu^-$,

$$\frac{\Gamma(V_t \rightarrow H^0\gamma)}{\Gamma(V_t \rightarrow \gamma' \rightarrow \mu^+\mu^-)} = \frac{G_F m_{V_t}^2}{2\pi v} \left( 1 - \frac{m_{H^0}^2}{m_{V_t}^2} \right),$$  \hfill (2.27)
Fig. 2.9 Dominant diagram contributing to toponium $V_L \rightarrow H^0 + \gamma$ decay

Fig. 2.10 Leading order ratio $\frac{\Gamma(V_L \rightarrow H^0 \gamma)}{\Gamma(V_L \rightarrow \gamma + \mu^+\mu^-)}$

which is plotted in Fig. 2.10. We consider two corrections to (2.27). The first is to modify the denominator to take into account $Z^0$ exchange:

$$\frac{\Gamma(V_L \rightarrow \gamma + Z^0 \rightarrow \gamma + \mu^+\mu^-)}{\Gamma(V_L \rightarrow \gamma + \mu^+\mu^-)} = \left[ 1 + \frac{1 - (2/3)x}{y^2} \Re x_Z^2 + \frac{1 - (2/3)x}{y^4} \left( \frac{(1-x)^2 + 1}{(2/3)^2} \right) |x_Z|^2 \right]$$

(2.28)

where

$$x_Z = \frac{m^2}{(m^2 - m_Z^2 + i\Gamma_Z m_Z)} ,$$

$$x = 4 \sin^2 \theta_W ,$$

$$y = 4 \sin \theta_W \cos \theta_W .$$

This correction factor is plotted as a function of $m_V$ in Fig. 2.11. Secondly, both the numerator and denominator have QCD corrections. From diagrams such as those in Fig. 2.12, Vysotsky [17] calculated the correction to the numerator to be

$$\Gamma(V_L \rightarrow H^0 \gamma) = \Gamma_V \left[ 1 - \frac{4\alpha_s}{3\pi} a(x) \right] ,$$

(2.29)
Fig. 2.11. Correction factor
\[ \Gamma(\nu_e \gamma, Z^{*} \rightarrow \mu^{+} \mu^{-}) / \Gamma(\nu_e \gamma, \mu^{+} \mu^{-}) \]

Fig. 2.12 Some diagrams contributing to the QCD radiative corrections for
\[ V^+_t \rightarrow H^0 + \gamma \] decay

where \( \kappa \equiv \frac{m^2_{H}}{m^2_{V^+_t}} \), and \( a(\kappa) \) is a mathematically non-trivial function [18]
ranging, typically, between 10 and 15 as seen in Fig. 2.13. There is a similar
correction [19] to the denominator:
\[ \Gamma(\nu_e \rightarrow \mu^{+} \mu^{-}) = \Gamma_0 \left(1 - \frac{16a_3}{3\pi} \right). \]  
(2.30)

Fig. 2.13 Reduced QCD radiative correction factor (2.29)
Combining Eqs. (2.29) and (2.30), we find

\[
\frac{\Gamma(V_t \rightarrow H^0\gamma)}{\Gamma(V_t \rightarrow \mu^+\mu^-)} = \frac{\Gamma_0(V_t \rightarrow H^0\gamma)}{\Gamma_0(V_t \rightarrow \mu^+\mu^-)} \left[ \frac{1 + \frac{\alpha_s}{3\pi} \left[ 16 - 4a_0(\kappa) \right] + O(\alpha_s^2)}{1} \right].
\]  

(2.31)

Taking into account QCD corrections to order \(\alpha_s\) has the unfortunate effect of reducing the rate by a factor of \(-2\). Table 2.2 shows the branching ratio with and without these corrections, and the corrected branching ratio is plotted in Fig. 2.14. The corrections [Eqs. (2.29) and (2.31)] become unreliable for \(m_{H^0}\) close to \(m_V\), when binding effects such as the possible mixing of the \(H^0\) with the \(3P_1(\bar{t}t)\) states become important [16, 20]. For estimates of these effects, see Refs. [21].

Another mechanism for \(H^0\) production in association with \(V_t\) is the reaction \(e^+e^- + V_t \rightarrow H^0\), through the diagram shown in Fig. 2.15, but the rate [22] even on the \(Z^0\) peak is very small \((-10^{-2}\) pb for \(m_{H^0} = 10\) GeV, \(m_{V_t} = 80\) GeV).

<table>
<thead>
<tr>
<th>(m_{H^0}) (GeV)</th>
<th>1.0</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without QCD correction</td>
<td>0.48</td>
<td>0.47</td>
<td>0.44</td>
<td>0.35</td>
<td>0.21</td>
</tr>
<tr>
<td>With QCD correction (\Lambda_{QCD} = 100) MeV</td>
<td>0.29</td>
<td>0.28</td>
<td>0.26</td>
<td>0.19</td>
<td>0.08</td>
</tr>
<tr>
<td>With QCD correction (\Lambda_{QCD} = 200) MeV</td>
<td>0.26</td>
<td>0.25</td>
<td>0.23</td>
<td>0.16</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Notes: Branching fractions for \(V_t \rightarrow H^0\gamma\) (\(m_{V_t} = 80\) GeV), without and with QCD corrections to the numerator and denominator. The numbers include the overall correction of 0.82 due to \(Z^0\) exchange in the denominator.
Fig. 2.14 Ratio \( \frac{\Gamma(V_t \rightarrow H^0 \gamma)}{\Gamma(V_t \rightarrow \gamma, Z^0 \rightarrow \mu^+ \mu^-)} \) including first-order QCD radiative corrections

\[ \frac{\Gamma(V_t \rightarrow H^0 \gamma)}{\Gamma(V_t \rightarrow \gamma, Z^0 \rightarrow \mu^+ \mu^-)} = \frac{\alpha^2}{\mu^2 \sin^2 \theta_w} \left( \frac{m_t}{m_Z} \right)^3 \frac{A^2}{[1 + (1 - 4 \sin^2 \theta_w)^2]} \]  

(2.32)

2.4.2 \( Z^0 \rightarrow H^0 \gamma \)

This process proceeds through loop diagrams such as those in Fig. 2.16. The general rate formula [23, 24] is

\[ \frac{\Gamma(Z^0 \rightarrow H^0 \gamma)}{\Gamma(Z^0 \rightarrow \mu^+ \mu^-)} = \frac{\alpha^2}{\mu^2 \sin^2 \theta_w} \left( \frac{m_t}{m_Z} \right)^3 \frac{A^2}{[1 + (1 - 4 \sin^2 \theta_w)^2]} \]  

(2.33)

Considering only \( W \) loops, a good approximation if \( m_t \ll m_W \), we find

\[ A = -\left[ 4.56 + 0.25 \left( \frac{m_H^2}{m_W^2} \right) \right] \]  

(2.34)

which leads to

\[ \frac{\Gamma(Z^0 \rightarrow H^0 \gamma)}{\Gamma(Z^0 \rightarrow \mu^+ \mu^-)} = 6.3 \times 10^{-5} \left[ 1 - \left( \frac{m_H^2}{m_Z^2} \right)^3 \right] \left[ 1 + 0.14 \left( \frac{m_H^2}{m_Z^2} \right) \right]. \]  

(2.35)

Fig. 2.16 Some diagrams contributing to \( Z^0 \rightarrow H^0 \gamma \) decay
Table 2.3
Number of $Z^0 \rightarrow H^0 \gamma$ events for $10^6 Z^0$: $\text{BR}(Z^0 \rightarrow \mu^+ \mu^-) = 3\%$

<table>
<thead>
<tr>
<th>Higgs mass (GeV)</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of events</td>
<td>1.8</td>
<td>1.8</td>
<td>1.7</td>
<td>1.5</td>
<td>1.4</td>
<td>1.2</td>
<td>1.1</td>
<td>0.9</td>
<td>0.7</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Fig. 2.17 Ratio $\Gamma(Z^0 \rightarrow H^0 \gamma)/\Gamma(Z^0 \rightarrow \mu^+ \mu^-)$ as a function of $m_{H^0}$

For a more complete discussion, see Ref. [24]. Using Eq. (2.34) we find the rates shown in Table 2.3, whilst the ratio $\Gamma(Z^0 \rightarrow H^0 \gamma)/\Gamma(Z^0 \rightarrow \mu^+ \mu^-)$ as a function of $m_{H^0}$ is shown in Fig. 2.17. Although the branching ratio is small, the large number of $Z^0$'s expected to be produced at LEP may make this process worth while. However, if there are generations of quarks and/or leptons with mass > $m_W$, the fermion loops must also be taken into account. Unfortunately, since fermion and boson loops interfere destructively, the rate is decreased [23] unless the number of heavy generations is large (~ 7). It has not yet been calculated how the extra fermions and bosons introduced by supersymmetry would affect this result.

Although we have not studied it in detail, the dominant background to this process is likely to come from radiative corrections to $e^+ e^- \rightarrow \ell \bar{\ell}$. This background is likely [1] to be more severe than for the process $Z^0 \rightarrow H^0 l^+ l^-$ discussed in the next subsection.
2.4.3 $e^+e^- \rightarrow (H^0 + Z^0) + H^0 \bar{f} \bar{f}$

This reaction takes advantage of the relatively large HZZ coupling (2.12). There are three distinguishable regions for the basic process,

i) $s \approx m_{Z^0}$, \hspace{1cm} $e^+e^- \rightarrow Z^0 \rightarrow H^0 + (Z^0 \rightarrow \bar{f} \bar{f})$

ii) $m_{Z^0} < s < m_{H^0} + 2m_{Z^0}$, \hspace{1cm} $e^+e^- \rightarrow Z^0 \rightarrow H^0 + (Z^0 \rightarrow \bar{f} \bar{f})$

iii) $s \gtrsim m_{Z^0} + 2m_{H^0}$, \hspace{1cm} $e^+e^- \rightarrow Z^0 \rightarrow H^0 + Z^0$

which proceed predominantly through the diagram shown in Fig. 2.18.

An exact analytic expression for the total cross-section of $e^+e^- \rightarrow H^0 l^+l^-$ is given by Refs. [25] to [27], which is plotted in Fig. 2.19 for $m_{H^0} = 10$ and 50 GeV as a function of $s$. Note that regions (i) and (iii) are clearly visible as the local peaks in the cross-section, and region (ii) as the dip between them. Near these maxima the rate is sizeable, and it is therefore useful to compare this with the other prize process $V_L \rightarrow H^0 \gamma$ as a function of the Higgs and toponium masses, as shown in Fig. 2.20.

Since initial-state radiative corrections are important for any process with large local peaks in the cross-section, they cannot be ignored here ([27] to [29]). Reference [27] derives a formalism valid for $e^+e^-$ annihilation into any intermediate vector particle ($\gamma$ and $Z^0$, for example) and fully describes the

![Fig. 2.18 Diagram for $e^+e^- \rightarrow H^0 l^+l^-$](image)

![Fig. 2.19 Cross-section for $e^+e^- \rightarrow H^0 l^+l^-$ as a function of the centre-of-mass energy $s$, for $m_{H^0} = 10$ and 50 GeV, indicating the regions (i) of $Z^0 \rightarrow H^0 l^+l^-$, (ii) of $e^+e^- \rightarrow Z^0 l^+l^-$, and (iii) of $e^+e^- \rightarrow Z^0 + H^0$](image)
Fig. 2.20 Regions of $(m_\gamma, m_\gamma)$ space where either $V_\gamma \rightarrow H^0 + \gamma$ or $Z^0 \rightarrow H^0 + 1^+ 1^-$ has the larger rate. The comparison assumes equal luminosities at the $V_\gamma$ and $Z^0$ peaks and includes radiative corrections in calculating the rates. A standard LEP energy resolution function (J. Jowett, private communication) has been assumed. This is only one of many uncertainties which enter into such a comparison.

construction of an event generator for this particular channel. The initial-state electromagnetic radiative corrected cross-section is given by Eq. (1.5) of the General Introduction (Section 1). We display in Fig. 2.21 a plot of $\delta$ versus $\sqrt{s}$, where the radiatively corrected cross-section $\sigma_B$ is related to the leading-order cross-section by

$$\sigma_B = \sigma_0 (1 + \delta) \tag{2.35}$$

for $m_{H^0} = 10$ and 50 GeV.

Fig. 2.21 Initial-state electromagnetic radiative corrections (1.5, 2.35) to $\sigma(e^+ e^- \rightarrow H^0 \gamma \gamma)$
In order to build an event generator, it is necessary to have expressions for the differential cross-section. Quite a complete set of formulae has been given in Refs. [25, 26]. They find that for unpolarized beams and vanishing lepton masses,

\[
\frac{\text{d} \sigma(e^+e^- \rightarrow H^0 l^+l^-)}{\text{d}(\cos \theta) \text{d}(m_L^2)} = \frac{m_L \Gamma_L(m_L)}{\pi \rho L(m_L)} \frac{\text{d} \sigma(e^+e^- \rightarrow H^0 Z^0)}{\text{d}(\cos \theta)}. \tag{2.36}
\]

Here, \( \theta \) is the dilepton production angle with respect to the beam axis, \( m_L \) is the invariant mass of the lepton pair,

\[
\Gamma_L(m) = C_L m/12 \pi \tag{2.37a}
\]

is the \( l^+l^- \) width for an off-shell \( Z^0 \) of mass \( m \), and

\[
D(m^2) = |m^2 - m_Z^2 + i \Gamma_L m_Z^2 |^2, \tag{2.37b}
\]

where \( \Gamma_L \) is the total width of the on-shell \( Z^0 \). Other auxiliary quantities are defined by

\[
C_L = g_V^2 + g_A^2, \quad g_V = \left( \frac{1}{4} - \sin \theta_W \right) \kappa, \quad g_A = \frac{1}{4} \kappa, \tag{2.37c}
\]

\[
g_H^0 = \kappa m_Z^0, \quad \kappa = e/\sin \theta_W \cos \theta_W. \]

In Eq. (2.36)

\[
\frac{\text{d} \sigma(e^+e^- \rightarrow HZ^0)}{\text{d}(\cos \theta)} = \frac{g_H^2 C_Q}{16\pi /s D(s)} \left( 1 + \frac{Q^2 \sin^2 \theta}{2m_L^2} \right), \tag{2.38}
\]

and after integration

\[
\sigma(e^+e^- \rightarrow HZ^0) = \frac{g_H^2 C_Q(3m_L^2 + Q^2)}{24\pi m_L^2 /s D(s)}, \tag{2.39}
\]

where \( Q = \lambda^{1/2} (s, m_L^2, m_H^2)/2s \) is the usual two-body phase-space function. We now discuss regions (i)-(iii) separately and in more detail.
2.4.4 $Z^0 \rightarrow H^0 \ell^+ \ell^-$

A very sizeable $Z^0$ rate is expected at LEP and the decay into $H^0 \ell^+ \ell^-$ may be observable. The dominant contribution comes from the diagram shown in Fig. 2.22, where the on-shell $Z^0$ emits the Higgs, then goes off-shell and decays into an $\tilde{t}\tilde{f}$ pair, such as two leptons. The total rate is given [30] by

$$\frac{1}{\Gamma(Z^0 \rightarrow \mu^+ \mu^-)} \frac{d\Gamma(Z^0 \rightarrow H^0 \ell^+ \ell^-)}{dx_{H^0}} = \frac{\alpha}{4\pi \sin^2 \theta_w \cos^2 \theta_w} \times \left[1 - x_H \left(\frac{x_H}{12} + \frac{2}{3} \left(\frac{m_H^2}{m_e^2}\right)\left[\frac{x_H^2}{2} - \left(\frac{2m_H^2}{m_e^2}\right)\right]^{1/2}\right) \right]$$

where $x_H = 2E_{H^0}/m_{H^0}$ has the kinematically allowed range

$$2m_e/m_Z < x_H < 1 + \left(\frac{m_e^2}{m_Z^2}\right).$$

Integrating Eq. (2.40), one obtains $\Gamma(Z^0 \rightarrow H^0 + \ell^+ \ell^-)$, which is plotted in Fig. 2.23. The rate per $10^6 Z^0$ is given in Table 2.4 assuming $B(Z^0 \rightarrow \mu^+ \mu^-) = 3%$.

Fig. 2.22 Diagram for $Z^0 \rightarrow H^0 + (Z^0 \rightarrow \ell \tilde{f})$ decay

Fig. 2.23 Rates for $Z^0 \rightarrow H^0 + \mu^+ \mu^-$ decay: (a) relative to $Z^0 \rightarrow \mu^+ \mu^-$ decay, (b) absolute branching ratio
Table 2.4
Number of $Z^0 \rightarrow H^0 \ell^+ \ell^-$ events for $10^6 Z^0$

<table>
<thead>
<tr>
<th>Higgs mass (GeV)</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of events</td>
<td>65.5</td>
<td>40.4</td>
<td>25.7</td>
<td>16.6</td>
<td>10.7</td>
<td>6.8</td>
<td>4.2</td>
<td>2.6</td>
<td>1.5</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Since the lepton pair comes from the virtual $Z^0$, the invariant mass will be peaked at high values as close as possible to $m_Z^2$, giving this channel a distinctive signature [31]. The integration of Eq. (2.36) gives the cross-section for $e^+e^- \rightarrow H^0 \ell^+ \ell^-$ per unit $m_L^2$:

$$\frac{d\sigma(e^+e^- \rightarrow H^0 \ell^+ \ell^-)}{d(m_L^2)} = \frac{12\pi f_L(f/s)}{D(s)} \frac{d\Gamma_{H^0 \ell^+ \ell^-}}{d(m_L^2)},$$

(2.42)

where

$$\frac{d\Gamma_{H^0 \ell^+ \ell^-}}{d(m_L^2)} = \frac{g^2 C Q(3m_L^2 + Q^2)}{288\pi^3 sD(m_L^2)}$$

(2.43)

is the differential width for the decay of a $Z^0^*$ of mass $s$ into $H^0 \ell^+ \ell^-$, which is equivalent to Eq. (2.40) for $s = m_Z^2$. Figure 2.24 shows the invariant mass

![Graph](image-url)

**Fig. 2.24** Shape of $m_{\ell^+ \ell^-}$ distribution in $Z^0 \rightarrow H^0 + \ell^+ \ell^-$ decay
spectrum of the lepton pair, peaked at high values since the virtual $Z^0$ is trying to be close to its mass-shell.

In order to establish the observability of this reaction, we [32] have made some detector simulations of it and compared them with possible backgrounds. The reaction $e^+e^- \rightarrow H^0 e^+ e^-$ followed by $H^0 \rightarrow b \bar{b}$ decay has been studied using Monte Carlo generated events in the mass range $20 < m_{H^0} < 50$ GeV. The final states were fragmented using the Lund Monte Carlo. Particles generated with this procedure were then traced using a detector simulation program that reproduced tracking and showering in a typical experimental set-up. The dominant background source for the above-mentioned reaction was found to be $e^+e^- + t\bar{t}$. In order to select this reaction on-line, a set of cuts were applied to the generated events that would simulate the trigger requirements and on-line processing. These cuts were:

i) an inclusive electron trigger;
ii) at least one electromagnetic cluster with energy $> 15$ GeV;
iii) missing energy in the event $< 15$ GeV;
iv) two electron candidates with opposite charge, one of them with energy $> 15$ GeV and the second with energy $> 5$ GeV; the sum of the two energies had to be $> 30$ GeV;
v) there should be no energy deposited in the hadron calorimeter behind the two electron candidates.

Further off-line cuts would require that the electrons should be isolated from neighbouring tracks, and that the missing energy in the event should not exceed 10 GeV. After applying the above-mentioned cuts, which had 65% efficiency, clear signals on top of a small background could be seen in the distribution of the mass recoiling against the final $e^+e^-$ system. Figure 2.25 shows the rate of

![Figure 2.25](image-url)
events in those signals as a function of the Higgs mass, for the case were \(10^7\) \(Z^0\)'s have been produced. The same figure also shows the background rates under the recoil mass peak. It can be seen that a clear \(H^0\) signal should be observable in the reaction \(e^+e^- + e^+e^- H^0\) for \(H^0\) masses of up to 50 GeV.

Hoping to take advantage of the larger rate [by a factor \(B(Z^0 + \bar{\nu}\nu)/B(Z^0 + e^+e^-) = 6\)], the reaction \(e^+e^- + \bar{\nu}\nu H^0\) has also been studied using Monte Carlo generated events for the mass range \(15 < M_{H^0} < 35\) GeV. Final-state particles were then traced, using the same detector simulation program as was mentioned earlier. Charged particles and photons with \(E_\gamma > 0.3\) GeV were then reconstructed. No effort was made to reconstruct long-lived neutral hadrons (\(n, K_{L}^0\)). Background sources were mainly events with long-lived energetic neutrals (\(K_{L}^0, n, \bar{n}\)). In order to reduce this background, the following cuts were imposed:

i) Only two-jet events were accepted.

ii) Since a large amount of energy in this reaction is carried away by the neutrinos, the following requirements have been imposed on it:
   a) missing energy > 50 GeV; b) missing momentum > 3 GeV; c) missing mass > 40 GeV.

iii) To ensure that the two jets originated from the decay of a Higgs with non-zero momentum, the jets were required to be non-collinear, by having an angle between them in the range 0.96 to 2.75 rad. Moreover, to ensure that this structure is not reproduced by widely spread multijets, the maximal angle between two energetic tracks (\(E > 2\) GeV) was required to be smaller than 2.6 rad.

iv) To ensure well-measured events (i.e. jets not going in the forward directions), the polar angle of each jet was required to exceed 0.192 rad.

Figures 2.26a, b show the reconstructed final-state masses for hypothetical

![Monte Carlo results for \(Z^0 + \nu\bar{\nu} + (H^0 \rightarrow \text{hadrons})\) compared with the multihadronic background: (a) \(m_{H^0} = 20\) GeV, (b) \(m_{H^0} = 30\) GeV]
Higgses of masses 20 and 30 GeV, respectively. In both bases clear peaks can be seen above the multihadronic background. The number of events generated has been normalized to a total of $10^5$ $Z^0$ events. From these figures it can be seen that the channel $e^+e^- + \nu\bar{\nu}H^0$ can be easily studied at LEP for $m_{H^0} < 30$ GeV.

The reaction $Z^0 + H^0 \rightarrow q\bar{q}$ could also be searched for, but would presumably be drowned by the multijet QCD background.

2.4.5 $e^+e^- \rightarrow Z^0 + H^0$

It is also important to consider what happens instead when the first $Z^0$ is off-shell, whilst the second $Z^0$ is on-shell, i.e. $e^+e^- \rightarrow Z^0^{\ast} \rightarrow Z^0 + H^0$ as shown in Fig. 2.27, which is a quasi-bremsstrahlung process. The differential and total cross-sections are given by Eqs. (2.36) to (2.39), with $m_{Z^0} = m_{Z^0}$. The total cross-section as a function of $E_{\text{beam}}$ is plotted in Fig. 2.28 for various Higgs masses, and the number of events expected per day is recorded in Table 2.5. We see that the cross-section is large enough to be observable for $m_{H^0}$ almost as high as the kinematic limit $\sqrt{s} - m_{Z^0}$, and may be the only way to see a Higgs weighing 0(100) GeV at LEP. As in the previous case of $Z^0 + H^0 \rightarrow \bar{f}f$ decay, the detection of $e^+e^- \rightarrow H^0 + (Z^0 \rightarrow l^+l^-)$ seems background-free, whilst $e^+e^- \rightarrow H^0 + Z^0$

---

**Fig. 2.27** Diagram for $e^+e^- \rightarrow Z^0 + H^0$

**Fig. 2.28** Cross-section for $e^+e^- \rightarrow Z^0 + H^0$ as a function of the beam energy, for different values of $m_{H^0}$
Table 2.5
Number of events per day for $e^+e^- + Z^0 + H^0$
(assuming a luminosity of $10^{31}$ cm$^{-2}$ s$^{-1}$)

<table>
<thead>
<tr>
<th>Mass of Higgs (GeV)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$ (GeV)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>12.33</td>
<td>8.55</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>140</td>
<td>5.17</td>
<td>4.72</td>
<td>3.92</td>
<td>2.51</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>160</td>
<td>2.70</td>
<td>2.58</td>
<td>2.37</td>
<td>2.08</td>
<td>1.66</td>
<td>1.03</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>180</td>
<td>1.60</td>
<td>1.56</td>
<td>1.48</td>
<td>1.37</td>
<td>1.23</td>
<td>1.05</td>
<td>0.83</td>
<td>0.51</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>200</td>
<td>1.04</td>
<td>1.02</td>
<td>0.99</td>
<td>0.94</td>
<td>0.87</td>
<td>0.79</td>
<td>0.70</td>
<td>0.59</td>
<td>0.46</td>
<td>0.28</td>
</tr>
<tr>
<td>220</td>
<td>0.72</td>
<td>0.71</td>
<td>0.69</td>
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<td>0.35</td>
</tr>
<tr>
<td>240</td>
<td>0.53</td>
<td>0.52</td>
<td>0.51</td>
<td>0.49</td>
<td>0.47</td>
<td>0.45</td>
<td>0.42</td>
<td>0.39</td>
<td>0.35</td>
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<td>260</td>
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<td>0.39</td>
<td>0.38</td>
<td>0.38</td>
<td>0.36</td>
<td>0.35</td>
<td>0.33</td>
<td>0.31</td>
<td>0.29</td>
<td>0.26</td>
</tr>
</tbody>
</table>

$(Z^0 + \nu\bar{\nu})$ is also promising. The reaction $e^+e^- + H^0 + (Z^0 + q\bar{q})$ has a QCD multi-jet background, as well as potential backgrounds from $e^+e^- \rightarrow W^+W^-$ or $Z^0Z^0$ decaying into jets, if $m_{H^0} = (80$ to $100)$ GeV. We have not investigated these very mass-dependent backgrounds in detail.

Note that the reaction $e^+e^- + H^0 + H^0 + Z^0$ is also interesting [33] from the point of view of testing gauge theories. However, the cross-section appears to be below the limit of detectability ($\sim 10^{-2}$ pb at peak for $m_{H^0} = 10$ GeV), making this process an unlikely way to detect the Higgs.

The reaction $e^+e^- + Z^0 + H^0$ at centre-of-mass energy 160 GeV and $m_{Z^0} = 50$ GeV has been studied using events generated by a Monte Carlo employing the Lund fragmentation model for jets, and a typical detector simulation. In these studies the mass of $Z^0$ was taken as 89 GeV. In the case of $m_{Z^0} = 93$ GeV the predicted background will be somewhat smaller.

We [34] study in detail the processes whose rates are given in Table 2.6:

1) $e^+e^- \rightarrow H^0 + (Z^0 \rightarrow e^+e^-)$
2) $e^+e^- \rightarrow H^0 + (Z^0 \rightarrow \mu^+\mu^-)$
3) $e^+e^- \rightarrow H^0 + (Z^0 \rightarrow \nu\bar{\nu})$
4) $e^+e^- \rightarrow H^0 + (Z^0 \rightarrow \text{jets})$
Table 2.6
Rates for different $H^0 + (Z^0 \rightarrow f\bar{f})$ final states

<table>
<thead>
<tr>
<th>Process</th>
<th>Events per 100 days at $L = 10^{32}$ cm$^{-2}$ s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow H^0 + (Z^0 \rightarrow e^+e^-)$</td>
<td>50</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow H^0 + (Z^0 \rightarrow \mu^+\mu^-)$</td>
<td>50</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow H^0 + (Z^0 \rightarrow \nu\bar{\nu})$</td>
<td>300</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow H^0 + (Z^0 \rightarrow$ jets)</td>
<td>1220</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow$ hadrons</td>
<td>39000</td>
</tr>
</tbody>
</table>

1) $e^+e^- \rightarrow H^0 + (Z^0 \rightarrow e^+e^-)$

Assuming that the $e^+$ and $e^-$ are detected by a tracking detector and electromagnetic calorimeter with resolutions of

$$\frac{\sigma_{p_T}}{p_T} = 0.14\% \ p_T \quad \text{(for charged particles)}$$

$$\frac{\sigma_E}{E} = \left( \frac{16\%}{\sqrt{E}} + 1\% \right) \quad \text{(for e/\gamma)}$$

Fig. 2.29 gives the invariant mass distribution of $e^+e^-$ for $Z^0 \rightarrow e^+e^-$ decay as shown by the shaded region. The $e^+e^-$ pairs from $H^0$ decays are also included. The events are required to have sphericity $\geq 0.05$ in order to reduce the QCD background.

Fig. 2.29 Results of a Monte Carlo analysis of the $e^+e^-$ invariant mass distribution from $\phi \rightarrow e^+e^-$ compared to the $e^+e^- \rightarrow t\bar{t}$ and $b\bar{b}$ backgrounds; 3000 $H^0 + (Z^0 \rightarrow e^+e^-)$ events were generated at $\sqrt{s} = 160$ GeV assuming $m_{Z^0} = 89$ GeV.

![Invariant mass distribution](attachment:Invariant_mass_distribution.png)
The QCD background is calculated from the processes

\[ e^+e^- + t\bar{t} (\text{+ gluons}) \quad (\text{assuming } m_t = 30 \text{ GeV}) \]

\[ \xrightarrow{L} e^+ + \ldots \]
\[ e^- + \ldots \]

and

\[ e^+e^- + b\bar{b} (\text{+ gluons}) \]

\[ \xrightarrow{L} e^+ + \ldots \]
\[ e^- + \ldots \]

Here the \( e^\pm \) includes the direct decays from \( t \) or \( b \), their cascade decays [for example \( t \rightarrow (b + e^+ + \ldots) \) etc.] and Dalitz decays of \( \pi^0 \).

Figure 2.29 shows a clear signal for the \( Z^0 \) and Fig. 2.30 gives the missing mass distribution calculated from the \( e^+e^- \) pairs. In Fig. 2.30a, the invariant mass of the \( e^+e^- \) pairs is restricted to be \( > 82 \) GeV and in Fig. 2.30b, in addition to the cut of \( M_{ee} > 82 \) GeV, we also constrain \( m_{e^+e^-} = m_{Z^0} \) and recalculate the recoil mass. Background-free signals for the \( H^0 \) are seen in both Figs. 2.30a and 2.30b.

2) \( e^+e^- \rightarrow H^0 + (Z^0 \rightarrow \mu^+\mu^-) \)

Similar results are obtained in this case as in the case \( e^+e^- \rightarrow H^0 + (Z^0 \rightarrow e^+e^-) \) discussed above.

3) \( e^+e^- \rightarrow H^0 + (Z^0 \rightarrow \nu\bar{\nu}) \)

The events from this process have large sphericity and large transverse-momentum imbalance. Figure 2.31a gives the missing mass distribution calculated

![Diagram](image_url)

Fig. 2.30 Missing mass distribution calculated from \( e^+e^- \) pairs with \( m_{e^+e^-} > 82 \) GeV and sphericity > 0.05: (a) without, (b) with the \( m_{e^+e^-} = m_{Z^0} \) constraint applied.
Fig. 2.31 Results of a Monte Carlo analysis of $e^+e^- \rightarrow H^0 + (Z^0 + \nu\bar{\nu})$: (a) distribution of missing mass recoiling against the observed hadrons, (b) distribution of hadronic invariant mass recoiling against missing masses above 82 GeV.

from the observed hadrons, as detected by the tracking detector and the electromagnetic calorimeter, both from $H^0$ decay and from QCD background. The following two cuts have been applied to the Monte Carlo generated events:

i) Sphericity of event $> 0.05$.

ii) $|E_1 \cdot \vec{p}_T| > 10 \text{ GeV/c} = |\text{missing } \vec{p}_T|$

where $\vec{p}_T$ = transverse momentum vector of the $i$th particle with respect to the beam direction.

Figure 2.31b shows the observed hadron invariant mass distribution with missing mass greater than 82 GeV. A clear $H^0$ peak is observed.

The QCD background under the $H^0$ peak in Fig. 2.31b is to a large extent due to energetic $K_L^0$ from $s, \bar{s}$ jets escaping the central detector. The clear signal to background ratio makes the reaction $e^+e^- \rightarrow H^0 + (Z^0 + \nu\bar{\nu})$ another tool to count the number of neutrinos. Note also that with the cut $|E_1 \cdot \vec{p}_T| > 10 \text{ GeV}$ the $\gamma\gamma + 2$ jets background is expected to be small.

4) $e^+e^- \rightarrow H^0 + (Z^0 + \text{jets})$

In this case, both the $Z^0$ and the $H^0$ decay dominantly into two jets. However the event generator also allows the $H^0$ and $Z^0$ to decay sometimes into three jets and four jets as in the QCD case. For each Monte Carlo generated event, a four-jet analysis is applied to the hadrons observed by the tracking detector and electromagnetic calorimeter using the method described in Ref. [34]. The 4-jet analysis first combines the observed particles into four groups. The direction of the vector sum of momenta in each group gives the direction of the jet. We take the velocities of the groups to be the velocities of the jets, and solve for the energies of the jets by imposing momentum and energy conservation. The definition of the velocity of a jet is
\[ \hat{\beta}_{\text{jet}} = \frac{\hat{p}_{\text{group}}}{E_{\text{group}}} \]

and the momentum-energy conservation equations become:

\[ \sum_i E_{\text{jet}} \hat{p}_{\text{jet}} i = 0 \]

\[ \sum_i E_{\text{jet}} i = E_{\text{cm}} \]

Then the four jets are ordered such that the invariant mass of the pair (12) gives a mass \( m_{12} \) closest to the \( Z^0 \) mass. Jet 3 and jet 4 are then the candidates for the two-jet decay of \( H^0 \). The invariant mass distribution was then calculated between jet 3 and jet 4 (\( m_{34} \)) using the energies and momenta of jets 3 and 4 and the opening angle between the two jets. The following cuts have been applied to Monte Carlo generated events:

i) sphericity of the event \( S > 0.1 \),

ii) minimum opening angle between any two of the four jets \( > 30^\circ \),

iii) maximum opening angle between any two of the four jets \( < 166^\circ \).

Figure 2.32a shows a clear mass peak at \( m_{H^0} \) mass above the QCD background. The event sample in Fig. 2.32a is further refined by selecting events with sphericity \( S > 0.3 \) and \( m_{12} \) between 80 and 100 GeV. For these events we constrain \( m_{12} \) to be \( m_{H^0} \) and recalculate the recoil mass (\( m_{34} \)) of jets 3 and 4. The final recoil mass distribution \( m_{34} \) is shown in Fig. 2.32b. In the range \( 46 < m < 54 \) GeV we obtain a signal to background ratio of about 1 to 1. Thus we think it not impossible to find the \( H^0 \) in this final state, though it would not be easy, and one might want confirmation of a small effect in some other channel.

Fig. 2.32 Results of a Monte Carlo analysis of \( e^+e^- \rightarrow H^0 + (Z^0 \rightarrow q\bar{q} \text{ jets}) \):
(a) after selecting 4-jet events with sphericity \( S > 0.1 \), and all 2-jet opening angles between \( 30^\circ \) and \( 166^\circ \), and (b) after further selecting events with sphericity \( S > 0.3 \) and one jet-jet mass in the range 80 to 100 GeV.
2.4.6 $e^+e^- + Z^0 + H^0(2Z^0 + f\bar{f})$

In the case where $m_{Z^0} < s < m_{H^0} + m_{H^0}$, and if $f\bar{f} = e^+e^-$, the dominant diagram for the process $e^+e^- + H^0 f\bar{f}$ involves two virtual $Z^0$ and the cross-section is given in Ref. [25]. As seen in region (ii) of Fig. 2.19, the cross-section is rather small. Note that this reaction can also have contributions due to $H^0$ bremsstrahlung from a final-state fermion line, although the contributions appear to be rather small, essentially because of the small $H^0 f\bar{f}$ coupling. (See Refs. [35] and [36] for some discussion.)

If the final-state leptons are electrons, there may be a contribution from the t-channel diagram [37, 38] shown in Fig. 2.33. Figure 2.34a shows cross-sections with and without the t-channel contribution. Note that the deviation is only significant where the cross-section appears too small to be observable.

![Diagram contributing to $e^+e^- + H^0$ involving t-channel $Z^0$ exchanges](image)

Fig. 2.33 Diagram contributing to $e^+e^- + H^0$ involving t-channel $Z^0$ exchanges

![Cross-sections](image)

Fig. 2.34 (a) Cross-sections for $e^+e^- + H^0 + e^+e^-$, both with and without the t-channel $Z^0$ exchange diagram of Fig. 2.33. (b) Cross-section for $e^+e^- + H^0 + \nu\bar{\nu}$, calculated with the corresponding t-channel $W^\pm$ exchange diagrams.
2.4.7 Other reactions involving $H^0$

a) $e^-e^+ \to H^0 + \bar{\nu}\nu$

The reaction [37, 38] is interesting both as a neutrino-counting experiment and as a way to produce very heavy Higgs bosons, although the cross-section is quite small ($<1$ pb) for $m_{H^0} > s - m_{\nu}$, as seen in Fig. 2.34b. The rate for heavy Higgs is dominated by the t-channel diagram as in the $e^-e^+ \to H^0 + e^-e^+$ process, except this time through $W^\pm$ exchange. It may be a good reaction to study after the existence and mass of the $H^0$ have been established by some other process.

b) Virtual effects of $H^0$

If the Higgs is too heavy to be produced at LEP, its presence may still be felt owing to higher-order corrections. Thus, through precision measurements, some evidence may be found for $H^0$. Such a calculation has been done [39] for the process $e^-e^+ \to W^+W^-$. Figure 2.35 shows the sensitivity of $\left[\frac{d\sigma}{d(cos \theta)}(e^-e^+ \to W^+W^-)\right]$ to $m_{H^0}$. The prospects for observing these small effects are discussed elsewhere [40] in this report.

![Fig. 2.35 The sensitivity of $d\sigma/e^-e^+ W^+W^-$ to $m_{H^0}$](image)

2.5 Non-minimal Higgses in the Standard Model

2.5.1 Introduction

Whilst the original Weinberg-Salam [4] formulation of the Standard Model uses a minimal Higgs mechanism, employing a single $Y = 1$ doublet, there is no obvious reason why the Higgs sector cannot be more complicated. Non-minimal Higgs sectors have been proposed as mechanisms for such varied phenomena as electroweak CP violation [41], the suppression of strong CP violation [42], and the generation of neutrino masses [43], and are an essential feature of supersymmetric theories [44]. All these models involve the introduction of extra multiplets which necessarily lead to singly charged and possibly even multiply charged [45] physical states, greatly enhancing Higgs boson phenomenology. Such models are constrained by experiment, not only by the direct (non?) observation of these Higgs bosons, but also indirectly by other measurements, such as the
value of the neutral-current $g$ parameter [46] and the absence of large flavour-changing neutral currents.

Since it is impossible to review the many varied proposals for unorthodox Higgs sectors, instead we briefly describe a class of models which is, in a sense, the minimal non-minimal (MNM) extension of the Standard Model, namely the addition of a second $Y = 1$ doublet. The discussion of an MNM model in its most general form will display the extra phenomenology of non-standard Higgs sectors (extra neutral and charged Higgs bosons) without leading us too far down the path of wild speculation.

The first thing to consider is the spectrum of the physical Higgs bosons in an MNM model. Taking two $Y = 1$ doublets

$$
\phi_1 = \begin{pmatrix} 
\phi_1^* \\
\phi_1 
\end{pmatrix}, \quad \phi_2 = \begin{pmatrix} 
\phi_2^* \\
\phi_2 
\end{pmatrix},
$$

with vacuum expectation values

$$
\langle \phi_1 \rangle = \begin{pmatrix} 0 \\
\nu_1 
\end{pmatrix}, \quad \langle \phi_2 \rangle = \begin{pmatrix} 0 \\
\nu_2 e^{i\xi} 
\end{pmatrix},
$$

the most general quartic Higgs potential is [47]

$$
V(\phi_1, \phi_2) = \lambda_1 (\phi_{1*}^\dagger \phi_1^2 - \nu_1^2)^2 + \lambda_2 (\phi_{2*}^\dagger \phi_2^2 - \nu_2^2)^2
$$

$$
+ \lambda_3 \left[ (\phi_{1*}^\dagger \phi_1 - \nu_1^2) + (\phi_{2*}^\dagger \phi_2 - \nu_2^2) \right]^2
$$

$$
+ \lambda_4 \left[ (\phi_{1*}^\dagger \phi_1) (\phi_{2*}^\dagger \phi_2) - (\phi_{1*}^\dagger \phi_2) (\phi_{2*}^\dagger \phi_1) \right]
$$

$$
+ \lambda_5 \left[ \text{Re} (\phi_{1*}^\dagger \phi_2) - \nu_1 \nu_2 \cos \xi \right]^2
$$

$$
+ \lambda_6 \left[ \text{Im} (\phi_{1*}^\dagger \phi_2) - \nu_1 \nu_2 \sin \xi \right]^2 + \lambda_7,
$$

where the $\lambda_i$ are real. Of the eight degrees of freedom introduced in Eq. (2.44), two charged and one neutral get 'eaten' as in the Standard Model, leaving three neutral and two charged scalar fields. To find the physical Higgs bosons and their couplings, the mass matrices must be diagonalized. Here two angles are needed since the charged and neutral fields cannot mix. One finds [47]:
\[ H^0 = -\phi_1^+ \sin \beta + \phi_2^+ \cos \beta , \]

\[ H_1^0 = \sqrt{2} \left[ (\text{Re} \ \phi_1^0 - \nu_1) \cos \alpha + (\text{Re} \ \phi_2^0 - \nu_2) \sin \alpha \right] , \] (2.47)

\[ H_2^0 = \sqrt{2} \left[ -(\text{Re} \ \phi_1^0 - \nu_1) \sin \alpha + (\text{Re} \ \phi_2^0 - \nu_2) \cos \alpha \right] , \]

\[ H_3^0 = \sqrt{2} \left[ -\sin \beta \ \text{Im} \ \phi_1^0 + \cos \beta \ \text{Im} \ \phi_2^0 \right] , \]

where \( H^0, H_1^0, \) and \( H_2^0 \) are scalars, whilst \( H_3^0 \) is a pseudoscalar. Their masses are

\[ m^2_{H^0} = \lambda_4 (v_1^2 + v_2^2) , \]

\[ m^2_{H_1^0,2} = \frac{1}{2} \left[ A + C \pm \sqrt{(A-C)^2 + 4B^2} \right] , \] (2.48)

\[ m^2_{H_3^0} = \lambda_6 (v_1^2 + v_2^2) , \]

where

\[ A = 4v_1^2 (\lambda_1 + \lambda_3) + v_2^2 \lambda_5 , \]

\[ B = (4\lambda_3 + \lambda_5) v_1 v_2 , \] (2.49)

\[ C = 4v_2^2 (\lambda_2 + \lambda_3) + v_1^2 \lambda_5 , \]

and

\[ \tan \beta \equiv \frac{v_2}{v_1} , \]

\[ \sin 2\alpha = \frac{2B}{\sqrt{(A-C)^2 + 4B^2}} , \] (2.50)

\[ \cos 2\alpha = \frac{(A-C)}{\sqrt{(A-C)^2 + 4B^2}} . \]

The couplings are also functions of the \( \lambda_i \) and of \( \alpha \) and \( \beta \) which can be found in Ref. [47]. To reduce the number of parameters, a specific model must be chosen. An example is provided by supersymmetry.
Fig. 2.36 Feynman rules (a) for $Z^0$-H-H, and (b) for H-f$ar{f}$ couplings in the minimal supersymmetric Standard Model.

Motivations for supersymmetry which come from the hierarchy and naturalness problems [44, 47] were discussed in Section 1. Minimal supersymmetric models require two Higgs doublets (of opposite hypercharge) to give mass both to the up-type and to the down-type quarks and leptons, respectively, giving rise to the same physical Higgs spectrum as the above MMH. Supersymmetry imposes symmetry relations on the Higgs potential (2.46), giving relations among the $\lambda_i$ [48] and hence relating the tree-level Higgs boson masses and couplings discussed earlier. Some couplings [47] in minimal SUSY models are shown in Fig. 2.36: they may have interesting enhancements over the minimal Standard Model for special choices of the mixing angles $\alpha, \beta$. The relations between Higgs masses and these angles are [47, 48]:

$$m_{H_3}^2 = m_{H_1}^2 - m_W^2,$$

$$m_{H_1, H_2}^2 = \frac{1}{2} \left[ m_{H_3}^2 + m_Z^2 \pm \sqrt{(m_{H_3}^2 + m_Z^2)^2 - 4m_Z^2 m_{H_3}^2 \cos^2 2\beta} \right], \quad (2.51)$$

$$\tan 2\alpha = \tan 2\beta \left( \frac{m_{H_1}^2 + m_{H_2}^2}{m_{H_3}^2 - m_Z^2} \right).$$
Table 2.7
Table of Higgs masses and SUSY parameters

<table>
<thead>
<tr>
<th>$m_2$ (GeV)</th>
<th>$m_3$ (GeV)</th>
<th>$m_{H^+}$</th>
<th>$m_1$</th>
<th>$\tan \beta$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\cos^2(\alpha-\beta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>83.4</td>
<td>93.8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>85.2</td>
<td>95.4</td>
<td>0.57</td>
<td>0.519</td>
<td>-0.538</td>
<td>0.24</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>88.1</td>
<td>98.0</td>
<td>0.70</td>
<td>0.608</td>
<td>-0.639</td>
<td>0.10</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>85.2</td>
<td>93.8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
<td>88.1</td>
<td>96.4</td>
<td>0.43</td>
<td>0.408</td>
<td>-0.458</td>
<td>0.42</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>88.1</td>
<td>93.8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>30</td>
<td>60</td>
<td>102.3</td>
<td>107.2</td>
<td>0.52</td>
<td>0.481</td>
<td>-0.643</td>
<td>0.187</td>
</tr>
<tr>
<td>30</td>
<td>90</td>
<td>122.3</td>
<td>126.5</td>
<td>0.61</td>
<td>0.552</td>
<td>-0.775</td>
<td>$5.8 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Note that the parameter $B$ [Eq. (2.49)] is necessarily negative in SUSY models, implying $-(\pi/2) < \alpha < 0$. Once two physical Higgs masses are fixed, all other parameters are determined. We can see from Eq. (2.51) that in general $m_{H^+} < m_{H^0}$, and one of the neutral Higgses $H_1^0$ is heavier than the $Z^0$, whilst $H_2^0$ is lighter than the $Z^0$. Table 2.7 records the various parameter values for different choices of $m_2$ and $m_3$. Note that specific supergravity models may make even more restrictive predictions [49].

The couplings in Fig. 2.36, group (b), are relevant to Higgs decays. The width of a scalar decay into two fermions with coupling $c = x + y\gamma_5$ is [50]

$$\Gamma(H \to ff') = \frac{N_c}{\delta x_m_H} \left[ (x^2 + y^2) (m_H^2 - m_f^2) - 2(x^2 - y^2) m_f^2 \right]$$

$$\times \left( 1 - \frac{2m_f^2}{m_H^2} - \frac{2m_f^2}{m_E^2} + \frac{m_f^4}{m_E^4} \right)^{1/2}, \quad (2.52)$$

where $N_c = 1$ for leptons and 3 for quarks. In the Standard Model, $x = gm_f/(2m_W)$ and $y = 0$, which reduces Eq. (2.52) to Eq. (2.15) in Section 2.2.1. One can read off the couplings from group (b) for whatever process is being considered. For example

$$\Gamma(H_2^0 \to uu) = \frac{3m_H}{\delta v} \left( \frac{q^2 m_u^2}{4m_w^2} \right) \left( \frac{\cos \alpha}{\sin \beta} \right)^2 \left[ 1 - \left( \frac{4m_u^2/m_H^2}{H_2^0} \right) \right]^{3/2}, \quad (2.53)$$
If we take $m_2 = 20$ GeV and $m_3 = 30$ GeV, then the factor involving $\alpha$ and $\beta$ gives an enhancement of more than a factor of 5. The couplings in group (b) will also have the corresponding effect on the Wilczek mechanism [11] discussed in subsection 2.4.1.

The presence of several additional Higgs bosons opens up some new production possibilities which we now describe. One might look for a $\tilde{W}^+ H^0 Z^0$ vertex giving rise to the reaction $e^+ e^- \rightarrow \tilde{W}^+ H^0$ as a good way of searching for charged Higgs bosons [1]. Unfortunately, it has been shown [46] that in models with any number of doublets--and only doublets--the $\tilde{W}^+ H^0$ coupling is identically zero. If multiplets other than doublets are added, this vertex is still very small, with a stringent upper bound given by the deviation of the $\rho$ parameter from 1 [46].

2.5.2 $e^+ e^- \rightarrow H^+ H^-$

The charged Higgs bosons couple both to the intermediate photon and to the $Z^0$. We can use the general formula (1.4) of Section 1 for the total scalar cross-section:

$$\sigma_{H^+ H^-} = \frac{1}{4} \beta^3 \sigma_0 \left( \frac{1}{2}, \frac{1}{2}, 1, 1, s \right),$$

(2.54)

where we use the quantum numbers $T_3 = 1/2$, $Y = +1$ as inputs to determine $C_{\nu}^0$ (recall that $C_{\nu}^0 = 0$ for scalars). In $\sigma_0$ the mass is set to zero, implying $\beta = 1$, although the prefactor $\beta$ in Eq. (2.54) is the usual $\beta = \sqrt{1 - 4m_{H^+}^2/s}$. This total cross-section is plotted in Fig. 2.37 for $m_{H^+} = 20$, 40, 60, 80 GeV. Notice the slow $\beta^3$ turn-on at threshold. If $2m_{H^+} < m_{Z^0}$, the rate will be substantial. Unfortunately, minimal SUSY (2.51) expects $m_{H^+} < m_{Z^0}$. The scalar $H^+$ produced in $e^+ e^-$

![Fig. 2.37 Tree-level cross-sections for $e^+ e^- \rightarrow H^+ H^-$](image-url)
Fig. 2.38 Initial-state electromagnetic radiative corrections (1.5, 2.35) to \( \sigma(e^+e^- \rightarrow H^- H^-) \): (a) for \( m_{H^-} = 10, 30 \) GeV, and (b) for \( m_{H^-} = 60 \) GeV

annihilation have a characteristic \( \sin^2 \theta \) angular distribution, to be contrasted with the \((1 + \alpha \cos \theta + \cos^2 \theta)\) angular distribution for fermions such as quarks. As in the case of \( H^0 \) production, we must consider initial-state radiation corrections to Eq. (2.54). Using formula (1.5), we plot in Fig. 2.38 the correction factor \( \delta \) as defined in Eq. (2.35). We see that the corrections can be large in some cases.

2.5.3 Other possible \( H^+ \) production mechanisms

a) \( Q \rightarrow H^+ q, V_t \rightarrow H^+ H^- b \bar{b} \)

The idea [51] here is to take advantage of the semiweak heavy quark–Higgs coupling (see Fig. 2.33) and hope the rate is not suppressed by mixing angles. The general expression for the decay rate, given a coupling of the form

\[
\Gamma(Q \rightarrow H^+ q) = \left(\frac{\lambda_{Q,q}^2}{16\pi m_Q}\right)^{1/2} \left[ (x^2+y^2)(m_Q^2-m_{H^+}^2-m_{H^-}^2) + 2(x^2-y^2)m_Q m_{H^-} \right],
\]

which is much larger than conventional weak \( Q + q \bar{q} \) decay unless \( x, y \ll 1 \). However, the decay \( Q \rightarrow H^+ q \) may be hard to distinguish from conventional heavy-quark decays in regular \( QQ \) events. It could have a substantial impact on heavy quarkonia if Eq. (2.55) is large enough. In this case, the heavyonium (e.g. \( V_t \)) will crumble rather than decay, as the individual quarks fall apart before they can find each other to annihilate as in conventional quarkonium decays. In this case all the usual expectations for quarkonium final states are completely altered, and conventional quarkonium search strategies may need to be rethought.
Note, however, that the UA1 Collaboration has observed [52] events consistent with conventional semileptonic $t \to b$ decay, which suggests $m_{H^+} > m_t - m_b$ and would rule out $\mathbb{V}_L + H^+ H^- b\bar{b}$ decay.

\[ b) \ e^+e^- \to H^+tb + H^-t\bar{b} \]

Even if $m_{H^+} > m_t - m_b$ and $2m_{H^0} > m_b$, it is still possible to produce a single charged Higgs in the reaction $e^+e^- \to H^+ tb$. There are [50] six amplitudes contributing to this process via $Z$ and $\gamma$ exchange into $t\bar{t}^*$, $b\bar{b}^*$, or $H^+H^0$ with the virtual particle ($^*$) decaying into the final state. Unfortunately, the cross-section is sizeable only when the $H^\pm$ are light enough to be pair-produced, making the reaction $e^+e^- \to H^+ tb$ a seemingly unprofitable way to search for charged Higgs bosons.

\[ c) \ z^0 + H^0 H^0 \]

This process does not occur at the tree level in models with elementary Higgs bosons. It can occur via higher-order loop diagrams, but these would give an unobservably small rate, much smaller than that in the composite Higgs models to be discussed later.

\[ d) \ e^+e^- \to z^0 + H^0 H^0 \]

In the minimal Higgs Standard Model, pair production $e^+e^- \to H^0 H^0$ is forbidden by Bose symmetry. However, in models containing more than one $H^0$, this reaction is possible and may well be important. In an NMH model, there are three neutral Higgs bosons, $H_i^0 (i = 1, 2, 3)$ and couplings for $z^0 + H_i^0 + H_j^0$ and $z^0 + H_i^0 + H_j^0$ as shown in Fig. 2.33a. The total cross-section for this type of reaction is again given by a formula such as (2.54), but with two modifications. First, the couplings are changed by factors reflecting the mixing of the neutral fields. Secondly, unlike the charged Higgs case where $m_{H^+} = m_{H^-}$, the masses of the neutral Higgs bosons will, in general, be different. Taking these into account, the total cross-section for $e^+e^- \to H_j^0 + H_k^0$ is

\[
\sigma = \frac{1}{4} \lambda^2 \left( \frac{m_j^2}{s}, \frac{m_k^2}{s}, \frac{m_{1,2}^2}{s} \right) \mathcal{F}_0 (0, C_{H_i}, C_{H'_i}, 1, s) , \tag{2.56}
\]

where $\lambda$ is the usual phase-space function and now

\[
C_{H_i} = \begin{cases} 
\frac{\cos (\alpha - \beta)}{2} & \text{for } e^+e^- \to H_i^0 H_i^0 , \\
\frac{\sin (\alpha - \beta)}{2} & \text{for } e^+e^- \to H_i^0 H_j^0 . 
\end{cases} \tag{2.57}
\]
Fig. 2.39 Cross-sections for $e^+e^- \rightarrow$ pairs of neutral Higgs bosons: (a) to (c) $H_2^0 H_2^0$ for different values of $m_{H_2}$, (d) $H_3^0 H_3^0$.

Figure 2.39 shows the total cross-section (2.54) for the production of pairs of neutral Higgs bosons, for some of the parameters shown in Table 2.7. We see that the rates may be large at the $Z^0$ peak. Since the total cross-section exhibits a large local maximum, initial-state bremsstrahlung corrections (1.5) will change the rate in a similar fashion to $e^+e^- \rightarrow H^0 H^0$.

The reaction $e^+e^- \rightarrow H^0 H^0$ has been studied [53] using Monte Carlo generated events for the mass range $7 < m_{H^0} < 10$ GeV and $15 < m_{H^0} < 50$ GeV. The decay channel searched for had $H^0 \rightarrow \gamma \gamma$ and $H^0 \rightarrow b \bar{b}$, where both final states were fragmented using the Lund Monte Carlo. Final-state particles were then tracked, using the same detector simulation program as in subsection 2.4.4. Charged particles and photons with $E_\gamma > 0.3$ GeV were then reconstructed. No effort has been made to reconstruct long-lived neutral hadrons ($n$, $K^0_s$). The main background sources for the above-mentioned reaction are the processes $e^+e^- \rightarrow t \bar{t}$ and $e^+e^- \rightarrow b \bar{b}$. In order to reduce these backgrounds, the following cuts were applied to both signal and background events:
a) Events were required to have three well-defined jets [i.e. \( b + \bar{b} + (H^0 + \tau \bar{\tau}) \)]. One of the \( \tau \)'s was required to decay leptonically, therefore the following additional requirements were imposed:
b) there had to be at least one isolated lepton in a cone of 0.25 rad with momentum exceeding 2 GeV;
c) the isolated lepton had to be in a jet that contained one or three additional particles;
d) the direction of the missing momentum in the event (carried mainly by the neutrinos of the \( \tau \bar{\tau} \) decays) had to agree with the direction of the isolated lepton to within 1.5 rad and also with the direction opposite to the \( b \bar{b} \) jets to within the same precision.

Making the optimistic assumption that the mixing angles are such that the coupling term \( \cos^2 (\alpha - \beta) = 1 \) in the vertex \( Z^0 H^0 H^{0'} \), the expected signals and backgrounds for this reaction were calculated. Figure 2.40a shows the reconstructed \( \bar{b}b \) mass for the above reaction for the case \( m_{H^0} = 10 \) GeV and \( m_{H^{0'}} = 15 \) GeV on top of the remaining multihadronic background, after applying the above-mentioned cuts. The events have been generated at the \( Z^0 \) peak and normalized to a total of \( 10^6 Z^0 \)'s. It can be seen that very clear signals stand on top of the background, making the reaction easily identifiable if \( \cos^2 (\alpha - \beta) \) is not very small. Figure 2.40b shows the observable rate of this reaction, after imposing the above-mentioned cuts for \( m_{H^0} = 10 \) GeV. Also in the same figure the corresponding background rates are shown. It can be seen that for the studied mass range, clear signals could be observable at LEP.

---

Fig. 2.40 (a) Reconstructed \( \bar{b}b \) invariant masses in \( e^+e^- \rightarrow H^0 (10 \text{ GeV} \rightarrow \tau \bar{\tau}) \) + \( H^0 (15 \text{ GeV} \rightarrow \bar{b}b) \), compared to the QCD background. (b) Observable rates for \( e^+e^- \rightarrow H^0 H^{0'} \) with \( m_{H^0} = 10 \) GeV, after making the cuts discussed in the text.
2.6 Techniopion processes

2.6.1 Introduction

It may be that electroweak symmetry breaking is due, not to the presence of some fundamental scalar such as the Higgs boson, but rather to the non-perturbative dynamics of some gauge interaction. This is the idea behind the so-called Technicolor theories [54]. A new non-Abelian gauge interaction analogous to that of SU(3) colour in QCD is postulated with a new strong interaction scale of about 1 TeV. The scalars in the theory are not fundamental, but are bound states of this new interaction and are referred to as technipions. In order to get masses for the quarks and leptons, the technicolor interactions must be extended (hence the term Extended Technicolor) in some way [55], producing as a by-product a spectrum [56] of technipions with masses between a few hundred MeV and a few hundred GeV, well below the 1 TeV scale. LEP is therefore a good place to look for these technipions.

A thorough report reviewing technicolor particles at LEP already exists [57]. Although Extended Technicolor theories are at present in trouble, mostly owing to negative experimental searches [58] and their tendency to predict large flavour-changing neutral currents [59], work on these theories continues [60]. A brief review of some of the phenomenology of light composite scalars at LEP is given in this subsection. We concentrate on the light isotriplet $P'$, $p^0$, $P^0$, and isosinglet $P^{0'}$ [56], neglecting the heavier coloured octet pseudoscalars which are expected to have mass $0(250 \text{ GeV})$ [56]. Most of the reactions discussed are analogous to the more promising of the Higgs processes discussed in the preceding subsections.

2.6.2 Pair production

a) $e^+e^- \rightarrow p^+p^-$

The charged technipions can couple both to a $\gamma^0$ and to a photon. Since $Q^2 \ll 1 \text{ TeV}$, the $P^\pm$ are effectively point-like particles [61]. The total cross-section is

$$\sigma(e^+e^- \rightarrow p^+p^-) = \frac{1}{4} \beta^3 a_0(1, 1, 1, 1, s),$$

(2.58)

i.e. with $T_3 = +1$ and $Y = 0$ as inputs to $C_Y$. Table 2.8 shows $R$ values for various centre-of-mass energies and technipion masses. We see that the cross-section is large enough to be observable, and expect the signatures to be similar to those of the charged Higgses $H^\pm$ discussed in subsection 2.5.2. Note, though, that the electroweak charges of the doublet $H^\pm$ and the triplet $p^\pm$ differ, so that their rates of production via the $\gamma^0$ are different.
Table 2.8
Values of $R = \sigma(e^+e^- \rightarrow P^-P^0)/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$

<table>
<thead>
<tr>
<th>$m_{P^\pm}$ (GeV)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s}$ (GeV)</td>
<td>80</td>
<td>1.11</td>
<td>0.80</td>
<td>0.36</td>
<td>0.00</td>
</tr>
<tr>
<td>120</td>
<td>1.29</td>
<td>1.12</td>
<td>0.87</td>
<td>0.56</td>
<td>0.23</td>
</tr>
<tr>
<td>160</td>
<td>0.82</td>
<td>0.76</td>
<td>0.67</td>
<td>0.54</td>
<td>0.40</td>
</tr>
<tr>
<td>200</td>
<td>0.70</td>
<td>0.67</td>
<td>0.61</td>
<td>0.55</td>
<td>0.46</td>
</tr>
</tbody>
</table>

b) $e^+e^- \rightarrow p^0p^0$

The process analogous to $e^+e^- \rightarrow H^0H^0$ does not occur to first order since both $T_3 = 0$ and $Y = 0$ for the $P^0$ and $P^0'$, meaning that the $Z^0p^0p^0'$ coupling is identically zero. The processes $e^+e^- \rightarrow Z^0F^0$ and $Z^0 \rightarrow p^01^+1^-$ which are analogous to promising ways to hunt for Higgs bosons (see subsection 2.4) also vanish to first order. The resulting small higher-order cross-sections are not expected [62, 63] to be observable.

2.6.3 $V_L \rightarrow F_0^0, F_0^0' \gamma$

These decays are analogous to the Wilczek mechanism of subsection 2.4.1, and the branching ratio is given by the same formula with an additional model-dependent factor [62],

$$\frac{\Gamma(V_L \rightarrow F_0^0\gamma)}{\Gamma(V_L \rightarrow \mu^+\mu^-)} = \frac{G_F M_V^2}{4\sqrt{2}\pi} R[1 - (m_{F_0^0}^2/m_{V_L}^2)]$$

(2.59)

where $R$ is guessed to be of the order of 1, and takes the value $3^{\pm1}$ in some simple models [62].

2.6.4 $Z^0 \rightarrow F^0F^0P^0$  

This process is primarily due to the $Z^0F^+F^-F^0$ vertex which is present in composite technicolour theories, but not at the tree level in elementary Higgs theories. The width is given [64] by
\[ \Gamma(Z^0 \rightarrow p^0 p^-) = \frac{1}{2m_Z} \left( \frac{2e \text{Tr } b^3}{f_{\gamma}} \right)^2 \times \int \frac{d^3k}{(2\pi)^3} \frac{d^3p}{2E_k} \frac{d^3q}{2E_p} \frac{d^3q}{2E_q} \frac{\delta(k+p+q-Q)(k \cdot n)^2}{(2\pi)^9 2E_k 2E_p 2E_q}, \]

where \( \text{Tr } b^3 = O(1) \) and \( f_{\gamma} = O(m_W/g_\gamma) \) are model-dependent factors.

To do the integration, we approximate \( m_W^0 = 0 \), to get

\[ \Gamma(Z^0 \rightarrow p^0 p^-) = \Gamma(Z^0 \rightarrow p^0 p^-) \bigg|_{m_W^0 = 0} \frac{x_m}{4} \int_0^x dx x^3 \left( \frac{x_m - x}{1 - x} \right)^{1/2}, \]

where

\[ \frac{\Gamma(Z^0 \rightarrow p^0 p^-)}{\Gamma(Z^0 \rightarrow \mu^+ \mu^-)} \bigg|_{m_W^0 = 0} = \frac{(\text{Tr } b^3)^2}{128\pi^2 \left( 1 - 4 \sin^2 \theta_W + 8 \sin^4 \theta_W \right)} \left( \frac{m_Z}{f_{\gamma}} \right)^2. \]

This decay width is shown in Fig. 2.41 taking [56]:

\[ \text{Tr } b^3 = 3^{-1/2}, \quad \frac{m_Z}{f_{\gamma}} = \frac{2e}{\sin 2\theta_W}. \]

We see that the process is likely to have a large enough cross-section to be observable unless \( 2m_{p^\pm} + m_{p^0} \) is close to \( m_Z^0 \). As this is a process which is absent for elementary Higgses, it is an interesting signature for Technicolor theories.

![Graph](image)

**Fig. 2.41** Number of events per \( 10^6 Z^0 \) for \( Z^0 \rightarrow p^0 p^- p^0 \), with \( m_{p^0} = 0 \) and \( m_{p^\pm} \) up to 45 GeV
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3. **SUPERSYMMETRY**

3.1 **Introduction**

3.1.1 **General outline**

The Standard SU(2)$_L \times$ U(1) Model has been very successful in its description of all the present data on the electroweak interactions. However, it leaves many fundamental parameters unexplained, such as the magnitudes of particle masses, charged-current mixing angles, the number of quark and lepton generations, the neutral weak mixing angle $\sin^2 \theta_W$, etc. To find origins for these parameters, one must go beyond the Standard Model. One popular extension of the Standard Model which attempts to understand the hierarchy of different mass scales in physics invokes supersymmetry, which interrelates particles with different spins.

Details of the supersymmetric models studied in this report are described in subsection 3.1.2. Supersymmetric theories predict many new particles with spins differing from those of the known particles by half a unit. In the limit of exact supersymmetry, the masses of particles and their supersymmetric partners would be equal. However, the negative experimental results from PETRA, PEP, and elsewhere tell us that supersymmetry must be broken. LEP will offer experimentalists a new opportunity to find supersymmetric particles if they exist, or at least improve dramatically the present limits on their masses.

Subsection 3.1.3 describes the experimental signatures expected for supersymmetry, and summarizes the relevant characteristics of a representative detector that we have assumed in our estimates of signal selection, backgrounds, etc. Subsection 3.1.4 sets out the strategy for finding sparticles advocated in this report. We divide the sparticles we seek into two broad categories: non-strongly interacting particles such as sleptons $\tilde{l}$, the photino $\tilde{\gamma}$, winos $\tilde{\omega}$ and the neutral zino $\tilde{z}$, and strongly interacting particles such as squarks $\tilde{q}$ and gluinos $\tilde{g}$. Searches for non-strongly interacting sparticles are described in subsection 3.2, which is in turn divided into subsections dealing with two-body final states (subsection 3.2.1) and three-body final states (subsection 3.2.2). The former may be used to look for sparticles weighing less than the beam energy, whereas the latter may be useful for looking beyond this limit in certain cases. Searches for strongly interacting sparticles are discussed in subsection 3.3.

There are Appendices containing some extra technical details of our calculations, and discussing the virtues of beam polarization as an analysis tool.

3.1.2 **Supersymmetric models**

Supersymmetry [1] is a novel type of symmetry which interrelates fermions and bosons. Theorists have long been intrigued by the new avenues towards unification of the fundamental interactions which it opens up [2]. Perhaps supersymmetry could be used to interrelate particles of matter (quarks and leptons)
with force particles (gauge and/or Higgs bosons)? Since supersymmetry can be made local [3] only by introducing as a gauge fermion the gravitino, the spin-3/2 supersymmetric partner of the graviton, perhaps supersymmetry plays an essential role in the eventual unification with gravity? It is known [4] that supersymmetry and supergravity theories have fewer divergences than conventional quantum field theories, so perhaps they may provide a consistent quantum theory of gravity? These theoretical motivations are all very fine, but what makes supersymmetry appear a particularly tempting target for LEP experiments is the phenomenological argument, already mentioned in the general introduction, that supersymmetry can help us understand [5] the hierarchy of different mass scales in physics if supersymmetric particles have masses $O(m_\nu)$.

It has already been emphasized that the only consistent way to provide masses for gauge bosons involves the introduction of one or more spin-0 particles of mass $O(m_\nu)$. However, elementary scalar masses are notoriously unstable [6], acquiring enormous corrections whenever they are integrated into a larger theoretical framework. For example, one-loop radiative corrections to elementary scalar masses diverge quadratically:

$$\delta m_H^2 = O\left(\frac{\alpha}{4\pi}\right)\Lambda^2,$$

(3.1)

where $\Lambda$ is a cut-off representing the appearance of new physics. The cut-off $\Lambda$ may be $O(m_\chi) \geq 10^{15}$ GeV in a grand unified theory, or $O(m_p) = 10^{15}$ GeV if no new physics intervenes before the Planck scale associated with quantum gravity. Even without calculating loop corrections, the unavoidable couplings of light Higgs bosons to heavy Higgses in grand unified theories (GUTs) yield

$$\delta m_H^2 = O(m_\chi^2),$$

(3.2)

whilst gravitational effects may yield

$$\delta m_H^2 = O(m_p^2).$$

(3.3)

These large corrections to the Higgs boson mass, which should be $O(m_\nu)$, raise problems at two levels.

The first is how to arrange, in the first place, for $m_H$ to be much smaller than other larger mass scales such as $m_\chi$ or $m_p$---the Hierarchy Problem. The other is how to avoid having corrections $\delta m_H^2$ to the Higgs boson mass squared which are much larger than $m_H^2$ itself---the Naturalness Problem.

Supersymmetry resolves the Naturalness Problem [5, 7], and some supergravity models [8] offer a solution to the Hierarchy Problem. Radiative corrections to scalar masses squared would be absent if supersymmetry were exact,
and are proportional to the differences between fermion and boson masses squared in theories where supersymmetry is broken:

\[ \delta m^2_H = O \left( \frac{\alpha}{\pi} \right) |m^2_B - m^2_P| . \]  

Comparing Eqs. (3.1) and (3.4) we see that the supersymmetry-breaking difference in mass squared acts as an effective cut-off \( \Lambda^2 \). For this to provide a solution to the Naturalness Problem, i.e. so that \( \delta m^2 \) [Eq. (3.4)] must be smaller than \( m^2_H = O(m^2_W) \), we need

\[ |m^2_B - m^2_P| \lesssim O(1 \text{ TeV})^2 . \]

Therefore supersymmetric particles cannot be very heavy if they are to solve the Naturalness Problem. The conventional particles of the Standard Model and their supersymmetric partners are listed in Table 3.1.

In recent years, a succession of different approaches [2] to the construction of models with small supersymmetry breaking [Eq. (3.5)] have been taken. In order to accommodate parity violation at low energies, attention has been focused on models with simple \( N = 1 \) supersymmetry. Models based on global supersymmetry which do not involve supergravity required either an extension of the low-energy \( SU(3) \times SU(2) \times U(1) \) gauge group, or the addition of extra gauge

| Table 3.1 |
| Supersymmetric particles. |
| The particle content of the Standard Model and their superpartners. |
| The sparticle states shown mix, in general, to give the mass eigenstates, as discussed in the text. |

| Particle | Spin | Sparticle | Spin |
| Quark \( q_{L,R} \) | 1/2, 1/2 | squark \( \tilde{q}_{L,R} \) | 0, 0 |
| Lepton \( \ell_{L,R} \) | 1/2, 1/2 | slepton \( \tilde{\ell}_{L,R} \) | 0, 0 |
| Photon \( \gamma \) | 1 | photino \( \tilde{\gamma} \) | 1/2 |
| Gluon \( g \) | 1 | gluino \( \tilde{g} \) | 1/2 |
| \( W^\pm \) | 1 | wino \( \tilde{W}^\pm \) | 1/2 |
| \( Z^0 \) | 1 | zino \( \tilde{Z}^0 \) | 1/2 |
| Higgs \( H \) | 0 | shiggs \( \tilde{H} \) | 1/2 |
singlet particles with bizarre couplings. In addition to their ugliness, these models also suffered from technical problems [9] and consequently were largely abandoned. In most current models, supersymmetry breaking is generated by supergravity via the super-Higgs mechanism [10]. This is the supersymmetric analogue of the conventional Higgs mechanism, whereby massless gauge bosons are combined with massless scalar bosons to provide massive gauge bosons. In the super-Higgs mechanism, the massless gravitino--the spin-3/2 gauge fermion--is combined with a massless spin-1/2 particle--the goldstino or Goldstone fermion--to provide a massive gravitino. Just as gauge bosons becoming massive signal the spontaneous breakdown of gauge symmetry, so the gravitino becoming massive signals the spontaneous breakdown of supersymmetry. Just as quarks and leptons acquire masses proportional to the gauge boson masses when gauge symmetry is spontaneously broken, so supersymmetric partners of conventional particles can get extra masses when supersymmetry is spontaneously broken. These may be as large as the gravitino mass, but could be significantly smaller. We will say no more here about the details of the mechanism of supersymmetry breaking, since only very general features of supersymmetry breaking are needed in our phenomenological analysis.

The couplings of supersymmetric particles are directly related to those of conventional particles. All one has to do is replace

\[ \tilde{f}g \rightarrow \tilde{f}\tilde{\phi} , \tilde{f}\tilde{f} , \tilde{f}\tilde{\phi} , \]  \hspace{1cm} (3.6)

where \( f \) is a matter fermion (quark or lepton), \( G \) a gauge boson, and their supersymmetric partners are denoted by tildes, and

\[ \tilde{f}\tilde{f}'H \rightarrow \tilde{f}\tilde{f}'\tilde{\phi} , \tilde{f}\tilde{f}'\tilde{\phi} , \]  \hspace{1cm} (3.7)

where \( H \) is a Higgs boson. The resulting vertices are shown in Fig. 3.1, where we see also that the sparticle couplings are the same as conventional gauge couplings \( g \) or Yukawa couplings \( \lambda \) up to factors of the 'spin Clebsch-Gordan coefficients' introduced by the SUSY transformation. It is easy to check that other trilinear couplings between particles and their sparticles are forbidden by the conservation of spin, baryon number \( B \), and lepton number \( L \). It is a general feature of the couplings (3.6) and (3.7) that they involve pairs of sparticles. Indeed, in most supersymmetric theories one can introduce a multiplicatively conserved quantum number \( R \) [11]:

\[ R = \begin{cases} +1 \text{ for conventional particles} \\ -1 \text{ for their supersymmetric partners}. \end{cases} \]  \hspace{1cm} (3.8)
The multiplicative conservation of $R$ has three important phenomenological consequences:

i) sparticles are always produced in pairs, e.g.
\[ e^+ e^- \rightarrow \tilde{\nu} \tilde{\nu}, \quad \tilde{e}^+ \tilde{e}^- \rightarrow \tilde{\nu} \nu, \quad (3.9) \]

ii) every sparticle decays into another sparticle, e.g.
\[ \tilde{e} \rightarrow e \tilde{\nu}, \quad \tilde{W} \rightarrow q \bar{q}' \gamma, \quad (3.10) \]

iii) the lightest sparticle is absolutely stable, since it has no allowed decay mode.

This last property imposes severe cosmological restrictions on the nature of the lightest supersymmetric particle (LSP) [12]. If it were electromagnetically charged or strongly interacting, the cosmological relic LSP would condense along with ordinary matter into galaxies, stars, and planets. LSPs would then bind with conventional stable particles to form anomalous heavy isotopes. One can estimate [13] the abundance of these to be $0(10^{-6}$ to $10^{-10})$ of conventional nuclides. The upper limit [14] on such anomalous heavy isotopes is $0(10^{-20})$ for relics lighter than 1 TeV, which seems to exclude a strongly or electromagnetically interacting LSP. Therefore, we are left with such weakly interacting
candidates for the LSP as the sneutrino $\tilde{\nu}$ of spin 0, the photino $\tilde{\gamma}$ or neutral
shiggs $\tilde{H}$ of spin 1/2, and the gravitino $\tilde{G}$ of spin 3/2. In most models the $\tilde{\nu}$
and $\tilde{G}$ have masses $O(m_\gamma)$, and this, in turn, is larger than the lightest mass
eigenstate among the mixtures of different neutral spin-1/2 sparticles. Cos-
monology disfavors the possibility $m_H < m_\gamma$, but does not exclude it. Unsuccess-
ful searches for $\tilde{\gamma} \to H + \gamma$ decays [15] in previous $e^+e^-$ experiments also rule
out $m_H < m_\gamma$ if $m_\gamma \sim O(40)$ GeV. Therefore, the favored candidate for the LSP is
the photino $\tilde{\gamma}$, but this is not unique, and the LSP could in particular contain a significant admixture of $\tilde{H}$. Most of the experimental signatures
discussed later are, in any case, valid whatever the identity of the LSP, as
long as it is stable, or at least lives longer than $O(10^{-8})$ seconds and escapes
out of the experiment before decaying.

The classic signature of supersymmetry is missing energy-momentum ($\not{p}$)
carried away by a pair of undetected, weakly interacting, neutral LSPs, e.g.

$$e^+e^- \to \mu^+\mu^- + (\mu^+\mu^-)(\mu^+\mu^-), \quad e^+e^- \to e^+e^- \to (e^+e^-)(e^+e^-). \quad (3.11)$$

Here the photinos are as difficult to see as neutrinos, since they have compar-
sably small cross-sections for interactions with detector components. The pros-
psects for detecting $\not{p}$ at LEP are better than those at hadron-hadron colliders
for at least two reasons. One is that in hadron-hadron collisions it is very
difficult to measure the components of the total final-state momentum parallel
to the incoming beams, because of particle losses down the beam tubes. This
means that in hadron-hadron collisions one can only measure the missing tran-
verse momentum $\not{p}_T$, whereas in $e^+e^-$ collisions one can measure the total missing
energy-momentum $\not{p}$. Another advantageous feature of $e^+e^-$ collisions is that in
$e^+e^- \to$ sparticle + antisparticle pair-production the two sparticles are each con-
strained to have the beam energy, whereas in hadron-hadron collisions they would
be variable.

We now specify our phenomenological model framework in more detail. As
discussed above, the gauge couplings of the SUSY Standard Model (for a given
Higgs field content) are completely fixed by $SU(3)_C \times SU(2)_L \times U(1)$ and super-
symmetry. In addition, arbitrary soft supersymmetry-breaking terms [16] are
induced in the effective low-energy theory when the model is coupled to super-
gravity [17]. The mass eigenstates, in general, are not the quanta of the fields
that enter into these couplings but mixtures thereof. These mixings are domi-
nantly governed in the minimal two-Higgs model (two Higgs fields are needed
to give both the up and the down quarks a mass and to cancel anomalies [7]) by
four parameters. These are: $m_0$, the common scalar mass; $v_1/v_2$, the ratio of the
Higgs field vacuum expectation values; $\mu$, the Higgs mixing term; and the tree-
level gaugino masses $\mu_1$, which we take to be related as in a GUT. For the super-
partners of the known quarks and leptons, except for the top quark [18], the coefficients of the trilinear and bilinear soft scalar couplings do not play a role.

a) Spin-0 mass matrix

We first consider the mass matrix for the spin-0 partners of conventional quarks and leptons (sfermions). The generic form for it is [19]

\[
\begin{pmatrix}
\hat{m}_L^2 & \hat{m}_F^2 \\
\hat{m}_L^2 & \hat{m}_R^2
\end{pmatrix}
\begin{pmatrix}
\hat{f}_L \\
\hat{f}_R
\end{pmatrix}.
\]

Here, \( m_0 \) is the typical scalar mass mentioned above (which must be \( \gtrsim 20 \) GeV), and \( m_F \) is the usual fermion mass (0.5 MeV to 50 GeV). The parameters \( L, R, \) and \( A \) are numerical coefficients which depend on one's model. We see that the mass eigenstates are mixtures of \( \hat{f}_L \) and \( \hat{f}_R \). There is, however, one great simplification that occurs when \( m_F/m_0 \ll 1 \), unless \( L = R \) to a high degree of precision. In this case \( L \neq R \), the mixing between \( \hat{f}_L \) and \( \hat{f}_R \) is negligible and the left- and right-handed sfermion states are approximate mass eigenstates. Here we do not consider the case of heavy quarks [18] and we will assume that \( \hat{f}_L \) and \( \hat{f}_R \) are the mass eigenstates.

b) Charged spin-1/2 mass matrix

We now turn to a discussion of the charged gaugino-shiggs system. As mentioned above and in subsection 2.5, supersymmetric theories contain at least two scalar doublets. The spin-1/2 mass eigenstates, after electroweak breaking, are mixtures of the wino and shiggs 'current' eigenstates. The mass terms for the charged wino-shiggs system are [19]

\[
(\lambda, \chi)
\begin{pmatrix}
\mu_2 & g
\end{pmatrix}
\begin{pmatrix}
v_2 \\
v_1
\end{pmatrix}
\begin{pmatrix}
1 - \gamma_5 \\
2
\end{pmatrix}
\begin{pmatrix}
\lambda \\
\chi
\end{pmatrix}
\]

+ h.c.,

with the Dirac spinors \( \lambda \) and \( \chi \) defined by

\[
\lambda = (\lambda_1 + i\lambda_2)/2, \quad \chi = \frac{1 - \gamma_5}{2} h_2 - \frac{1 + \gamma_5}{2} h_1^C,
\]

where \( h_1 \) and \( h_2 \) are the shiggs fields, the superpartners of the Higgs fields with vacuum expectation values (v.e.v.'s) \( v_1 \) and \( v_2 \) respectively, and \( \mu_2 \) and \( \mu \) are the SU(2) gaugino mass and shiggs mixing term respectively. Thus, apart from a correction due to \( \mu_2 \), which we assume to be small, the mixing is governed by the ratio \( v_1/v_2 \) of the v.e.v.'s. Unless otherwise stated, we take \( v_1/v_2 \geq 1 \),
a value favoured by a class of supergravity models [20]. Then, the eigenvalues \( m_\pm \) of the charged wino/shiggs (chargino) system are given by

\[
(m_+ - \mu_2)(m_- + \mu_2) = m_w^2.
\]  

(3.15)

For small values of \( \mu_2 \) we see that there is one state lighter than \( m_w \) (\( m_+ \)) and one state heavier (\( m_- \)). We refer to these as the wino (\( \tilde{W} \)) and the heavy wino (\( \tilde{W}_h \)), respectively. We may take \( m_\pm = m_w \) as an input parameter, i.e. we can eliminate \( \mu \) in favour of the physical mass of the wino. The mass of the heavy wino is then determined. As we will see later, the other parameter \( \mu_2 \) is fixed in terms of \( m_- \). Finally, the mass eigenstates are related to the current eigenstates by

\[
\gamma_5 \tilde{W} = f_+ \lambda_+ - f_- \lambda_-,
\]

\[
\tilde{W}_h = f_- \lambda_+ + f_+ \lambda_-,
\]

(3.16)

with \( \lambda_+ \) and \( \lambda_- \) as previously defined. The mixing parameters \( f_\pm \) are determined in terms of the input parameters \( m_\pm \) and \( \mu_2 \) by

\[
f_\pm = \frac{m_\pm \pm \mu_2}{m_+ + m_-}.
\]

(3.17)

Note that the \( \gamma_5 \) factor in Eq. (3.16) has been introduced in order to make the eigenvalues of the mass matrix positive.

c) Neutral spin-1/2 mass matrix

We now turn to a discussion of the neutral gaugino-shiggs (neutralino) sector. This is necessarily more complicated because, in addition to the two neutral gauginos, there are the neutral shiggses. In the minimal model of the type we are discussing there are just two of these, although models with more complicated Higgs sectors can also be written down. In particular, there exist models with singlet Higgs fields [21], although the simplest of these seem to suffer from technical difficulties [22]. For the discussion of the photino and zino states, or more accurately, the states which are dominantly the photino and the zino, these details are not of importance, and we will consider the minimal case. In the (Majorana) \( h, h', \lambda_3, \lambda_0 \) basis, the neutralino mass matrix has the form [19]
where \( \mu_2 \) and \( \mu_1 \) are the SU(2) and U(1) tree-level gaugino masses. In the case \( \mu_1 = \mu_2 = 0 \), the matrix can be exactly diagonalized to yield a zero-mass photino, a zino \( (\tilde{Z}) \), a heavy zino \( (\tilde{Z}_h) \), and another neutral state which we will ignore. In this limiting case the masses \( \mu_- \) and \( \mu_+ \) of the zino and the heavy zino are related by

\[
\mu_+ \mu_- = m_Z^2,
\]

with the mass eigenstates being given by

\[
\tilde{Z}^{(0)} = \sin \theta_W \lambda_3 + \cos \theta_W \lambda_0,
\]

\[
\tilde{Z}_h^{(0)} = N_1 \left( -h+h'+\sqrt{2} \frac{m_2}{\mu_-} \cos \theta_W \lambda_3 - \sqrt{2} \frac{m_2}{\mu_-} \sin \theta_W \lambda_0 \right),
\]

\[
\tilde{Z}_h^{(0)} = N_2 \left( h-h'+\sqrt{2} \frac{m_2}{\mu_+} \cos \theta_W \lambda_3 - \sqrt{2} \frac{m_2}{\mu_+} \sin \theta_W \lambda_0 \right),
\]

with

\[
N_1 = \sqrt{\frac{\mu_-}{2(\mu_+ + \mu_-)}} \quad \text{and} \quad N_2 = \sqrt{\frac{\mu_+}{2(\mu_+ + \mu_-)}}.
\]

The superscript \((0)\) indicates that these are pure eigenstates if \( \mu_1 = \mu_2 = 0 \).

If we now consider the case of small gaugino masses, treating them as a perturbation, then the photino acquires a mass which is approximately

\[
m_{\tilde{Z}} = |\mu_2| \sin^2 \theta_W + |\mu_1| \cos^2 \theta_W
\]

whilst the zino and the heavy zino masses become

\[
m_{\tilde{Z}_h} = \mu_1 - \frac{\mu_+}{\mu_+ + \mu_-} \left( \mu_1 \sin^2 \theta_W + \mu_2 \cos^2 \theta_W \right)
\]

and
\[ m_{\tilde{h}}^2 = \mu_+ + \frac{\mu_-}{\mu_+ + \mu_-} (\mu_1 \sin^2 \theta_W + \mu_2 \cos^2 \theta_W) . \] (3.24)

The mass eigenstates are related to those for \( \mu_1 = \mu_2 = 0 \) by

\[ \begin{pmatrix} z(0) \\ z_\tilde{h}(0) \\ -i \gamma_5 z_\tilde{h}(0) \end{pmatrix} = \begin{pmatrix} 1 & \delta & \epsilon_1 \\ -\delta & 1 & \epsilon_2 \\ -\epsilon_1 & -\epsilon_2 & 1 \end{pmatrix} \begin{pmatrix} z(0) \\ z_{\tilde{h}}(0) \\ -i \gamma_5 z_{\tilde{h}}(0) \end{pmatrix} \] (3.25)

with

\[ \epsilon_1 \equiv \sqrt{2} \frac{m_{\tilde{h}}^2}{\mu_+} \sin \theta_W \cos \theta_W (\mu_2 - \mu_1) , \] (3.26)

\[ \epsilon_2 \equiv -\sqrt{2} \frac{m_{\tilde{h}}^2}{\mu_-} (\mu_2 - \mu_1) , \] (3.27)

and

\[ \delta \equiv \frac{2N_c N_f}{\mu_+ + \mu_-} (\mu_2 \cos^2 \theta_W + \mu_1 \sin^2 \theta_W) . \] (3.28)

The couplings of these mass eigenstates can then be calculated. In practice, \( \epsilon_2 \) and \( \delta \) are small and we have ignored them in numerical calculations.

Since we have treated the gaugino masses as a perturbation, our analysis is restricted to \( m_\gamma \lesssim 10 \text{ GeV} \). In most cases we have chosen \( |\mu_2| = 10 \text{ GeV} \) corresponding to \( m_\gamma \approx 6 \text{ GeV} \). This value is for illustrative purposes only. We take \( \mu_+ \) (or \( m_\tilde{h} \)) as an input parameter as well as \( \mu_2 \) (or \( m_\gamma \)). (For technical reasons, in the convention we have used the input value of \( \mu_2 \) should be negative.) In the simplest GUT, \( \mu_1 \) is determined in terms of \( \mu_2 \) by

\[ \frac{\mu_1}{\mu_2} = \frac{5}{3} \tan^2 \theta_W . \] (3.29)

Finally, we remark that the \( \gamma_5 \) factors in Eq. (3.25) are inserted to get positive eigenvalues of the mass matrix, and the factor \( i \) to ensure that the fields \( z, \tilde{z}, \gamma \), and \( \tilde{z}_h \) are Majorana. This is, of course just a matter of convention.

d) Current experimental limits

Electron-positron colliders are ideal for new particle searches because the production mechanisms only involve the electroweak interactions, and the initial state is very simple. Therefore, the cross-sections can be estimated
quite reliably, as can final-state distributions. Thus the negative results of sparticle searches at PETRA and PEP can be interpreted as lower limits for sparticle masses. Table 3.2 is a compilation of the limits available as this report was prepared [23]. The interpretation of sparticle mass limits coming from hadron-hadron collisions is more difficult. Ultimately, the cleanliness of

<table>
<thead>
<tr>
<th>Sparticle</th>
<th>MARK J</th>
<th>CELLO</th>
<th>JADE</th>
<th>TASSO</th>
<th>MARK II</th>
<th>MAC</th>
<th>ASP</th>
<th>Model assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>photon $\gamma$</td>
<td>20.5</td>
<td>20</td>
<td>18</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td>Unstable $\gamma$, e.g. $\gamma + \gamma \rightarrow \ell^+ \ell^-$ Goldstino; $m_\gamma = 50$ GeV in o(e$^+ e^- + \gamma \gamma$)</td>
</tr>
<tr>
<td>selectron $\tilde{e}$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\tilde{e} \rightarrow \ell + \gamma$ decay; $m_\ell = 0$</td>
</tr>
<tr>
<td>smuon $\tilde{\mu}$</td>
<td>20</td>
<td>16</td>
<td>20.3</td>
<td>16.4</td>
<td></td>
<td></td>
<td></td>
<td>$\tilde{\mu} \rightarrow \tilde{\tau} + \gamma$ decay</td>
</tr>
<tr>
<td>stau $\tilde{\tau}$</td>
<td>17</td>
<td>15.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\tilde{\tau} \rightarrow \tilde{\tau} + \gamma$ decay</td>
</tr>
<tr>
<td>charged shiggs $H^\pm$</td>
<td>22.5</td>
<td>22.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$m_\pm = 4$ GeV</td>
</tr>
<tr>
<td>wino $\tilde{\omega}$</td>
<td>35</td>
<td>35</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$m_\omega &lt; 2$ GeV; $m_\omega &lt; 40$ GeV</td>
</tr>
<tr>
<td>charged wino $\tilde{\omega}^c$</td>
<td>25</td>
<td>22.5</td>
<td>22.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\tilde{\omega}^c$ decays leptonically</td>
</tr>
<tr>
<td>squarks $q$</td>
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<td></td>
<td></td>
<td></td>
<td>$m_\tilde{q} = m_\tilde{q}^c$</td>
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<tr>
<td>(charge 2/3)</td>
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<td></td>
<td>$m_\tilde{q} = m_\tilde{q}^c$, $\bar{q} \rightarrow \ell + \gamma$; $m_\tilde{q} = 3$ GeV, $m_\tilde{q}^c &lt; 10$ GeV</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>$m_\tilde{q} = m_\tilde{q}^c$, $\tilde{q} \rightarrow \tilde{\nu} + \gamma$; $m_\tilde{q} = 0.5$ GeV</td>
</tr>
</tbody>
</table>

Comments:
a) Here and elsewhere, the reaction used has been indicated where appropriate. We assume $m_\ell = m_\omega$.
b) Neutral shiggsino mixing can reduce the production rate and hence the lower mass limit.
c) Charged shiggsino mixing changes the pair-production cross-section only slightly. The decay of a charged shiggs is expected either i) to proceed via $W$-exchange, giving a signal identical to that of $\tilde{\nu} \tilde{\nu}$ pair-production, or ii) if the sneutrino is light, to give predominantly $\ell + \tilde{\nu}$ or $\ell + \gamma$, which would yield acoplanar $\ell$ pairs. These final states were looked for in the $\tilde{\tau} \tilde{\nu}$ search. In either case, there is a solid lower limit of 22.5 GeV on the masses of both the charged wino $\tilde{\omega}^c$ and the charged shiggs $H^\pm$.

References:

These papers contain references to earlier publications by the same Collaborations.
e^+e^- collisions should enable one to deduce the natures of the couplings of any sparticles observed, allowing one to test whether they fit into any one of the theoretical schemes proposed.

We present here the results of a detailed study of the characteristic signatures of the production of some supersymmetric particles and the use of them to distinguish these particles from the backgrounds. The calculations of the production of these particles, when they are not readily available in the literature, have been carried out by ourselves for this report, and are described in detail along with some analysis methods for separating the super-symmetric particles from the backgrounds. The requirement of eliminating the backgrounds puts important constraints on the design of the detectors to be used in the searches for these particles.

3.1.3 Detector model

In order to get quantitative results for observable cross-sections, we have assumed a large solid-angle detector with the following properties:
1) good photon/electron/muon energy-momentum measurement with a resolution of 1% over the regions specified below;
2) the photon/electron detector covers polar angles between 5° and 175° with respect to the beams;
3) the muon acceptance covers polar angles from 10° to 170°.

The selection criteria for the supersymmetry candidates, typically involving ranges of polar angles as defined above, minimum values of missing p_T (\vec{p}_T), and acoplanarity angle cuts, are also specified in the captions to each of the graphs that follow. In certain cases, e.g. the searches for strongly interacting particles discussed in subsection 3.3, we supplement the assumptions (1) to (3) above with additional assumptions on hadronic energy-momentum measurements.

3.1.4 Experimental strategy

As discussed in subsection 3.1.2, sparticles are always produced in pairs. Thus, unless a sparticle is produced in association with another very light one such as the photino, the mass of the sparticle is obviously limited to the beam energy. Before proceeding to discuss the rates and signatures for the various processes, we briefly outline our strategy in the search for weakly interacting sparticles which is followed in subsection 3.2. We first concentrate on pair-production processes which have the lowest thresholds (subsection 3.2.1). If sleptons are lighter than the gauginos (\tilde{\mu}, \tilde{\tau}), these would be copiously produced [24-26] if E_{beam} > \tilde{m}. These would then decay 100% of the time via \tilde{\tau} → l\gamma. On the other hand, if the winos and zinos were lighter than the sleptons, the threshold for e^+e^- → \tilde{\mu}\tilde{\mu} [27] would be crossed first. Winos of such a mass would decay via the three-body modes \tilde{\mu} → q\bar{q}\gamma or \tilde{\mu} → l\bar{\nu}\gamma [28]. It could also be that
both slepton and wino pair-productions are inaccessible. In that case the only allowed process could well be $e^+e^- \rightarrow \tilde{\chi}^0$ [29] since the photino has been assumed to be light. Recall that for light photinos, $m_{\tilde{\chi}} < m_\chi$ [30]. A zino lighter than the slepton would decay via the three-body modes [28] $\tilde{\chi} \rightarrow q\bar{q} \gamma$ or $\tilde{\chi} \rightarrow l\bar{l} \gamma$.

Thus in our computations for slepton pair-production, we have assumed that the sleptons decay 100% via $\tilde{l} \rightarrow l\bar{l} \gamma$, whereas in those for gaugino pair-production we have assumed that the gauginos decay via the three-body mode.

In addition to the two-body processes of subsection 3.2.1, we have also considered in subsection 3.2.2 several three-body processes in which a sparticle is produced in association with a photino [31] (or sneutrino [32]) and another particle. Such processes are of interest since if the photino (or sneutrino) is relatively light, a larger region of sparticle masses can be explored, just as in the case of the reaction $e^+e^- \rightarrow \tilde{\chi}^0$. The price paid is that all the cross-sections are suppressed by an extra power of the fine-structure constant. Also of interest are the processes $e^+e^- \rightarrow \chi\gamma \gamma$ [33], or $e^+e^- \rightarrow \tilde{\nu}\nu\gamma$ [34], which have a suppressed cross-section for the same reason. However, they provide a characteristic signature, namely a single photon recoiling against nothing, much like that suggested for neutrino counting experiments [35].

The event topology we look for in the case of the three sparticle pair-production processes discussed is a pair of charged leptons acoplanar with the beam, a substantial amount of missing energy $\not{E_T}$ and missing transverse momentum $p_T$ carried off by the photinos, and an absence of hadronic activity. Obviously, in the case of gaugino pair-production there will also be events with jets + $p_T$ and jets + lepton + $p_T$, corresponding to the hadronic decay modes of the zino and the wino. Here we will focus only on the leptonic modes.

The Standard Model background to these processes comes from

i) $e^+e^- \rightarrow \tau^+\tau^-$ with the $\tau$'s decaying leptonically,

ii) $e^+e^- \rightarrow l^+l^-\gamma$ with the $\gamma$ going down the beam-pipe and where $l = e, \mu, \tau$, and

iii) $e^+e^- \rightarrow l^+l^-e^+e^-$ where the e+e- pair goes down the beam-pipe.

At LEP I, in particular, the process $e^+e^- \rightarrow \tau^+\tau^-$ has a much larger rate than the $e^+e^- \rightarrow \tilde{\chi}^0$ process [29] since the latter takes place only via t-channel selectron exchange, and then is not resonance-enhanced like the $\tau^+\tau^-$ process. Fortunately, it is possible to eliminate $\tau^+\tau^-$ events kinematically with the following cuts [29]:

*) It could be that $m_\tau/2 < m_\tilde{\chi} < m_\chi$, in which case $\tilde{\chi}^0$ production could take place at LEP I without slepton pair-production. The zino would then decay via the two-body modes $\tilde{\chi} \rightarrow q\bar{q}, l\bar{l}, \text{ or } v\bar{v}$. The branching fraction into charged leptons is then the same as in the three-body case and so the rates we present would not be altered although some details of the distributions may be.
a) The energy of each lepton should exceed a preassigned value $E_0$.

b) For a given value of $E_0$, the opening angle between the leptons should be less than $\theta_M$ given by

$$\theta_M = 180^\circ - 2 \arccos \left( \frac{\sqrt{s} E_0 - m^2}{\sqrt{s} E_0 \beta^2} \right).$$

(3.30)

where

$$\beta^2 = \left(1 - 4m^2/s\right)^{1/2}.$$  

The QED processes (i) and (iii) discussed earlier have been studied using the Berends-Dawidveldt-Kleiss code [36] and references therein. The resulting $p_T$ spectrum for the standard QED process $e^+e^- \leftrightarrow l^+l^-\gamma$ is shown in Fig. 3.2 along with a typical spectrum for a two-body SUSY process. As expected, a $p_T$ cut, $p_T > \sin 5^\circ \times \sqrt{s}/2 - 0.05/s$ essentially eliminates these events without substantially reducing the SUSY signal. Thus, such a $p_T$ cut and the $\tau$ cuts (a) and

![Graph showing lepton pair distributions](image)

Fig. 3.2 A comparison of the $p_T$ distributions of lepton pairs from selectron pair-production and from radiative corrections to pair production in the Standard Model, where the photon either goes down the beam pipe or has an energy less than 1 GeV so that it escapes detection. The backgrounds from two-photon processes are even smaller. Other supersymmetric processes have similar $p_T$ distributions, as may be expected from the fact that the photinos are a genuine source of missing $p_T$. 
(b) discussed earlier, in conjunction, are sufficient to remove the Standard Model backgrounds for all the two-body supersymmetry processes leading to leptonic final states.

Searches for hadronic final states from sparticles (e.g. $e^+e^- \rightarrow q\bar{q} + q\bar{q} + \phi$) are subject to additional backgrounds from heavy-flavour production ($e^+e^- \rightarrow c\bar{c}, b\bar{b}, t\bar{t}$, etc.) and decay ($c \rightarrow s\ell^-\nu, b \rightarrow c\ell^-\nu, t \rightarrow b\ell^-\nu$) where the $l^\pm$ are not seen but the $\ell^\mp$ carry off $p_T$ and $p_T^\phi$. Our calculations suggest that simple topological cuts remove the $e^+e^- \rightarrow c\bar{c}, b\bar{b}$, and possibly $t\bar{t}$ backgrounds, and vertex detectors may also be useful for rejecting them. Above the $e^+e^- \rightarrow W^+W^-$ threshold there are also $\ell$ and $p_T$ backgrounds from $W^\pm \rightarrow l^\mp\nu$ decays. Whilst $\sigma(e^+e^- \rightarrow W^+W^-)$ is large, these backgrounds can probably also be beaten down, although we have not studied this problem in detail.

In the rest of this section we quote cross-section formulae for all two-body, most three-body, and one four-body sparticle production process. In some cases the cross-section formulae or final-state distributions are too complicated to be written out explicitly, in which cases we refer readers to the appropriate file in the Electronic Yellow Book (EYB).

3.2 Production of non-strongly interacting sparticles

3.2.1 Two-body final states

a) Slepion pair-production

The production of like-handed sparticles takes place via the s-channel exchange of the photon and the $Z^0$. Like-handed selectron pairs can also be produced via t-channel exchanges of the photino and zinos. The t-channel exchanges also lead to $\tilde{e}_L^c\tilde{e}_R^c + \tilde{e}_R^c\tilde{e}_L^c$ production, but this is always $\lesssim 20\%$ of $\tilde{e}_L^c\tilde{e}_L^c + \tilde{e}_R^c\tilde{e}_R^c$ production [24, 25] and is negligible at LEP I. There is no production of $\tilde{\nu}_L^c\tilde{\nu}_R^c$ pairs.

The differential cross-section for like-handed slepton pairs is given by [25, 37, 38]

$$\frac{d\sigma}{dz} = \frac{1}{128\pi} \frac{F_z^3}{s E} \phi(z), \quad (3.31)$$

with

$$\phi_L/R (z) = (1-z^2) \left[ \frac{8\alpha^4}{s} + \frac{2\alpha^2(\alpha^2+\beta^2)\beta}{s} + \frac{8\alpha^3\lambda(s-m^2)}{(s-m^2_\beta)^2 + m^2_{\beta/2}^2} \right], \quad (3.32)$$

*) By a left- (right-) handed slepton is meant the superpartner of the corresponding left- (right-) handed lepton.
\[
\phi^{\pm}_{L/R}(z) = \phi^{\pm}_{L/R}(z) + \mathcal{B}_{L/R} \frac{B^4 s(2E\xi - \zeta^2 - m_e^2)}{p^2 (2E\xi - m_e^2 + m_e^2)^2} \\
+ \mathcal{C}_{L/R} \frac{C^4 s(2E\xi - \zeta^2 - m_e^2)}{p^2 (2E\xi - m_e^2 + m_e^2)^2} + \mathcal{D}_{L/R} \frac{D^4 s(2E\xi - \zeta^2 - m_e^2)}{p^2 (2E\xi - m_e^2 + m_e^2)^2} \\
- \frac{4e^2 C^2 (1 - z^2)}{(2E\xi - m_e^2 + m_e^2)} - \frac{4e^2 B^2 (1 - z^2)}{(2E\xi - m_e^2 + m_e^2)} - \frac{4e^2 D^2 (1 - z^2)}{(2E\xi - m_e^2 + m_e^2)} \\
+ \frac{2eA^2 (\alpha + \beta) (s - m_e^2)}{[s - m_e^2 + m_e^2] (2E\xi + m_e^2 - m_e^2)} \\
- \frac{2eB^2 (\alpha + \beta) (s - m_e^2)}{[s - m_e^2 + m_e^2] (2E\xi + m_e^2 - m_e^2)} \\
- \frac{2eA^2 (\alpha - \beta) (s - m_e^2)}{[s - m_e^2 + m_e^2] (2E\xi + m_e^2 - m_e^2)} \\
+ \frac{B^2 C^2 s (1 - z^2)}{(2E\xi + m_e^2 - m_e^2) (2E\xi + m_e^2 - m_e^2)} + \frac{B^2 D^2 s (1 - z^2)}{(2E\xi + m_e^2 - m_e^2) (2E\xi + m_e^2 - m_e^2)} \\
+ \frac{C^2 D^2 s (1 - z^2)}{(2E\xi + m_e^2 + m_e^2) (2E\xi + m_e^2 + m_e^2)}.
\] 

where \(E\) is the beam energy, \(s = 4E^2\), \(p\) is the slepton momentum, \(z = \cos \theta\) (\(\theta\) being the angle between the beam electron and the negative slepton), and \(\zeta = E - pz\). In Eq. (3.33), the upper (lower) signs refer to left- (right-) handed sleptons. We have suppressed the subscripts \(L\) and \(R\) on the constants \(A\), \(B\), \(C\), and \(D\) which are defined below along with the other relevant constants. We have

\[
\alpha = \frac{1}{4} (3 \tan \theta_w - \cot \theta_w), \quad \beta = \frac{1}{4} (\cot \theta_w + \tan \theta_w),
\]

\[
A_L = 4e (\tan \theta_w - \cot \theta_w), \quad A_R = 2e \tan \theta_w,
\]
\begin{align}
B_L &= \frac{\sqrt{2}e}{2\mu_-} \left( 1 - \frac{N_1m_Z}{\mu_-} (\tan \theta_w - \cot \theta_w) \epsilon_1 \right), \\
B_R &= \frac{\sqrt{2}e}{2\mu_-} \left( 1 - \frac{N_1m_Z}{\mu_-} \tan \theta_w \epsilon_1 \right), \\
C_L &= \frac{N_1m_Z}{\mu_-} e(\tan \theta_w - \cot \theta_w) + \frac{\sqrt{2}e\epsilon_1}{2}, \\
C_R &= 2 \frac{N_1m_Z}{\mu_-} e \tan \theta_w + \frac{\sqrt{2}e\epsilon_1}{2}, \\
D_L &= -\frac{N_1m_Z}{\mu_-} e(\tan \theta_w - \cot \theta_w), \\
D_R &= 2 \frac{N_1m_Z}{\mu_-} e \tan \theta_w.
\end{align}

Here $\mu_-$ is an input parameter which reduces to the zino mass in the absence of tree-level gaugino masses. The other parameters that enter in the definition of the parameters $A$ to $D$ are defined by

$$
\mu_+ \equiv \frac{m^2_Z}{\mu_-} \quad \text{and} \quad N_{1,2} \equiv \left[ \frac{\mu_+}{2(\mu_+ + \mu_-)} \right]^{1/2}.
$$

The tree-level gaugino masses are $(-\mu_1)$ and $(-\mu_2)$, which are related by grand unification as $\mu_1/\mu_2 = (5/3) \tan^2 \theta_w$. One of these is chosen as an input to determine the photino mass $m_\gamma = -\mu_1 \cos^2 \theta_\gamma + \mu_2 \sin^2 \theta_\gamma$. As in section 3.1.2, light and heavy zino masses are related by $m^2_Z = \mu_2 - [\mu_1/(\mu_+ + \mu_-)] \mu_1 \sin^2 \theta_w + \mu_2 \cos^2 \theta_w$. Finally, $\epsilon_1 \equiv (N_1m_Z/\sqrt{2\mu_-}) \sin 2\theta_w (\mu_2 - \mu_1)$. The terms involving $\epsilon_1$ arise precisely because of the additional mixings appearing due to non-zero tree-level masses of the gaugino.

The cross-section for $\tilde{e}_L \tilde{\nu}_R$ production is again given by Eq. (3.31) with [37, 38]

$$
\left( \frac{P_z^2}{s} \right) \tilde{e}_L \tilde{\nu}_R (z) = \left( \frac{P_z^2}{s} \right) \tilde{e}_R \tilde{\nu}_L (z) =
$$

$$
\frac{B^2_\gamma B^2_\gamma m^2_\gamma}{L/R \gamma} + \frac{C^2_\gamma C^2_\gamma m^2_\gamma}{L/R \gamma} + \frac{D^2_\gamma D^2_\gamma m^2_\gamma}{L/R \gamma} + \frac{2B_\gamma B_\gamma C_\gamma C_\gamma m_\gamma}{L/R \gamma} + \frac{2C_\gamma C_\gamma D_\gamma D_\gamma m_\gamma}{L/R \gamma} + \frac{2B_\gamma B_\gamma D_\gamma D_\gamma m_\gamma}{L/R \gamma} + \frac{2B_\gamma B_\gamma C_\gamma C_\gamma m_\gamma}{L/R \gamma} + \frac{2C_\gamma C_\gamma D_\gamma D_\gamma m_\gamma}{L/R \gamma} + \frac{2B_\gamma B_\gamma C_\gamma C_\gamma m_\gamma}{L/R \gamma} + \frac{2C_\gamma C_\gamma D_\gamma D_\gamma m_\gamma}{L/R \gamma} + \frac{2B_\gamma B_\gamma D_\gamma D_\gamma m_\gamma}{L/R \gamma} \left( 2E \bar{z}_e^2 - m^2_\gamma \right) \left( 2E \bar{z}_e^2 - m^2_\gamma \right) \left( 2E \bar{z}_e^2 - m^2_\gamma \right)
$$

$$
\left( 2E \bar{z}_e^2 - m^2_\gamma \right) \left( 2E \bar{z}_e^2 - m^2_\gamma \right) \left( 2E \bar{z}_e^2 - m^2_\gamma \right)
$$

(3.36)
For the final state we have (as discussed earlier) assumed a 100% branching fraction for the $\tilde{\tau}^+ \rightarrow \tilde{\tau}_L^+ \tilde{\tau}_R$ mode. We now turn to the rates and distributions for the final states obtained using Eqs. (3.33) and incorporating the cuts and resolutions discussed earlier.

**$e^+e^-$**

Shown in Fig. 3.3a is the total cross-section including acceptance and $p_T$ cuts for selectron pair-production. Also shown separately is the cross-section for opposite-handed selectron pairs. A few comments regarding these cross-sections are in order:

i) The $\tilde{e}^-L_L^- + \tilde{e}^-R_R^-$ cross-sections are almost independent of the zino and photino masses (for $0 \leq m_\gamma \leq 10$ GeV) although the individual component cross-sections depend on $m_\gamma$, primarily through the altered mixings induced by non-zero gaugino masses. Throughout this paper, we have chosen $|\mu_\gamma| = 10$ GeV corresponding to $m_\gamma = 5.9$ GeV.

ii) The production of opposite-handed selectrons occurs only via $t$-channel exchanges of the photino and the zinos, the corresponding amplitude being proportional to the mass of the exchanged particle. Thus this cross-section is quite sensitive to the photino and zino masses [37]. For $0 \leq m_\gamma \leq 10$ GeV and for a wide range of zino masses, it is always $\leq 25\%$ of that of the $\tilde{e}^-L_L^- + \tilde{e}^-R_R^-$ cross-section and is negligible at LEP I.

**Fig. 3.3** The total cross-sections for $e^+e^- \rightarrow \tilde{\tau}_L^+ \tilde{\tau}_R^-$ for (a) $l = e$ and (b) $l = \mu$, with the acceptance and $p_T$ cut incorporated, as functions of the centre-of-mass energy $\sqrt{s}$. The hollow points refer to the total cross-section, whereas the solid points in (b) refer to the cross-section for opposite-handed selectron pairs as discussed in the text. The $t$-cut has not been included. These graphs have been calculated with $m_\gamma = 6$ GeV, $m_\zeta = 70$ GeV.
The cross-section for smuon pair-production—in this case there is only $\tilde{\mu}_L\tilde{\mu}_L$ and $\tilde{\mu}_R\tilde{\mu}_R$ production, since only s-channel graphs contribute—is shown in Fig. 3.3b. This cross-section is obviously independent of the gaugino masses, except for a small dependence on $m_\tau$ due to the effect of our cuts on the altered phase space in the decay of the smuon.

It is clear from Figs. 3.3a and 3.3b that slepton pair-production, if kinematically allowed, would lead to several hundred to several tens of thousands of events per 100 pb$^{-1}$ at LEP, the precise number depending on the mass of the slepton and the beam energy. The only Standard Model background we have not removed in these graphs is from $\tau$ pairs with both $\tau$'s decaying leptonically. The total cross-section for this reaction with both $\tau$'s decaying into the same flavour of lepton at LEP I is between ~5% (for $m_\tau \lesssim 25$ GeV) to ~20% (for $m_\tau \lesssim 40$ GeV) of the slepton pair-production cross-section. Moreover, as discussed earlier, a cut on the opening angle between the leptons completely eliminates the $\tau$'s [29] but reduces the slepton cross-section by only a maximum of 2% at LEP I and 7% at LEP II [37].

The angular distributions for the electrons (muons) coming from the selectron (smuon) decays are shown in Fig. 3.4. We see that the electrons are sharply peaked in the forward direction (except at $s = m_\tau$), whereas the muon distribution is comparatively flat and symmetric. This is just a reflection of the t-channel exchange of a (light) photino that is present in the first case.

Fig. 3.4 The angular distributions of the electrons (muons) coming from selectron (smuon) pair-production. Here $\theta$ is the angle between the incoming electron and the outgoing negative lepton. The acceptances and cuts are the same as in Fig. 3.3.
alone. This sharp peak obviously emphasizes the need for detection capability close to the beam-pipe.

The energy distributions of the leptons are absolutely flat [25] with

$$\frac{(m_1^2 - m_2^2)}{2(E+p)} \leq E_1 \leq \frac{(m_1^2 - m_2^2)}{2(E-p)}.$$  \hspace{1cm} (3.37)

This follows from the fact that the leptons are produced by the two-body decay of a spinless particle and is independent of any model.

A study of the correlations between the two leptons is also of interest for two reasons. i) The nature of the correlations can obviously provide clues to the nature of the underlying dynamics. ii) A study of the end-points of suitable spectre, after taking into account the resolutions and cuts, can yield the masses of the various sparticles, e.g. the missing mass is greater than $2m_\gamma$. Details of these distributions will not be presented here, but can be found in Ref. [37].

\[\tau^+\tau^-\]

Stau production resembles smuon production quite strongly. Both have the same cross-section, and the decays are anticipated to be $\tau(\mu) + \tau(\mu) + \gamma$. However, the signature for staus is somewhat different. As the $\tau$ decays into at least one neutrino, there is missing energy already involved in standard $\tau$-production. Whereas the magnitude of the energy from the neutrinos in $\tau$-decays is similar to that of the $\gamma$ in $\tilde{\nu}$ decays, it has a different angular distribution. At LEP energies the visible decay products from the $\tau$ have nearly the same direction as the $\tau$ itself, whilst the visible decay products from a stau have quite a different direction from that of the original stau. Thus standard $\tau$-pair-production leads to collinear $\tau$-decay products but the visible remnants from stau production are rather acollinear. The $\tau$ pairs are not collinear either in processes like $e^+e^- + \tau^+\tau^-\gamma$ and $e^+e^- + e^+e^-\tau^+\tau^-$. However, all LEP detectors are able to tag the $\gamma$ or the $e^\pm$ and $e^\pm$ over a large range of solid angle. The particles along the beam-pipe which evade detection can lead to pairs which are acollinear in space, but do not influence the momenta in the directions perpendicular to the beam.

These arguments suggest that one uses the acollinearity in the plane transverse to the beam to separate staus from conventional $\tau$ pair-production. For the present study the following properties were required:

i) Two well-separated clusters of particles had to be present. Summing up the momenta $\vec{p}_\alpha$ of charged and neutral particles in each cluster leads to $\vec{p}_{\alpha,C}$ ($\alpha = 1, 2$). The opening angle between the two momenta should be larger than 45°.

ii) To ensure that the clusters are coming from taus, the two clusters had to consist of either one or three charged particles. In the case that both clusters only contained one particle, the two particles had to be different
kinds, e.g. one an electron, the other a muon, or at least one of them a hadron. In this way, events in which only \( e^+e^- \) or \( \mu^+\mu^- \) pairs are detected were rejected.

iii) To allow for an appropriate determination of the \( \tau \) direction, a minimum visible energy of 2 GeV was required for each \( \tau \).

iv) The difference in azimuthal angle of the two clusters \( \Delta \phi = |\phi_1 - \phi_2| \) had to be \( \Delta \phi < \pi - 0.5 \). This criterion is equivalent to the acoplanarity cut used to set limits on stau pairs at PETRA.

To simulate the detector, we assumed that electromagnetic energy could be detected within \( 0.15 < \theta < \pi - 0.15 \) radians with respect to the beam axis. In addition, electrons and muons could be positively identified, as well as the momentum or energy of charged and neutral particles measured, within \( |\cos \theta| < 0.95 \). The momentum resolution was taken as \( \sigma(p)/p = \sqrt{(0.002p)^2 + (0.02)^2} \), and the electromagnetic energy resolution as \( \sigma(E)/E = 0.05/E + 0.01 \). The angular resolution was assumed to be 5 mrad, and good two-particle separation was assumed if the particles were separated by more than 2 mm in space.

Applying the above cuts and using such a detector parametrization reduces the acceptance of stau pairs to about 30%, the precise value depending on the centre-of-mass energy and on the assumed mass of the stau. For a fixed mass the visible energy cut (iii) becomes slightly less important with increasing c.m. energy, but the \( \Delta \phi \) cut (i) becomes more significant.

Increasing the mass of the stau reduces the overall cross-section, but the acceptance increases because the typical values of \( \Delta \phi \) are larger. The accepted cross-section as a function of energy for stau masses of 30 and 60 GeV can be seen in Fig. 3.5. Obviously the accepted rates should allow detection even if the \( \bar{\tau} \) is heavier than 75% of the beam energy.

The background within these cuts from radiative processes and from two-photon collisions is negligible. The \( \tau \gamma \gamma \) background is more than two orders of magnitude smaller than the accepted \( \bar{\tau} \bar{\tau} \) production. The two-photon \( \tau \) pair-

---

* Note that the \( \Delta \phi \) distributions of the background processes are rather independent of the c.m. energy.
production process has a high cross-section, but its contribution can be further suppressed by demanding that neither the electron nor the positron is detected in the typical $2\gamma$ taggers of the LEP detectors ($45 < \theta < 115$ mrad), and in particular that the invariant mass of the detected $\tau^+\tau^-$ decay products exceed $m_{\min} = 0.075$ GeV. These additional cuts hardly affect the stau pair acceptance but reduce the background from $e^+e^- \rightarrow e^+e^-\tau^+-\tau^-$ to less than $10^{-2}$ of the possible $\tau\tau$ signal.

Thus, staus can be detected at LEP with a unique signature up to masses of at least 60 GeV.

b) Wino pair-production

As discussed earlier, if sleptons are heavier than winos, wino pair-production may well be the most prolific source of sparticles at LEP. In this case, the winos would decay via the three-body modes $\tilde{W} \rightarrow q\bar{q}\gamma$, $\tilde{W} \rightarrow l\nu\gamma$. It is worth pointing out that in almost all super-symmetric models with a light photino, the wino is expected to be lighter than the $W$ boson and hence should be produced, at least at LEP II.

Wino pair-production in $e^+e^-$ collisions takes place via the s-channel exchange of the photon and the $Z^0$ and the t-channel exchange of the sneutrino [27, 39]. The cross-section is given by [39]:

$$\sigma(e^+e^- \rightarrow \tilde{W}\tilde{W}) = \frac{e^4}{128\pi s} \frac{P}{E}$$

$$\times \left( \frac{2}{3} \left[ \frac{16}{3} \left[ \frac{2s}{(s-m_Z^2)} \cot^2 \theta_W \left( \frac{1}{2} \cot^2 \theta_W \right) \left( \cot^2 \theta_W + 5 \tan^2 \theta_W - 2 \right) \right] + \frac{8(s-m_Z^2) \cot \theta_W}{(s-m_Z^2) + m_{\tilde{W}}^2} \left( 1 - \frac{2}{3} \cot^2 \theta_W \left( \cot^2 \theta_W - 3 \tan \theta_W \right) \right) \right]$$

$$+ \frac{f^4}{\sin^4 \theta_W} \left[ 2 - \frac{m_{\tilde{W}}^2 - m_{\tilde{W}}^2}{pE} \ln \frac{2E(E+p) + m_{\tilde{W}}^2 - m_{\tilde{W}}^2}{2E(E-p) + m_{\tilde{W}}^2 - m_{\tilde{W}}^2} + \frac{2(s-m_{\tilde{W}}^2)^2}{(2E^2 + m_{\tilde{W}}^2 - m_{\tilde{W}}^2) - s \left( \frac{1}{4}s-m_{\tilde{W}}^2 \right)} \right]$$

$$- \left[ 1 - \frac{2}{s} \frac{(s-m_{\tilde{W}}^2)^2 + m_{\tilde{W}}^2 s}{2Eps} \ln \frac{2E(E+p) + m_{\tilde{W}}^2 - m_{\tilde{W}}^2}{2E(E-p) + m_{\tilde{W}}^2 - m_{\tilde{W}}^2} \right]$$

$$\times \left[ \frac{4f^2}{\sin^2 \theta_W} + 2(\cot^2 \theta_W - 1) \frac{f^2}{\sin^2 \theta_W} \left[ \frac{1}{2} \cot^2 \theta_W \left( 1 - \frac{2}{3} \cot^2 \theta_W \right) \right] \frac{s(s-m_{\tilde{W}}^2)}{(s-m_{\tilde{W}}^2) + m_{\tilde{W}}^2} \right]$$

(3.38)
where
\[ \begin{align*}
\mathcal{K}^2 &= \frac{m_w^2 - \mu_2}{m_w^2 + m_w}, \\
\mathcal{T}^2 &= \frac{m_w^2 + \mu_2}{m_w^2 + m_w}
\end{align*} \]
and \( \mu_2 \) is as defined in subsection 3.1.2. The wino mass is a free parameter and the mass of the heavier wino is related to this by
\[ m_{\tilde{w}}^2 = \mu_2 + \frac{m_{\tilde{w}}^2}{(m_{\tilde{w}}^2 + \mu_2)}; \]

\( E \) is the beam energy \( \sqrt{s}/2 \), and \( p \) is the momentum \( \sqrt{E^2 - m_w^2} \). The above formulae for the mixings \( f \) between the gaugino and higgsino to give the wino mass eigenstate are for the class of supersymmetry models in which the two Higgs vacuum expectation values are equal.

An estimate for this cross-section can be obtained using Eq. (3.38) (see Fig. 3.6a). In order to incorporate the cuts and acceptances discussed earlier, we have performed a Monte Carlo study [37] for this reaction. We have weighted

![Graphs](image)

Fig. 3.6 a) The total cross-section for wino pair-production as a function of \( \sqrt{s} \) without any acceptance corrections or cuts, for \( m_w = 10 \text{ GeV} \) and various wino masses. b) The cross-section times (branching ratio)² for \( e^+ e^- \rightarrow WW + l\nu\nu\gamma\gamma \) for \( l = e \) with the acceptance and \( p \) cut incorporated, plotted as a function of the centre-of-mass energy, taking \( m_Z^2 < 6 \text{ GeV} \), \( m_w = 70 \text{ GeV} \), \( m_w = 70 \text{ GeV} \) and various wino masses. The \( \tau \) cut has not been included.
the events with the square of the complete production and decay matrix element,
\[ M_p(e^+e^-\to \tilde{W}^+\tilde{W}^-) \times M_d(\tilde{W}^+ + 1\nu) \times M_d(1\nu) \] so as to take into account correctly the correlations arising due to the wino spin. The formula for this is too lengthy to be reproduced here but can be found in the EYB [40].

The width and branching ratio for \( \tilde{W} \to 1\nu \) assuming \( m_\tilde{W} = m_\gamma \) (and \( m_\gamma = 0 \)) \(^4\) are given by [27, 40]

\[
\Gamma(\tilde{W} \to 1\nu) = \frac{e^4 f_\gamma^2 m_\tilde{W}^2}{256 \pi^3 \sin^2 \theta_\tilde{W}} \left[ 5 \beta^2 - 4 - \frac{1}{3} \beta^2 + \frac{1}{\beta^2} - (3\beta^4 - 4\beta^2 + 1) \ln \left( \frac{\beta^2}{\beta^2 - 1} \right) + \left( \frac{1-\beta^2}{\beta^2} \right) \ln \left( \frac{1-\beta^2}{\beta^2} \right) - 2 \beta^2 \left( 1-\beta^2 \right) \right] \frac{\beta^2 \xi}{\beta^2 (\beta^2 - 1)} \ln \left( \frac{\beta^2}{\beta^2 - 1} \right),
\]

(3.39a)

where \( \beta = m_\tilde{e}/m_\tilde{W} \) and \( \beta = m_\tilde{W}/m_\tilde{W} \),

and

\[
\frac{\Gamma(\tilde{W} \to 1\nu)}{\Gamma(\tilde{W} \to \text{everything})} = \left( 9 - \frac{8}{3} \beta \right)^{-1},
\]

(3.39b)

respectively, with

\[ r = \frac{\Gamma(\tilde{W} \to 1\nu)}{\Gamma(\tilde{W} \to \text{everything})}, \]

(3.40)

and

\[
\Gamma_1 = \frac{e^4 f_\gamma^2 m_\tilde{W}^2}{256 \pi^3 \sin^2 \theta_\tilde{W}} \left[ 3 \beta^2 - \frac{5}{2} - (3\beta^4 - 4\beta^2 + 1) \ln \left( \frac{\beta^2}{\beta^2 - 1} \right) \right].
\]

(3.41)

In deriving (3.39b) we have assumed that \( m_\tilde{W} > m_\gamma \) so that the decay \( \tilde{W} \to q\bar{q}q \) is kinematically suppressed. The \( 1\nu \) branching fraction is between 1/7 and 1/9 over a range of wino and slepton masses. For most values of these masses it is close to 1/9 (which is also a lower bound) and approaches 1/7 only when \( m_\tilde{W} = m_\gamma \).

As in the case of slepton pairs, the experimental signatures are a pair of acoplanar leptons plus \( \not{P}_T \) and \( \not{P}_T \).

The product of cross-section and branching ratios for electron pairs [37] is shown in Fig. 3.6b. We recall that, depending on the mass values, the electron branching ratio varies between 0.10 and 0.14. One can also replace the di-electron final states by dimuon with a similar rate, up to a correction factor which is close to one because of the slight differences in the muon and electron

\(^4\) Only the branching ratio is relevant in the calculation, and we have assumed that it is independent of \( m_\gamma \) for \( 0 \leq m_\gamma \leq 10 \) GeV.
acceptances. We see that at the $Z^0$ peak the rate is very large. This is mostly due to the fact that the wino contains (for $v_1 = v_2$) a substantial gaugino ($I = 1$) component which has a large coupling to the $Z$. We also note that the cross-section is not very sensitive to the photino mass: at $s = 180$ GeV, varying $m_\gamma$ from 0 to 10 GeV changes the cross-section by $\pm 5\%$. For a wino mass of 60 GeV the accepted cross-sections are of the order of 0.04 pb and are not shown in Fig. 3.6b. Thus a substantial number of events of the type

$$e^+e^- + \bar{\nu} + \bar{\nu} + l\bar{\nu} + \bar{\nu}$$

may be expected at LEP. The largeness of the wino cross-section allows us to focus on the clean (though suppressed) leptonic decay of the wino. This process can be easily distinguished from slepton pair-production in that unlike the $\tilde{t}$ or $\tilde{\mu}$ cases the energy distribution of the leptons is not flat, and also there are almost the same number of $e^+\mu^- + \mu^+e^-$ pairs as of $e^+e^- + \mu^+\mu^-$ pairs [27].

We now turn to a study of the energy and angular distributions of the outgoing leptons. These are shown in Figs. 3.7a and 3.7b respectively.

A few comments are in order:

i) As stated above, the energy distributions are not flat, unlike the slepton case.

ii) Although the wino distributions are forward-backward symmetric at the $Z^0$ pole, the spin correlation referred to earlier causes the lepton to come out in the forward direction.

iii) Off the $Z^0$ pole, the distributions are governed not only by the spin effects but also by the interference between the s-channel $Z^0$ and $\gamma$ exchanges and the t-channel sneutrino exchange. In fact, below the pole, due to the change in sign of this interference, the distributions actually peak backward.

iv) Off the $Z^0$ pole, the peaking of the angular distributions is not as pronounced as in the slepton case, reflecting the fact that the sneutrino mass is much greater than the photino mass.

As has already been mentioned, the formulae for obtaining these distributions may be found in EYB [40]. A study of the correlations between the leptons is also of interest. For reasons of brevity, we have, as in the slepton case, not included any curves for this but refer the reader to Ref. [37] for results on these.

Before concluding this section, we note that in addition to the dilepton events discussed here, wino pair-production will also lead (via hadronic decays of the wino) to mono (or di) jet + $p_T$ events and mono (or di) jet + lepton + $p_T$ events at a much higher rate. We have chosen to focus on the clean dileptonic signature, but emphasize that for every such event there will be several events
Fig. 3.7a The energy distribution of the charged lepton arising from the three-body decay of the wino for various wino masses.

Fig. 3.7b The angular distributions of the electrons coming from wino pair-production. Here $\theta$ is the angle between the incoming electron and the outgoing negative lepton. The acceptance and cuts are the same as in Fig. 3.3.

with jets, which will serve to distinguish wino pair-production from either slepton pair-production (no jet events) or zino-photino production, as discussed in the next paragraph (no jet + lepton events). Further discussions of this can also be found in Ref. [37].
c) Zino-photino production

As has already been noted, if \( m_\chi \) (and \( m_\gamma \)) and \( m_\gamma /2 \), the zino-photino channel may be the first two-body supersymmetry process to be accessible at LEP [29]. This is of particular interest since \( m_\gamma < m_\gamma \) in a wide class of models. Moreover, it has been argued [29] that this process, because of its characteristic signature, is free of Standard Model backgrounds.

The total cross-section for the process is given by [37, 41]

\[
\sigma = \frac{k}{32 \pi s} \frac{1}{E} \left( B^2 c^2 + B^2 c^2 \right) \times \left[ 1 + \frac{(m_\gamma^2 + m_\gamma^2 - 2m_\gamma^2)}{2k/s} \ln \frac{(s - m_\gamma^2 + m_\gamma^2) + 2m_\gamma^2 + 2k/s}{(s - m_\gamma^2 + m_\gamma^2) + 2m_\gamma^2 - 2k/s} \right] \frac{1}{2} \left( m_\gamma^2 + m_\gamma^2 \right) - k^2 s
\]

\[
- \frac{m_\gamma m_\gamma + s}{k/s} \ln \frac{(s - m_\gamma^2 + m_\gamma^2) + 2m_\gamma^2 + 2k/s}{(s - m_\gamma^2 + m_\gamma^2) + 2m_\gamma^2 - 2k/s}, \quad (3.42)
\]

whereas the angular distribution is given by [37, 41]

\[
\frac{d\sigma}{d \cos \theta} = \frac{1}{512 \pi s} \frac{k}{E} \left( B^2 c^2 + B^2 c^2 \right) \times \left[ s^2 - \frac{(m_\gamma^2 + m_\gamma^2)^2 + 4s^2 \cos^2 \theta + 4s^2}{2} \frac{k \cos \theta}{s} \right]
\]

\[
+ \left[ \frac{1}{2} (s - m_\gamma^2 - m_\gamma^2) + k \cos \theta + m_\gamma^2 \right]^2
\]

\[
- \frac{8m_\gamma m_\gamma m_\gamma}{2} \ln \frac{1}{2} \frac{(s - m_\gamma^2 - m_\gamma^2) - k \cos \theta + m_\gamma^2}{(s - m_\gamma^2 - m_\gamma^2) - k \cos \theta + m_\gamma^2 - k^2 s}
\]

\[
(3.43)
\]

with \( k \), the zino momentum, being

\[
k = \frac{1}{2s} \left[ s^2 + \frac{(m_\gamma^2 + m_\gamma^2)^2}{2} - 2s \left( m_\gamma^2 + m_\gamma^2 \right) \right]^{1/2}. \quad (3.44)
\]
In deriving the Monte Carlo distributions for the final state, we have assumed a factorization of the production and decay processes, i.e., we have ignored correlations due to the zino spin. The relevant squared and spin-averaged matrix element for the decay is given by \[37\]

\[
\frac{1}{2} |M(\tilde{z} \to l^+ l^-)|^2 = 2(B^2_{L^+ L^-} + 2 B^2_{R^+ R^-})
\]

\[
\times \left[ \frac{(P \cdot p_{\gamma})(p_{\gamma} \cdot p_{-})}{[(P \cdot p_{-})^2 - m^2_{\tilde{\ell}}]^2} + \frac{m^2_{\tilde{\ell}}}{[(P \cdot p_{-})^2 - m^2_{\tilde{\ell}}][(P \cdot p_{-})^2 - m^2_{\ell}]} \right]
\]

(3.45)

where \(P, p_{\gamma}, p_{-},\) and \(p_{-}\) are the four-momenta of the parent zino, the positron, the electron, and the photino, respectively. Finally, the branching ratio (for \(m^2_{\tilde{\ell}} = 0\)) per lepton flavour is given by \[29\]

\[
BR = \left[ (1 - \tan^2 \theta_w)^2 + 4 \tan^4 \theta_w \right]^{1/2}
\]

\[
\left[ n_1 \left( (1 - \tan^2 \theta_w)^2 + 4 \tan^4 \theta_w \right) + 3n_u \left[ \frac{14}{9} \left( 1 - \frac{1}{3} \tan^2 \theta_w \right)^2 + \frac{64}{61} \tan^4 \theta_w \right] \right]
\]

\[
+ 3n_d \left[ 9 \left( 1 + \frac{1}{3} \tan^2 \theta_w \right)^2 + \frac{4}{61} \tan^4 \theta_w \right] \right].
\]

(3.46)

For \(n_1 = n_d = 3, n_u = 2,\) this is 13.2%.

The results for the total cross-section, including the acceptance, \(p_T\) and \(\tau\) cuts discussed earlier, are shown in Fig. 3.8. We emphasize that although the

Fig. 3.8 The cross-section for \(e^+ e^- + Z\gamma\) followed by \(Z \to l^+ l^-\) as a function of \(s\) for varying zino masses. The cross-section shown has been corrected for the acceptance and has both the \(p_T\) and the \(\tau\)-cuts discussed in the text. We have taken \(m_\gamma = 40\) GeV, \(m_\tilde{\ell} = 6\) GeV. The expected cross-section from \(Z \to \mu^+ \mu^- \gamma\) is, except for the slight difference in the acceptance for muons, identical to this and so the total event rate expected (neglecting taus) is about twice that shown. All the curves were calculated with a leptonic branching ratio of 11%.
cross-sections are very small, the events are 'gold-plated' in that all Standard Model backgrounds have been eliminated. We have chosen $E_0$ [see Eq. (3.30)] so as to maximize the signal. For $\sqrt{s} = 93$ GeV (LEP I), we find $E_0 = 3$ GeV for $m_Z = 60$ GeV [37]. For lighter zinos, a slightly lower value of $E_0$ is preferred since the leptons have smaller energies but the gain in efficiency is negligible. For $\sqrt{s} = 180$ GeV, the optimal value is $E_0 = 4$ GeV [37].

As before, we have fixed $m_V = 6$ GeV and have also fixed $m_e = 70$ GeV. It is clear that increasing the beam energy is of no advantage. The smallness of the cross-section is due to the largeness of the mass of the selectron that is exchanged in the t-channel and not due to phase space.

The angular distribution of the outgoing leptons is flat, reflecting the rather isotropic production of the zino. More characteristic is the distribution in the opening angle between the leptons. These are, roughly speaking, determined by $m_Z^2/\sqrt{s}$. For small values, the zino is fast moving and the leptons emerge with a relatively small angle between them, whereas for larger values of the zino mass the distribution peaks at larger values of the opening angle.

Thus, in principle, zino production can lead to the characteristic signature of two leptons in one hemisphere recoiling against 'nothing' [29]. These distributions are shown in Fig. 3.9 at the centre-of-mass energies of 70, 93.8, and 180 GeV for $m_Z = 30$ GeV. As already mentioned, the angular distributions for other zino masses can be obtained by scaling in $m_Z^2/\sqrt{s}$.

![Fig. 3.9](image.png)

Fig. 3.9 The distribution in opening angle between the leptons for the zino-photon process. In this figure, $m_Z$ is fixed at 40 GeV. We see that as $m_Z^2/\sqrt{s}$ decreases the leptons tend more and more to be in one hemisphere, as discussed in the text. The distributions for other zino masses can be inferred because to a great extent they depend only on $m_Z^2/\sqrt{s}$. 
We note here that a study of the missing energy and invariant mass distributions may also be of interest since kinematics requires

\[
\left( \frac{(s + m^2)}{t} \right)^2 - \frac{m_Z^2}{2s} \tag{3.47a}
\]

and

\[
m_{\omega^+} < m_Z - m_\gamma. \tag{3.47b}
\]

Thus, apart from the effects of cuts and resolutions, a study of these endpoints may give us an idea of the sparticle masses.

We note that in addition to the leptonic decay modes of the zino, the hadronic decays will yield jet + $p_T$ and dijet + $p_T$ events depending on whether the jets are resolved or not—an experimental question that has not been studied in detail. It is worth emphasizing that the handful of leptonic events that emerge from zino decay must be accompanied by about three to four times as many hadronic events which may thus provide a corroboratio of the zino signal.

3.2.2 Three-body final states

a) Radiative processes: photino and sneutrino searches

Two interesting supersymmetric particles which can still be searched for even within the PETRA-PEP energy region are the photino and sneutrino. They can be pair-produced, and the events tagged by a bremsstrahlung photon in the final state. Here one uses the photon accompanied by no other detectable particles in the final state to identify the production of non-interacting neutrino-like particles in $e^+e^-$ annihilation, in our cases

\[
e^+ + e^- \rightarrow \gamma + \tilde{\nu} + \tilde{\nu} \tag{3.48}
\]

\[
\rightarrow \gamma + \tilde{\nu} + \tilde{\nu}. \tag{3.49}
\]

The details of the calculation of the production of photino pairs can be found in Ref. [33] and we will not repeat the details here. The relevant formulae for the cross-sections are listed in Appendix A [42].

A complete and reliable calculation of sneutrino pair-production is not available in the literature [43]. Thus we present here details of our own calculations [44] for this process. The Feynman diagrams used for our calculations are shown in Fig. 3.10. The amplitudes are listed in Appendix B and have been calculated numerically for several sets of parameter values. In Fig. 3.11a we show the cross-sections as functions of the sneutrino mass for several values of the wino mass and centre-of-mass energy. The cross-section with vanishing sneutrino mass is shown in Fig. 3.11b as a function of the wino mass.
Fig. 3.10 The Feynman diagrams for the production of photino and sneutrino pairs in association with a radiated photon, i.e. reactions (3.46) and (3.49)

Fig. 3.11a The production cross-sections of sneutrino pairs together with a photon, as functions of the sneutrino mass for three different sets of the centre-of-mass energies and wino masses.

Fig. 3.11b The production cross-sections of sneutrino pairs together with a photon, as functions of the sneutrino mass at centre-of-mass energies \( \sqrt{s} = 44, 93 \) and 160 GeV.
Fig. 3.12 The expected numbers of single high $p_T$ photon events per 100 pb$^{-1}$ of running, as functions of the centre-of-mass energy, for the pair-production of photinos, sneutrinos and 3 types of neutrinos in association with a photon, for the two different sets of cuts indicated in (a) and (b).

Finally, the expected rates for the production of photino pairs, sneutrino pairs, and neutrino pairs with the experimental photon acceptance described in subsection 3.1.3 are shown in Fig. 3.12a as functions of the centre-of-mass energy. To demonstrate the sensitivity of the rates to the acceptance, we show in Fig. 3.12b the rates with slightly different cuts. One sees that even the peaks of the rates depend critically on the acceptance. One can also see from Fig. 3.12 that to search for sneutrinos one would like to run the machine energy below the $Z^0$ peak, say around the 70 to 80 GeV region where the background from neutrinos is not too large. To search for photinos, the PETRA energy around 40 GeV is the most favourable region. The experimental signature for all three processes is an excess of events with a single high-$p_T$ photon. Since photino, sneutrino, and neutrino production cross-sections have very different energy dependences, if one observes such a signal one must measure the energy dependence of the cross-section in order to pin down the source.

b) Associated production

If the masses of the supersymmetric particles are larger than the beam energy, one can still look for sparticles via associated production processes where one
heavy sparticle is produced together with a lighter but hard to detect neutral sparticle such as a photino or sneutrino. Other reactions involve three final-state particles in order to conserve quantum numbers. Two reactions which are particularly interesting but very tedious to calculate are

\[ e^+ + e^- \rightarrow e + \bar{e} + \gamma, \]  

and

\[ e^+ + e^- \rightarrow e + \tilde{\nu} + \nu, \]  

where each reaction involves a dozen or so diagrams with both s- and t-channel contributions. The experimental signals of these processes depend on the centre-of-mass energies as we now describe.

1) At centre-of-mass energies far away from the \( Z^0 \) peak, the reaction is dominated by the t-channel contribution. The final-state electron often escapes detection because it is predominantly produced in the small-angle region. Therefore the most important signature in a typical detector is the production of a single high-\( p_T \) lepton without other detectable particles.

2) Near the \( Z^0 \) peak, these processes are often dominated by virtual pair-production intermediate states in the s-channel. As a result, the rate of coplanar lepton pairs, similar to what were described in subsection 3.2.1 on pair-production, becomes larger than that of the single lepton. This explains why the relative rates of the single-lepton channel and the coplanar di-lepton channels depend on whether the centre-of-mass energy is near the \( Z^0 \).

This reaction (3.50) is important mainly in the case that the photino mass is much smaller than the selectron mass. In these circumstances the main decay channel of the selectron is the \( e\gamma \) final state. The Feynman diagrams for this reaction in the energy region we are interested in are shown in Fig. 3.13, and the amplitudes are given in Appendix C. We present the rates for two separate classes of events distinguished by the final observable particles, i.e. i) the class of events with a single electron only, and ii) the class of events with a pair of coplanar electrons. Figure 3.14a shows the expected rates for the associated production of \( e\gamma \) with zero photino mass at the \( Z^0 \) peak. Similarly, Figs. 3.14b and 3.14c show the rates at 44 and 160 GeV. To illustrate the dependence on the photino mass, we show in Fig. 3.15...
Fig. 3.14 The expected rates for both the single-\(e\) and the acoplanar dielectron final states from reaction (3.50), calculated with zero photino mass as functions of the selectron mass at a centre-of-mass energy of (a) 93 GeV, (b) 44 GeV, and (c) 160 GeV.

Cut conditions

1. For single electron events
   a) \( E \geq 0.2 E_{\text{cm}} \)
   b) \( P \geq \sin 5^\circ \times E_{\text{cm}} \)

2. For double electron events
   a) \( E \geq 0.2 E_{\text{cm}} \)
   b) \( P \geq \sin 5^\circ \times E_{\text{cm}} \)
   c) Acoplanar angle \( \geq 30^\circ \)
   d) Acollinear angle \( \geq 20^\circ \)

Fig. 3.15 Similar to Fig. 3.14a, but with a photino mass of 10 GeV.
the rate at the $Z^0$ peak for a photino mass of 10 GeV. As can be seen from Figs. 3.14 and 3.15, with an integrated luminosity of 100 pb$^{-1}$ at the $Z^0$ peak one can search for the selectron up to a mass of 57 (51) GeV for the case of a photino mass of 0 (10) GeV, respectively.

\[ e^- W^- \]

The complete set of Feynman diagrams included in our calculation of reaction (3.31) is shown in Fig. 3.16. All the 12 diagrams and a total of 78 terms are computed, and the cross-section formulae are given in Appendix D. They do not take into account spin correlations, which introduce negligible error in the total cross-section but may induce small errors in differential final-state distributions. The calculations are first carried out under the idealized assumption that there is no mixing between the shiggs and the wino [46, 47]. In this special case, there are no free parameters and the calculations yield definite predictions. For zero sneutrino mass, the expected rates for events with one muon only and for the $e^- \mu$ type of events are shown in Fig. 3.17a, as a function of the wino mass at a centre-of-mass energy of 93 GeV. The selection criteria for these types of events are shown in the caption to Fig. 3.17. One can also replace the muon by an electron and thus obtain the rate of single-electron and dielectron events, up to a correction factor close to one owing to the slight difference between the muon and electron acceptances. Similarly, Fig. 3.17b, 3.17c, and 3.17d show the expected rates as functions of the wino mass for centre-of-mass energies 44, 110, and 160 GeV, respectively. We see that the single-$\mu$ rate is higher than the $e^- \mu$ rate outside the $Z^0$ region. At the $Z^0$ peak, however, the s-channel diagrams dominate and the $e^- \mu$ rate becomes significantly higher than the single-$\mu$ rate. To study the dependence on the sneutrino

![Feynman diagrams](image)

**Fig. 3.16** Feynman diagrams for the reaction $e^+ + e^- \rightarrow e + \tilde{w} + \tilde{\nu}$
Fig. 3.17 The expected number of single-μ events and of acoplanar e-μ events through reaction (3.51), the associated production of a wino together with a sneutrino and an electron, as functions of the wino mass at a centre-of-mass energy of 93 GeV. The points are for the general case when $v_3$ is not equal to $v_2$ as described in the text. One can replace the muon with an electron and obtain the event rates of single-e and acoplanar di-electron events, after a minor correction factor due to the slight difference in detector acceptances for muons and electrons. The graphs are for $\sqrt{s} =$ (a) 93 GeV, (b) 44 GeV, (c) 110 GeV and (d) 160 GeV.
mass, we show the rate of events at the $Z^0$ peak in Fig. 3.18 for a sneutrino mass of 10 GeV. As seen from Figs. 3.17 and 3.18, at the $Z^0$ peak with 100 pb$^{-1}$ integrated luminosity, one is sensitive to a wino mass of about 70 GeV if $m_\chi = 0$, significantly above the beam energy. This result suggests the importance of the associated production of supersymmetric particles.

**ewW: general case of unequal Higgs v.e.v.'s**

The calculation [37] in this case is very involved. The Lagrangian for the relevant interactions is described in Appendix E [47]. We note the following important modifications to the couplings.

1) There is now a new axial-vector coupling constant in the vertex $ZWW$, characterized by the factor

$$ A_C = \sec^2 \theta_W (\cos^2 \gamma_L - \cos^2 \gamma_R)/4 \ , \quad (3.52) $$

and there is an additional constant in the vector coupling characterized by the factor

$$ V_C = \sec^2 \theta_W (\cos^2 \gamma_L + \cos^2 \gamma_R)/4 \ . \quad (3.53) $$

2) The coupling at the $WW$ vertex is modified by a constant

$$ f_\sigma = \sin \gamma_R \ . \quad (3.54) $$

We now use the following relationships between the masses of the selectron, the photino, the wino, and sneutrino, and the parameters $m_e$, $\mu_1$, $\mu_2$, and $\kappa$
### Table 3.3

Coupling constants and parameters in terms of physical masses

<table>
<thead>
<tr>
<th>Input masses (GeV)</th>
<th>Output parameters</th>
<th>Couplings*</th>
<th>$\alpha'_V/\alpha'_V \gamma'_V/\gamma'_V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$</td>
<td>$x_0$ / $x_0$</td>
<td>$x_0$ / $x_0$</td>
<td>$x_0$ / $x_0$</td>
</tr>
<tr>
<td>$x_0$ / $x_0$</td>
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<td>$x_0$ / $x_0$</td>
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<td>$x_0$ / $x_0$</td>
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<td>$x_0$ / $x_0$</td>
<td>$x_0$ / $x_0$</td>
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<td>$x_0$ / $x_0$</td>
<td>$x_0$ / $x_0$</td>
<td>$x_0$ / $x_0$</td>
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<td>$x_0$ / $x_0$</td>
</tr>
<tr>
<td>$x_0$ / $x_0$</td>
<td>$x_0$ / $x_0$</td>
<td>$x_0$ / $x_0$</td>
<td>$x_0$ / $x_0$</td>
</tr>
</tbody>
</table>

* Note that corresponding to each set of masses, in general there are two sets of coupling constants, of which we only show one here [37].
We ignore the small contribution from the high-mass wino. The parameters $r = \frac{v_1^2 - v_2^2}{v_1^2 + v_2^2}$ and $m_0, \mu_1, \mu_2$ are all defined in subsection 3.1.2 and in Appendix E. The expressions for all the parameters in terms of physical masses can also be found in Appendix E. We can see from the above mass relationships that when $r$ is significantly greater than 0, say

$$r = 2m_2/m_0^2,$$

the mass of the sneutrino can be much smaller than the mass of the selectron, and the associated production process (3.51) becomes a powerful channel to search for SUSY particles such as the wino.

With a given set of masses for the sneutrino, wino, photino, and selectron as shown in the leftmost four columns of Table 3.3, we can calculate the following quantities up to a quadratic ambiguity discussed in Ref. [37]:
1) all the parameters $r, a, m_1, \gamma_1, \gamma_1', \gamma_1''$, which are shown in the next six columns of Table 3.3;
2) the coupling constants $f, A_\gamma, \tilde{V}_c = 1 - V_c$, which are shown in the columns on the right-hand side of Table 3.3;
3) the ratio of the cross-section with respect to that of the previous results where $v_1 = v_2$ was assumed. This ratio can be approximated by the following formula:

$$\frac{\sigma(v_1 = v_2)}{\sigma(v_1 = v_2)} \approx (1 - V_c - A_c)^2$$

which is shown in the rightmost column in Table 3.3. This ratio can be compared with the results of a rigorous calculation for several important cases as described below.

If $A_c$ and $V_c$ are small and $f$ is close to one, the calculation simplifies to the previous special case $v_1 = v_2$, as can be seen in the simplified ratio shown in the rightmost column of Table 3.3. For a few cases when $A_c$ and $V_c$ are large and $f$ is far away from 1, we have carried out a rigorous calculation and we show
Table 3.4
Cross-section for $e^+e^- \rightarrow e^\pm \nu \ell^\mp$ at $\sqrt{s} = 93$ GeV

<table>
<thead>
<tr>
<th>$\theta^e_\gamma$</th>
<th>$\theta^e_\nu$</th>
<th>$\theta^e_\ell$</th>
<th>$\theta^e_R$</th>
<th>$\tau^e_\gamma$</th>
<th>$\tau^e_\nu$</th>
<th>$\tau^e_\ell$</th>
<th>$\tau^e_R$</th>
<th>$\sigma^e_{\text{tot}}$</th>
<th>$\sigma^e_{\text{cut}}$</th>
<th>$\text{Ratio}_{\text{cut}}$</th>
<th>$\text{Estimated}_{\text{cut}}^{c)}$</th>
<th>$f_+^e$</th>
<th>$\lambda^e_c$</th>
<th>$\tilde{\nu}^e_c$</th>
<th>(\pm 1-V_{e_c}^{c)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>70</td>
<td>-</td>
<td>90</td>
<td>90</td>
<td>0.301(2)</td>
<td>0.236(1)</td>
<td>1</td>
<td>1</td>
<td>1.0</td>
<td>1</td>
<td>0.10</td>
<td>0.80</td>
<td>0.00</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>133.0</td>
<td>113.7</td>
<td>0.110(2)</td>
<td>0.101(1)</td>
<td>0.37</td>
<td>0.41</td>
<td>0.92</td>
<td>0.09</td>
<td>0.09</td>
<td>0.56</td>
<td>0.80</td>
<td>0.00</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>30 (3)</td>
<td>30</td>
<td>23.57</td>
<td>42.36</td>
<td>0.0170(2)</td>
<td>0.0138(1)</td>
<td>0.56</td>
<td>0.10</td>
<td>0.67</td>
<td>0.09</td>
<td>0.09</td>
<td>0.56</td>
<td>0.80</td>
<td>0.00</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>40 (3)</td>
<td>60</td>
<td>173.6</td>
<td>58.6</td>
<td>0.0202(1)</td>
<td>0.0148(1)</td>
<td>0.67</td>
<td>0.09</td>
<td>0.85</td>
<td>0.23</td>
<td>0.23</td>
<td>0.60</td>
<td>0.80</td>
<td>0.00</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>60</td>
<td>-</td>
<td>90</td>
<td>90</td>
<td>1.54(1)</td>
<td>1.50(1)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.91</td>
<td>0.04</td>
<td>0.06</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>121.7</td>
<td>113.8</td>
<td>0.856(4)</td>
<td>0.828(4)</td>
<td>0.55</td>
<td>0.56</td>
<td>0.91</td>
<td>0.04</td>
<td>0.04</td>
<td>0.86</td>
<td>0.80</td>
<td>0.00</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>30 (3)</td>
<td>30</td>
<td>29.37</td>
<td>41.25</td>
<td>0.152(1)</td>
<td>0.139(1)</td>
<td>0.099</td>
<td>0.12</td>
<td>0.66</td>
<td>0.06</td>
<td>0.06</td>
<td>0.58</td>
<td>0.80</td>
<td>0.00</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>40 (3)</td>
<td>60</td>
<td>41.13</td>
<td>52.87</td>
<td>0.156(1)</td>
<td>0.139(1)</td>
<td>0.10</td>
<td>0.09</td>
<td>0.80</td>
<td>0.20</td>
<td>0.20</td>
<td>0.57</td>
<td>0.80</td>
<td>0.00</td>
<td>0.60</td>
<td></td>
</tr>
</tbody>
</table>

a) The cut cross-section has $5^\circ \leq \theta^e \leq 175^\circ$.
b) Result of rigorous calculation.
c) Simple ratio $\alpha^e_{\text{cut}}/\alpha^e_{\text{cut}}$.
d) No mixing between spino and Higgsino has been included in this case.
e) Note that the diagonalization of the neutral supersymmetric fermion mass matrix which has been used here is strictly valid only for $m^e_\gamma << m^e_\nu$.

the results in Table 3.4. We see from this table that the simple ratios are in the most cases within 20% of the values obtained with the rigorous calculation.

We conclude that this associated production mechanism is very useful for searching for a wino with mass above the beam energy in the case where the sneutrino is not too heavy.

3.3 Production of strongly interacting sparticles

3.3.1 General framework

We now turn to the possibility of detecting squarks and gluinos at LEP. In the squark search we will assume that the winos and zinos are heavier than the squarks, so that the decays $\tilde{q} \rightarrow q\tilde{\nu}$ and $\tilde{q} \rightarrow q\tilde{\ell}$ are kinematically disallowed, i.e. $\tilde{q} \rightarrow q\tilde{\nu}$ and probably $\tilde{q} \rightarrow q\tilde{\ell}$ are the only allowed decays of the squark. We first turn to the simplest reaction, i.e. squark pair-production. The possibilities of gluin pair-production through a loop diagram and the production of gluinos in association with quarks are discussed subsequently.

3.3.2 Squark pair-production

This is the simplest reaction to look for squarks. It proceeds via direct-channel $\gamma$ and $Z^0$ exchange and has the differential cross-section:
\[
\frac{d\sigma}{d\cos \theta} = \frac{1}{32\pi} \frac{(E^2 - m^2_q)^{3/2}}{4E} \sin^2 \theta \times \left[ \frac{8q^2 e^2}{s^2} + \frac{2e^2 A^2 (m^2 + \rho^2)}{s^2} - \frac{8ag^2 A(s - m^2)}{s[(s - m^2)^2 + m^2 r^2]} \right],
\]

where \( E \) is the beam energy, \( s = 4E^2 \), \( m_q \) is the squark mass, \( e \) is the unit of electromagnetic charge, \( q \) is the squark charge, \( m_\gamma \) is the \( \gamma \) mass, \( \Gamma_\gamma \) is the total \( \gamma \) decay width, and

\[
\alpha = \frac{1}{4} (\cot \theta_w - 3 \tan \theta_w), \quad \beta = -\frac{1}{4} (\cot \theta_w + \tan \theta_w).
\]

The values of \( q \) and of the constant \( A \) are given in Table 3.5 for different-handed squarks. Note that formula (3.58) applies to either \( \tilde{q}_L \tilde{q}_L \) or \( \tilde{q}_R \tilde{q}_R \) production, whilst there is no production of \( \tilde{q}_L \tilde{q}_R + \tilde{q}_R \tilde{q}_L \). The energy-dependence of the total cross-section is shown in Fig. 3.19 for a squark mass of 40 GeV. With an integrated luminosity of \( 3 \times 10^{37} \text{ cm}^{-2} \) (30 pb\(^{-1}\)), one expects 8800 \( \tilde{\mu} \) events and 11,800 \( \tilde{d} \) events if the left- and right-handed squarks are degenerate.

### Table 3.5

**Squark couplings**

<table>
<thead>
<tr>
<th>Squark flavour/helicity</th>
<th>Charge</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_L )</td>
<td>2/3</td>
<td>(-\cot \theta_w + 1/3 \tan \theta_w)</td>
</tr>
<tr>
<td>( u_R )</td>
<td>2/3</td>
<td>(4/3 \tan \theta_w)</td>
</tr>
<tr>
<td>( d_L )</td>
<td>-1/3</td>
<td>(\cot \theta_w + 1/3 \tan \theta_w)</td>
</tr>
<tr>
<td>( d_R )</td>
<td>-1/3</td>
<td>(-2/3 \tan \theta_w)</td>
</tr>
</tbody>
</table>

**Fig. 3.19** Squark-antisquark cross-sections (cm\(^{-2}\)) versus beam energy (GeV)
The preferred mode of $\tilde{q}$ decay depends on the gluino mass. If $m_{\tilde{g}} < m_{\tilde{q}}$, one expects $\tilde{q} \to q + \tilde{g}$ to dominate, but if $m_{\tilde{g}} > m_{\tilde{q}}$ the decay $\tilde{q} \to q + \tilde{\gamma}$ may dominate. In the former case, one expects the $\tilde{g}$ to decay subsequently into $q + \tilde{\gamma}$. For the case of $\tilde{q} \to q + \tilde{\gamma}$ decay one expects

- two jets + O(50)% missing energy,
- moderate sphericity,
whereas in the case of $\tilde{q} \to q + \tilde{q}$ decay one expects

- many jets + less missing energy,
- a more spherical event structure.

Clearly the first case is more favourable for experimental detection.

The dominant background comes from the process $e^+ e^- \to q\bar{q}$, which is expected to give low sphericity and small missing energy. Possible sources of the latter are energetic neutrinos from heavy-quark decays and particles which escape detection or are mismeasured. A crucial background is $e^+ e^- \to t\bar{t}$ events, which tend to have more missing energy and larger sphericity than lighter flavours. With an integrated luminosity of $3 \times 10^{-37}$ cm$^{-2}$ (30 pb$^{-1}$) at the $Z^0$ peak, one expects a total of 750,000 $t\bar{t}$ events if $m_t = 40$ GeV.

Squark-antisquark and $q\bar{q}$ events have been simulated using a Monte Carlo program in which the top mass and the squark mass are both set to 40 GeV. The squarks are generated with their characteristic $\sin^2 \theta$ angular distribution (3.58) and then allowed to decay into quarks. Quark fragmentation is handled using the Lund Monte Carlo model. Gluinos are also allowed to decay before fragmentation, which is perhaps not the correct physical picture but is unlikely to induce large errors. To simulate the acceptance of the experimental apparatus, the final-state particles are assumed to be observed only in a limited angular range. Measurement errors are simulated by Gaussian errors. Table 3.6 summarizes the angular cuts and Gaussian widths assumed for different particle species.

Figure 3.20 is a scatter plot of sphericity versus missing energy. Clearly one can cut on these variables in such a way as to separate signal and back-

<table>
<thead>
<tr>
<th>Particle species</th>
<th>Angular cut</th>
<th>Resolution $\sigma(E)/E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charged</td>
<td>$</td>
<td>\cos \theta</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$</td>
<td>\cos \theta</td>
</tr>
<tr>
<td>Neutral hadrons</td>
<td>$</td>
<td>\cos \theta</td>
</tr>
</tbody>
</table>
Fig. 3.20 Sphericity versus missing energy scatter plots for (a) $q\bar{q}: q + q + \gamma$ events, (b) $qq: q + q + g$ events, (c) $qq$ events excluding $tt$ production
Table 3.7
Numbers of missing energy events on the $Z^0$ peak

<table>
<thead>
<tr>
<th>Missing energy (GeV)</th>
<th>Sphericity</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{q}\tilde{q}$: $\tilde{q} + q + \gamma$ events</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td></td>
<td>16 500</td>
<td>16 000</td>
<td>14 500</td>
<td>11 900</td>
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<tr>
<td>0.3</td>
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<td>13 000</td>
<td>12 600</td>
<td>11 300</td>
<td>9 200</td>
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<tr>
<td>0.4</td>
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<td>9 000</td>
<td>8 700</td>
<td>7 700</td>
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</tr>
<tr>
<td>$\bar{q}\bar{q}$: $\bar{q} + q + \bar{\gamma}$ events</td>
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<td>9 700</td>
<td>4 700</td>
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<td>8 100</td>
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<td>0.4</td>
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<td>$\bar{q}q$ events</td>
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</table>

The table shows the numbers of events surviving different choices of cuts on the missing energy and sphericity. If the decays $\tilde{q} + q + \gamma$ dominate, their detection does not seem very difficult, but if $\tilde{q} + q + \bar{\gamma}$ decays dominate, things are not so easy.

It is clear that the main source of background is $e^+e^- + t\bar{t}$. Therefore, if these events can be selected out, the squark search becomes much easier. LEP experiments are considering the installation of microvertex detectors (made, for example, of silicon microstrips), which would enable heavy-quark events to be
identified from the different impact parameter distributions of their decay products. A Monte Carlo study [48] of the performance of such a detector indicates that it would be used to remove 80% of the $t\bar{t}$ background without a large reduction in the $\tilde{q}\tilde{q}$ signal. Such a removal of the $t\bar{t}$ background is not essential to the $\tilde{q}$ search if $\tilde{q} + q + \gamma$, but would be necessary if $\tilde{q} + q + \tilde{\gamma}$.

### 3.3.3 $e^+e^- + q\bar{q}q\bar{q}$

This reaction [49] has contributions from diagrams such as those in Fig. 3.21. If gluinos are relatively light and squarks are quite heavy, gluinos may be the only strongly interacting sparticles produced at LEP and we may neglect the contribution to the $q\bar{q}q\bar{q}$ cross-section from the diagrams of class (b) in Fig. 3.21. This avoids the uncertainties in the squark masses. To find the cross-section due to the diagrams of class (a), the familiar QCD diagrams [50] are cannibalized and rescaled:

$$\sum_q \sigma(e^+e^- + q\bar{q}q\bar{q}) = \frac{1}{2} R_C \sum_q \sigma(e^+e^- + q\bar{q}q') ,$$

(3.60)

where the ratio of colour factors $R_C = 6$, and the factor of $1/2$ reflects the fact that gluinos are Majorana fermions. The cross-section formula (3.60) is shown, normalized by $\Gamma_q \sigma(e^+e^-+q\bar{q})$, in Fig. 3.22 as a function of $m_{\tilde{q}}$. The small cross-section and lack of a clear signature may hinder detection of $\tilde{q}$ through this reaction.

![Fig. 3.21 Diagrams contributing to $e^+e^- + q\bar{q}q\bar{q}$](image)

![Fig. 3.22 Cross-section for $e^+e^- + q\bar{q}q\bar{q}$ relative to $e^+e^- + q\bar{q}$](image)
3.3.4 $e^+e^- \rightarrow \tilde{g}\tilde{g}, Z^0 \rightarrow \tilde{g}\tilde{g}$

A possibly promising way to hunt for gluinos is through the loop diagrams of Fig. 3.23 which give a $\tilde{g}\tilde{g}$ final state [51, 52]. By charge conjugation, the vector coupling to $\tilde{g}\tilde{g}$ always vanishes, but this is not true for the axial part of the Z coupling since $m_u \neq m_d$ and therefore $C_A \propto T_3$. In practice, the only large contribution comes from the t-b quark mass differences. To search for a large rate, we consider this process at $\sqrt{s} = m_Z$ and neglect the photon contribution.

The $Z^0 \rightarrow \tilde{g}\tilde{g}$ ratio is then given by [52]

$$\frac{\Gamma(Z^0 \rightarrow \tilde{g}\tilde{g})}{\Gamma(Z^0 \rightarrow f\bar{f})} = \left(\frac{q_m}{2\pi^2}\right) \beta^3 \left| \sum_q \sum_{L,R} (\pm 1)_{L,R} \left(Q^q_{L,R} F^1_{L,R} + Q^q_{R,L} F^2_{L,R}\right) \right|^2$$

$$\times \left[ 2n_v + \left[ 1 + (1 - 4\sin^2 \theta_w)^2 \right] n_1 + 3 \left[ 1 + \left( 1 - \frac{8}{3} \sin^2 \theta_w \right)^2 \right] n_u ight]^{-1} + 3 \left[ 1 + \left( 1 - \frac{4}{3} \sin^2 \theta_w \right)^2 \right] n_d \right]^{-1},$$

(3.61)

where

$$F^1 = \int_0^1 dx \int_0^{1-x} dy \left[ \ln \left( \zeta(s, m_q^2, m_g^2) / \zeta(s, m_{q'}^2, m_g^2) \right) \right.$$

$$- \zeta^{-1}(s, m_q^2, m_g^2) \left[ xy - (1-x-y)^2 m_g^2 \right] \left. \right],$$

$$F^2 = \int_0^1 dx \int_0^{1-x} dy \left[ - \zeta^{-1}(s, m_q^2, m_g^2) \times (m_q^2) \right],$$

and

$$\zeta(s, m^2, m^2) = (x+y)(1-x-y)m^2 + xy + (x+y)(m^2 - m^2) - m^2 + i\epsilon.$$
Fig. 3.24 Branching ratio for $Z^0 \to \tilde{g}\tilde{g}$ via the diagrams of Fig. 3.23, assuming the squark masses quoted in the text

The branching ratio (3.61) is shown in Fig. 3.24 in the limit that $m_q = 200$ GeV independent of chirality. We expect that the left-right mass splitting is sizeable only for the stop squark. We see that the branching ratio is quite small ($\lesssim 10^{-4}$), whilst the missing-energy signature is expected to be intermediate in magnitude between the $\tilde{q}\tilde{q}$: $\tilde{q} \rightarrow q + \tilde{\gamma}$ and $\tilde{q} \rightarrow q + \tilde{g}$ cases discussed in subsection 3.3.2. We are not very optimistic about detection of the $\tilde{q}$ in this channel.

3.3.5 Quarkonium decays

There are many possible mechanisms for producing strongly interacting sparticles in toponium decay, which are covered in the contribution to this report by the Toponium Study Group [53].
APPENDIX A: MATRIX ELEMENT AND CROSS-SECTION FOR $e^+e^- \rightarrow \gamma \gamma \gamma \gamma$

The Feynman diagrams contributing to this process in leading order are shown in Fig. A.1. The matrix element due to either $\tilde{e}_R$ or $\tilde{e}_L$ exchange can be written [42, 54] in the form

$$M = (M_1 + M_2 + M_3) - \text{(exchange contribution)}, \quad (A.1)$$

where

$$M_1 = 2e^3 \left[ \bar{v}(p_{z}) \left( \frac{1 - \gamma_5}{2} \right) v(p'_z) \right] \frac{1}{2N_{22}q \cdot p_1} \left[ \bar{u}(p'_1) \left( \frac{1 + \gamma_5}{2} \right) u(p_1) \right],$$

$$M_2 = 2e^3 \left[ \bar{v}(p_{z}) \left( \frac{1 - \gamma_5}{2} \right) v(p'_z) \right] \frac{1}{2N_{11}q \cdot p_2} \left[ \bar{u}(p'_1) \left( \frac{1 + \gamma_5}{2} \right) u(p_1) \right], \quad (A.2)$$

$$M_3 = 2e^3 \left[ \bar{v}(p_{z}) \left( \frac{1 - \gamma_5}{2} \right) v(p'_z) \right] \frac{\epsilon \cdot (2p_1 - 2p'_1 - q)}{N_{11}N_{22}} \left[ \bar{u}(p'_1) \left( \frac{1 + \gamma_5}{2} \right) u(p_1) \right],$$

are the contributions from Fig. A.1a, and the exchange contribution from Fig. A.1b has the same form with $p'_1$ and $p'_2$ interchanged. In Eq. (A.2),

$$N_{11} = \Delta^2 + 2p_1 \cdot p'_1, \quad N_{22} = \Delta^2 + 2p_2 \cdot p'_2, \quad \Delta^2 = m^2_{\tilde{e}_R \text{ or } \tilde{e}_L} - m^2_\gamma. \quad (A.3)$$

The differential cross-section can be obtained from Eqs. (A.1, 2, 3). It takes the following form in the $e^+e^-$ centre-of-mass frame:

$$\frac{d\sigma}{dx \, d\cos \theta} =$$

$$\frac{\alpha^2}{16\pi} \int \left[ \left( T_1 + T_2 + T_3 + T_4 + T_5 + T_6 \right) + \left( p'_1 \cdot p'_2 \right) \right] \left[ T_{14} + (p'_1 \cdot p'_2) + T_{36} \right]$$

$$\times \left( 1 - \frac{4m^2_\gamma}{5} \right)^{1/2} \frac{d\cos \theta \, d\phi}{d\phi'}, \quad (A.4)$$

where $s' = (p'_1 + p'_2)^2$ is the invariant mass of the photino pair, $0 < x \leq (E_\gamma / E) < 1 - m^2_\gamma / E^2$, where $E$ is the beam energy, $E_\gamma$ is the energy of the detected $\gamma$, and $\theta$ is the angle it makes with the beam direction. The angles $\theta'$ and $\phi'$ are the polar and azimuthal angles of one $\gamma$ in the $\gamma - \gamma$ centre-of-mass frame, with the $z$-axis defined opposite to the $\gamma$ direction. The $\gamma$ momenta in the $e^+e^-$ centre-of-mass frame can be written in terms of these angles as
\[ p_1' = \left( (\gamma E_1' - \beta p_1' \cos \theta'), p_1' \sin \theta \cos \varphi', \right), p_1' \sin \theta \sin \varphi', \cos \theta \cos \varphi', \right), \]
\[ (p_1' \sin \theta \sin \varphi' \sin \phi' + \cos \theta (\beta E_1' - \gamma p_1' \cos \theta')) \sin \theta, \]
\[ -\sin \theta (p_1' \sin \theta \sin \varphi' + \cos \theta (\beta E_1' - \gamma p_1' \cos \theta')) \right], \]
\[ (A.5) \]
\[ p_2' = \left( (\gamma E_1' + \beta p_1' \cos \theta'), -p_1' \sin \theta \cos \varphi', \right), -p_1' \sin \theta \sin \varphi', \cos \theta \cos \varphi', \right), \]
\[ (-p_1' \sin \theta \sin \varphi' \sin \phi' + \cos \theta (\beta E_1' + \gamma p_1' \cos \theta')) \sin \theta, \]
\[ +\sin \theta (p_1' \sin \theta \sin \varphi' + \cos \theta (\beta E_1' + \gamma p_1' \cos \theta')) \right], \]

where \( E_1' \) and \( p_1' \) are the energy and momentum of one of the \( \gamma \) in the \( \gamma - \gamma \) centre-of-mass frame, and
\[ \beta \equiv \frac{s - s'}{2ss'}, \quad \gamma \equiv \frac{s + s'}{2ss'}, \quad p_1' \equiv |p_1'| = \frac{1}{2} \sqrt{s' - 4m_\gamma^2}, \quad E_1' = \frac{s'}{2}. \quad (A.6) \]

The quantities \( T_{ij} \) and \( T_{1j} \) in Eq. (A.4) are
\[ T_1 = \frac{p_2 \cdot p_2 q \cdot p_1'}{q \cdot p_1' N_{22}}, \]
\[ T_2 = \frac{p_1' \cdot p_1 q \cdot p_2'}{q \cdot p_2 N_{11}}, \]
\[ T_3 = \frac{-2p_1' \cdot p_2' \cdot p_2'}{N_{11} N_{22}} \left[ \frac{2(m_\gamma^2 - 2p_1' \cdot q \cdot p_1' + q \cdot p_1')}{q' \cdot p_1 N_{22}} - \frac{(2p_1' \cdot q \cdot p_1')}{q' \cdot p_1 q' \cdot p_2} \right], \]
\[ -\frac{(2p_1' \cdot p_2' - 2p_1' \cdot p_2 - q' \cdot p_2)}{q' \cdot p_2 N_{11}} \right], \]
\[ T_{13} = \frac{2p_2 \cdot p_2'}{q' \cdot p_1 N_{22}} \left( \frac{m_\gamma^2 q \cdot p_1' - 2p_1' \cdot q \cdot p_1'}{N_{11} N_{22}} - \frac{p_1' \cdot p_1 q' \cdot p_2' + p_1' \cdot p_2 q' \cdot p_1' - q' \cdot p_1' \cdot p_2'}{2q' \cdot p_2 N_{11}} \right), \]
\[ T_{23} = \frac{-2p_1'}{q' \cdot p_2 N_{11}} \left[ \frac{q' \cdot p_2' \left( p_1' \cdot p_2' - p_1' \cdot p_2 \right) + p_2 \cdot p_2' \left(q \cdot p_1' - q \cdot p_1'\right) - q \cdot p_2 \left(p_1' \cdot p_2' - p_1 \cdot p_2\right)}{N_{11} N_{22}} \right], \]
\[ + \frac{p_1' \cdot p_2' \cdot q' \cdot p_2 + p_2 \cdot p_2' \left(q \cdot p_1' - q \cdot p_1'\right)}{2q' \cdot p_1 N_{22}} \right], \quad (A.7) \]
\[ T_{16} = \frac{m_{\gamma}^2}{q'p_{12}N_{22}} \left[ \frac{q'p_{2}p_{1}p_{2}' + p_{2}q'p_{2}'p_{1}p_{2}'}{N_{21}N_{12}} - \frac{q'p_{1}p_{2}p_{2}'}{N_{12}} + \frac{p_{1}p_{2}}{N_{21}} \right], \]

\[ T_{26} = \frac{m_{\gamma}^2}{q'p_{2}N_{11}} \left[ \frac{(q'p_{1} - q'p_{2}')p_{1}p_{2} + (p_{1}p_{2}' - p_{2}'p_{1})q'p_{1} + q'p_{2}p_{1}p_{2}'}{N_{12}N_{21}} + \frac{p_{1}p_{2}}{N_{21}} \right], \]

\[ T_{14} = \frac{-m_{\gamma}^2q'p_{2}}{q'p_{1}N_{21}N_{22}}, \]

\[ T_{36} = m_{\gamma}^2p_{1}p_{2} \left[ \frac{2}{N_{11}N_{22}} \left( \frac{2p_{1}p_{2} + q'p_{1}p_{2} - 2p_{1}p_{2} - q'p_{2}p_{1}}{N_{11}N_{12}} \right) \right. \]

\[ - \left[ \frac{1}{N_{11}N_{22}} \left( \frac{2p_{1}p_{2} + q'p_{1}p_{2} - 2p_{1}p_{2} - q'p_{2}p_{1}}{N_{11}N_{12}} \right) \right] + \left( p_{1}' + p_{2}' \right) \]

\[ - \frac{p_{1}p_{2}}{q'p_{1}q'p_{2}} \left( \frac{1}{N_{12}N_{22}} + \frac{1}{N_{11}N_{21}} \right) \]

where \( T_{ij} \equiv |M_{ij}|^2 \) and \( T_{ij} = 2 \text{ Re} (M_{ij}M_{ij}^*) \).

For completeness, we note that the cross-section for \( e^+ e^- \rightarrow \gamma \nu \bar{\nu} \) can be written as

\[
\frac{d\sigma}{dx \, d\cos \theta} = \frac{g_{\nu}^2}{6\pi} \left( \frac{m_{\nu}^4}{s(1-x) - m_{\nu}^2} + 2 \right) \left[ \frac{s(1-x)}{x \sin^2 \theta} \left( 1 - \frac{x}{2} \right)^2 \right] + \frac{1}{4} x^2 \cos^2 \theta \right],
\]

(A.8)

where

\[ g_{\nu} = -\frac{1}{2} + 2 \sin^2 \theta, \quad g_A = -\frac{1}{2}. \]  

(A.9)
Fig. A.1 Feynman diagrams for $e^+e^- \rightarrow \gamma\gamma$
APPENDIX B: AMPLITUDES FOR $e^+e^- \to \gamma vv$

In general, mass eigenstates of SUSY particles are not weak eigenstates. The scalar partners of left-handed and right-handed quarks (and leptons) are mixed together, as are the fermionic partners of gauge bosons and Higgs bosons. Indeed the mass eigenstates for supersymmetric fermions are mixtures of shiggles (the partners of the Higgs bosons) and winos and zinos (the partners of gauge bosons) with mixing angles which depend on the relative strengths of the different supersymmetry and SU(2) × U(1) breaking parameters.

In a model with no right-handed neutrinos, however, the partner of the left-handed neutrino, the sneutrino $\tilde{\nu}_L$, is a mass eigenstate. Furthermore, in models where the sneutrino is the lightest supersymmetric particle, one ends up with a light chargino state (charged gauge fermion) which is almost a weak eigenstate--a wino--and a charged heavy chargino which is an almost pure shiggs state. The latter decouples for any practical purpose from electron interactions, so that only the light wino plays a role in our process.

The amplitudes [37, 43, 44] that participate in the process $e^+e^- \to \gamma vv$ are displayed in Fig. B.1. The first three amplitudes involve, apart from the electromagnetic vertex, the interaction

$$l = ig \cos \varphi \tilde{\nu}^C L W^0 W^0 \tilde{\nu}_L + h.c.,$$

where $W$ is the (light) wino field and $\varphi$ is the mixing angle, which from now on we shall take such that $\cos \varphi = 1$.

The last two amplitudes in Fig. B.1 involve the $Z vv$ vertex

$$l' = \frac{ig}{2 \cos \varphi} \tilde{\nu}_L W^0 W^0 \tilde{\nu}_L Z^0,$$

and the familiar Standard Model couplings to electrons.

It is now straightforward to write down the explicit formulae for the five amplitudes. They read:

$$T_1 = \frac{-ie^2 \left[ e^+ (p_1) \bar{p}^0_2 (p_2) p_1 e^- (p_3) \right]}{[(p_3 - p_1)^2 - m_e^2] [(p_2 - p_4)^2 - m_w^2]},$$

$$T_2 = \frac{-ie^2 \left[ e^+ (p_1) \bar{p}^0_2 (p_2) p_1 e^- (p_3) \right]}{[(p_3 - p_1)^2 - m_e^2] [(p_2 - p_3)^2 - m_w^2]},$$
\[ T_3 = \frac{-ie^2 [e^+(p_1) P_R (p_3 + m_W^2) \bar{\gamma}^3 (\gamma^3 P_L) e^-(p_2) - (p_2 - p_1)^2 - m_W^2] }{[2 \cos^2 \theta_W] [2 \cos^2 \theta_W] } , \]

\[ T_4 = \frac{i e g^2}{2 \cos^2 \theta_W} \left[ e^+(p_1) \bar{\gamma}^3 (\gamma^3 P_L) \bar{e}^-(p_2) \right] \left[ (p_3 - p_1)^2 - m_Z^2 \right] \left[ (p_4 + p_5)^2 - m_Z^2 + i \epsilon \right] , \]

\[ T_5 = \frac{i e g^2}{2 \cos^2 \theta_W} \left[ e^+(p_1) \bar{\gamma}^3 (\gamma^3 P_L) \bar{e}^-(p_2) \right] \left[ (p_3 - p_2)^2 - m_Z^2 \right] \left[ (p_4 + p_5)^2 - m_Z^2 + i \epsilon \right] , \]

where

\[ P_{R,L} = \frac{1}{2} (1 + \gamma_5) , \quad L_e = \sin^2 \theta_W - \frac{1}{2} , \quad C_e = \sin^2 \theta_W , \]

\( e^\mu \) is the polarization vector of the photon, and \( \epsilon = m_Z^2 \).

The generalization [37] to significant mixing between the \( \tilde{W}^\pm \) and \( \tilde{H}^\pm \) is straightforward, and the general result is given in the Electronic Yellow Book [44].

Fig. B.1 Feynman diagrams for \( e^+ e^- \rightarrow \gamma \nu \bar{\nu} \)
APPENDIX C: MATRIX ELEMENTS FOR $e^+e^-\rightarrow e^+e^-$

There are eight diagrams not involving the $Z^0$ which contribute to this process, shown in Fig. C.1, as well as two with direct-channel $Z^0$ poles, shown in Fig. C.2.

The matrix elements [31, 45] for production of the $\tilde{e}_R$ by the first eight diagrams of Fig. C.1 are as follows:

\begin{align}
M^{(1)} &= e^2 g [\bar{\nu}(p_1) \gamma_\mu \nu(p_2)] q^{-2} \left[ \bar{u}(k_3) \frac{1}{2} (1+\gamma_5) (\not{k}_3 + \not{q})^{-1} \gamma^\mu u(k_1) \right], \\
M^{(2)} &= e^2 g [\bar{\nu}(p_1) \gamma_\mu \nu(p_2)] q^{-2} \left[ \bar{u}(k_3) \frac{1}{2} (1+\gamma_5) \times \{(2k_3 - q)^\mu / [(k_3 - q)^2 - m^2_e] \} u(k_1) \right], \\
M^{(3)} &= -e^2 g [\bar{\nu}(p_1) \gamma_\mu u(k_1)] \frac{1}{4} \frac{E^{-2}}{E_{\text{beam}}} \left[ \bar{u}(k_3) \frac{1}{2} (1+\gamma_5) (\not{k}_3 + \not{q})^{-1} \gamma^\mu \nu(p_2) \right], \\
M^{(4)} &= e^2 g [\bar{\nu}(p_1) \gamma_\mu u(k_1)] \frac{1}{4} \frac{E^{-2}}{E_{\text{beam}}} \left[ \bar{u}(k_3) \frac{1}{2} (1+\gamma_5) \times \{(p_2 + k_3 - q)^\mu / [(p_2 + k_3)^2 - m^2_e] \} \nu(p_2) \right], \\
M^{(5)} &= g^3 \left[ \bar{\nu}(p_3) (\not{k}_1 - \not{k}_4)^{-1} \frac{1}{2} (1+\gamma_5) u(k_1) \right] [(k_3 + p_2)^2 - m^2_e]^{-1} \left[ \bar{u}(k_3) \frac{1}{2} (1+\gamma_5) \nu(p_2) \right], \\
M^{(6)} &= -g^3 \left[ \bar{\nu}(p_3) (\not{k}_1 + \not{p}_2)^{-1} \frac{1}{2} (1+\gamma_5) \nu(p_2) \right] [(k_3 - k_1)^2 - m^2_e]^{-1} \left[ \bar{u}(k_3) \frac{1}{2} (1+\gamma_5) u(k_1) \right], \\
M^{(7)} &= g^3 \left[ \nu(p_2) \frac{1}{2} (1+\gamma_5) [\gamma_0 (\not{k}_1 - \not{k}_4)^{-1}] u(k_1) \right] [(p_1 - k_3)^2 - m^2_e]^{-1} \left[ \bar{\nu}(p_1) \frac{1}{2} (1+\gamma_5) \nu(k_3) \right], \\
M^{(8)} &= g^3 \left[ \nu(p_2) \frac{1}{2} (1+\gamma_5) [\gamma_0 (\not{p}_2 + \not{k}_4)^{-1}] u(k_1) \right] [(p_1 - k_3)^2 - m^2_e]^{-1} \left[ \bar{\nu}(p_1) \frac{1}{2} (1+\gamma_5) \nu(k_3) \right],
\end{align}

where $g = \sqrt{2}e$. Matrix elements for producing $\tilde{e}_L$ can be obtained from Eqs. (C.1) by replacing $\gamma_5 \rightarrow -\gamma_5$.

The corresponding matrix elements [31, 45] for the production of $\tilde{e}_{L,R} \rightarrow e^+e^-$ via the $Z^0$ diagrams of Fig. C.2 are
\[ T_{1L} = -i\alpha C_L [\bar{e}^+(p_1) \gamma^\mu (C_L P_L + C_R P_R) e^-(p_2)] \times [\gamma(p_3) P_L (\not{p}_3 + \not{p}_5) \gamma_\mu e^+(p_4)] , \]
\[ T_{1R} = i\alpha C_R [\bar{e}^+(p_1) \gamma^\mu (C_L P_L + C_R P_R) e^-(p_2)] \times [\gamma(p_3) P_R (\not{p}_3 + \not{p}_5) \gamma_\mu e^+(p_4)] , \]

where
\[ \alpha = \frac{e^3/2}{(\sin \theta_w \cos \theta_w)^2 (p_1 + p_2)^2 - m_Z^2 + i\epsilon (p_5 + p_3)^2 - m_e^2} , \]

and
\[ T_{2L} = -i\beta C_L [\bar{e}^+(p_1) \gamma^\mu (C_L P_L + C_R P_R) e^-(p_2)] [\gamma(p_3) P_L e^+(p_4)] (p_5 - p_3 - p_4)_\mu \]
\[ T_{2R} = i\beta C_R [\bar{e}^+(p_1) \gamma^\mu (C_L P_L + C_R P_R) e^-(p_2)] [\gamma(p_3) P_R e^+(p_4)] (p_5 - p_3 - p_4)_\nu \]

where
\[ \beta = \frac{e^3/2}{(\sin \theta_w \cos \theta_w)^2 (p_1 + p_2)^2 - m_Z^2 + i\epsilon (p_3 + p_4)^2 - m_e^2} \]

with
\[ p_{L,R} = \frac{1}{2} (1 + \gamma_5) , \quad \text{and} \quad C_L = \sin^2 \theta_w - \frac{1}{2} , \quad C_R = \sin^2 \theta_w . \]
Fig. C.1 Feynman diagrams for $e^+e^- \to e^+e^-\gamma$ not involving the $Z^0$

![Feynman diagrams for $e^+e^- \to e^+e^-\gamma$ not involving the $Z^0$](image)

Fig. C.2 Feynman diagrams for $e^+e^- \to e^+e^-\gamma$ via a direct-channel $Z^0$

![Feynman diagrams for $e^+e^- \to e^+e^-\gamma$ via a direct-channel $Z^0$](image)
APPENDIX D: MATRIX ELEMENTS FOR $e^+e^- \rightarrow e^+\nu^-\nu$ WHEN $v_1 = v_2$

In order to compute the $e^+e^- \rightarrow e^+\nu^-\nu$ cross-section we can use subsets of the 12 diagrams which contribute to the total cross-section. One subset is appropriate for energies much below the $Z^0$ pole and the other one can be used for energies near or beyond the $Z^0$. We show here the output of the REDUCE computation of these subsets [46, 47].

a) Low energies (Figs. D.1a-f)

Introducing the notation

$$\begin{align*}
P_{ij} &\equiv p_i \cdot p_j \quad (D.1) \\
(p_1, p_2, p_3) &\equiv (p_{11}, p_{12}, p_{13}) + (p_{21}, p_{22}, p_{23}) \quad (D.2) \\
(S_1, S_2, S_3) &\equiv (p_{45}, p_{23}, p_{13}) + (p_{23}, p_{13}, p_{34}) \quad (D.3)
\end{align*}$$

for $S_k = \pm, 0, -$ ($k = 1, 2, 3$) we have for the squared matrix elements

$$\begin{align*}
A(1a, 1a) &= 16C_1 \left[ \frac{m_W^2}{2} (p_{12} (2p_{25} + p_{35}) + p_{23} (2p_{15} + p_{14})) - m_V^2 (p_{34} p_{12} + p_{23} p_{14}) \\
&\quad + 2p_{45} (p_{35} p_{12} + p_{23} p_{15}) \right], \\ 
A(1b, 1b) &= 16C_2 \left[ \frac{m_W^2}{2} (p_{34} p_{12} + 2p_{25} p_{13} + p_{23} p_{14}) - m_V^2 (p_{34} p_{12} + p_{23} p_{14}) \\
&\quad + 2p_{45} (p_{35} p_{12} + p_{23} p_{15}) \right], \\ 
A(1c, 1c) &= A(1a, 1a; p_2 \leftrightarrow p_3), \\ 
A(1d, 1d) &= A(1b, 1b; p_2 \leftrightarrow p_3), \\ 
A(1e, 1e) &= -4C_2 \left[ \frac{m_W^2}{2} (p_{24} p_{13} - 2p_{25} p_{24} p_{15}) \right], \\ 
A(1f, 1f) &= A(1e, 1e; p_2 \leftrightarrow p_3), \\ 
A(1a, 1b) &= 32C_1 \left[ \frac{m_W^2}{2} (p_{12} (p_{35} + p_{34}) + p_{23} (p_{15} + p_{14}) + p_{23} p_{13}) + (p_5 \leftrightarrow p_3) + p_{45} (p_{35} p_{12} + p_{23} p_{15}) \\
&\quad + p_{35} (p_{25} p_{14} - 2p_{24} p_{15}) + p_{34} p_{25} p_{15} \right], \\ 
A(1a, 1c) &= 32C_1 \left[ \frac{m_W^2}{2} (p_{23} (-2p_{15} - p_{14}) + m_V^2 p_{23} p_{14} - 2p_{45} p_{23} p_{15}) \right]. 
\end{align*}$$
\[ A(1a,1d) = 16C_1 \{m_w^2 [-p_{35}^2 + p_{25}^2 + p_{12}^2 + p_{13}^2 - p_{23}^2 (3p_{15}^2 + 2p_{14}^2)] + (p_5^{++}) + 2(-+) \} , \] (D.4i)

\[ A(1a,1e) = 8C_3 \{(p_4^{-}) + (p_5^{-}) + 2(-+)\} , \] (D.4j)

\[ A(1a,1f) = 8C_3 \{(p_4^{-}) + (p_5^{-}) + 2(-+)\} , \] (D.4k)

\[ A(1b,1c) = 16C_1 \{m_w^2 [p_{35}^2 p_{12}^2 - p_{25}^2 p_{13}^2 - p_{23}^2 (3p_{15}^2 + 2p_{14}^2)] + (p_5^{++}) + 2(-+) \} , \] (D.41)

\[ A(1b,1d) = 16C_1 \{-2m_w^2 p_{23}^2 (p_{14}^2 + p_{15}^2) + (p_5^{--}) + 2(--+)\} , \] (D.4m)

\[ A(1b,1e) = 8C_3 \{(p_4^{-}) + (p_5^{--}) + 2(--+)\} , \] (D.4n)

\[ A(1b,1f) = 8C_3 \{(p_4^{-}) - 2m_w^2 p_{34}^2 p_{12}^2 + 4(00+)\} , \] (D.4o)

\[ A(1c,1d) = 32C_1 \{m_w^2 [p_{35}^2 p_{12}^2 + p_{25}^2 (p_{25}^2 + p_{24}^2) + p_{23}^2 (p_{15}^2 + p_{14}^2)] + (p_5^{--}) \]
\[ + p_{45}^2 (p_{25}^2 p_{13}^2 + p_{23}^2 p_{15}^2) + p_{35}^2 (p_{25}^2 p_{14}^2 + p_{24}^2 p_{15}^2) - 2p_{34}^2 p_{25}^2 p_{15}^2 \} , \] (D.4p)

\[ A(1c,1e) = 8C_3 \{(p_4^{--}) + (p_5^{--}) + 2(--+)\} , \] (D.4q)

\[ A(1c,1f) = 8C_3 \{(p_4^{--}) + (p_5^{--}) + 2(--+)\} , \] (D.4r)

\[ A(1d,1e) = 8C_3 \{(p_4^{--}) - 2m_w^2 p_{24}^2 p_{13}^2 + 4(00+)\} , \] (D.4s)

\[ A(1d,1f) = 8C_3 \{(p_4^{--}) + (p_5^{--}) + 2(--+)\} , \] (D.4t)

\[ A(1e,1f) = 4C_2 \{(p_5^{--}) + 2(--+)\} , \] (D.4u)

where we have used the following definitions:

\[ C_1 \equiv K_1^2 , \quad C_2 \equiv K_2^2 , \quad C_3 \equiv K_1 K_2 , \quad \text{with} \]
\[ K_1 \equiv (e)^{3/2}/\sin \theta_w , \quad (D.5) \]
\[ K_2 \equiv K_1/\sin^2 \theta_w . \]

Notice that propagators are defined including the global sign in front of the 'i' of each amplitude; this means that they include the global sign of the diagram.
b) LEP energies (Figs. D.1a,b,e; D.2c,d,f)

Defining now,

\[
(L_{S_1 S_2 S_3 S_4}) = \mathbb{R}_{S_1 S_2 S_3 S_4}^{\text{S}}(S_{P_{15}} P_{12} + S_{P_{25}} P_{13} + S_{P_{23}} P_{14} + S_{P_{23}} P_{15}), \tag{D.6a}
\]

\[
(R_{S_1 S_2 S_3 S_4}) = \mathbb{R}_{S_1 S_2 S_3 S_4}^{\text{R}}(S_{P_{15}} P_{12} + S_{P_{25}} P_{13} + S_{P_{24}} P_{13} + S_{P_{23}} P_{15}), \tag{D.6b}
\]

\[
(L_{S_1 S_2 S_3}) = S_{P_{45}} P_{23} P_{15} + S_{P_{25}} P_{13} + S_{P_{23}} P_{15}, \tag{D.6c}
\]

\[
(R_{S_1 S_2 S_3}) = S_{P_{45}} P_{25} P_{13} + S_{P_{25}} P_{14} + S_{P_{34}} P_{25} P_{15}, \tag{D.6d}
\]

for \( S_K = -, 0, +, 2, 3 \) \((k = 1, 2, 3, 4)\) we have

\[
A(2c,2c) = -16C_2^2 \left\{ \mathcal{L}^2 \left[ (p_5000) - (L0020) + 2(L000) \right] + \beta^2 \left[ (p_5000) - (R020) \right] + 2(R000) \right\}, \tag{D.7a}
\]

\[
A(2d,2d) = -16C_2^2 \left\{ \mathcal{L}^2 \left[ (p_5000) - (L---) + 2(L---) \right] + \beta^2 \left[ (p_5000) - (R---) \right] + 2(R000) \right\}, \tag{D.7b}
\]

\[
A(2f,2f) = -16C_2^2 \left\{ \left( \alpha^2 \beta^2 \right) p_{34} \left( \alpha^2 p_{12} - \beta^2 p_{25} p_{15} \right) \right\}, \tag{D.7c}
\]

\[
A(2c,2d) = 16C_2^2 C_2^2 \left\{ \mathcal{L}^2 \left[ (p_5+++0) - (L+++0) + 2(L++0) \right] + \beta^2 \left[ (p_5+++0) - (R+++0) \right] + 2(R++0) \right\}, \tag{D.7d}
\]

\[
A(2d,2f) = -16C_2^2 C_2^2 \left\{ \mathcal{L}^2 \left[ (p_5+++0) - (L++0) + 2(L++0) \right] + \beta^2 \left[ (p_5+++0) - (R++0) \right] + 2(R++0) \right\}, \tag{D.7e}
\]

\[
A(2c,2f) = 16C_2^2 C_2^2 \left\{ \mathcal{L}^2 \left[ (p_5+++0) - (L+++0) + 2(L++0) \right] + \beta^2 \left[ (p_5+++0) - (R++0) \right] + 2(R++0) \right\}, \tag{D.7f}
\]

\[
A(1a,2c) = 32a C_2^2 K_1 \left\{ R_{2p_{23}} \left( \mathcal{L}^2 p_{14} + \mathcal{L}^2 p_{25} - 2p_{25} p_{15} \right) \right\} / p_2, \tag{D.7g}
\]

\[
A(1a,2d) = -16a C_2^2 K_1 \left\{ C_2^2 \mathcal{L}^2 \left[ (p_5+++0) - (L+++0) + 2(L++0) \right] \right\} / p_2, \tag{D.7h}
\]
\[ A(1a,2f) = 16a_{c2f}^2 \left\{ G_2 + R_Z \left[ (p_1^{++}) + (L^{+--}) + 2(L^{+-}) \right] \right\}/P_Z , \quad (D.7i) \]

\[ A(1b,2c) = -16a_{c2c}^2 \left\{ G_2 + R_Z \left[ (p_1^{--}) + (L^{-+32}) + 2(L^{+++}) \right] \right\}/P_Z , \quad (D.7j) \]

\[ A(1b,2d) = -16a_{c2d}^2 \left\{ G_2 + R_Z \left[ (p_1^{++}) + (LO022) + 2(L^{+-}) \right] \right\}/P_Z , \quad (D.7k) \]

\[ A(1b,2f) = 16a_{c2f}^2 \left\{ G_2 + R_Z \left[ (p_2^{00}) + (L^{+--}) + 4(L^{0+-}) \right] \right\}/P_Z , \quad (D.7l) \]

\[ A(1e,2c) = -8a_{c2c}^2 \left\{ G_2 + R_Z \left[ (p_5^{+++}) + (L^{+--}) + 2(L^{++-}) \right] \right\}/P_Z , \quad (D.7m) \]

\[ A(1e,2d) = -8a_{c2d}^2 \left\{ G_2 + R_Z \left[ (p_5^{020}) + (L^{+--}) + 4(L^{0+-}) \right] \right\}/P_Z , \quad (D.7n) \]

\[ A(1e,2f) = -8a_{c2f}^2 \left\{ G_2 + R_Z \left[ (p_5^{--}) + 2(L^{+++}) \right] \right\}/P_Z , \quad (D.7o) \]

where

\[ C_{2c} \equiv \alpha K_1 , \quad C_{2d} \equiv K_1 / \beta , \quad C_{2f} \equiv K_1 / \sin 2 \theta_w , \quad (D.8a) \]

\[ R_Z \equiv (p_1 + p_2)^2 - m_Z^2 , \quad P_Z \equiv (R_Z + I_Z)^{1/2} , \quad I_Z \equiv (m_Z^2 - m_2^2 - 2m_1 s_1) \quad (D.8b) \]

\[ \hat{G}_e \equiv -m^2 \gamma \rho \rho^\nu \rho_\mu \rho_\nu, \quad G_e \equiv -(m_2^2 - m_1^2 + 2m_1 s_1) \gamma \rho \rho^\nu \rho_\mu \rho_\nu \quad (D.8c) \]

Here the propagators are again defined including the global sign in front of the 'i' of each amplitude. In the \( Z^0 \) propagators, only the modulus must be included, because the phase has been already included in the trace calculation.
APPENDIX E: MATRIX ELEMENTS FOR \( e^+e^- \rightarrow \bar{e}^+\bar{\nu} \) IN THE CASE \( v_1 \neq v_2 \)

For the general case \( v_1 \neq v_2 \), most of the interactions remain the same as in the case \( v_1 = v_2 \) described in Appendix D [37, 47]. The only ones that are different are as follows:

1) The vertex \( \bar{Z}W^+ \) is now

\[
L_{ZWW} = -e \cot \theta_w \bar{W} \bar{\nu} \gamma^\mu \left( 1 - \frac{1}{2} \sec^2 \theta_w \cdot v_c - \gamma_5 \frac{1}{2} \sec^2 \theta_w \cdot A_c \right) \bar{W} Z_\mu ,
\]

(E.1)

where

\[
v_c = (\cos^2 \gamma_L + \cos^2 \gamma_R)^{1/2},
\]

and

\[
A_c = (\cos^2 \gamma_R - \cos^2 \gamma_L)^{1/2}.
\]

(E.2)

2) The vertex \( \bar{\nu}lW \) is now

\[
L_{\bar{\nu}lW} = g \sin \gamma_R \bar{\nu} \gamma^\mu \bar{W} \frac{1 - \gamma_5}{2} e - g \sin \gamma_R \bar{\nu} \gamma^\mu \bar{W} \frac{1 - \gamma_5}{2} \nu ,
\]

(E.3)

where \( \sin \gamma_R = f_+ \). These couplings can be expressed in terms of the parameters \( m_1 \) and \( \mu_2 \) (with \( m_1 > 0 \) and \( \mu_2 < 0 \)), the mixing angle \( \alpha \): \( \tan \alpha = v_2/v_1 \), and \( \tan \gamma_L \) and \( \tan \gamma_R \).

We can express these parameters in terms of the physical masses of the \( \tilde{\nu} \), \( \tilde{\nu} \), \( \tilde{e} \) and \( \tilde{W} \) as follows:

\[
\mu_2 = -\frac{3}{8 \sin^2 \theta_w} m_\tilde{\nu} ,
\]

(E.4)

\[
m_0^2 = m_\tilde{\nu}^2 + \frac{1}{2} \left( \frac{\mu_2}{\mu_1 - m_\tilde{\nu}} \right)^2 \left( \frac{\sin^2 \theta_w + \frac{1}{2} \mu_2}{\sin^2 \theta_w + \frac{1}{2}} \right) ,
\]

(E.5)

\[
\rho = \frac{m_\tilde{e} - m_\tilde{\nu}^2}{m_\tilde{\nu}^2 \left( \sin^2 \theta_w + \frac{1}{2} \right) } ,
\]

(E.6)

and

\[
m_1 = \frac{\mu_2 m_\tilde{\nu}^2 \sin 2\alpha + m_\tilde{\nu}^2 \Delta^{1/2}}{2(\mu_2^2 - m_\tilde{\nu}^2)} ,
\]

(E.7)
with

$$\Delta = (m_w^2 - m_W^2)^2 + 2\mu_2^2 (m_W^2 - m_W^2) \mu_2^4 - m_W^4 \cos^2 2\alpha . \quad (E.8)$$

With given sets of the $\tilde{\gamma}$, $\tilde{c}$, $\tilde{v}$, and $\tilde{\tilde{w}}$ masses, we have calculated the parameters $\mu_2$, $m_0$, $\gamma$, $\alpha$, and $m_1$ and the vertex factors $\sin \gamma_R$, $V_C$ and $A_C$, which are given in Table 3.3. We see that in most cases $\sin \gamma_R = 1$, $V_C = A_C = 0$, and the interactions reduce to the previous special case described in Appendix D. For the other cases, the cross-sections can be very different. We have calculated them for the cases shown in Table 3.4. Also shown for comparison are the cross-sections that would have been obtained using the results of Appendix D, although in these comparison cases the full mass relations of broken supersymmetry have not been used.
APPENDIX F: POLARIZATION AND SUPERSYMMETRY

Thanks to recent progress [55] in the understanding of spin motion in storage rings, today it appears possible to get highly polarized \( e^+ e^- \) beams in the first phase of LEP, and perhaps even at higher energies. Depolarization effects could be controlled up to energies in the 50 GeV range. Longitudinal polarization could be obtained using spin rotators, although we appreciate that their construction would not be cheap.

One may wonder whether \( e^\pm \) beams could play a significant role in the hunt for exotic particles. It is already known that the comparison of cross-sections for left- and right-handedly polarized incident beams would provide the relative sign of the vector and axial electroweak couplings. This effect is a consequence of the chiral properties of usual fermions under the SU(2)\(_L\) \( \times \) U(1) gauge group. In supersymmetric theories, the main difference between usual particles and their supersymmetric partners lies, by definition, in their different spins. A simple comparison of the one-photon annihilation cross-sections for \( e^+ e^- \rightarrow f\bar{f} \) and \( e^+ e^- \rightarrow \tilde{f}\tilde{f} \) with transversely polarized \( e^\pm \) beams shows an opposite sign in the \( \cos 2\phi \) azimuthal dependence [56], i.e.

\[
\frac{\mathrm{d}\sigma}{\mathrm{d}t} (e^+ e^- \rightarrow f\bar{f}) = \frac{\alpha^2 Q_f^2 \beta}{4s} \left[ 2 - \beta^2 \sin^2 \theta + \beta^2 \sin^2 \theta (P^r_1 P^r_2 \cos 2\phi) \right], \quad (F.1)
\]

\[
\frac{\mathrm{d}\sigma}{\mathrm{d}t} (e^+ e^- \rightarrow \tilde{f}\tilde{f}) = \frac{\alpha^2 Q_{\tilde{f}}^2 \beta^3}{8s} \sin^2 \theta \left[ 1 - P^r_1 P^r_2 \cos 2\phi \right]. \quad (F.2)
\]

This property remains valid if one includes the \( Z^0 \) boson, and is independent of the signs of the coupling constants.

This change of sign in the azimuthal angular dependences of these annihilation cross-sections is worth a theoretical comment. Remembering that the \( e^+ e^- \) annihilation cross-section is related to the renormalization factor for boson propagators, one can recognize in this property an aspect, and therefore a possible experimental test, of the non-renormalization theorem [4] which is based on the change of signs between fermion and boson loops. It can easily be checked that, summing the different production cross-sections \( e^+ e^- \rightarrow f\bar{f}, \tilde{f}\bar{\tilde{f}}, \tilde{f}\bar{\tilde{f}} \), and \( \tilde{f}\bar{\tilde{f}} \) for a given fermion \( f \) and its corresponding spinless companions \( \tilde{f} \) and \( \tilde{f} \), and neglecting the mass difference, the total transverse asymmetry cancels [57].

We now consider in some detail the production, from polarized \( e^\pm \) beams, of squarks and sleptons, before paying some attention to the production of the fermionic partners of the gauge bosons.
Slepton and squark production [58, 59]

The general form for $e^+e^-$ cross-sections with polarized $e^+$ incident beams but unpolarized outgoing particles reads:

$$\frac{d\sigma}{d\Omega} = (1-P^L_\parallel P^R_\parallel)X_1(\theta) + (1+P^L_\parallel P^R_\parallel)X_1'(\theta)$$

$$+ (P^L_\perp P^R_\perp)X_2(\theta) + (P^L_\perp P^R_\perp)X_2'(\theta)$$

$$+ P^L_\perp P^R_\perp [X_3(\theta) \cos 2\varphi + X_4(\theta) \sin 2\varphi], \quad (F.3)$$

where $P^L_\parallel$ and $P^L_\perp$ are the longitudinal and transverse degrees of polarization, and $\theta$ and $\varphi$ are the polar and azimuthal angles of the final centre-of-mass momentum of the scalar $\tilde{t}$, the Oz axis being chosen along the $e^-$ beam. The coefficients $X_i(\theta)$ ($i = 1, \ldots, 4$) and $X'_i(\theta)$ ($j = 1, 2$) are combinations of the helicity amplitudes $F^\lambda_{\ell^+}, F^\lambda_{\ell^-} (\theta)$:

$$X_1 = |F_{++}|^2 + |F_{-+}|^2, \quad X'_1 = |F_{++}|^2 + |F_{--}|^2,$$

$$X_2 = |F_{+-}|^2 - |F_{-+}|^2, \quad X'_2 = |F_{++}|^2 - |F_{--}|^2, \quad (F.4)$$

$$X_3 = 2 \text{ Re } (F_{-+} F^*_{-+}) \quad \text{and} \quad X_4 = 2 \text{ Im } (F_{-+} F^*_{-+}).$$

The production of a scalar pair via $\gamma$ or $Z$ formation involves only $F^\lambda_{\ell^+}, F^\lambda_{\ell^-}$ amplitudes. Scattering processes with gaugino exchange (e.g. $\tilde{\gamma}, \tilde{Z}$ for $e^+e^- \to \tilde{e}^+\tilde{e}^-$ or $\tilde{W}$ for $e^+e^- \to \tilde{\nu}_e \tilde{\nu}_e$) could contribute to $F^\lambda_{\ell^+}, F^\lambda_{\ell^-}$ amplitudes, and then give rise to non-zero contributions for all $X_i$ and $X'_j$ coefficients.

Pure annihilation processes will provide left-handed scalar pairs $e^+e^- \to \tilde{\tau}_L^+ \tilde{\tau}_L^-$ as well as right-handed ones $e^+e^- \to \tilde{\tau}_R^+ \tilde{\tau}_R^-$ from $e^+e^- \to \tilde{e}_L^+ \tilde{e}_L^-$ or $e^+e^- \to \tilde{e}_R^+ \tilde{e}_R^-$ helicity states. It then follows that $X'_1 = X'_2 = 0$, whilst

$$X_1 = \frac{a^2 p^3 \sin^2 \theta}{s} \left( \frac{g^r}{s} - \frac{g^r a}{D_Z} e^2 + \frac{g^r b}{D_Z} e^2 \right),$$

$$X_2 = - \frac{2a^2 p^3 \sin^2 \theta}{s} \text{ Re } \left[ \left( \frac{g^r}{s} - \frac{g^r a}{D_Z} e^2 \right) \left( \frac{g^r b}{D_Z} e^2 \right) \right], \quad (F.5)$$

$$X_3 = - \frac{a^2 p^3 \sin^2 \theta}{s} \left( \frac{g^r}{s} - \frac{g^r a}{D_Z} e^2 - \frac{g^r b}{D_Z} e^2 \right),$$
where \(s\) is the total c.m. energy and \(D Z = s - m^2 + i m^2 s\). The \(g^Z\) coupling is, according to the considered chirality, either \(g^Z_L = a_f + b_f\) or \(g^Z_R = a_f - b_f\), where \(a_f\) and \(b_f\) are the standard electroweak couplings associated with the fermion \(f\), and \(g^Y = Q_f\) for \(f = L\) or \(R\). Taking as an example the case \(e^+ e^- \rightarrow \mu^+ \mu^-\), we will have \(g^Y = -1\), whilst

\[
a_{\mu} = a_{e} = \frac{1}{2 \sin 2\theta_W} (-1 + 4 \sin^2 \theta_W); \quad b_{\mu} = b_{e} = -\frac{1}{2 \sin 2\theta_W} \tag{F.6}
\]

Returning to the polarized cross-section, the two observable polarization quantities are [60]

i) the longitudinal asymmetry, defined by

\[
\hat{A}_L = \frac{\tilde{\sigma}(P^{++}, P^{--}) - \tilde{\sigma}(P^{--}, P^{++})}{\tilde{\sigma}(P^{+}, P^{+}) + \tilde{\sigma}(P^{--}, P^{--})} = \frac{X_1 - X_2}{X_1 + X_2}, \tag{F.7}
\]

which can be detected even if just one \(e^+\) beam is longitudinally polarized, and

ii) the transverse or azimuthal asymmetry, defined by

\[
\hat{A}_T = \frac{X_1 \cos 2\phi + X_2 \sin 2\phi}{X_1 + X_2}, \tag{F.8}
\]

which comes from the interference of amplitudes with opposite helicities \((P_{\lambda \lambda'}, F_{\lambda + \lambda'})\) and vanishes when only one helicity combination occurs.

In the case of \(\mu^-\mu^+\) production, the two longitudinal asymmetries \(A_L\) for \(\tilde{\mu}_L\) and \(\tilde{\mu}_R\)—obtained from \(A_L\) after the sequential decays \(\tilde{\mu}^+ \rightarrow \mu^-\gamma\) are integrated over the decay angles \(\theta^-\)—are large, very different from the standard \(e^+ e^-\) case, and mainly opposite in sign. They will therefore constitute a good signal for scalar muon production if \(\tilde{\mu}_L\) and \(\tilde{\mu}_R\) are not degenerate in mass. If the \(\tilde{\mu}_L\) and \(\tilde{\mu}_R\) are degenerate, the sum of both contributions will give the same result as \(e^+ e^-\), \(\mu^+ \mu^-\) (see Fig. F.1a). The transverse asymmetry \(A_T\) can usefully be studied at \(\phi = 0\) since there the \(X_4\) term is negligible compared with the \(2\phi\) coefficient. As already remarked at the beginning of this Appendix, it has the opposite sign with respect to the case \(e^+ e^-\), \(\mu^+ \mu^-\), and this effect is very pronounced around \(s = 100\) GeV if the scalar muon is not too heavy (see Fig. F.1b).

For the processes \(e^+ e^- \rightarrow \tilde{e}^+ \tilde{e}^- + e^+ e^- \tilde{\gamma}\), one has to add the t-channel gaugino exchange contributions, and one also has reactions of the type \(e^+ e^- \rightarrow \tilde{\nu}_R^+ \tilde{\nu}_R^-\) and
$e^+_L e^-_L \rightarrow \tilde{e}^+_L \tilde{e}^-_L$. There are again clear differences from the standard $e^+e^- \rightarrow e^+e^-$ reaction. Figure F.2 shows the longitudinal asymmetry $A_L$ as a function of $\theta_e$ at $\sqrt{s} = 92 \text{ GeV}$; the sleptons $\tilde{e}_L$ and $\tilde{e}_R$ are here assumed to be degenerate, and a summation over all the four reactions corresponding to the different possible chiralities of incident $e^\pm$ beams has been performed. Notice the sizeable effect of the photino mass in the forward direction $\theta_e \lesssim 45^\circ$. However, the results are almost insensitive to the zino mass at this energy. Left- and right-handed selectrons again provide longitudinal asymmetries of opposite sign. The transverse asymmetry $A_T$ is presented in Fig. F.3 and compared with the usual $e^+e^- \rightarrow e^+e^-$ one; in this case, again the sensitivity to $m_{\tilde{e}}^2$ requires large energies ($\sqrt{s} \gtrsim 150 \text{ GeV}$).

We therefore see that in both the $\tilde{e}$ and $\tilde{\mu}$ cases, polarization provides a useful tool for distinguishing between the left- and right-handed sleptons.

**Gaugino production [61]**

Three types of diagrams describe gaugino pair-production $e^+e^- \rightarrow \tilde{X}(p)\tilde{X}'(p')$ at the tree level: the annihilation one via $\gamma$ and $Z$ formation, and two exchange diagrams involving the exchange of a selectron $\tilde{e}_L, \tilde{e}_R$ or a sneutrino $\tilde{\nu}_L$. The corresponding cross-sections have a general form analogous to, and even simpler than, the scalar production ones, i.e.

$$
\frac{d\sigma}{dQ} = (1 - P^2_{\tilde{X}}) X_1(\theta) + (P^+_{\tilde{X}} - P^-_{\tilde{X}}) X_2(\theta)
+ P^+_{\tilde{X}} P^-_{\tilde{X}} \left[ X_3(\theta) \cos 2\varphi + X_4(\theta) \sin 2\varphi \right].
$$

Once again, a longitudinal asymmetry can be observed even if only the $e^+$ beam is longitudinally polarized:

$$
\tilde{A}_L = -\frac{X_2}{X_1},
$$

whilst the transverse asymmetry,

$$
\tilde{A}_T = \frac{X_3 \cos 2\varphi + X_4 \sin 2\varphi}{X_1},
$$

needs both $e^\pm$ beams to be transversely polarized.

Physical gaugino states are, up to now, unknown combinations of pure gaugino and shiggs states. In this Appendix we denote by $\tilde{w}^\pm$ and $\tilde{z}$ the Weyl spinors associated with the massless $w^\pm$ and $Z$ bosons and by $(\tilde{h}_1^0, \tilde{h}_1^-)_L$ and $(\tilde{h}_2^0, \tilde{h}_2^-)_L$ the two Weyl shiggs doublets involved in the minimal supersymmetric
SU(2)_{L} \times U(1) model. Then three limiting situations can be selected for the charged four-component Dirac winos, namely

I) the pure wino and Higgsino case:

\[ \tilde{w} = \begin{pmatrix} w_{-} \\ \bar{w} \\ \end{pmatrix} , \quad \tilde{w}' = \begin{pmatrix} \tilde{w}^{*} \\ \bar{H} \end{pmatrix} \] (F.12)

II) the maximal mixing case where 'Wiggsons' appear at the Weyl level:

\[ \tilde{w}_{a,b} = \frac{1}{\sqrt{2}} \begin{pmatrix} \tilde{w}^{*} \\ \bar{w} \pm \bar{H} \end{pmatrix} \] (F.13)

or, III) at the Dirac level

\[ \tilde{w}_{a} = \begin{pmatrix} \tilde{w} \\ \bar{H} \end{pmatrix} , \quad \tilde{w}_{b} = \begin{pmatrix} \tilde{w}^{*} \\ \bar{H} \end{pmatrix}. \] (F.14)

These three cases reduce to two for the neutral (Majorana) zinos:

I') \[ \tilde{z}_{a} = \begin{pmatrix} \tilde{z} \\ \bar{z} \end{pmatrix} , \quad \tilde{z}_{b} = \begin{pmatrix} \tilde{H}^{0} \\ \bar{z} \end{pmatrix} \] (F.15)

II') \[ \tilde{z}_{a,b} = \frac{1}{\sqrt{2}} \begin{pmatrix} \tilde{z} \pm \tilde{H}^{0} \\ \bar{z} \end{pmatrix} \] (F.16)

with \[ \tilde{H}^{0} = (v_{1}^{2} + v_{2}^{2})^{-1/2} (v_{1}^{2} H_{1}^{0} - v_{2}^{2} H_{2}^{0}) \], \( v_{1} \) and \( v_{2} \) being the vacuum expectation values of the scalar partners of \( H_{1}^{0} \) and \( H_{2}^{0} \).

Among the different gaugino pair-production processes \( e^{+} e^{-} \rightarrow \tilde{w}_{W} \tilde{w}_{W} \), \( \tilde{\nu}_{\tau}, \tilde{\nu}_{\tau}, \) and \( \tilde{\tau}_{\tau} \), the charged process appears most promising. Indeed, its cross-section \( \sigma \) is much higher than the other ones, reaching a few nanobarns at the \( Z \) peak. The cross-section \( e^{+} e^{-} \rightarrow l^{+} l^{-} \) for the pair-production of heavy leptons with the same mass as the wino is one order of magnitude lower and therefore should not be a dangerous background.

If \( 2m_{W} < m_{\tilde{w}} \), wino pair-production will be accessible at the first stage of LEP before \( \tilde{w}_{W} \tilde{w}_{W} \) pair-production itself, allowing one to test the \( Z \tilde{w}_{W} \tilde{w}_{W} \) Yang-Mills coupling before the classical \( Z \tilde{w}_{W} \tilde{w}_{W} \) one. In this case the annihilation diagram which involves the coupling of two gauginos to a gauge boson dominates, in contrast to the case \( e^{+} e^{-} \rightarrow \tilde{w}_{W} \tilde{w}_{W} \) where it is the t-channel exchange contribution which dominates over the annihilation diagrams.

A study of the \( \theta_{W} \) dependence of \( \lambda_{\tilde{w}} \) at \( s = m_{Z} \) gives very different results for the different models (see Fig. F.4). Models I and II are \( \theta_{W} \) independent.
since there is then no axial coupling of the $Z^0$ to the wino pair. This axial coupling changes sign when going from model IIIa to model IIIb. When integrated over the scattering angle $\theta$, the longitudinal asymmetry $A_L$ as well as the transverse one $A_T$ is rather model-independent. However, one finds for $A_L$ a large sensitivity to the mass of the exchanged sneutrino (see Fig. F.5).

Fig. F.1. a) Longitudinal asymmetry $A_L$ versus $\sqrt{s}$ for muons coming from $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \mu^-\mu^-$. b) Transverse asymmetry $A_T$ versus $\sqrt{s}$ for muons produced at $\phi = 0$ integrated over the polar angle $\theta$. Full curve: muon coming from $\mu$ with mass 20 GeV. Dot-dashed curve: same with $m_\mu = 40$ GeV. Dashed curve: $e^+e^- \rightarrow \mu^+\mu^-$. 
Fig. F.2 Longitudinal asymmetry $A_L$ for electrons coming from $e^+e^- \to e^+e^-$ at $\sqrt{s} = 92$ GeV. For the solid curves $m_\gamma = 20$ GeV with $m_\gamma = 0$ (1) and $m_\gamma = 15$ GeV (2). For the dot-dashed curve, $m_\gamma = 40$ GeV and $m_\gamma = 0$. For all cases $m_\gamma \leq 100$ GeV. Dashed curve: $e^+e^- \to e^+e^-$. 

Fig. F.3 Transverse asymmetry $A_T$ for electrons coming from $e^+e^- \to e^+e^-$ (with $m_\gamma = 20$ GeV) at $\theta_\gamma = 0$ as a function of $\sqrt{s}$ and integrated over the polar angle $90^o \leq \theta \leq 140^o$. The dot-dashed curves correspond to the absence of a zino contribution, the solid curves to one zino with $m_\gamma = 100$ GeV, and the double dot-dashed curve to two zinos with $m_\gamma = 77$ GeV and $m_\gamma = 107$ GeV. In the first two cases (1) means $m_\gamma = 0$ and (2) means $m_\gamma = 152$ GeV. Dashed curve: $e^+e^- \to e^+e^-$ for comparison.
Fig. F.4 The asymmetry $\tilde{A}_H$ for $e^+e^- \rightarrow \tilde{W}^+\tilde{W}^-$ at $s = m_Z$ for $m_{\tilde{W}} = 40$ GeV as a function of $\theta_{\tilde{W}}$ in different models.

Fig. F.5 The spin asymmetries as a function of $\sqrt{s}$ for $e^+e^- \rightarrow \tilde{W}^+\tilde{W}^-$ with $m_{\tilde{W}} = 30$ GeV. Full curve: $\tilde{A}_H$ with $\nu_L$ exchange and $m_{\tilde{W}} = 20$ GeV; dot-dashed curve: $\tilde{A}_H$ with $m_{\tilde{W}} = 60$ GeV; dashed curve: $\tilde{A}_H$ without $\nu_L$ exchange. $\tilde{A}_L$ is represented by the thick line.
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4. OTHER NEW PARTICLES

4.1 Introduction

In Sections 2 and 3 we have discussed some new particles whose existence with masses accessible to LEP has strong theoretical motivation from the hierarchy and naturalness problems. In this section we review some 'classical' examples of new particles which should be sought at LEP, even if there is no strong motivation for their existence. This would make their discovery all the more interesting.

4.2 Heavy leptons

The total cross-section for pair-producing non-electron-type heavy leptons is just

$$\sigma_{LL} = \sigma(T, T_L, T_R, \beta, s)$$

in the notation of Section 1. In general in the Standard Model, right-handed lepton components are assigned to SU(2) singlets ($T_R = 0$) to prevent them from participating in charged-current interactions. This may not be true for new leptons as yet unobserved. In particular, for Majorana neutrinos, $T_R = -T_L$ whilst also $\sigma_{LL} = \sigma_{LL}/2$ because the final-state particles are identical.

Figure 4.1 shows, as a function of beam energy, the ratio of massive lepton-pair production to muon-pair production for lepton masses of 25, 40, 60, and 80 GeV and a range of $(Q, T_L)$ assignments. Contours corresponding approximately to production of 100, 1000, and 10000 events per year, assuming 100 pb$^{-1}$ per year and neglecting radiative corrections, are also indicated. In most cases rates are substantial, even for 80 GeV leptons at LEP II.

A fourth-generation sequential charged lepton, $(Q, T_L) = (-1, -1/2)$, whose associated neutrino $L^0$ is relatively light, will decay through $L^\pm + L^0$ + real or virtual $W^\mp$ [1]. From the branching ratios of the subsequent real or virtual $W$ decays, the rates in the channels $L^\pm + L^0$, $L^0\nu\bar{\nu}$, $L^0\nu\bar{\nu}$, $L^0\bar{\nu}d$, $L^0\bar{\nu}cs$ will be approximately in the ratio 1:1:1:3:3. Thus the $L^\pm L^\mp$ events will be seen as $l_jl_j$ events (7%), $l_lj_j$ events (39%), or $j_jj_jj_j$ events (54%), where $l$ stands for an $e$ or $\mu$, $J$ for a hadron jet, and $\not{E}_T$ for the missing transverse momentum carried off by the $L^0$ and other neutrinos. Half of the $l_jl_jj_j$ events, i.e. 3-4% of all events, will be $e^-\mu^+\not{E}_T$ or $e^+\mu^-\not{E}_T$ events without hadrons: the other half will be $e^-e^-\not{E}_T$ or $\mu^+\mu^-\not{E}_T$ events. The acollinearity of the observed leptons depends on the $L^\pm$ mass and will typically be much larger than for the $\tau^+\tau^-$ background (Fig. 4.2a, b); so will the momentum component of one lepton out of the plane defined by the beam and the other lepton (Fig. 4.2c), as will the jet-jet opening angle and invariant mass (Figs. 4.2d, e, f, g) in the $l_lj_jj_jj_j$ events. Thus the signal will be loud and clear.
Fig. 4.1 Ratios relative to $\sigma(e^+e^-\gamma^*,Z\rightarrow\mu^+\mu^-)$ of the cross-sections for $e^+e^-\gamma^*,Z\rightarrow$ heavy lepton pairs of masses (a) 25 GeV, (b) 40 GeV, (c) 60 GeV and (d) 80 GeV. The numbers in parentheses on the right-hand sides of the figures are the charge $Q$ and the third component of isospin $T_3$ for the different species of heavy lepton considered. The dashed lines are contours of equal cross-sections: 1, 10 and 100 pb.
Fig. 4.2: Final-state distributions from $e^+e^- \rightarrow L^+L^-$ heavy lepton pair events. (a) Opening angles between decay leptons from L$^+L^-$ production at the Z$^0$ peak. (b) Ditto, but at $\sqrt{s} = 180$ GeV with $m_L = 60$ GeV. (c) Momentum component of one lepton out of the plane defined by the beam and the other lepton. (d) Distributions in the normal jet-jet opening angles from L$^+L^-$ production at the Z$^0$ peak. (e) Ditto, but at $\sqrt{s} = 180$ GeV with $m_L = 60$ GeV. (f) Jet-jet invariant mass distributions from L$^+L^-$ production at the Z$^0$ peak. (g) Ditto, but at $\sqrt{s} = 180$ GeV with $m_L = 60$ GeV.
For completeness, we quote here the spin-correlated cross-section for pair-production and decay of heavy leptons on the $Z^0$ pole [2]. The spin-summed squared matrix element is given by

$$|m|^2 = |G|^2 2^{22} (x_1 + x_2 + x_3 + x_4),$$  \hspace{1cm} (4.2)

where

$$x_1 = \left[ (a_1^2 + b_1^2) a_2 b_2 \mu(q_e \bar{q}_L \bar{q}_L - q_L \bar{q}_e \bar{q}_e) \left( q_e \bar{L} \bar{L}, \bar{e} \right) \left( \mu, \nu q_L \bar{q}_L \bar{q}_L \right) \right],$$  \hspace{1cm} (4.3a)

$$x_2 = \left[ \left( a_1^2 + b_1^2 \right) a_2 b_2 \mu^2 q(q_e \bar{q}_L \bar{q}_L - q_L \bar{q}_e \bar{q}_e) q \bar{L} \right] \left( q_e \bar{L} \bar{L}, \bar{e} \right) \left( \mu, \nu q_L \bar{q}_L \bar{q}_L \right),$$  \hspace{1cm} (4.3b)

$$x_3 = -\left[ (a_1^2 + b_1^2) a_2 b_2 \mu q_e q_L \bar{q}_L - q_L \bar{q}_e q L \bar{q}_e \right] q \bar{L}$$

$$+ \left( b_1^2 \mu^2 - a_1^2 \mu^2 \right) q_e \bar{L} q_L \bar{q}_L - m_L^2 q_e q_L \bar{q}_L,$$

$$+ \left( a_1^2 + b_1^2 \right) a_2 b_2 \mu^2 q \left( q_L \bar{q}_e q_e - q_L \bar{q}_e q_e \right) \left( \mu, \nu q_L \bar{q}_L \bar{q}_L \right) \left( q_e \bar{L} \bar{L}, \bar{e} \right),$$  \hspace{1cm} (4.3c)

$$x_4 = \left( q_e \bar{L} \bar{L}, \bar{e} \right) \left( \mu, \nu_q \right) \left[ (a_1^2 + b_1^2) a_2 b_2 \mu^2 q \left( q_e \bar{L} \bar{L}, \bar{e} \right) \left( q_L \bar{q}_e q_e - q_L \bar{q}_e q_e \right) \left( \mu, \nu q_L \bar{q}_L \bar{q}_L \right) \left( q_e \bar{L} \bar{L}, \bar{e} \right),$$  \hspace{1cm} (4.3d)
and

$$|G|^2 = 8G_F^2 m_W^2 |D_0(W^2_1)|^2 |D_2(W_2^2)|^2 |D_Z(s)|^2 \bigg( \frac{m_L}{m_F} \bigg)^2 \delta(L^2 - m_L^2) \delta(L^2 - m_L^2). \quad (4.3e)$$

In this formula, particle labels are used to denote four-momenta, and

$$D_V(Q^2) = \frac{1}{(Q^2 - m_V^2)^2 + m_V^2 \Gamma_N^2}, \quad (4.3f)$$

$$a = \frac{g_V + g_A}{2} \quad \text{and} \quad b = \frac{g_V - g_A}{2}, \quad (4.3g)$$

where \( g_V = (T_3 - 2Qx_w) \) and \( g_A = \frac{1}{2} T_3 \). \quad (4.3h)

The term \( x_1 \) is just the product of production and decay squared matrix elements; it integrates to the total cross-section. In the terms \( x_2 \) and \( x_3 \), one decay of \( L \) or \( L^- \) factorizes, whilst in the term \( x_4 \) there is no factorization of production and decay. The terms \( x_2, x_3, \) and \( x_4 \) each integrate to give no net contribution to the total cross-section.

Equations (4.2) and (4.3) can be easily adapted to heavy-lepton production through a virtual photon by the following substitutions:

$$32G_F^2 m_Z^4 \rightarrow e^4$$

$$g_V^{\hat{L}, f} = Q_{\hat{L}, f}$$

$$g_A^{\hat{L}, f} \rightarrow 0$$

$$|D_Z(s)|^2 \rightarrow 1/s^2,$$ \quad (4.4)

where \( Q_{\hat{L}, f} \) is the charge of the initial/final fermion in units of e. Production of conventional \( L^+L^- \) pairs is dominantly through the \( Z^0 \) for \( 70 < s < 120 \text{ GeV} \); for other energies, production through a virtual photon is dominant.

A massive neutral lepton \( L^0 \), \( (Q, T_{3L}) = (0, 1/2) \), whose associated charged lepton is even more massive will perforce decay into other charged leptons such as \( L^0 \rightarrow \nu^+ L^- \) at rates dependent on the relevant \( (L^0, \nu_e) \) mixing parameters. Unless all mixings are so small \( |U_{Li}|^2 \ll 10^{-7} \) that all decays occur outside the detector, very distinctive events will again result. These will either contain just four leptons of otherwise random flavour plus missing \( p_T \), or have hadron jets replacing either or both of the leptons from \( \nu^- \) decay together with its associated neutrino.
Fig. 4.3 Cross-sections for $e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$ via $V \pm A$ couplings (a) at $\sqrt{s} = 100$ GeV, and (b) at 200 GeV.

For production of possible charged or neutral heavy leptons carrying electron number, t-channel diagrams are also important. The most interesting case, because of the high cross-section and mass reach, is the reaction $e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$. The cross-section through W exchange in the case of maximal coupling is given by [3]

$$
\sigma(s, m_\nu^0, m_W) = \frac{G^2_F}{2\pi} \left( 1 - \frac{m_\nu^0}{s} \right)^2 \frac{m_W^2}{s}
$$

and is plotted versus $m_\nu^0$ for $\sqrt{s} = 100$ and 200 GeV in Fig. 4.3. Such an object would decay through $e^+ e^- \rightarrow e^+ W^-$ followed by $W \rightarrow l\nu$ or $q_1 \bar{q}_2$, giving characteristic $e^+ l^+$ events or $e^+ JJ$ events with missing energy. It would be observable for $m_\nu^0$ up to the kinematic limit at LEP II.

4.3 New heavy quarks

In addition to fourth-generation colour-triplet quarks, massive quarks may also exist with higher colour representations [4], such as sextets, octets, etc., of SU(3)$_C$. The cross-sections for such particles are given by formula (1.6) et
They are larger than for triplet quarks because of the larger statistical factor $N$. If present with masses $< m_{Q_3}/2$ they could cause a significant increase in its total width and a consequent reduction in its leptonic decay branching ratio. Such exotic quarks would combine with conventional quarks and gluons to form colour-singlet hadrons, e.g. $Q_6 \bar{q}q$ or $Q_8 g$. At least one new hadron would be stable against conventional weak decay, since the $W^\pm$ and $Z^0$ are colour singlets. However, the exotic quarks might be able to decay through some new interaction, perhaps with a long lifetime. Depending on the unknown electromagnetic charge of the exotic quark, some of the hadrons might have non-integral electromagnetic charges, in which case the lightest such object would automatically be absolutely stable. However, cosmological estimates [5] of the likely abundance of stable, heavy, electromagnetically charged particles conflict with experimental upper limits [6] on anomalous heavy isotopes with masses $< O(1)$ TeV. It therefore seems unlikely that such a stable charged hadron could be accessible to LEP.

The different strengths of the QCD interactions of colour non-triplet quarks have distinctive effects on their quarkonium spectroscopy [7]. Figure 4.4 compares the mass differences between the ground state and the lowest orbital and radial excitations for colour triplet and octet quarkonia. The upper set of curves corresponds to the Richardson [8] potential, and the lower set to the Kühn-Ono [9] potential. Both potentials adequately reproduce both charmonium and bottomonium spectroscopy. They differ only at very small radii, $< 0.1$ fm, where the potential is not effectively probed by the charmonium and bottomonium states. Figure 4.5 taken from Ref. 8 compares the Richardson potential,

![Fig. 4.4 Mass splittings between higher quarkonium states and the lightest 1S state, for triplet quarks $Q_6$ and octet 'quarks' $Q_8$. Also indicated is the range of model-dependence suggested by the different potentials discussed in the text.](image1)

![Fig. 4.5 Three potentials which reproduce (cc) and (bb) spectra but have different extrapolations into the short-distance region. The classical turning point is shown for various quark masses.](image2)
\[ V(r) = \frac{8\pi}{33 - 2n_f} \Lambda \left( \Lambda r - \frac{f(\Lambda r)}{\Lambda r} \right), \quad \left[ \Lambda = 398 \text{ MeV}, \quad n_f = 4, \right. \]

where

\[ f(\tau) = \frac{4}{\pi} \int_0^\infty dq \frac{\sin(q\tau)}{q} \left[ \frac{1}{\ln(1+q^2)} - \frac{1}{q^2} \right] \]

\[ = \left[ 1 - 4 \int_1^\infty dq \frac{e^{qt}}{q} \right] \left[ \int_0^\infty dq \frac{e^{qt}}{[\ln(q^2-1)] + w^2} \right], \]

with the Kühn-Ono potential [9]

\[ V_T(r) = -\frac{16\pi}{25} \frac{1}{f(r)} \left[ 1 + \frac{2\gamma_E + 53/75}{462 \ln f(r)} - \frac{625 \ln f(r)}{625 f(r)} \right] + a/\tau + c \]

\[ f(r) \equiv \ln \left[ \frac{1}{(\Lambda_{\overline{\text{MS}}} r)^2 + b} \right], \]

using

\[ \Lambda_{\overline{\text{MS}}} = 140 \text{ MeV}, \]

\[ a = 0.63 \text{ GeV}^{3/2}; \quad b = 20; \quad c = -1.39 \text{ GeV}. \]

Clearly the model-dependence of the calculations shown in Fig. 4.5 is quite large. For the octet case the whole potential, including the confining piece, has been scaled by the ratio of the octet and triplet Casimirs, i.e. by the factor \( 3/2 \). The effect of the stronger QCD couplings of the higher representation quarks is clearly large and easily measurable.

4.4 Free quarks

Free quarks have been hunted at all new accelerators. Figure 4.6 [10] shows the limits currently achieved at PETRA and PEP. Searches have also been conducted recently in muon [11] and neutrino [12] scattering of nuclei at the CERN Super Proton Synchrotron (SPS) and in pp collisions [13] at the CERN pp Collider. Clearly sustained running on the \( Z^0 \) peak at LEP will provide an ideal opportunity to continue the search.

These experiments limit also the fractionally charged hadrons and colour singlet quarks arising in more exotic schemes (see above) as well as 'standard' quarks. Whilst exact QCD (presumably) confines 'standard' quarks permanently, slightly broken QCD need not do so. Some years ago, De Rujula, Giles and Jaffe [14] speculated on the effects of slightly breaking QCD by giving gluons a small
mass in the context of a minimally modified MIT bag model. They concluded that free quarks and gluons should then exist. They should be massive, large-radius objects with correspondingly strongly suppressed production cross-sections but large strong interaction cross-sections for absorbing ('eating') any nucleons in their path, thus changing their charge and baryon number, until their potential well ('appetite') has been filled. Table 4.1 gives the calculated values of the main free-quark parameters as a function of the gluon mass assumed. The production cross-sections predicted [14] are exceedingly small (Fig. 4.7), but Bjorken [15] has emphasized the very large uncertainties inherent in such a calculation.
Fig. 4.7 Guesses of the inclusive free quark production rates for different quark masses, assuming a dipole or monopole form factor, as functions of $q^2 = E^2_{CM}$

Theoretical arguments [16, 17] have been advanced against the scheme of Ref. [14] but do not appear to be quite conclusive. Georgi [16] pointed out that the gauge symmetry would be dynamically restored if the transition to the broken-symmetry phase were first-order. But it may not be [18]. Okun' and Shifman [17] pointed out that if the Higgs particles giving masses to the gluons were colour triplets (or $6, 15$, etc.) then low-mass fractional-charge hadrons ('hidrons') would arise which are experimentally excluded. However, if they were colour octets (or $10, 27$, etc.) the 'hidrons' would have integral charge and so might still be experimentally acceptable.

Although there are no reasons for optimism, the large number of $Z^0$ decays with $Q^2 = 0(10^4)$ GeV$^2$ offer a unique and clean opportunity to see whether the 'bags' containing quarks can be split open. The search for free quarks will be an essential aspect of LEP physics.

* * *

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