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ON THE ADDITION OF VECTOR-LIKE QUARKS TO THE STANDARD MODEL

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ABSTRACT

We analyze some of the implications of adding vector-like isosinglet quarks to the standard model and its simplest extension based on the low-energy gauge group $SU(3)_C \times SU(2) \times U(1)^m$, which naturally arises in some grand unified theories as well as in some versions of superstring theories. Some of the novel features of this class of models are pointed out: non-unitarity of the Cabibbo-Kobayashi-Maskawa matrix, new CP-violating phases, flavour-changing neutral currents. We derive the CP invariance restrictions on the various quark mass terms and propose a parametrization of the quark mixing matrix which is particularly suitable for models with vector-like quarks. Constraints on these models are derived from rare kaon decays and the value of the K_L-K_S mass difference.

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1. - INTRODUCTION

The $SU(3)_C \times SU(2) \times U(1)$ model has had impressive success when subjected to experimental tests [for a recent review, see, for example, Ref. 1)]. However, it is unlikely that the standard model is the ultimate theory, since it has too many free parameters and leaves unanswered too many fundamental questions. Therefore, it is important to investigate what may lie beyond the standard model, awaiting discovery by the next generation of accelerators.

One of the simplest additions to the standard model consists of introducing isosinglet vector-like quarks and/or leptons. There is a variety of theoretical motivations for considering vector-like fermions²⁾. For example, in some superstring theories³⁾, one starts from $E_8 \times E_8$ in ten dimensions, and the requirement of having an unbroken $N = 1$ supersymmetry in four dimensions naturally leads to an E_6 gauge group in four dimensions, with each fermion generation placed in a 27-dimensional representation. In this case, for each standard fermion generation one would have new fermions which include an extra vector-like isosinglet charge $-1/3$ quark, together with its antiquark.

In this paper we analyze some of the implications of isosinglet vector-like quarks, with special emphasis on some novel features of CP violation which arise when vector-like quarks are present⁴⁾. We will keep our discussion as model-independent as possible. In the next section, we derive the restrictions that CP invariance implies for the various quark mass matrices. These restrictions are written in a conveniently-chosen weak basis. In Section 3, we derive some invariant conditions necessary for CP invariance. These conditions are a generalization of those previously derived⁵⁾ in the context of the standard model with an arbitrary number of generations^{*}), and are specially useful in building and testing models of CP violation, since they are written in terms of quantities which are invariant under a change of weak basis. Therefore the invariant conditions can be applied directly to the various quark mass matrices, without the need to diagonalize them. In Section 4, we analyze the various sources of CP violation. In particular, we point out that with the introduction of vector-like quarks, the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix is no longer unitary, and therefore the analysis of CP violation through charged and neutral current interactions requires special care. We suggest a parametrization of the quark mixing matrices which is particularly suitable for models with vector-like

*) For the case of three generations, see also C. Jarlskog in Ref. 6).

isosinglet quarks. An approximate diagonalization of the quark mass matrices is carried out in Section 5, where we also emphasize that, although the class of models considered here does not conform to the principle of natural flavour conservation⁷⁾ of Glashow and Weinberg, the flavour-changing neutral currents (FCNC) are naturally suppressed provided that the $\Delta I = 0$ quark masses are substantially larger than the corresponding $\Delta I = \frac{1}{2}$ masses. In Section 6, we analyse some of the phenomenological implications of models with isosinglet vector-like quarks, and in the last section we draw our conclusions.

2. - CP RESTRICTIONS ON QUARK MASS MATRICES

In order to widen the applicability of our considerations, we will assume that the low-energy gauge group is $SU(3) \times SU(2)_L \times U(1)^m$. Such low-energy gauge groups naturally arise in certain grand unified theories as well as in some versions of the currently popular superstring theories³⁾. Moreover, we will assume that quarks with the same helicity, electric charge ($-1/3$ or $2/3$) and weak isospin (0 or $\frac{1}{2}$) will have equal value of all the $U(1)$ hypercharges. Finally, all right-handed quarks will be assumed to be singlets under $SU(2)_L$. Under these assumptions, the quark field content of the theory will be denoted in the following way:

$$\begin{array}{ll}
 (u \ d)_{Li} & i = 1 \dots n \\
 D_{Lp} & p = 1 \dots N_d \\
 U_{Lq} & q = 1 \dots N_u \\
 D_{R\alpha} & \alpha = 1 \dots (n+N_d) \\
 U_{R\beta} & \beta = 1 \dots (n+N_u)
 \end{array} \tag{2.1}$$

We are thus considering the existence of n left-handed doublets, N_d left-handed charge $-1/3$ isosinglet quarks, etc. The gauge interactions can be written in terms of quark weak eigenstates as:

$$\frac{g}{\sqrt{2}} \left(\bar{u}_{iL} \gamma^\mu d_{iL} W_\mu^+ + \text{h.c.} \right) + \sum_{k=1}^m \left(a_k \bar{u}_{iL} \gamma^\mu u_{iL} + b_k \bar{d}_{iL} \gamma^\mu d_{iL} + J_k^\mu \right) Z_\mu^k - e J_{em}^\mu A_\mu \tag{2.2a}$$

$$J_K^\mu = \chi_K^{(1)} \bar{D}_{R\alpha} \gamma^\mu D_{R\alpha} + \chi_K^{(2)} \bar{U}_{R\beta} \gamma^\mu U_{R\beta} + \\ + \chi_K^{(3)} (\bar{u}_{Li} \gamma^\mu u_{Li} + \bar{U}_{Lq} \gamma^\mu U_{Lq}) + \chi_K^{(4)} (\bar{d}_{Li} \gamma^\mu d_{Li} + \bar{D}_{Lp} \gamma^\mu D_{Lp}) \quad (2.2b)$$

where χ_k^j are arbitrary real coefficients and a summation over repeated indices is understood. The defining feature of J_k^μ is that they keep their diagonal form when written in terms of quark mass eigenstates. This is, of course, due to the fact that the summations entering into J_k^μ run over all quarks of given charge and helicity. As a result, the currents J_k^μ will not play any rôle in our discussions.

In the present work, we will be interested only in CP violation arising from charged and neutral gauge interactions, and therefore we do not need to specify the mechanism responsible for the generation of quark masses, which will be written as:

$$\bar{u}_{Li} (m_u)_{i\beta} U_{R\beta} + \bar{U}_{Lq} (M_u)_{q\beta} U_{R\beta} + \bar{d}_{Li} (m_d)_{i\alpha} D_{R\alpha} + \\ + \bar{D}_{Lp} (M_D)_{p\alpha} D_{R\alpha} \quad (2.3)$$

where the dimensions of the four mass matrices are implicit through the use of the index range convention of (2.1). Next we will derive CP invariance constraints on quark mass matrices, by noting that CP symmetry requires the existence of transformations for the quark fields of the type

$$u_L \rightarrow V_L C u_L^* ; U_L \rightarrow W_L^U C U_L^* ; U_R \rightarrow W_R^U C U_R^* \\ d_L \rightarrow V_L C d_L^* ; D_L \rightarrow W_L^D C D_L^* ; D_R \rightarrow W_R^D C D_R^* \quad (2.4)$$

which leave the Lagrangian invariant. In (2.4), $V_L, W_{L,R}^{U,D}$ are unitary matrices acting in generation space and C is the charge conjugation matrix. In order for the Lagrangian to be invariant under the set of transformations (2.4), the quark mass matrices must satisfy the conditions:

$$V_L^\dagger m_u W_R^U = m_u^* \quad (2.5a)$$

$$V_L^\dagger m_d W_R^D = m_d^* \quad (2.5b)$$

$$(W_L^D)^\dagger M_D W_R^D = M_D^* \quad (2.5c)$$

$$(W_L^U)^\dagger M_U W_R^U = M_U^* \quad (2.5d)$$

The set of relations (2.5) constitutes the necessary and sufficient conditions for having CP invariance in the charged and neutral gauge interactions. It can easily be shown that these conditions are weak-basis-independent, in the sense that if there are unitary matrices $V_L, W_{L,R}^{U,D}$ satisfying (2.5) in a particular weak basis, there will also exist unitary matrices $V_L', W_{L,R}'^{U,D}$ which satisfy (2.5) in another weak basis.

We will now deduce the constraints that Eq. (2.5) imply for the four mass matrices in a weak basis where these matrices take a particularly simple form. It can be shown that, without loss of generality, there is a weak basis where the mass matrices are

$$\begin{aligned} m_u &= \begin{pmatrix} \tilde{m}_u & 0 \end{pmatrix} & m_d &= \begin{pmatrix} G & J \end{pmatrix} \\ M_U &= \begin{pmatrix} L & \tilde{M}_U \end{pmatrix} & M_D &= \begin{pmatrix} 0 & \tilde{M}_D \end{pmatrix} \end{aligned} \quad (2.6)$$

where $\tilde{m}_u, \tilde{M}_D, \tilde{M}_U$ are diagonal real positive matrices of dimensions n, N_d, N_u respectively, while L, G and J are arbitrary matrices of dimensions $N_u \times n, n \times n$ and $n \times N_d$ respectively. From Eq. (2.5a), it follows that

$$V_L^\dagger (m_u m_u^\dagger) V_L = (m_u m_u^\dagger)^* \quad (2.7)$$

which in view of (2.6) implies

$$V_L = \text{diag.} [\exp(i\alpha_i)] \quad (2.8)$$

Here and in the following we use the index range conventions of Eq. (2.1). Similarly, Eqs. (2.6), (2.5a) and (2.5d) constrain W_R^U , W_L^U to be of the form

$$W_R^U = \text{diag.} [\exp(i\delta_\beta)] \quad (2.9)$$

$$W_L^U = \text{diag.} [\exp i\delta_{n+q}] \quad (2.10)$$

with $\delta_i = \alpha_i$ ($i = 1, \dots, n$). Equations (2.9) and (2.10), together with Eq. (2.5d) constrain the matrix L to have phases

$$\arg. L_{qi} = \frac{1}{2} (\delta_{n+q} - \alpha_i) \quad (2.11)$$

We turn now to Eq. (2.5c), and note that M_D constrains W_R^D to be of the form

$$W_R^D = \left(\begin{array}{c|c} Y & 0 \\ \hline 0 & \text{diag.} [\exp i\gamma_p] \end{array} \right) \quad (2.12)$$

where Y is a unitary $n \times n$ matrix. Finally, taking into account the form of m_d , one deduces from Eq. (2.5b)

$$\arg J_{ip} = \frac{1}{2} (\alpha_i - \gamma_p) \quad (2.13)$$

$$\text{diag.} [\exp(-i\alpha_i)] G Y = G^* \quad (2.14)$$

which constrains GG^+ to be a matrix with cyclic phases.

At this point, it is worthwhile summarizing the constraints that CP invariance implies for the various quark mass matrices. The phases appearing in GG^+ , J , L are constrained to be such that

$$\begin{aligned} \arg (GG^+)_{ii'} &= \frac{1}{2} (\alpha_i - \alpha_{i'}) \\ \arg L_{qi} &= \frac{1}{2} (\delta_{n+q} - \alpha_i) \\ \arg J_{ip} &= \frac{1}{2} (\alpha_i - \delta_p) \end{aligned} \quad (2.15)$$

It is straightforward to convince oneself that Eqs. (2.15) mean the following number of independent constraints (N_c) on the mass matrices:

$$N_c = \frac{1}{2} (n-1) [(n-2) + 2 (N_u + N_d)] \quad (2.16)$$

The conditions expressed in Eqs. (2.15) are necessary and sufficient for CP invariance. However, they are of little practical use, due to the fact that they are written in a special weak basis where the quark mass matrices have the form (2.6). In the next section, we will deduce necessary conditions for CP invariance which are invariant under a change of weak basis, and can therefore be used for testing and constructing models of CP violation.

3. - INVARIANT CONDITIONS FOR CP SYMMETRY

We will now derive a set of conditions necessary for CP invariance, written in terms of quantities which are invariant under a change of weak basis. From Eqs. (2.5), one obtains

$$\begin{aligned} V_L^\dagger h_d V_L &= h_d^T ; & W_R^{D\dagger} h'_d W_R^D &= h_d'^T \\ V_L^\dagger h_u V_L &= h_u^T ; & W_R^{U\dagger} h'_u W_R^U &= h_u'^T \\ W_R^{D\dagger} H_D W_R^D &= H_D^T ; & W_R^{U\dagger} H_U W_R^U &= H_U^T \end{aligned} \quad (3.1)$$

where

$$h_{d,u} = \begin{pmatrix} m_{d,u} & m_{d,u}^\dagger \end{pmatrix}; \quad h'_{d,u} = \begin{pmatrix} m_{d,u}^\dagger & m_{d,u} \end{pmatrix}; \quad H_{D,U} = \begin{pmatrix} M_{D,U}^\dagger & M_{D,U} \end{pmatrix} \quad (3.2)$$

Multiplying Eqs. (3.1) by themselves and evaluating appropriate commutators, one easily derives the following set of invariant conditions:

$$\begin{aligned} \text{tr} [h_d^a, h_u^b]^r &= 0 \\ \text{tr} [h_d'^a, H_D^b]^r &= 0 \\ \text{tr} [h_u'^a, H_U^b] &= 0 \end{aligned} \quad (3.3)$$

where r is odd and a, b are arbitrary positive integers.

It is obvious that the conditions of Eq. (3.3) are weak-basis-independent. For instance, if under a change of weak basis

$$\begin{pmatrix} u \\ d \end{pmatrix}_i \longrightarrow A_{ii'} \begin{pmatrix} u \\ d \end{pmatrix}_{i'} \quad (3.4)$$

with A an n -dimensional unitary matrix, $h_{d,u}$ transform as

$$h_d(u) \longrightarrow A h_d(u) A^\dagger \quad (3.5)$$

and therefore the first of Eqs. (3.3) remains invariant. Similar arguments apply to the other two conditions.

The relations (3.3) are necessary conditions for CP invariance and they are useful for constructing and testing models of CP violation with vector-like quarks, since they are weak-basis-independent and do not require diagonalization of any quark mass matrices. The violation of any of the conditions (3.3) implies CP non-conservation. However, it should be pointed out that the conditions of Eq. (3.3) are not sufficient for CP invariance. This is to be expected since, in the case of the standard model, the analogous invariant conditions are also insufficient when there are more than three generations⁵⁾.

4. - PARAMETRIZATION OF QUARK MIXING MATRICES

We will next analyze the mixing angles and CP-violating phases appearing in the charged and neutral currents, when these are expressed in terms of quark mass eigenstates. We will first consider the special case where there are only charge $-1/3$ vector-like quarks (i.e., $N_u = 0$), as happens for instance in the E_6 model.

- 1) The case $N_u = 0$, $N_d \neq 0$
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In this case, the discussion is simplified by the fact that there is a weak basis where the up quark mass matrix is diagonal. In that weak basis, the down quark mass matrix is diagonalized by a bi-unitary transformation, and

$$\begin{pmatrix} d \\ D \end{pmatrix}_L = X_D \begin{pmatrix} D' \end{pmatrix}_L \quad (4.1)$$

where the D'_L are the $n + N_d$ left-handed charge $-1/3$ quark mass eigenstates. It is convenient to write X_D as

$$X_D = \begin{pmatrix} A_D \\ -B_D \end{pmatrix} \quad (4.2)$$

where A_D , B_D are $n \times (n+N_d)$, $N_d \times (n+N_d)$ matrices respectively. The charged and neutral current interactions can be written

$$\begin{aligned} & \left(g \bar{u}_L \gamma^\mu A_D D'_L W_\mu^+ + h.c. \right) + \sum_{k=1}^m Z_\mu^k \left(a_k \bar{u}_L \gamma^\mu u_L + \right. \\ & \left. + b_k \bar{D}'_L \gamma^\mu A_D^\dagger A_D D'_L + J_k^\mu \right) \end{aligned} \quad (4.3)$$

It is clear that A_D is the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which is no longer unitary. The neutral current involving charge $-1/3$ quarks is not diagonal any more, and its structure is dictated by the matrix $A_D^\dagger A_D$. Note that the CKM matrix completely determines the neutral current mixing matrix. Therefore, experimental bounds on the strength of flavour-changing neutral currents (FCNC) imply constraints on the CKM matrix, and vice versa. At this point, one may ask what will be the number of CP-violating phases appearing in the CKM matrix A_D . The crucial point is that A_D is not a unitary matrix, but rather the first n rows of the unitary matrix X_D . We will

adopt a convenient way of parametrizing X_D , which will enable us to have only the physically meaningful phases in the CKM matrix.

Let us write⁸⁾ the $N = (n+N_d)$ -dimensional unitary matrix X_D as a product of "complex rotations" Ω_{ij} :

$$X_D = \prod_{i=N-1}^1 \prod_{j=i+1}^N \Omega_{ij} \rightarrow \begin{aligned} N=3 : X_D &= \Omega_{23} \Omega_{12} \Omega_{13} \\ N=4 : X_D &= \Omega_{34} \Omega_{23} \Omega_{24} \Omega_{12} \Omega_{13} \Omega_{14} \end{aligned} \quad (4.4)$$

where Ω_{ij} are the unitary matrices which mix the i 'th and j 'th families. If one introduces the orthogonal matrices

$$O_{ij} = \begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & c_{ij} & -s_{ij} \\ & & s_{ij} & c_{ij} \\ & & & & \ddots & \\ & & & & & 1 \end{pmatrix} \begin{matrix} \leftarrow i \\ \leftarrow j \end{matrix} \quad (4.5)$$

and the phase transformations

$$I_i(\delta_k) = \begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & e^{i\delta_k} & \\ & & & \ddots \end{pmatrix} \leftarrow i \quad (4.6)$$

then it can be shown that the matrix X_D can be written as

$$X_D = O_{N-1,N} I_N O_{N-2,N-1} O_{N-2,N} I_{N-1} I_N \dots O_{23} \dots \dots O_{2N} I_3 \dots I_N O_{12} \dots O_{1N} \quad (4.7)$$

where we have omitted phase factors appearing on the left and on the right of all other matrices, since they can be absorbed through redefinition of quark fields. It is clear that X_D in (4.7) has the correct number of orthogonal rotations and physical phases.

A straightforward but tedious calculation shows that the N -dimensional unitary matrix X_D , when parametrized as (4.7), contains in its first n rows

$\frac{1}{2}(n-1)(n-2 + 2N_d)$ phases ($N_d = \int N - n$). These are the only physically relevant CP-violating phases and their number coincides with the number of CP invariance restrictions which we have previously derived [(2.16)]. For example, in the E_6 model with $n = 27$ multiplets, since there is a vector-like charge $-1/3$ quark per family, there will be $\frac{1}{2}(n-1)(3n-2)$ CP-violating phases in the weak charged current mixing matrix. In the E_6 model with three generations, the six-dimensional X_D matrix,

$$X_D = O_{56} I_6(\delta_{10}) O_{45} O_{46} I_5(\delta_9) I_6(\delta_8) \dots O_{26} I_3(\delta_4) \dots \dots I_6(\delta_1) O_{12} \dots O_{16} \quad (4.8)$$

will have its phases distributed along the rows in the following way:

$$\left[\begin{array}{cccc} \delta_1 & \delta_2 & \delta_3 & \delta_4 \\ \delta_1 & \dots & \dots & \delta_7 \\ \delta_1 & \dots & \dots & \delta_9 \\ \delta_1 & \dots & \dots & \delta_{10} \\ \delta_1 & \dots & \dots & \delta_{10} \end{array} \right] \quad (4.9)$$

i.e., the first row is real, the second one has four phases, and so on. Of course, only the first seven phases, appearing in the first three rows, are physically relevant.

2) The case $N_u \neq 0, N_d \neq 0$

We will now briefly discuss the case where both N_u and N_d are non-vanishing. The important difference is that in this case it is no longer possible to find a weak basis where the charge $2/3$ quark mass matrix is diagonal. The simplest weak basis form for the mass matrices is that of Eq. (2.6). Then, besides (4.1), one has

$$\begin{pmatrix} u \\ U \end{pmatrix}_L = X_U \begin{pmatrix} U' \end{pmatrix}_L \quad (4.10)$$

where U'_L are the $(n+N_u)$ left-handed charge $2/3$ quark mass eigenstates. We will write X_U as:

$$X_U = \begin{pmatrix} A_U \\ -\frac{A_U}{B_U} \end{pmatrix} \quad (4.11)$$

where A_U, B_U are $n \times (n+N_U)$, $N_U \times (n+N_U)$ matrices respectively. The charged and neutral current interactions are then given by:

$$g \left(\bar{U}'_L \gamma^\mu A_u^\dagger A_D D'_L W_\mu^+ + h.c. \right) + \sum_k \left[a_k \bar{U}'_L \gamma^\mu (A_u^\dagger A_u) U'_L + b_k \bar{D}'_L \gamma^\mu (A_D^\dagger A_D) D'_L + J_k^\mu \right] \quad (4.12)$$

The CKM matrix is now given by $A_U^\dagger A_D$, and in general there will be flavour-changing couplings in the neutral currents of both charge $-1/3$ and charge $2/3$ quarks. Again, a convenient parametrization of $X_{U,D}$ is that of Eq. (4.4).

5. - THE LIMIT OF SMALL MIXING ANGLES

So far, our discussion has been quite general and we have not taken into account the phenomenological constraints on models with vector-like quarks. It is obvious that these models do not conform to the principle of natural flavour conservation⁷⁾ of Glashow and Weinberg, and there are, as we have seen, flavour-changing neutral currents at tree level. Furthermore, the CKM matrix is no longer unitary, as we have previously emphasized. It turns out that both the experimental limits on the strength of FCNC and on deviations from unitarity of the CKM matrix can be satisfied, provided the mixing between vector-like and standard quarks is small.

In this connection, the parametrization we have introduced in (4.7) proves to be very useful. For definiteness, let us consider again the example of the E_6 model with three generations of 27 multiplets. The CKM matrix is the upper 3×6 part of a unitary six-dimensional matrix X_D . If we parametrize X_D as in (4.8), and further assume that the mixing angles $\phi_{\ell f}$ between vector-like and standard quarks are small:

$$\begin{aligned} \phi_{\ell f} &\ll 1 \quad ; \quad \ell = 1, 2, 3 \quad ; \quad f = 4, 5, 6 \\ \cos \phi_{\ell f} &\equiv C_{\ell f} \approx 1 \quad ; \quad \sin \phi_{\ell f} \equiv S_{\ell f} \approx \phi_{\ell f} \end{aligned} \quad (5.1)$$

Then, if we keep only the leading terms in $\phi_{\ell f}$, the CKM matrix can be written:

$$A_D = (K \mid R) \quad (5.2)$$

where

$$K = \begin{pmatrix} c_{12} c_{13} & -s_{12} & -c_{12} s_{13} \\ s_{12} c_{13} c_{23} - s_{13} s_{23} e^{i\delta_4} & c_{12} c_{23} & -s_{12} s_{13} c_{23} - c_{13} s_{23} e^{i\delta_4} \\ s_{12} c_{13} s_{23} + s_{13} c_{23} e^{i\delta_4} & c_{12} s_{23} & -s_{12} s_{13} s_{23} + c_{13} c_{23} e^{i\delta_4} \end{pmatrix} \quad (5.3)$$

while the three columns of R are

$$R_{i1} = \begin{pmatrix} -c_{12} c_{13} \phi_{14} \\ -s_{12} c_{13} c_{23} \phi_{14} + s_{13} s_{23} \phi_{14} e^{i\delta_4} - c_{23} \phi_{24} e^{i\delta_3} \\ -s_{12} c_{13} s_{23} \phi_{14} - s_{13} c_{23} \phi_{14} e^{i\delta_4} - s_{23} \phi_{24} e^{i\delta_3} - \phi_{34} e^{i(\delta_3 + \delta_7)} \end{pmatrix} \quad (5.4a)$$

and R_{i2} , R_{i3} are obtained from R_{i1} with the changes

$$\begin{aligned} R_{i2} &: \phi_{14} \rightarrow \phi_{15} ; \delta_3 \rightarrow \delta_2 ; \delta_7 \rightarrow \delta_6 \\ R_{i3} &: \phi_{14} \rightarrow \phi_{16} ; \delta_3 \rightarrow \delta_1 ; \delta_7 \rightarrow \delta_5 \end{aligned} \quad (5.4b)$$

We see that, to leading order in ϕ_{lf} , the mixing of the standard families is dictated by a unitary matrix K with the single phase δ_4 playing the rôle of the usual KM phase. All other six CP-violating phases appear only in R and have their effects suppressed by the small mixing angles ϕ_{lf} . With this parametrization, it becomes especially simple to place an upper bound on ϕ_{lf} since, up to $O(\phi_{lf}^2)$,

$$1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 = |V_{ud}|^2 (\phi_{14}^2 + \phi_{15}^2 + \phi_{16}^2) \quad (5.5)$$

and therefore a limit on the left-hand side of this equation can be translated into a limit on the ϕ_{lf} . We will return to this question later.

Next we will show how in the class of models considered here, provided the $\Delta I = 0$ masses are significantly larger than the $\Delta I = \frac{1}{2}$ masses, the mixing angles are naturally suppressed. For simplicity, we will assume $N_u = 0$. We work in the

weak basis defined by (2.6), with M_U non-existent. Let us write

$$\mathcal{M}_d = \left(\begin{array}{c|c} m_d & \\ \hline & M_D \end{array} \right) = \left(\begin{array}{c|c} G & J \\ \hline O & \tilde{M}_D \end{array} \right) \quad (5.6)$$

As we know, X_D is the matrix which diagonalizes $\mathcal{M}_d \mathcal{M}_d^\dagger$:

$$\mathcal{M}_d \mathcal{M}_d^\dagger X_D = X_D \begin{pmatrix} \bar{m}^2 & 0 \\ 0 & \bar{M}^2 \end{pmatrix} \quad (5.7)$$

where $\bar{m}^2$, $\bar{M}^2$ are diagonal real positive matrices of dimension n , N_d respectively. If we write X_D in block form

$$X_D = \left(\begin{array}{c|c} A_D & \\ \hline & B_D \end{array} \right) = \left(\begin{array}{c|c} K & R \\ \hline S & T \end{array} \right) \quad (5.8)$$

Eq. (5.7) gives us

$$\begin{aligned} (GG^\dagger + JJ^\dagger)K + (J\tilde{M}_D)S &= K\bar{m}^2 \\ (GG^\dagger + JJ^\dagger)R + (J\tilde{M}_D)T &= R\bar{M}^2 \\ (\tilde{M}_D J^\dagger)K + \tilde{M}_D^2 S &= S\bar{m}^2 \\ (\tilde{M}_D J^\dagger)R + \tilde{M}_D^2 T &= T\bar{M}^2 \end{aligned} \quad (5.9)$$

while the unitarity constraints on X_D are:

$$\begin{aligned} K^\dagger K + S^\dagger S &= \mathbb{1}_n \quad ; \quad R^\dagger R + T^\dagger T = \mathbb{1}_{N_d} \\ K^\dagger R + S^\dagger T &= 0 \end{aligned} \quad (5.10)$$

where $\mathbb{1}_n$, $\mathbb{1}_{N_d}$ designate unit matrices of dimension n , N_d respectively. So far, no approximations have been made. If we now take into account that G and J are $\Delta I = \frac{1}{2}$ mass terms ($\approx m$) while $\tilde{M}_D$ is $\Delta I = 0$ ($\approx M$) and assume that $M \gg m$, an

approximate solution to X_D and $\bar{M}$ can be easily found, with

$$T = \mathbb{I} N_d ; S = -(\tilde{M}_D^{-1} J^\dagger) K ; R = J \tilde{M}_D^{-1} ; \bar{M}^2 = \tilde{M}^2 \quad (5.11)$$

while K in this approximation [up to $O(m^2/M^2)$] is the unitary matrix which diagonalizes $GG^\dagger$:

$$K^\dagger (GG^\dagger) K = \bar{m}^2 \quad (5.12)$$

At this point, we recall that K , R are the matrices which enter into the physical CKM matrix. We have thus shown that the elements of R are of order m/M , and therefore $\phi_{\lambda f} \approx m/M$. It follows also from (5.12) that in this approximation there will be as many phases in K as in the standard model with n families, as is to be expected, since we are in the limit of small mixing angles. The matrix R will contain the phases of J which constitute the extra $(n-1)N_d$ phases arising in a model with N_d vector-like quarks.

6. - PHENOMENOLOGICAL IMPLICATIONS

We will analyse in this section some of the phenomenological consequences of the class of models considered here.

a) Deviations from unitarity

As we have emphasized, one of the consequences of adding vector-like quarks to the standard model is to render the CKM matrix non-unitary. Recently, Marciano and Sirlin⁹⁾ have summed the leading log radiative corrections to β decay and derived the matrix element:

$$V_{ud} = .9729 \pm .0012 \quad (6.1)$$

An analysis of $Ke3$ and hyperon decays by Leutwyler and Roos¹⁰⁾, giving

$$|V_{us}| = .221 \pm .002 \quad (6.2)$$

was then used, together with a bound on $|V_{ub}|$ ($|V_{ub}| < 0.008$), to derive

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = .9954 \pm .0025 \quad (6.3)$$

Although (6.3) is consistent with unitarity at the two-sigma level, it may also be an indication of new physics. We will use (6.3) to place an upper bound on the mixing angles between vector-like and standard quarks. For example, in the case of an E_6 GUT, from Eqs. (5.5) and (6.3) one obtains

$$\phi_{14}^2 + \phi_{15}^2 + \phi_{16}^2 \leq .0075 \quad (6.4)$$

If one takes into account that $\phi_{\lambda f}$ are $O(m/M)$, from (6.4) one derives a rough lower bound on the mass of vector-like quarks:

$$M \geq 12 \text{ GeV} \quad (6.5)$$

In (6.5) and in the following analysis, we will take, somewhat arbitrarily, $m \approx 1 \text{ GeV}$.

At this point, it should be added that although deviations from unitarity naturally arise in the context of models with vector-like quarks, a relatively large deviation, such as that of Eq. (6.3) is unlikely to occur within this class of models. The point is that such large deviations would require significant mixing between vector-like and standard quarks, which in turn would imply too large a rate for rare kaon decays like $K_L \rightarrow \mu^+ \mu^-$, as will be seen in the following.

It is obviously premature to interpret Eq. (6.3) as a clear indication for deviations from unitarity, especially taking into account the possibility that the uncertainties in the evaluation $|V_{us}|$ may be larger than those of Eq. (6.2). A rather conservative explanation for Eq. (6.3) is to assume the

existence of a fourth generation, with relatively large mixing⁹⁾ ($V_{ub}' \approx 0.088$). A more exciting possibility^{*}) is to interpret the deviations from unitarity of Eq. (6.3) as being due to the effect of radiative corrections to μ and β decay arising from supersymmetric particle exchanges¹¹⁾.

b) $K_L - K_S$ mass difference

There will be new contributions to the $K^0 - \bar{K}^0$ transition, arising from the fact that the neutral gauge mesons couple in general to flavour-changing neutral currents. If we write the strangeness-changing coupling of the standard neutral gauge meson as:

$$\mathcal{L}_{N.C.}^{|\Delta S|=1} = \frac{g}{2 \cos \theta_W} \left[\beta_{ds} \bar{d}_L \gamma_\mu s_L \right] Z^\mu + h.c. \quad (6.6)$$

then the contribution to the $\Delta S = 2$ effective Lagrangian arising from the diagram of Fig. 1 will be given by

$$\mathcal{L}_{eff}^{\Delta S=2} = \frac{G_F}{2\sqrt{2}} \beta_{ds}^2 \left[\bar{d} \gamma^\mu (1-\gamma_5) s \bar{d} \gamma_\mu (1-\gamma_5) s \right] \quad (6.7)$$

Using this equation, one obtains for the Z-exchange contribution to the $K_L - K_S$ mass difference:

$$\frac{m_L - m_S}{m_K} = \frac{G_F}{\sqrt{2}} \frac{2}{3} f_K^2 \beta_{ds}^2 \quad (6.8)$$

where we have used the vacuum insertion approximation. From the experimental value of the left-hand side of Eq. (6.8), one derives the following bound:

$$\beta_{ds}^2 \leq 2 \times 10^{-7} \quad (6.9)$$

The value of β_{ds} will depend on the particular model considered. For example, in the E_6 GUT, if one expresses the down quark neutral current in terms of physical quark fields:

*) W.J. Marciano, private communication.

$$\sum_{i=1}^3 \bar{d}_{iL}^0 \gamma_{\mu} d_{iL}^0 = \sum_{i=1}^3 \left[\sum_{k,l=1}^6 (X_D^{\dagger})_{ki} (X_D)_{il} (\bar{D}'_L)_k \gamma_{\mu} (D'_L)_l \right] \quad (6.10)$$

and uses the unitarity of X_D :

$$\sum_{i=1}^3 (X_D^{\dagger})_{ki} (X_D)_{il} = - \sum_{i=4}^6 (X_D^{\dagger})_{ki} (X_D)_{il} + \delta_{lk} \quad (6.11)$$

together with (5.8) and (5.11), one obtains

$$\beta_{ds} \approx \frac{1}{M_1^2} m_{14} m_{24}^* + \frac{1}{M_2^2} m_{15} m_{25}^* + \frac{1}{M_3^2} m_{16} m_{26}^* \quad (6.12)$$

where we have kept only the non-Cabibbo suppressed couplings. From Eq. (6.9) and taking into account that $\beta_{ds} = O(m^2/M^2)$, one obtains the bound:

$$M \geq 47 \text{ GeV} \quad (6.13)$$

c) $K_L \rightarrow \mu^+ \mu^-$

The strangeness-changing neutral current interaction of Eq. (6.5) contributes through the tree diagram of Fig. 2 to the decay rate of $K_L \rightarrow \mu^+ \mu^-$, with

$$\Gamma(K_L \rightarrow \mu^+ \mu^-) = \frac{G_F^2}{8\pi} f_k^2 m_{\mu}^2 m_K \left(1 - \frac{4m_{\mu}^2}{m_K^2} \right)^{1/2} (\text{Re} \beta_{ds})^2 \quad (6.14)$$

From the experimental value of the branching ratio of $K_L \rightarrow \mu^+ \mu^-$, one deduces

$$\text{Re} \beta_{ds} < 10^{-5} \quad (6.15)$$

which in turn implies

$$M \geq 300 \text{ GeV} \quad (6.16)$$

d) $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

This decay has special interest, since there is a planned experiment at Brookhaven where there will be a search for $K^+ \rightarrow \pi^+ X^0 \bar{X}^0$, where X^0 is any neutral, light and weakly-interacting particle. The sensitivity of this proposed experiment will be of 2×10^{-10} , which is three orders of magnitude higher than the present limit. In the class of models considered here, there will be a new contribution to this decay arising from the diagram of Fig. 3. If one assumes that there are three light neutrinos, one obtains

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 3\beta_{ds}^2 \frac{1}{\sin^2 \theta_c} Br(K^+ \rightarrow \pi^+ e^+ \nu) \quad (6.17)$$

Using now the bound of Eq. (6.15), one concludes that in this class of models one can have a branching ratio of the order of

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \approx 4 \times 10^{-10} \quad (6.18)$$

This branching ratio is of the same order of magnitude as the corresponding ratio in the standard model with three generations¹¹⁾. This is to be contrasted with the case of a fourth standard generation where a much larger branching ratio can be obtained¹²⁾.

e) ε'/ε ratio

We have seen that in this class of models CP violation arises in general from both the usual W-box diagram and Z exchange. Therefore one cannot make a clear-cut prediction for ε'/ε without further assumptions. However, in the context of specific models of this class, it has been shown^{*)} that one can have $\varepsilon'/\varepsilon \approx 10^{-3}$.

*) See F. del Aguila et al., Ref. 4).

7. - CONCLUSIONS

We have derived the restrictions that CP invariance imposes on the various quark mass matrices, when vector-like quarks are added to the standard model. These restrictions were then used to write invariant necessary conditions for CP invariance which can be used to test the existence of CP violation in any specific model with vector-like quarks, without the need of diagonalizing quark mass matrices. We have pointed out some of the subtleties which arise in the analysis of CP violating phases appearing in the CKM matrix, and have proposed a parametrization of the quark mixing matrix which is specially useful for models with vector-like quarks. Finally, we have shown that models with vector-like quarks can naturally conform with all the present phenomenological constraints, even if vector-like quarks are relatively light (10^2 - 10^3 GeV).

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FIGURE CAPTIONS

Fig. 1 Tree diagram contribution to the K^0 - $\bar{K}^0$ transition.

Fig. 2 Tree diagram contribution to the decay $K^+ \rightarrow \mu^+ \mu^-$ decay.

Fig. 3 Tree diagram contribution to the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.

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Fig.1

Fig.2

Fig.3