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The Heterotic String and Supersymmetric Counterparts of the Lorentz Chern-Simons Terms

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THE HETEROTIC STRING AND SUPERSYMMETRIC
COUNTERPARTS OF THE LORENTZ CHERN-SIMONS TERMS

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Abstract

The heterotic string 3-point amplitude, which involves two fermionic and one bosonic states determines some of the terms that must be added to the action of the $N=1$ supergravity coupled to $N=1$ supersymmetric Yang-Mills in ten dimensions to preserve supersymmetry, if the Lorentz Chern-Simons term is included in the definition of the field strength $H_{\mu\nu\rho}$ for the cancellation of anomalies. Our calculation reveals an interesting structure, which suggests that the relevant H-Bianchi for the analysis of possible heterotic string compactifications involves a spin connection possibly with torsion.

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It is still an open question whether a supersymmetric anomaly-free theory of $N=1$ supergravity coupled to $N=1$ supersymmetric Yang-Mills theory can be constructed in a closed form in ten dimensions. If such a theory exists, it is presumably obtained as a low energy limit (appropriately defined) of a consistent (anomaly free) superstring theory [1] which has in its spectrum $N=1$ supergravity and $N=1$ supersymmetric Yang-Mills multiplets. If we like to construct such a theory, a good starting point will be the Chapline-Manton theory [2] with gauge groups $SO(32)$ or $E_8 \times E_8$ and with modified Chern-Simons terms needed to cancel out the anomalies [3] in gauge and Lorentz transformations arising from fermion loops. It must be stressed, however, that simply adding ω_{3L} to $H = dB + \omega_{3Y}$ breaks the supersymmetry of the Chapline-Manton Lagrangian. Finding the supersymmetric completion of such terms as well as the necessary modification (if any) to the original supersymmetry transformations, is a non-trivial task [4,5,6,7].

It has been demonstrated recently [8] that the bosonic part of the Chapline-Manton action with the antisymmetric tensor field strength H modified by the Lorentz Chern-Simons term can be obtained from the 3-point bosonic tree amplitude of the heterotic string. In addition, in the same paper [8], it is shown that there is a higher order derivative term coupling the dilaton field to $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$. A safe way, therefore, of finding supersymmetric counterparts to the Lorentz Chern-Simons term is to calculate more heterotic string tree amplitudes especially the ones which include bosonic as well as fermionic states.

In our letter we show that the mixed (two fermions and one boson) 3-point tree amplitude gives already important information about possible supersymmetric counterparts to ω_{3L} . We follow the approach of [8,11,12], where one matches amplitudes for S-matrix elements calculated in the first quantized heterotic string theory with those calculated in the corresponding low energy ten dimensional field theory. Our results include all the terms conjectured in [4], but we find more structure. Moreover we confirm that the heterotic string gives all the terms, with the appropriate numerical coefficients, needed in the analysis of [5] where the first steps in the construction of a supersymmetric, anomaly-free theory of $N=1$ supergravity coupled to $N=1$ supersymmetric Yang-Mills theory in ten dimensions have been undertaken. Moreover, there is extra structure emerging from our calculation, which is very suggestive in connection with the non-uniqueness

of the effective ten dimensional field theory arising from the requirement of anomaly cancellation and supersymmetry [9]. Finally our results in conjunction with [5] show that the analysis of Candelas, Horowitz, Strominger and Witten [11] on possible vacuum configurations for the heterotic string is reliable only to first order in α' (the inverse of string tension) for configurations with $H=0$.

On account of duality the heterotic string amplitude of interest has the form

$$A(k_1, k_2, k_3) = \langle 1 | V_B^{\text{GRAV}}(2, \xi_2) | 3 \rangle \quad (1)$$

where $V_B^{\text{GRAV}}(\xi)$ is the vertex for the emission of a graviton (traceless, symmetric $\xi^{\mu\nu}$), antisymmetric tensor (antisymmetric $\xi^{\mu\nu}$), or dilaton (the trace of $\xi^{\mu\nu}$) given by eq (II.11) of [8]. The authors of [8] worked in a system of units where $\alpha' = \frac{1}{2}$. The fermionic states $|1\rangle$ and $|3\rangle$ are given, in the light cone formalism [1] of the heterotic string [8], by

$$\begin{aligned} \langle 1 | &= \frac{1}{8} \frac{1}{(k_1^+)^{\frac{1}{2}}} \langle 0 |_R \times \langle 0 |_L \alpha_1^\ell \tilde{\alpha}_1^j (\bar{S}_o \Gamma_\ell)^a U_1^{aj} \\ |3\rangle &= \frac{1}{8} \frac{1}{(k_3^+)^{\frac{1}{2}}} \bar{U}_3^{aj} (\Gamma_\ell S_o)^a \alpha_{-1}^\ell \tilde{\alpha}_{-1}^j |0\rangle_R \times |0\rangle_L \end{aligned} \quad (2)$$

where S_o^a is the fermion zero mode oscillator.

The calculation of (1) is straightforward but tedious. It yields terms up to order $(\alpha')^2$. The zero order terms correspond to the usual trilinear couplings of the Chapline-Manton theory [2] involving the Majorana-Weyl gravitino ψ_μ and the Majorana-Weyl spinor λ of the $N=1$ supergravity multiplet. The $O(\alpha')$ corrections are the terms we are interested in. Recall that in the calculation of Gross, Harvey, Martinec and Rohm [8], the Lorentz Chern-Simons term as well as the $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$ term are all $O(\alpha')$ terms.

The $O(\alpha')$ terms of the string calculation, by adding extra terms which vanish on mass shell, yield after covariantisation

(3)

$$\begin{aligned} & \frac{1}{8} g \alpha' \{ \bar{U}_3 [\sigma \xi_2]_{\mu\nu} k_{2\rho} k_{2\sigma} k_1^\nu \Gamma^{\mu\rho\lambda} U_{1\lambda} \} + \\ & + 8 \bar{U}_3^\rho k_{2\rho} k_{2\sigma} \Gamma^\mu \frac{1}{2} (\xi_{2\mu\nu} + \xi_{2\nu\mu}) k_1^\nu U_1^\sigma \} \end{aligned} \quad (3)$$

where $\Gamma^{\mu\rho\lambda}$ is the totally antisymmetric product of 3 γ matrices and g is the dimensionless string coupling constant. Indices between $[\dots]$ are not included in the antisymmetrization. In (3) the polarisation tensor $\xi^{\mu\nu}(k)$ and spinor $U^{\mu a}(k)$ describe the on shell supergravity multiplet. In the light cone gauge

$$\begin{aligned} \xi^{+\mu}(k) &= \xi^{\mu+}(k) = k_\mu \xi^{\mu\nu}(k) = k_\nu \xi^{\mu\nu}(k) = 0 \\ k_\mu U^\mu(k) &= k U^\mu(k) = \Gamma^- U^\mu(k) = \frac{1}{2} (1 - \Gamma^{11}) U^\mu(k) = 0 \quad (k^+ \neq 0) \end{aligned} \quad (4)$$

The supersymmetry transformations that relate the fermionic, V_F , and bosonic V_B , vertices of the heterotic string relate also $U^{\mu a}$ and $\xi^{\mu\nu}$ through the relation

$$U_\mu^a = \frac{1}{2} (\Gamma^{\nu\lambda} k_\lambda \xi_{\nu\mu} \epsilon)^a \quad (5)$$

Using, now, the decomposition of $\xi^{\mu\nu}$ into the symmetric, antisymmetric, and trace (or dilaton) parts given in [11,12] one can justify the following decomposition of $U^{\mu a}$ into the gravitino and dilatino parts which is compatible with the canonical form of Chapline-Manton action [2],

$$U^{\mu a} = U_G^{\mu a} + \frac{1}{8\sqrt{2}} \Gamma^\mu U_D^a \quad (6)$$

Taking the symmetric part, $h_{\mu\nu}$, of $\xi_{\mu\nu}$ in (3) with the help of (6) we deduce that such an amplitude may arise (up to on shell terms) from a term in the action of ten-dimensional field theory given by

$$\begin{aligned} & \frac{g \alpha' \Delta}{8\kappa_{10}} \int d^{10}x \{ -\frac{3}{2} \bar{\psi} [\sigma \Gamma^{\mu\rho\lambda} \partial_\nu \psi_\lambda] a^{\sigma\nu}{}_{\mu\rho} + \\ & + \frac{\bar{\lambda}}{\sqrt{2}} \Gamma^{\mu\rho} \partial [\nu \psi^\sigma] a_{\mu\rho\nu\sigma} \} \end{aligned} \quad (7)$$

where

$$a_{\mu\rho\nu\sigma} = \frac{1}{2} (g_{\mu\nu,\rho\sigma} - g_{\mu\sigma,\rho\nu} - g_{\rho\nu,\mu\sigma} + g_{\rho\sigma,\mu\nu}) \quad (8)$$

and $g_{\mu\nu}$ in the linearised approximation is given by

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa_{10} h_{\mu\nu} \quad (9)$$

with κ_{10} the ten dimensional gravitational coupling. It is expected that the appropriate four-point and higher string amplitudes will complete (8) into $R_{\mu\rho\nu\sigma}(\omega)$, the Riemann tensor corresponding to the usual Christoffel spin connection, convert ordinary derivatives into covariant ones and couple the dilaton field D to (7). Furthermore in the effective field theory we can make the substitution $\left[\frac{g\alpha'\Delta}{\kappa_{10}} \right] \rightarrow \frac{\kappa_{10}^2}{g_{10}^2}$, where g_{10} is the ten dimensional gauge coupling, and rescale the spinor fields $\psi \rightarrow \frac{1}{\sqrt{2}\kappa_{10}} \psi$, $\lambda \rightarrow$

$$\begin{aligned} & \frac{1}{\sqrt{2}\kappa_{10}} \lambda \text{ to obt} \\ & \frac{1}{g_{10}^2} \int d^{10}x e^{-\frac{\kappa_{10} D}{\sqrt{2}}} \left\{ -\frac{3}{2} \bar{\psi} [\sigma^{\mu\rho\lambda} \nabla_\nu \psi_\lambda] R^{\sigma\nu}{}_{\mu\rho}(\omega) + \right. \\ & \quad \left. + \frac{1}{\sqrt{2}} \bar{\lambda} \Gamma^{\mu\rho} \nabla_\nu [\psi^\sigma] R_{\mu\rho\nu\sigma}(\omega) \right\} \end{aligned} \quad (10)$$

+ terms vanishing on shell.

It is clear from (10) that there is no on shell dilatino-dilatino-graviton coupling, and the structure of terms in (10) are of the type advocated in [4]. Also in place of $R_{\mu\nu\rho\sigma}$ we can have extra terms involving the Ricci tensor $R_{\mu\nu}$, and the scalar curvature R , which give zero on shell contributions. Thus in general we can replace

$$R^{\sigma\nu}{}_{\mu\rho} \rightarrow R^{\sigma\nu}{}_{\mu\rho} + \alpha \delta \left[\begin{smallmatrix} \sigma \\ \mu \end{smallmatrix} R^{\nu}{}_{\rho} \right] + \beta \delta^\sigma{}_\mu \delta^\nu{}_\rho R \quad (11)$$

Expression (10) in conjunction with (11) gives precisely the terms needed in the first steps of the analysis of Romans and Warner [5], where the coefficients α and β are determined by off shell supersymmetry.

Projecting out the antisymmetric part of $\xi^{\mu\nu}$ in (3) and going through the same analysis as before we finally obtain

$$\frac{1}{g_{10}^2} \int d^{10}x \left\{ -\frac{3}{2} \bar{\psi} [\sigma^{\mu\rho\lambda} \partial_\nu \psi_\lambda] h^{\sigma\nu}_{\mu\rho} + \frac{1}{\sqrt{2}} \bar{\lambda} \Gamma^{\mu\rho} \partial [\psi_\sigma] h_{\mu\rho\nu\sigma} \right\} \quad (12)$$

where

$$b_{\sigma\nu\mu\rho} = -\frac{\sqrt{2}}{2} [B_{\sigma\mu, \nu\rho} - B_{\sigma\rho, \nu\mu} - B_{\nu\mu, \sigma\rho} + B_{\nu\rho, \sigma\mu}] \quad (13)$$

The second term in (12) arises from a term in the string amplitude, which vanishes identically, when we project out the antisymmetric part of $\xi^{\mu\nu}$. This term, however, can be amended by torsion terms which vanish on shell. It is then possible to obtain in the effective ten dimensional field theory a trilinear coupling involving dilatino-gravitino-torsion curvature tensor.

Now, it is anticipated that higher order string amplitudes will complete (13) into the part of a generalized Riemann tensor corresponding to torsion $T_{\mu\nu\rho}$ [12,13], namely

$$\nabla_\lambda T_{\nu\rho\sigma} - \nabla_\rho T_{\nu\lambda\sigma} + T^\mu_{\nu\rho} T_{\nu\lambda\sigma} - T^\mu_{\nu\lambda} T_{\mu\rho\sigma} \quad (14)$$

where, as it was first pointed out by Scherk and Schwarz [11],

$$T_{\mu\nu\rho} = \frac{3\sqrt{2}}{2} e^{-\frac{\kappa_{10} D}{\sqrt{2}}} H_{\mu\nu\rho} \quad (15)$$

In (15) $H_{\mu\nu\rho}$ is the field strength associated with the antisymmetric tensor field $B_{\mu\nu}$ plus terms of higher order in α' and D is the dilaton field of reference [8].

Finally, by projecting in (3) the trace of $\xi^{\mu\nu}$ corresponding to the dilaton, one can derive the possible terms in the ten dimensional field theory which involve derivatives acting on the dilaton field. It is easy to show that all such terms give vanishing on shell contributions.

The results of our analysis can, now, be conveniently summarized as follows. The $O(\alpha')$ part of the heterotic string amplitude (involving two fermions and

one boson) suggests that there is a term in the effective ten dimensional theory with the following structure.

$$\begin{aligned} \frac{1}{g_{10}^2} \int d^{10}x e^{-\frac{\kappa_{10}^D}{\sqrt{2}}} \left\{ -\frac{3}{2} \bar{\psi} [\sigma^{\mu\rho\lambda} \nabla_{\nu} \psi_{\lambda}] R^{\sigma\nu}_{\mu\rho}(\Omega) + \right. \\ \left. + \frac{1}{\sqrt{2}} \bar{\lambda} \Gamma^{\mu\rho} \nabla_{\nu} [\psi^{\sigma}] R_{\mu\rho\nu\sigma}(\Omega) \right\} \end{aligned} \quad (16)$$

where Ω is a connection which involves in general the usual Christoffel connection, the dilaton part [11,12], as well as the torsion which is proportional to $H_{\mu\nu\rho}$ (and possibly ϕ -torsion). The Riemann curvature $R_{\sigma\nu\mu\rho}(\Omega)$ that appears in (16) may be amended by terms similar to the ones in (11) with the Ricci and scalar curvatures defined with respect to Ω . The calculation of 3-point bosonic amplitude [8,12] has given to $O(\alpha')$ terms proportional to the combination.

$$e^{-\frac{\kappa_{10}^D}{\sqrt{2}}} (R^{\mu\nu\rho\sigma}(\Omega) R_{\rho\sigma\mu\nu}(\Omega) - 4R^{\mu\nu}(\Omega) R_{\nu\mu}(\Omega) + R(\Omega)^2)$$

where the relevant coefficients ensure unitarity as pointed out by Zwiebach [14]. The terms involving Ricci and scalar curvatures give vanishing on shell contributions.

The structure of (16) is very suggestive. Taken together with the analysis of Nepomechie [12] and the observation that the 3-point bosonic amplitude involving three antisymmetric tensors vanishes on shell it suggests that the relevant H-Bianchi (in the language of forms) is given by

$$dH = \text{tr} F^2 - \text{tr} R^2(\Omega) \quad (17)$$

This is in accordance with the conclusions of reference [6]. It is of great interest, of course, to find out whether or not equation (17) has solutions with $H \neq 0$, which are at the same time of finite order in α' . This is non-trivial, since any solution is expected to be found through iteration, because H appears on both sides of (17). To $O(\alpha')$ H changes by the usual Lorentz Chern-Simons term proposed by Green and Schwarz [3] and this can be used as the starting point of iteration of (17).

In a recent paper Hull [9] has argued that it is not necessary to use a torsion-free spin connection in the anomaly cancellation. The spin

connection with torsion, Ω , can be used instead. This is very reassuring, when one tries to find solutions of (17) with $H \neq 0$ in order to investigate possible heterotic string compactifications, other than the ones corresponding to Calabi-Yau spaces found in [9].

Finally, we would like to remark that our analysis, based on direct calculation of heterotic string amplitudes, taken together with the analysis of Romans and Warner [5] shows that the Chapline-Manton supersymmetry transformations (appropriately modified to $O(\alpha')$ on account of ω_{3L} terms) can be reliably used to investigate the supersymmetry of the heterotic string (and consequently the supersymmetry of the effective ten dimensional theory obtained from it) to $O(\alpha')$ in a background with $H=0$. The Chapline-Manton supersymmetry transformations are certainly not reliable for $H \neq 0$, even to order α' .

So far we have not dealt with the Yang-Mills sector of the theory. The 3-point amplitude analysis of the heterotic string gives some of the $O(\alpha'=0)$ terms in H^2 in the action that justify the modification of H by gluino terms $\text{Tr} \bar{\chi} \Gamma_{\mu\nu\rho} \chi$ [15]. Higher order terms in α' are to appear in higher-point amplitudes.

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