B
CETWO-BODY ABSORPTION OF PIONS IN ${}^3\text{He}$ TRI-PP-85-6;
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Abstract

Pion absorption by ${}^3\text{He}$ with two-body kinematics is examined in a simple model consisting of rescattering through $\Delta(3,3)$ or nucleon intermediate states and a contact rescattering term based on the phenomenological Lagrangian of Koltun and Reitan. The model does not include either initial state π — ${}^3\text{He}$ correlations or final NN correlations and treats the intermediate baryon propagators in a local, static approximation. With this model integrated cross sections and angular distributions are obtained for the reactions ${}^3\text{He}(\pi^+, pp)p$ and ${}^3\text{He}(\pi^-, pn)n$ with two of the outgoing nucleons carrying all of the momentum and the third acting as a spectator. It is found that the calculated cross sections are strongly isospin dependent both in their magnitude and energy dependence. Inclusion of rescattering through nucleon intermediate states significantly increases the π^- cross section, thereby decreasing the ratio $R = \sigma(\pi^+)/\sigma(\pi^-)$ from the value obtained with Δ rescattering alone. The model reproduces the observed energy dependence of the cross sections fairly well and also yields angular distributions of the correct shape, except in the case of π^- absorption at low energy. On the other hand, the overall normalization of the cross sections is overestimated, especially for π^- absorption. Several possibilities as to the origin of this difficulty are briefly considered.

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1. Introduction

A microscopic understanding of the pion absorption mechanism is essential for the detailed description of pion-nucleus interactions. The simplest and most frequently studied example of pion absorption is pion absorption by the deuteron, $\pi d + NN$. Examination of this reaction was motivated by the hope that a good understanding of pion absorption by a simple nucleus could provide the basis for analysis involving more complicated nuclei. Because of the kinematics of $\pi d + NN$, which require a large momentum transfer between the two baryons, it is to be expected that two-nucleon processes are very important in the absorption mechanism. The shape of the empirical cross section as a function of T_π , the laboratory kinetic energy of the pion, suggests also that the two-nucleon absorption processes are dominated by the excitation of an intermediate $\Delta(3,3)$ resonance. Both of these expectations are born out by detailed microscopic calculations¹⁻⁵).

The nucleon pair in the deuteron has quantum numbers $I=0$, $S=1$, and thus, only isoscalar absorption of pions can be studied in the reaction $\pi d + NN$. To obtain information on the absorption of pions by an $I=1$ nucleon pair, one must consider heavier target nuclei. The simplest of these is ${}^3\text{He}$. Recently, the reactions ${}^3\text{He}(\pi^+, pp)p$ and ${}^3\text{He}(\pi^-, pn)n$ have been the subject of experimental studies at SIN, LAMPF, and TRIUMF in the energy span from threshold through the $\Delta(3,3)$ resonance excitation region⁶⁻⁹. In these experiments two outgoing nucleons (either pp or pn) are detected in coincidence. If a spectrum of such events is examined, those events corresponding to the free $\pi d + NN$ kinematics form a peak, which is associated with two-nucleon absorption processes with the third nucleon in the target acting as a spectator. Since π^- absorption must

involve an initial pp pair ($I=1$) in the target to yield a final pn pair, whereas π^+ absorption involves an initial pn pair ($I=0$ or 1), the ratio

$$R = d\sigma(\pi^+)/d\sigma(\pi^-) \quad (1.1)$$

yields information on the isospin dependence of nuclear pion absorption. For $T_\pi \lesssim m_\pi$, a relatively large ratio $R \sim 10-20$ has been found. This ratio cannot be accounted for on the basis of isospin geometry alone and clearly indicates a strong isospin dependence in pion absorption. A dynamical understanding of this isospin dependence is of fundamental interest and is important for a microscopic description of π -nucleus dynamics.

To date, theoretical studies of pion absorption by ^3He have concentrated on rescattering contributions through an intermediate $\Delta(3,3)$ resonance^{10,11}. These studies substantially overestimate the ratio R , which can be qualitatively understood as follows: in the $I=0$ channel, the absorption process is dominated by the direct Δ -rescattering contribution (diagram A in fig. 1). This is not the case in the $I=1$ channel, however, since the absorption of a p-wave pion by an $I=1$, $S=0$, $L=0$ nucleon pair would lead to an intermediate NA state with quantum numbers $I=1$ and $J^\pi=1^+$, which cannot be accommodated by the outgoing NN pair. Consequently, two-nucleon absorption contributions with intermediate Δ resonances are dynamically suppressed relative to other contributions, so that if only the Δ contributions are included, the π^- absorption cross section will be substantially underestimated and the ratio R overestimated.

In a unitary πNN interaction model, Silbar and Piasetzky calculated the ratio R including intermediate NP_{11} states, as well as NA states¹².

The inclusion of NP_{11} states drastically reduces R , especially at low pion energies, but the resulting value of the ratio is still too large by about a factor 2. The calculation of these authors, although preserving unitarity at the three-body level, cannot be regarded as complete in that Fermi motion of the target nucleons is neglected and ρ -meson exchange is not included.

In view of the theoretical situation, we have undertaken a study of pion absorption by ^3He in which the effects of various two-body absorption contributions to the cross sections are examined in some detail. In addition to the usual Δ -rescattering contributions, these include rescattering through intermediate nucleon states and a contact rescattering term constructed from the phenomenological πNN s-wave vertex introduced by Koltun and Reitan¹³.

A number of simplifying approximations have been made in the evaluation of the transition amplitude in order to make the calculations tractable and to keep the model as transparent as possible. Intermediate baryon propagators are treated in a local approximation that makes their form especially simple, and in the propagators associated with meson exchanges, a fixed energy transfer is assumed. We also suppose that both the incident pion and the outgoing two-nucleon state are adequately represented by plane waves, so that neither final state correlations nor π - ^3He correlations prior to absorption are accounted for. These approximations are discussed more fully in the next section, which is devoted to a detailed description of the model.

Numerical results obtained within the model are presented and discussed in sec. 3. Because of the approximations mentioned above, these results should be interpreted in a more semi-quantitative than

quantitative fashion. Our main concern in this study has been to examine the role of various absorption mechanisms and their sensitivity to different parameters that appear in the model, rather than obtain a precise fit to the available data.

Section 4 contains a brief summary and a few concluding remarks.

2. The model

The reaction model employed in the present work is depicted diagrammatically in fig. 1. For either π^+ or π^- absorption, the major contributions to the two-body absorption cross section above threshold arise from the rescattering processes represented by diagrams A-D. These all involve a p-wave pion absorption vertex, an intermediate nucleon or $\Delta(3,3)$ propagator, and a meson exchange interaction that transfers energy and momentum between the two active nucleons. Absorption of the incident pion may occur either before or after the intermediate baryon propagation, leading to what we will subsequently refer to as the direct (A and C) and crossed (B and D) contributions respectively. Note that in a model containing single nucleon processes, where the pion is absorbed on a single nucleon with no explicit meson exchange, the direct and crossed nucleon terms would be automatically included through incorporation of initial and final state correlations in the model. However, single nucleon processes contribute not only to the two-body cross section but also to the three-body background that underlies it, and thus, should not be included in full in a calculation that treats only the two-body part. In a crude sense, the direct and crossed nucleon terms included in the present model represent that part of the full single nucleon amplitude that contributes mainly to the two-body cross section.

In position space the transition amplitudes corresponding to diagrams A-D may be written schematically

$$T_d(\omega, \vec{k}, \vec{r}) = e^{i\vec{k} \cdot \vec{r}/2} v_{12}(\vec{r}) iG_d(\omega, \vec{k}) v_1^{\dagger}(\vec{k}) + e^{-i\vec{k} \cdot \vec{r}/2} v_{21}(-\vec{r}) iG_d(\omega, \vec{k}) v_2^{\dagger}(\vec{k}) \quad (2.1)$$

$$T_c(\omega, \vec{k}, \vec{r}) = e^{i\vec{k} \cdot \vec{r}/2} v_1(\vec{k}) iG_c(\omega, \vec{k}) v_{12}^{\dagger}(\vec{r}) + e^{-i\vec{k} \cdot \vec{r}/2} v_2(\vec{k}) iG_c(\omega, \vec{k}) v_{21}^{\dagger}(-\vec{r}) , \quad (2.2)$$

where ω , k are the laboratory energy and momentum of the incident pion; r if the relative two-nucleon coordinate; v , G , and V designate the pion absorption vertex, intermediate baryon propagator, and meson exchange potential respectively; and the subscripts d and c refer to the direct diagrams (A and C in fig. 1) and crossed diagrams (B and D). The two terms in each of these expressions correspond to absorption of the incident pion by either member of the active nucleon pair.

Explicit expressions for the absorption vertices and meson exchange potentials are derived from the interaction Hamiltonian densities,

$$H_{\pi NN}(\vec{k}) = i \frac{f_{\pi NN}}{m_{\pi}} u_{\pi NN}(k) \sum_{\alpha} N^{\dagger} \tau_{\alpha} \vec{\sigma} \cdot \vec{k} N_{\alpha}(\alpha, \vec{k}) + h.c., \quad (2.3)$$

$$H_{\pi NA}(\vec{k}) = i \frac{f_{\pi NA}}{m_{\pi}} u_{\pi NA}(k) \sum_{\alpha} \Delta^{\dagger} \tau_{\alpha} \vec{\sigma} \cdot \vec{k} N_{\alpha}(\alpha, \vec{k}) + h.c., \quad (2.4)$$

$$H_{\rho NN}(\vec{k}) = i \frac{f_{\rho NN}}{m_{\pi}} u_{\rho NN}(k) \sum_{\alpha} N^{\dagger} \tau_{\alpha} \vec{\sigma} \cdot \vec{k} \hat{N}_{\alpha}(\alpha, \vec{k}) + h.c., \quad (2.5)$$

$$H_{\rho NA}(\vec{k}) = i \frac{f_{\rho NA}}{m_{\pi}} u_{\rho NA}(k) \sum_{\alpha} \Delta^{\dagger} \tau_{\alpha} \vec{\sigma} \cdot \vec{k} \hat{N}_{\alpha}(\alpha, \vec{k}) + h.c., \quad (2.6)$$

$$H_{\omega NN}(\vec{k}) = i \frac{f_{\omega NN}}{m_\pi} u_{\omega NN}(k) N^+(\vec{p}+\vec{p}') \cdot \hat{e}_\omega N_{\omega}(k) + h.c., \quad (2.7)$$

$$H_{\sigma NN}(\vec{k}) = i \frac{f_{\sigma NN}}{m_\pi} u_{\sigma NN}(k) N^+ N_{\sigma}(k) + h.c., \quad (2.8)$$

in which the operator $a_\lambda(\alpha, \vec{k})$ annihilates a λ -meson of charge α and momentum k , S^+ and T^+ designate the $1/2 + 3/2$ spin and isospin transition operators, $f_{\lambda NN}$ and $f_{\lambda N\Delta}$ denote the λNN and $\lambda N\Delta$ coupling strengths on-shell, and the u -functions are vertex structure functions (form factors). For the meson-nucleon-delta vertices, the particular non-relativistic forms here involving the spin and isospin transition matrices are only valid in the meson-nucleon cm frame. Thus, in the external $\pi N\Delta$ vertices, the πN cm momentum,

$$k_{\pi N} = \frac{M_N}{\sqrt{m_\pi^2 + M_N^2} + 2\omega M_N} \vec{k} \quad (2.9)$$

should be used rather than the laboratory momentum $\vec{k}$. For the sake of consistency the πN cm momentum is also employed in the external πNN vertices. With this choice and with the structure functions normalized to one on-shell, we then obtain

$$v_1^{NN\pi}(\vec{k}) = -i \left(\frac{f_{\pi NN}}{m_\pi} \right) \tau_{\alpha 1} \vec{\sigma}_1 \cdot \vec{k}_{\pi N} a_1^+(N) a_1(N) \quad (2.10)$$

$$v_1^{N\Delta\pi}(\vec{k}) = -i \left(\frac{f_{\pi N\Delta}}{m_\pi} \right) \tau_{\alpha 1}^+ \vec{S}_1 \cdot \vec{k}_{\pi N} a_1^+(\Delta) a_1(N) \quad (2.11)$$

for the external πNN and $\pi N\Delta$ vertices involving absorption of a pion of charge α by nucleon i ($i=1$ or 2).

Recoil terms and the energy dependence of the coupling strengths

have both been neglected in these expressions. At the energies of interest

in the present study, the latter dependence is probably unimportant and in any case is not known quantitatively. Recoil effects were examined in ref. 3) in the context of πd absorption and found to be numerically important; however, eq. (B.45) and (B.46) of that reference, upon which the recoil results were based, contain an erroneous phase factor. More recent results employing the correct phase factor suggest that recoil effects are not significant at moderate pion energies, either in absorption by the deuteron or by heavier nuclei^{14,15}.

In momentum space the contributions from exchange of λ -mesons to the NN interaction and the $N\Delta + NN$ transition interaction assume the forms,

$$v_{12}^{NN\lambda}(q_0, \vec{q}) = v_2^{NN\lambda}(\vec{q}) G_\lambda(q_0, \vec{q}) v_1^{NN\lambda}(-\vec{q}), \quad (2.12)$$

$$v_{12}^{N\Delta\lambda}(q_0, \vec{q}) = v_2^{N\Delta\lambda}(\vec{q}) G_\lambda(q_0, \vec{q}) v_1^{N\Delta\lambda}(-\vec{q}), \quad (2.13)$$

where q_0 and $\vec{q}$ specify the energy and momentum transferred by the λ -meson, and G_λ designates the meson propagator. If a $\Delta(3,3)$ resonance is excited in the intermediate state, only isovector meson exchanges are possible ($\lambda=\pi$ or ρ); while for intermediate nucleon states, isoscalar mesons — in particular, the σ and the ω — may be exchanged as well. The various $NN\lambda$ and $N\Delta\lambda$ interaction vertices corresponding to these possibilities are obtained from the Hamiltonian densities, eq. (2.3) — (2.8), in the same manner as the pion absorption vertices. This yields expressions analogous to eq. (2.10) and (2.11) but supplemented with vertex form factors to take account of the highly off-shell character of the exchanged mesons. No attempt is made to derive these form factors from more fundamental considerations (such as a bag model); rather the simple phenomenological form

$$u_\lambda(q_0, \vec{q}) = \frac{\Lambda_\lambda^2 - m_\lambda^2}{\Lambda_\lambda^2 - q^2} \quad (2.14)$$

is employed with $q^2 = q_0^2 - \vec{q}^2$. In principle, the form factor masses associated with the $NN\lambda$ and $N\lambda\lambda$ vertices need not be the same; however, we take them equal in order to reduce the number of parameters in the model. Λ_λ then depends only on the identity of the exchanged meson.

For the meson propagators, we employ the simple form

$$G_\lambda^{-1}(q_0, \vec{q}) = q^2 - m_\lambda^2, \quad (2.15)$$

where again $q^2 = q_0^2 - \vec{q}^2$. Combining these meson propagators with the form factor-modified $NN\lambda$ and $N\lambda\lambda$ vertices yields the explicit momentum space expressions,

$$V_{12}^{NN\pi}(q_0, \vec{q}) = \left(\frac{f_{\pi NN}}{m_\pi} \right) \left(\frac{f_{\pi NN}}{m_\pi} \right) \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q^2} \frac{\vec{q}_1 \cdot \vec{q}_2}{q^2 - m_\pi^2} \vec{t}_1 \cdot \vec{t}_2 \quad (2.16)$$

$$V_{12}^{NA\rho}(q_0, \vec{q}) = \left(\frac{f_{\rho NA}}{m_\rho} \right) \left(\frac{f_{\rho NN}}{m_\rho} \right) \frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - q^2} \frac{\vec{s}_1 \cdot \vec{q}_1 \vec{q}_2 \cdot \vec{xq}}{q^2 - m_\rho^2} \frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - q^2} \vec{t}_1 \cdot \vec{t}_2 \quad (2.17)$$

for the $NA \rightarrow NN$ transition potentials, and

$$V_{12}^{NN\pi}(q_0, \vec{q}) = \left(\frac{f_{\pi NN}}{m_\pi} \right)^2 \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q^2} \frac{\vec{q}_1 \cdot \vec{q}_2}{q^2 - m_\pi^2} \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q^2} \vec{t}_1 \cdot \vec{t}_2 \quad (2.18)$$

$$V_{12}^{NN\rho}(q_0, \vec{q}) = \left(\frac{f_{\rho NN}}{m_\rho} \right)^2 \frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - q^2} \frac{\vec{q}_1 \cdot \vec{q}_2}{q^2 - m_\rho^2} \frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - q^2} \vec{t}_1 \cdot \vec{t}_2 \quad (2.19)$$

$$V_{12}^{NN\omega}(q_0, \vec{q}) = \left(\frac{f_{\omega NN}}{m_\omega} \right)^2 \frac{\Lambda_\omega^2 - m_\omega^2}{\Lambda_\omega^2 - q^2} \frac{4M_N^2 + \frac{1}{2} q^2 + 2i(\vec{q} \cdot \vec{xq})}{q^2 - m_\omega^2} \sum \frac{\Lambda_\omega^2 - m_\omega^2}{\Lambda_\omega^2 - q^2} \vec{t}_1 \cdot \vec{t}_2 \quad (2.20)$$

$$V_{12}^{NN\omega}(q_0, \vec{q}) = \left(\frac{f_{\omega NN}}{m_\omega} \right)^2 \frac{\Lambda_\omega^2 - m_\omega^2}{\Lambda_\omega^2 - q^2} \frac{-4M_N^2 + \frac{1}{2} q^2 + i(\vec{q} \cdot \vec{xq}) \cdot \sum \vec{q}_1 \cdot \vec{xq} \cdot \vec{q}_2 \cdot \vec{xq}}{q^2 - m_\omega^2} \times \left(\frac{\Lambda_\omega^2 - m_\omega^2}{\Lambda_\omega^2 - q^2} \right) \quad (2.21)$$

for the NN potentials, where M_N is the nucleon mass and $\vec{p} = \vec{p}_1 + \vec{p}_2$ and $\sum = 1/2(\vec{q}_1 + \vec{q}_2)$ are the two nucleon cm momentum and spin, respectively. Particular choices for the coupling strengths and form factor masses appearing here are discussed in the next section.

To obtain expressions appropriate for insertion in eq. (2.1) and (2.2) for the transition amplitudes, it is necessary to Fourier transform eq. (2.16)-(2.21) to position space. For fixed energy transfer, the Fourier transformations are straightforward. q_0 depends not only on the incident pion energy, however, but also upon the ^3He momentum distribution and thus, is a variable which in principle must be integrated over. To avoid this complication, we note that since the mean relative nucleon momentum in ^3He is small compared with the incident pion momentum (except near threshold) and since $q_0 \ll |\vec{q}|$, it is reasonable, for purposes of evaluating q_0 , to treat the incident nucleons in ^3He as static. In that approximation q_0 is fixed and up to terms of order $|\vec{k}|/M_N$, where $\vec{k}$ is the laboratory pion momentum, is given approximately by

$$q_0 = \frac{1}{2} \omega; \quad (2.22)$$

i.e., the incident pion energy ω is shared equally by the two active nucleons in the process. With q_0 fixed, eq. (2.16)-(2.21) can be expanded in partial fraction series, and each term in these series

transformed separately. The resulting position space expressions for the NN and $N\Delta \rightarrow NN$ potentials are listed in appendix A.

The final ingredients required to assemble the p-wave rescattering amplitudes are the intermediate state propagators corresponding to the direct and crossed diagrams. In the present work these are approximated by the local non-relativistic expressions,

$$[G_d(\omega, \vec{k})]^{-1} = E_I - M_N - \omega - i\Gamma_I/2, \quad (2.23)$$

$$[G_c(\omega, \vec{k})]^{-1} = E_I - M_N + q_0, \quad (2.24)$$

where M_I , P_I , and $E_I = M_I + P_I^2/2M_I$ are the mass, momentum, and energy of the intermediate baryon ($I=N$ or Δ) and Γ_I is the intermediate baryon width. Like the energy transfer, the intermediate baryon momentum and energy depend on the ${}^3\text{He}$ momentum distribution as well as on the incident pion momentum and energy. However, if we employ the same approximation used to obtain (2.22) — i.e., treat the incident nucleons as static — the dependence on the ${}^3\text{He}$ momentum distribution disappears, and the intermediate baryon momentum reduces to

$$P_I^2 = k^2 \quad (2.25)$$

for the direct terms and

$$P_I^2 = |\vec{q}|^2 \equiv \omega M_N \quad (2.26)$$

for the crossed terms, where the last approximate equality is valid up to terms of order $|\vec{k}|/M_N$. Note here that the static approximation has been used to fix both q_0 and $|\vec{q}|$, in contrast with the meson exchange potentials where q_0 alone is fixed and the $\vec{q}$ -dependence eliminated through Fourier transformation to position space. In eq. (2.23) for G_d , the

width is zero for an intermediate nucleon and is given in the Born approximation by

$$\Gamma_\Delta = \frac{1}{4\pi} \left(\frac{f_\pi N\Delta}{m_\pi} \right)^2 \frac{2}{3} k_N^3 \quad (2.27)$$

for an intermediate $\Delta(3,3)$. The width does not appear in the crossed diagram version of the $\Delta(3,3)$ propagator, since in that diagram the intermediate resonance can never be on-shell and G_c has no imaginary component.

The rescattering contributions discussed thus far in this section all involve a p-wave pion absorption vertex and thus vanish in the limit of zero incident momentum. To reproduce the finite absorption cross section observed at threshold, it is necessary to include in the transition amplitude the contact rescattering process depicted by diagram E in fig. 1. This term involves an effective $\pi\pi NN$ vertex where the incident pion is absorbed, a pion propagator, and a p-wave πNN vertex on the other nucleon line. The $\pi\pi NN$ vertex is obtained from the phenomenological s-wave Lagrangian introduced by Koltun and Reitan¹⁶,

$$L_S = -4\pi \left[\frac{\lambda_0}{m_\pi} \vec{\nabla}\vec{\phi} \cdot \vec{\phi} N + \frac{\lambda}{m_\pi^2} \vec{\nabla}\vec{\tau} \cdot \vec{\phi} \vec{\pi} N \right], \quad (2.28)$$

and $\vec{\phi}$ and $\vec{\pi}$ designate the pion field and its conjugate momentum. Combining this expression with eq. (2.10) and (2.15) (with $\lambda=\pi$) for the p-wave πNN vertex and the pion propagator and transforming to position space yields the s-wave rescattering amplitude,

$$T_S(\omega, \vec{k}, \vec{r}) = 2i \left(\frac{f_\pi NN}{m_\pi} \right) \left(\frac{\mu_\pi}{m_\pi} \right) \left(1 + \frac{1}{\mu_\pi r} \right) e^{-\mu_\pi r/r} \times \left[e^{-i\vec{k} \cdot \vec{r}/2} (\vec{\sigma}_1 \cdot \vec{\Delta})(\vec{\sigma}_2 \cdot \vec{\Delta})(\vec{\sigma}_1 \cdot \vec{\Delta}) - e^{i\vec{k} \cdot \vec{r}/2} (\vec{\sigma}_2 \cdot \vec{\Delta})(\vec{\sigma}_1 \cdot \vec{\Delta}) \right] \quad (2.29)$$

with $\hat{\phi} = \vec{\phi}/|\vec{\phi}|$, $\mu_n^2 = m_n^2 - q_0^2 = m_n^2 - 1/4\omega^2$, and

$$\vec{\sigma}_{12} \equiv \lambda_0 \vec{\sigma}_1 + \frac{1}{2} \lambda_1 (\omega + q_0) / m_{\pi} \vec{\tau}_1 \times \vec{\tau}_2 . \quad (2.30)$$

On shell, the isoscalar and isovector couplings, λ_0 and λ_1 , are related to the πN s-wave scattering lengths, a_1 and a_3 , through the expressions,

$$\lambda_0 = -\frac{1}{6} m_{\pi} (a_1 + 2a_3) \quad (2.31)$$

$$\lambda_1 = \frac{1}{6} m_{\pi} (a_1 - a_3) , \quad (2.32)$$

which yield $\lambda_0=0.0012$ and $\lambda_1=0.050$ for the values of a_1 and a_3 given in ref. 1). However, as for the p-wave rescattering diagrams, the off-shell nature of the exchanged pion here requires that the on-shell vertices be modified by momentum dependent form factors. For the $\pi\pi NN$ vertex, simple dynamical models for the isoscalar and isovector contributions to the vertex discussed in ref. 3), lead to the off-shell prescription,

$$\lambda_0(q) = \lambda_{gr} + \lambda_0 \frac{m_0^2}{m_0^2 + 3/4\omega^2 - m_{\pi}^2 - q^2} , \quad (2.33)$$

$$\lambda_1(q) = \lambda_0 \frac{m_p^2}{m_0^2 + 3/4\omega^2 - m_{\pi}^2 - q^2} , \quad (2.34)$$

with $\lambda_{gr}=0.111$, $\lambda_0 = \lambda_0 - \lambda_{gr} = -0.11$, and $\lambda_p=\lambda_1=0.050$. Inclusion of these form factors, together with a monopole form factor [eq. (2.14)] at the p-wave πNN vertex, yields a modified form of the s-wave rescattering amplitude in momentum space that can be Fourier transformed after expanding in partial fractions. The resulting position space expression has

the same form as eq. (2.29) but contains additional terms with ranges related to the various form factor masses appearing in eq. (2.14), (2.33), and (2.34).

The two-body angular distribution for pion absorption by ${}^3\text{He}$ is most conveniently evaluated in the laboratory frame where the ${}^3\text{He}$ nucleus is at rest. Up to terms of order $\epsilon = k^2/E^2$, the angular distribution in this frame is given approximately by

$$\frac{d\sigma}{d\Omega} \Big|_{\text{lab}} \approx \frac{1}{(4\pi)^2} \frac{PE}{2k} \sum |\langle \vec{\epsilon} | \vec{T} | I \rangle|^2 \quad (2.35)$$

where E is the total energy in the lab of the initial two-nucleon, one-pion state, $\vec{p}$ is the relative momentum in the final two-nucleon state, and $\sum$ represents a sum over final spins and average over initial spins. It should be noted here that $d\Omega$ is not defined with respect to the laboratory scattering angle, but with respect to the angle θ_{pk} between $\vec{p}$ and $\vec{k}$. The latter angle is related to the scattering angle in the two-nucleon, one-pion cm frame by the expression

$$\cos^2 \theta_{pk} = \frac{\cos^2 \theta_{cm}}{1 - \epsilon \sin^2 \theta_{cm}} \quad (2.36)$$

If we neglect the binding energy of ${}^3\text{He}$ compared with the incident pion energy ω , then

$$E \approx 2M_N + \omega , \quad (2.37)$$

so that in the momentum range of interest, ϵ is of the order of a percent or less. Thus, θ_{pk} and θ_{cm} are nearly equal, and eq. (2.35) is a good approximation to the angular distribution in the cm frame. In the same approximation,

$$p^2 \equiv \omega_{M_N}^2 + \frac{1}{4} m_\pi^2 \quad (2.38)$$

The transition amplitudes discussed previously incorporate the wave function of the incident pion, so that the matrix elements in eq. (2.35) must be evaluated with two-nucleon configurations in both the initial and final states. Initially, the two nucleons are bound in the ${}^3\text{He}$ nucleus. If we suppose both nucleons occupy single particle ${}^1S_{1/2}$ states, then the orbital angular momentum of the bound nucleon pair is zero, and the two-nucleon wave function assumes the form,

$$|1\rangle = \frac{1}{\sqrt{4\pi}} \phi(r) |S_1 M_1\rangle |I_1 \mu_1\rangle, \quad (2.39)$$

where S_1 and I_1 are the spin and isospin of the pair and M_1 and μ_1 the corresponding projections. Since this wave function must be antisymmetric under interchange of the two nucleons, only the combinations $S_1=0$, $I_1=1$ and $S_1=1$, $I_1=0$ are allowed. For π^- absorption, where the initial state consists of two protons, just the first combination occurs; while for π^+ absorption, both combinations are possible. The radial wave function employed here is just the correlated Gaussian given in ref. ¹⁸ with the dependence on the spectator coordinates factored out; in particular,

$$\phi(r) = N e^{-3/2 \alpha^2 r^2} [1 - c e^{-\beta^2 r^2}] \quad (2.40)$$

with $\alpha=0.318 \text{ fm}^{-1}$, $\beta=1.176 \text{ fm}^{-1}$, $c=0.925$, and the normalization N defined by the requirement,

$$\int_0^\infty r^2 |\phi(r)|^2 dr = 1. \quad (2.41)$$

The final state consists of a two-nucleon scattering state, which we assume to be sufficiently well-represented by a plane wave. In principle, final state correlations could be included by expanding the plane

wave in spherical Bessel functions, adopting some NN interaction, and replacing the free wave function in each partial wave by the corresponding solution to the Schrödinger equation. It would not really be consistent, however, to include correlations between the active nucleons in the final state, while neglecting those that involve the spectator nucleon, so we have omitted final state correlations altogether. Such correlations should not be too important in any case because of the large relative momentum between the outgoing nucleons. With final state correlations omitted, the antisymmetrized final state wave function is given by

$$|f\rangle = \frac{1}{\sqrt{2}} \left[e^{i\vec{p} \cdot \vec{r}} + (-)^{I+S_F+I_F} e^{-i\vec{p} \cdot \vec{r}} \right] |S_F M_F\rangle |I_F \mu_F\rangle \quad (2.42)$$

in a notation analogous to that of eq. (2.39).

Incorporating the constraints imposed by antisymmetry, the spin-summed squared matrix elements assume the form

$$\sum |\langle f | T | i \rangle|^2 = \frac{1}{2} \sum_{M_F} |\sum_{I_F} \langle I_F, 0 | \langle S_F=1, M_F | \hat{O} | S_1=0 \rangle | I_1=1, 1 \rangle|^2 \quad (2.43)$$

for π^- absorption, and

$$\sum |\langle f | T | i \rangle|^2 = \frac{1}{2} \sum_{S_F M_F S_1 M_1} |\langle I_F=1, 1 | \langle S_F M_F | \hat{O} | S_1 M_1 \rangle | I_1, 0 \rangle|^2 \quad (2.44)$$

for π^+ absorption, where $\hat{O}$ represents a spatial matrix element, and in the second expression, the initial isospin is fixed by the requirement, $I_1 + S_1 = 1$. Evaluation of the spin-spatial matrix elements is most conveniently accomplished within the LS coupling scheme after expanding the pion and final state wave functions in partial waves. The resulting expressions consist of linear combinations of radial integrals and $SU(2)$ matrix elements and are given in appendix B.

3. Results and discussion

Using the model described in the previous section, we have obtained integrated cross sections and angular distributions for the two-body absorption of pions by ^3He over a range of pion laboratory energies. The coupling strengths required in these calculations were fixed by the relations

$$\frac{f_{\lambda NN}^2}{m_\lambda^2} = \frac{g_\lambda^2}{4M_N^2} \quad \text{for } \lambda = \pi, \sigma \quad (3.1)$$

$$\frac{f_{\lambda NN}^2}{m_\lambda^2} = \frac{g_\lambda^2}{4M_N^2} (1 + \kappa_\lambda)^2 \quad \text{for } \lambda = \rho, \omega \quad (3.2)$$

$$f_{\lambda N\Delta} = \alpha_\lambda f_{\lambda NN} \quad \text{for } \lambda = \pi, \rho, \quad (3.3)$$

with values of g_λ^2 chosen consistent with one boson exchange (OBE) models of the nucleon-nucleon interaction¹⁹. In particular,

$$g_\pi^2/4\pi = 14.3 \quad (3.4)$$

$$g_\sigma^2/4\pi = 0.52 \quad (3.5)$$

$$g_\rho^2/4\pi = 8.2 \quad (3.6)$$

$$g_\omega^2/4\pi = 14.6 \quad (3.7)$$

Note that these coupling strengths are defined at $t=m_\lambda^2$, rather than $t=0$, since it is on the meson mass shells where the vertex form factors given by eq. (2.14) are normalized to one. For the ω -meson, the tensor part of the NN vertex is very small and can be ignored; i.e., $\kappa_\omega=0$. By contrast, the ρ -meson tensor coupling is not small and in fact, dominates the vector coupling. Recent phase shift analyses suggest a value in the range 6.0-7.0 for $\kappa_\rho^{20,21}$. Here we adopt the value $\kappa_\rho=6.6$. Within

the SU(6) symmetry scheme, the $\pi N\Delta$ and $\rho N\Delta$ coupling strengths are related to the corresponding NN coupling strengths by the factors

$$\alpha_\pi^2 = \alpha_\rho^2 = 72/25. \quad (3.8)$$

A more phenomenological approach to the $\pi N\Delta$ vertex, which relates the $\pi N\Delta$ strength to the Δ width, yields the value $\alpha_\pi=2.0^{22}$; however, there are diagrams which contribute to the πN interaction (diagrams with the pions crossed) that have no analogue in the Δ width. Consequently, we prefer to use the SU(6) values, eq.(3.8), for both α_π and α_ρ .

In addition to the overall coupling strengths, the form factor masses associated with the various meson-baryon vertices must be specified. At the energies considered here, the numerical results are not very sensitive to the ρ , σ , or ω form factors, provided the form factor masses are not chosen too small. Consequently, the particular values adopted for these masses are not important. For the σ and ω masses, we employ the values

$$A_\sigma = 1.5 \text{ GeV} \quad (3.9)$$

$$A_\omega = 1.8 \text{ GeV} \quad (3.10)$$

which are within the range employed in OBE NN models¹⁹; while for the ρ -meson, we use a value,

$$A_\rho = 1.8 \text{ GeV}, \quad (3.11)$$

that is more typical of those employed in πd calculations¹⁻⁵). The particular choice made for the pion form factor mass, on the other hand, does affect the pion absorption results, especially the overall normalization of the integrated cross sections. As a representative value for

this mass, we make the choice

$$\Lambda_\pi = 1.2 \text{ GeV}, \quad (3.12)$$

which is again typical of those employed in πd calculations, but we have also examined the integrated cross sections when somewhat different values are adopted.

The integrated cross section obtained within the full model is displayed as a function of the laboratory kinetic energy in fig. 2 for π^+ absorption and in fig. 3 for π^- absorption. Also shown are the contributions to the cross sections from particular two nucleon partial waves in the final state. The π^+ cross section exhibits the characteristic peak associated with Δ resonance excitation that is seen also in πd absorption. As in the latter case, transitions to the $I=1$ 1D_2 final state dominate the cross section, but transitions to $I=1$ 3P states are also important, especially at larger energies. The small magnitude of the 3P contributions to π^+ absorption results from destructive interference between the contact rescattering term (diagram E in fig. 1) and those involving intermediate Δ 's (diagrams A and B).

In contrast to π^+ absorption, the π^- absorption cross section is relatively flat as a function of energy, reflecting the reduced significance of intermediate Δ resonance excitation in this reaction. Transitions to D-wave final states again dominate the absorption cross section. At higher energies, transitions to P-wave states are more important than in π^+ absorption, while transitions to F-wave states are less important.

The significance of particular contributions to the reaction mechanism is indicated in fig. 4 and 5, which exhibit the ratio of the integrated cross sections for various versions of the model. As seen in

fig. 4, the inclusion of nucleon intermediate states in addition to Δ intermediate states greatly reduces the cross section ratio from the result obtained with the Δ states alone. Inclusion of the contact rescattering terms further reduces the ratio (except near threshold), indicating that such terms have a more pronounced effect in π^- absorption than in π^+ absorption.

Fig. 5 illustrates the role of different meson exchanges in the NN and $NA \rightarrow NN$ transition potentials. As is well known, inclusion of ρ -meson exchange reduces the tensor part of the one pion exchange (OPE) potential, while increasing the spin-spin part. In the Δ rescattering terms, which dominate π^+ absorption, the former effect is by far the more important one. Reduction of the OPE tensor potential is also the dominant effect in the nucleon rescattering terms; but for these terms, the spin-spin potential plays a more important role than for the Δ terms so that the inclusion of ρ -meson exchange affects π^- absorption less than π^+ absorption, as evidenced by the reduced ratio in fig. 5. Inclusion of σ and ω exchange has little influence on the π^+ cross section since neither σ nor ω exchange contributes to the $NA \rightarrow NN$ transition potential, but it increases the π^- cross section, thereby further decreasing the cross section ratio.

All of the numerical results reported thus far were obtained with the choice $\Lambda_\pi = 1.2 \text{ GeV}$. We have explored the sensitivity of our results to this parameter by recalculating the integrated cross sections using somewhat different values of Λ_π . The results are illustrated in fig. 6 and 7, which also include the most recent empirical results from experiments performed at TRIUMF and LAMPF^{6,9}). For either π^+ or π^- absorption, it can be seen that variation of Λ_π mainly influences the overall

normalization of the cross section without affecting its energy dependence much. The relative change in cross section resulting from a given change in Λ_π is comparable in the two cases, so that variation of Λ_π has very little effect on the cross section ratio.

In comparison with the data, our results are generally too large, especially for π^- absorption. By choosing Λ_π small enough we could reproduce the π^+ data over most of the energy range of interest, but such small values of Λ_π are difficult to justify since ρ -meson exchange is explicitly included in the model. Even with quite small values of Λ_π we obtain π^- cross sections that are too large. Moreover, because the π^- cross section is always overestimated by a larger factor than the π^+ cross section in our model, we always underestimate the cross section ratio, regardless of the value adopted for Λ_π . Both the cross section ratio and the magnitude of the π^- cross section can be improved by suppressing σ and ω exchange in the NN interaction, but there does not appear to be any compelling physical argument to justify such a suppression.

The remaining two figures display angular distributions obtained within the full model using $\Lambda_\pi=1.2$ GeV. In fig. 8 the π^+ angular distribution is plotted for a laboratory kinetic energy of 65 MeV, together with recent data from TRIUMF⁹). Although there is a small discrepancy in the overall normalizations, it can be seen that the theoretical curve reproduces the shape of the empirical angular distribution quite satisfactorily. This agreement between the shapes of the theoretical and empirical distributions for π^+ absorption is also obtained at higher energies, but then the discrepancy in normalizations becomes more

pronounced, as expected from the integrated cross section results shown in fig. 6.

In contrast with the π^+ results, the angular distributions obtained for π^- absorption in the present model do not reproduce the shape of the empirical distribution over the full energy range considered. This is indicated in fig. 9 where the π^- angular distributions for two different energies have been plotted together with data from TRIUMF⁹) and LAMPF⁶). To facilitate comparison with the shape of the empirical angular distributions, the theoretical results have been reduced in magnitude by over-all normalization factors chosen to bring the integrated cross sections in agreement with the data. Both of the theoretical distributions so obtained are somewhat forward peaked. The empirical distributions, on the other hand, are peaked at backward angles, with the degree of peaking decreasing as a function of energy. At 65 MeV, the backward peaking in the empirical distribution is quite evident so that there is a definite discrepancy between the shapes of the empirical and theoretical distributions. At 165 MeV, the empirical distribution is more symmetric and, within the range of angles for which measurements have been, appears to be consistent with the shape of the theoretical distribution.

The forward-backward asymmetry in the π^- angular distribution results from interference between even and odd partial waves in the final two nucleon state and on the theoretical level, is quite model dependent. By suppressing the ρ -exchange contribution to the NN interaction, it is possible to reverse the sign of the forward-backward asymmetry so as to make the calculated π^- angular distribution backward peaked. The shapes of the theoretical and empirical distributions then agree reasonably well at 65 MeV. Unfortunately, the angular distributions calculated for

larger energies with this prescription become more and more backward peaked as the energy increases, in clear disagreement with the energy dependence of the data.

4. Summary and conclusions

We have attempted in the work described here to construct a simple model which incorporates the major reaction mechanisms that contribute to the two-body absorption of pions by ^3He . In addition to the traditional mechanism, rescattering through intermediate $\Delta(3,3)$ states, which dominates absorption by $I=0$ nucleon pairs, these include rescattering through nucleon intermediate states, which is important in the $I=1$ initial state channel where the leading Δ terms are suppressed by the Pauli principle. The significance of the latter contributions is borne out qualitatively by a comparison of our numerical results with the available data. Without the nucleon rescattering terms, the calculated π^- cross section is much too small, making the π^+/π^- cross section ratio too large. When the nucleon terms are included, the π^- cross section increases substantially, while the π^+ cross section is only moderately affected. Thus, inclusion of these terms reduces the cross section ratio to a value more in accord with experiment. The nucleon rescattering contributions also yield a rather flat energy dependence, as seen in the π^- data. These two observations suggest that the overall picture of pion absorption physics that underlies the present model is essentially correct.

A more detailed comparison with the data, however, yields mixed conclusions. On the one hand, the model accounts for the overall energy dependence of the integrated cross sections reasonably well, yielding, in particular, both the resonance peak in the π^+ cross section and the flat

energy dependence of the π^- cross section. The model also reproduces the shape of the empirical angular distribution for π^+ absorption and, to a lesser extent, for π^- absorption over most of the energy range considered, although there is some difficulty accounting for the observed forward-backward asymmetry in the π^- angular distributions at low energies.

On the other hand, the overall normalization of the integrated cross sections is not predicted correctly in the model. In the case of π^+ absorption, it is observed empirically that the cross section for two-body absorption by ^3He is roughly just the πd absorption cross section scaled upward by a factor equal to the number of $I=0$ np pairs in ^3He . If we compare our model results for ^3He with results obtained previously for πd absorption in a model similar to the present one³), we find that the empirical relation is not satisfied by the calculated cross sections. This reveals that the normalization of the two-body cross sections is rather sensitive to particular details of the reaction model. For example, in ref. 3) it was found that single nucleon absorption terms with final state correlations included make significant contributions to the integrated πd cross section through destructive interference with the dominant Δ rescattering terms. In the model described here, such single nucleon terms are not included, for reasons outlined in sec. 2.

Another aspect of the reaction model that affects cross section normalizations is the manner in which ρ -meson exchange is treated. In the πd calculation mentioned above, a distributed mass ρ -exchange interaction is employed which explicitly includes certain two pion box diagrams with intermediate $N\Delta$ states that are not normally included in ρ exchange. The inclusion of these additional diagrams makes the ρ tensor

potential more effective in cutting down the OPE tensor potential than it would be otherwise. In the ^3He calculation, on the other hand, the ρ -meson has a single fixed mass and these box diagrams are not included.

In the case of π^- absorption by ^3He , the discrepancy between the empirical and calculated cross sections is much more serious than for π^- absorption. Part of this discrepancy could have its origins in the various approximations employed in the calculations. For example, no attempt has been made to take account of $\pi^-^3\text{He}$ correlations prior to absorption. Presumably, inclusion of such correlations would reduce the π^- cross section, perhaps significantly; however, if experience with π^+ absorption is any guide, the reduction obtained from this source would not be nearly enough to bring the calculated cross section into agreement with the empirical one. Other approximation which could be important are the neglect of interactions in intermediate two-baryon states and the use of baryon propagators that are both local and static. Unfortunately, it is necessary to perform calculations in a more complete model to fully assess the consequences of these approximations.

It is possible that the normalization difficulty experienced with the π^- cross section is not so much the result of particular approximations employed in the calculations, but has more to do with an inherent ambiguity in the way the two and three-body contributions are separated in a simple model like the present one. Conceivably, diagrams like C and D in fig. 1 contribute a significant fraction of their strength to the three-body background underlying the two-body peak in the spectrum, rather than to the peak itself. If that were the case, then a straightforward calculation of these diagrams would overestimate the two-body cross section. For π^+ absorption, this is not an important consideration

since in that case the two-body peak lies at least an order of magnitude above the background. For π^- absorption, however, the ratio of peak to background is only two or three, so that the calculational ambiguities mentioned here could be significant.

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Appendix A - NN and NA + NN potentials

The Fourier transforms which convert the meson exchange interactions expressed in momentum space, eq. (2.16)-(2.21), to position space expressions are defined by the integral,

$$V_{12}(\vec{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} V_{12}(q_0 = \frac{1}{2}\omega, \vec{q}), \quad (\text{A.1})$$

which can be evaluated analytically for each interaction after the momentum space forms are expanded in partial fractions. The resulting position space interactions are conveniently expressed in the form

$$V_{12}^{NN\lambda}(\vec{r}) = \frac{1}{4\pi} \left(\frac{f_{\lambda NN}}{m_\lambda} \right)^2 \mu_\lambda^2 \mathcal{J}_\lambda^N(\vec{r}), \quad (\text{A.2})$$

$$V_{12}^{NA\lambda}(\vec{r}) = \frac{1}{4\pi} \left(\frac{f_{\lambda NN}}{m_\lambda} \right) \left(\frac{f_{\lambda NA}}{m_\lambda} \right) \mu_\lambda^2 \mathcal{J}_\lambda^A(\vec{r}), \quad (\text{A.3})$$

where $\mu_\lambda^2 = m_\lambda^2 - \omega^2/4$, and the functions $\mathcal{J}_\lambda^N$ and $\mathcal{J}_\lambda^A$ are just linear combinations of radial structure functions and angular momentum operators multiplied by isospin operators. In particular,

$$\mathcal{J}_\pi^N(\vec{r}) = [\vec{F}_T^N(r) S_{12}(\hat{r}) + 1/3 \vec{F}_S^N(r) \vec{\sigma}_1 \cdot \vec{\sigma}_2] \vec{t}_1 \cdot \vec{t}_2, \quad (\text{A.4})$$

$$\mathcal{J}_\rho^N(\vec{r}) = [-\vec{F}_T^N(r) S_{12}(\hat{r}) + 2/3 \vec{F}_S^N(r) \vec{\sigma}_1 \cdot \vec{\sigma}_2] \vec{t}_1 \cdot \vec{t}_2, \quad (\text{A.5})$$

$$\mathcal{J}_\omega^N(\vec{r}) = [\vec{F}_S^N(r) - \vec{F}_C^N(r)] \vec{1} - 2\vec{F}_{LS}^N(r) \vec{L} \cdot \vec{\Sigma}, \quad (\text{A.6})$$

$$\begin{aligned} \mathcal{J}_\omega^A(\vec{r}) = & -\vec{F}_T^A(r) S_{12}(\hat{r}) + 2/3 \vec{F}_S^A(r) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \\ & + [2\vec{F}_S^A(r) + \vec{F}_C^A(r)] \vec{1} - 6\vec{F}_{LS}^A(r) \vec{L} \cdot \vec{\Sigma}, \end{aligned} \quad (\text{A.7})$$

with $\vec{\Sigma} = 1/2(\vec{\sigma}_1 + \vec{\sigma}_2)$, $\vec{L}$ denoting the orbital angular momentum operator,

and

$$S_{12}(\hat{r}) = \vec{\sigma}_1 \cdot \hat{r} \vec{\sigma}_2 \cdot \hat{r} - 1/3 \vec{\sigma}_1 \cdot \vec{\sigma}_2. \quad (\text{A.8})$$

The functions $\mathcal{J}_\pi^A$ and $\mathcal{J}_\rho^A$ are given by eq. (A.4) and (A.5) with the operators $\vec{\sigma}_1$ and $\vec{t}_1$ replaced by the corresponding spin and isospin transition operators, $\vec{S}_1$ and $\vec{T}_1$. For the radial structure functions, we have

$$\vec{F}_S^\lambda(r) = Y_0(\mu_\lambda r) + [\alpha_\lambda(r) - 1] Y_0(\Omega_\lambda r), \quad (\text{A.9})$$

$$\vec{F}_T^\lambda(r) = Y_2(\mu_\lambda r) - \frac{\Omega_\lambda^2}{\mu_\lambda^2} Y_2(\Omega_\lambda r) + \alpha_\lambda(r) Y_1(\Omega_\lambda r), \quad (\text{A.10})$$

$$\vec{F}_C^\lambda(r) = \left(\frac{2M_N}{\mu_\lambda} \right)^2 \left\{ Y_0(\mu_\lambda r) + \left[\frac{\mu_\lambda^2}{\Omega_\lambda^2} \alpha_\lambda(r) - 1 \right] Y_0(\Omega_\lambda r) \right\}, \quad (\text{A.11})$$

$$\vec{F}_{LS}^\lambda(r) = \frac{1}{\mu_\lambda r} \left[Y_1(\mu_\lambda r) - \frac{\Omega_\lambda}{\mu_\lambda} Y_1(\Omega_\lambda r) + \frac{\mu_\lambda}{\Omega_\lambda} \alpha_\lambda(r) Y_0(\Omega_\lambda r) \right], \quad (\text{A.12})$$

with $\Omega_\lambda^2 = \Lambda_\lambda^2 - \omega^2/4$,

$$\alpha_\lambda(r) = 1/2 \Omega_\lambda r (1 - \Omega_\lambda^2/\mu_\lambda^2), \quad (\text{A.13})$$

$$Y_0(\mu r) = e^{-\mu r/r}, \quad (\text{A.14})$$

$$Y_1(\mu r) = (1 + 1/\mu r) Y_0(\mu r) \quad (\text{A.15})$$

$$Y_2(\mu r) = \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2} \right) Y_0(\mu r). \quad (\text{A.16})$$

Note in these expressions that only the first term of each structure function survives in the limit $\Lambda_\lambda \rightarrow \infty$, and for the choice $\Lambda_\lambda = m_\lambda$, all the structure functions vanish, as they should.

Appendix B - matrix elements

To evaluate the spatial matrix elements, it is convenient to couple the pion plane waves occurring in the transition amplitude expressions, eq. (2.1) and (2.2), with the initial ^3He wave function, eq. (2.39). After expanding the exponentials in partial waves, this yields

$$e^{\pm i\mathbf{k}\cdot\mathbf{r}/2} |1\rangle = \sum_{\ell} (\pm 1)^{\ell} \sqrt{2\ell+1} j_{\ell}(kr/2) \phi(r) \sum_{j_1} (\ell 0 S_1 M_1 | j_1 M_1) \times |(\ell S_1) j_1 M_1\rangle |1 \mu_1\rangle, \quad (\text{B.1})$$

where a coordinate system with $\hat{z}$ along $\hat{k}$ has been employed in the partial wave expansion. For the final two-nucleon state we obtain in the LS coupling scheme,

$$\langle f | = 4\pi \sum_{L_M} (-1)^L \frac{1}{\sqrt{2}} \left[1 + (-1)^{1+I_f+S_f+L} \right] j_L(pr) Y_{LM}(\hat{p}) \times \sum_{J_f M_J} (L M_S f M_f | J_f M_J) \langle (L S_f) J_f M_J | \langle I_f \mu_f f | \quad (\text{B.2})$$

The spin-isospin matrix elements of the transition amplitude can be expressed in a general form which includes absorption terms involving either of the two active nucleons. For the p-wave rescattering terms with nucleon intermediate states, we have

$$\langle I_f S_f | T_N | I_i S_i \rangle = \frac{1}{4\pi} \frac{f_{\pi NN}}{m_{\pi}} \left[e^{i\mathbf{k}\cdot\mathbf{r}/2} {}_{+}(-) 1 + I_f S_f e^{-i\mathbf{k}\cdot\mathbf{r}/2} \right] \times \sum_{\lambda=\pi, \rho, \sigma, \omega} \left(\frac{f_{\lambda NN}}{m_{\lambda}} \right)^2 \mu_{\lambda}^2 \left[G_{\Delta}^N(\omega, \hat{k}) \langle I_f S_f | \mathcal{D}_{\lambda}^N(\hat{r}) (\hat{\sigma}_1 \cdot \hat{k})^{\dagger} \hat{1}_1 \cdot \hat{\phi} | I_i S_i \rangle + G_C^N(\omega, \hat{k}) \langle I_f S_f | \hat{1}_1 \cdot \hat{\phi} (\hat{\sigma}_1 \cdot \hat{k}) \mathcal{D}_{\lambda}^N(\hat{r}) | I_i S_i \rangle \right] \quad (\text{B.3})$$

with all quantities defined as in sec. 2 and appendix A. Combining this expression with eq. (B.1) and (B.2) yields for the full matrix element,

$$\langle f | T_N | i \rangle = \frac{4}{\sqrt{2}} \sum_{\ell L} (-) (\ell-L) / 2 b_{\ell L}^f \sqrt{2\ell+1} \sum_{j_1 j_f} (\ell 0 S_1 M_1 | j_1 M_1) \times (L M_1^{-} M_f S_f M_f | j_f M_f) Y_{LM_1^{-} M_f}(\hat{p}) \int_0^{\infty} dr r^2 j_{\ell}(kr/2) \phi(r) j_L(pr) \times \langle (L S_f) j_f M_f; I_f \mu_f | \mathcal{J}_N | (\ell S_1) j_1 M_1; I_i \mu_i \rangle, \quad (\text{B.4})$$

where

$$b_{\ell L}^f = 1 \quad \text{for } (-)^{1+I_f+S_f+L} = (-)^{\ell+L} = 1 \\ = 0 \quad \text{otherwise,} \quad (\text{B.5})$$

and

$$\mathcal{J}_N = \frac{f_{\pi NN}}{m_{\pi}} \sum_{\lambda=\pi, \rho, \sigma, \omega} \left(\frac{f_{\lambda NN}}{m_{\lambda}} \right)^2 \mu_{\lambda}^2 \left[G_{\Delta}^N(\omega, \hat{k}) \mathcal{D}_{\lambda}^N(\hat{r}) (\hat{\sigma}_1 \cdot \hat{k})^{\dagger} \hat{1}_1 \cdot \hat{\phi} + G_C^N(\omega, \hat{k}) \hat{1}_1 \cdot \hat{\phi} (\hat{\sigma}_1 \cdot \hat{k}) \mathcal{D}_{\lambda}^N(\hat{r}) \right] \quad (\text{B.6})$$

To obtain the corresponding expression for Δ intermediate states, we need only substitute

$$\mathcal{J}_{\Delta} = \frac{f_{\pi N\Delta}}{m_{\pi}} \sum_{\lambda=\pi, \rho} \left(\frac{f_{\lambda N\Delta}}{m_{\lambda}} \right)^2 \mu_{\lambda}^2 \left[G_{\Delta}^{\Delta}(\omega, \hat{k}) \mathcal{D}_{\lambda}^{\Delta}(\hat{r}) (\hat{\sigma}_1^{\dagger} \cdot \hat{k})^{\dagger} \hat{1}_1 \cdot \hat{\phi} + G_C^{\Delta}(\omega, \hat{k}) \hat{1}_1 \cdot \hat{\phi} (\hat{\sigma}_1 \cdot \hat{k}) \mathcal{D}_{\lambda}^{\Delta}(\hat{r}) \right] \quad (\text{B.7})$$

in place of $\mathcal{J}_N$ in eq. (B.4). For the s-wave rescattering term, a similar procedure based on eq. (2.28) yields the matrix element

$$\langle f | T_S | i \rangle = \frac{4}{\sqrt{2}} \sum_{\ell L} (-) (\ell+L-1) / 2 v_{\ell L}^f \sqrt{2\ell+1} \sum_{j_1 j_f} (\ell 0 S_1 M_1 | j_1 M_1) \times (L M_1^{-} M_f S_f M_f | j_f M_f) Y_{LM_1^{-} M_f}(\hat{p}) \int_0^{\infty} dr r^2 j_{\ell}(kr/2) \phi(r) j_L(pr) \times \langle (L S_f) j_f M_f; I_f \mu_f | \mathcal{J}_S | (\ell S_1) j_1 M_1; I_i \mu_i \rangle, \quad (\text{B.8})$$

where

$$\gamma_{\ell L}^f = 1 \quad \text{for } (-)^{1+I_f+S_f+L} = (-)^{\ell+L+1} = 1 \quad (\text{B.9})$$

$$= 0 \quad \text{otherwise,}$$

and

$$\mathcal{Y}_s = 8\pi \left(\frac{f_{\pi NN}}{m_\pi} \right) \frac{\mu_\pi}{m_\pi} \left(1 + \frac{1}{\mu_\pi r} \right) e^{-\mu_\pi r / r} (\vec{\sigma}_1 \cdot \hat{r}) (\vec{\sigma}_2 \cdot \hat{r}) \quad (\text{B.10})$$

The matrix elements of the $\mathcal{Y}$ operators consist of sums of products of radial functions, angular momentum matrix elements, and isospin matrix elements and may be evaluated through standard procedures.

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Figure captions

1. Contributions to the reaction amplitude.
2. Contributions to the integrated cross section for ${}^3\text{He}(\pi^+, pp)p$ from different partial waves in the final two nucleon state. T_n is the pion kinetic energy in the lab.
3. Same as fig. 2 for ${}^3\text{He}(\pi^-, pn)n$.
4. $\sigma(\pi^+)/\sigma(\pi^-)$ vs. T_n for various reaction models. The curve labelled "A ONLY" was obtained with diagrams A and B in fig. 1; the curve "N+A" includes in addition diagrams C and D; and the curve "FULL MODEL" was obtained with the full set of diagrams. In every case all possible meson exchanges were included.
5. $\sigma(\pi^+)/\sigma(\pi^-)$ vs. T_n for the inclusion of various meson exchanges in the NN and NA + NN transition potentials. The top three curves were obtained with diagrams A-D in fig. 1, while the lower curve includes also diagram E.
6. The integrated cross section for ${}^3\text{He}(\pi^+, pp)p$ vs. T_n for various values of Λ_π in GeV. The filled circle data points are from ref. 9), while the cross point is from ref. 6).
7. Same as fig. 6 for ${}^3\text{He}(\pi^-, pn)n$.
8. $d\sigma/d\Omega$ vs. the cm scattering angle for ${}^3\text{He}(\pi^+, pp)p$ at $T_n=65$ MeV. The data points are the 62.5 MeV points from ref. 9).
9. $d\sigma/d\Omega$ vs. the cm scattering angle for ${}^3\text{He}(\pi^-, pn)n$ at $T_n=65$ MeV (upper curve) and $T_n=165$ MeV (lower curve). Each curve has been multiplied by the ratio of the empirical to calculated cross section at the appropriate energy. The filled circle points represent 62.5 MeV data from ref. 9), while the cross points are 165 MeV data from ref. 6).