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WEAK-BASIS INVARIANT CONDITIONS FOR CP CONSERVATION

IN $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ MODELS

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ABSTRACT

We derive weak-basis invariant necessary conditions for CP conservation in the framework of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models, with an arbitrary number of generations. In the case of two and three generations, we select a set of conditions which are both necessary and sufficient for CP invariance. The case of degenerate or vanishing fermion masses is also analyzed, with emphasis on some novel features which arise in left-right models.

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One of the most attractive ways¹⁾ of introducing CP violation in unified gauge theories is through physical phases appearing in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. The CP violating phases may originate in complex Yukawa couplings and/or complex neutral Higgs vacuum expectation value (VEV) which in turn lead to complex fermion mass matrices. In this letter, we will derive a set of necessary and sufficient conditions for CP invariance in the context of the left-right (LR) model²⁾ based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. These conditions will be expressed in terms of quantities which are invariant under a change of a weak basis and therefore can be directly applied to the fermion mass matrices, without the need to diagonalize them.

The LR model provides an attractive framework to search for a better understanding of the origin of the breaking of P and CP invariance. Indeed, one of the original motivations for the suggestion of the LR model was the fact that parity can be a symmetry of the Lagrangian, spontaneously broken by the vacuum. It turns out that CP can also be spontaneously broken³⁾ in the LR model, although achieving a natural CP breaking vacuum requires a careful choice⁴⁾ of the Higgs system. On a more phenomenological level⁵⁾, the LR model is of special interest in view of the rather small experimental value for the ϵ'/ϵ ratio obtained both by the Chicago-Saclay collaboration⁶⁾ [$\epsilon'/\epsilon = (-4.6 \pm 5.3 \pm 2.4)10^{-3}$] and the Yale-BNL collaboration⁷⁾ [$\epsilon'/\epsilon = (1.7 \pm 8.2)10^{-3}$]. Indeed, prior to these experimental results, it was pointed out⁸⁾ that in LR models a rather small value for ϵ'/ϵ can be obtained [$|\epsilon'/\epsilon| \approx 10^{-3}$].

In the present work, we will be only interested in CP violation through the left-handed (LH) and right-handed (RH) charged currents and therefore all our considerations will be valid, independently of whether CP is spontaneously broken or not. Although most of our analysis will be dealing with CP violation in the quark sector, our results can be trivially extended to the leptonic sector with massive Dirac neutrinos. Having this extension in mind, we will discuss the case where some of the fermion masses are degenerate or vanishing, as it may be the case for neutrinos. In particular, we will show that in the LR model with Dirac neutrinos, there will be CP violation even in the limit where $m_{\nu_e} = m_{\nu_\mu} = 0$, but $m_{\nu_\tau} \neq 0$. This is to be contrasted with the situation in the standard model, where all CP violation effects due to the Kobayashi-Maskawa (KM) mechanism disappear in the above limit. It should be emphasized that even for the quark sector, the limit of degenerate quarks is not only of academic interest. The point is that if one considers CP violation at high energies^{9),10)}, some of the quark masses may become effectively degenerate. Indeed, it has been pointed out

recently¹⁰⁾ that if one tries to see CP violation at the Z^0 peak through an asymmetry measured by $\alpha \equiv [\Gamma(s\bar{b}) - \Gamma(\bar{s}b)] / [\Gamma(s\bar{b}) + \Gamma(\bar{s}b)]$, the value of α in the standard model with three generations is very small due to the effective mass degeneracy of the u and c quarks at higher energies. We will show that in the LR model there is CP violation even in the above limit, thus opening the possibility of a larger effect.

In the $SU(2)_L \times SU(2)_R \times U(1)$ gauge model, after spontaneous gauge symmetry breaking, the mass terms and the hadronic charged current interactions are:

$$\begin{aligned} \mathcal{L} = & g_L \bar{u}_L \gamma_\mu d_L W_L^\mu + g_R \bar{u}_R \gamma_\mu d_R W_R^\mu + \\ & + \bar{u}_L M_u u_R + \bar{d}_L M_d d_R + h.c. \end{aligned} \quad (1)$$

where u, d stand for charge 2/3 (up) and -1/3 (down) quark weak eigenstates, each representing a n-component vector in generation space. The mass matrices M_u , M_d are n-dimensional arbitrary complex matrices. In (1) we have omitted all other terms of the Lagrangian, since they will not play any rôle in our discussion. The Lagrangian of Eq. (1) will conserve CP if and only if there are CP transformations of the type:

$$\begin{aligned} u_L &\rightarrow V_L C u_L^* & ; & & u_R &\rightarrow V_R C u_R^* \\ d_L &\rightarrow V_L C d_L^* & ; & & d_R &\rightarrow V_R C d_R^* \end{aligned} \quad (2)$$

which leave the Lagrangian invariant. In Eq. (2), V_L , V_R are n-dimensional unitary matrices acting in generation space and C is the Dirac charge conjugation matrix. From Eq. (2), one sees the rôle played by the charged current interactions in dictating whether CP is conserved or not. For example, it is the existence of both LH and RH charged currents which constrains each one of the pairs (u_L, d_L) , (u_R, d_R) to transform in the same way under CP. In the standard model, where there are no RH charged currents, u_R , d_R may transform in a different way under CP. This is the crucial difference between the LR model and the standard model, as far as CP violation is concerned. In order for the Lagrangian to be invariant under (2) the quark mass matrices M_u , M_d have to satisfy the following conditions:

$$V_L^\dagger M_u V_R = M_u^* \quad (3a)$$

$$V_L^\dagger M_d V_R = M_d^* \quad (3b)$$

The conditions expressed by Eqs. (3) are weak-basis-independent. Indeed, if in a particular weak basis where the mass matrices are M_u , M_d , there are unitary matrices V_L , V_R satisfying those equations, in another weak basis, where the transformed matrices are $M'_{u,d} = U_{L,u,d} M_{u,d} U_{R,u,d}^{-1}$, there will also exist a solution of Eqs. (3) given by the unitary matrices $V'_L = U_{L,u,d} V_L U_{L,u,d}^T$, $V'_R = U_{R,u,d} V_R U_{R,u,d}^T$. Therefore, we conclude that Eqs. (3) constitute weak-basis independent necessary and sufficient conditions for CP invariance. However, it is clear that in their present form, they are not very useful to test the existence of CP violation in a specific model. Next we are going to derive a set of necessary conditions for CP invariance expressed in terms of functions of the quark mass matrices which are invariant under a change of weak basis. These invariant conditions are specially useful in building and testing models of CP violation, since they can be applied directly to the various quark mass matrices, without the need to diagonalize them. In the cases of two and three generations, we will also derive a set of invariant conditions which are both necessary and sufficient for CP invariance.

INVARIANT CP CONDITIONS

If one multiplies Eqs. (3) by their Hermitian conjugate an arbitrary number of times, one easily obtains a variety of invariant CP necessary conditions such as:

$$\begin{aligned} \text{tr} [H_u^p, H_d^q]^r &= 0 \\ \text{tr} [H_u^p, H_d^q]^r &= 0 ; \quad \text{tr} [(M_u M_d^\dagger)^p] = \text{tr} [(M_d M_u^\dagger)^p] \\ \text{tr} [(M_u M_d^\dagger)^p (H_u)^q]^s &= \text{tr} [(H_u)^q (M_d M_u^\dagger)^p]^s \\ \text{tr} [(M_u M_d^\dagger)^p (H_d)^q]^s &= \text{tr} [(H_d)^q (M_d M_u^\dagger)^p]^s \end{aligned} \quad (4)$$

$$\begin{aligned} \det [(M_u M_d^\dagger)^p] &= \det [(M_d M_u^\dagger)^p] \\ \det [(M_u M_d^\dagger)^p, H_u^q] &= (-1)^n \det [(M_d M_u^\dagger)^p, H_u^q] \end{aligned} \quad \begin{array}{l} (4) \\ \text{cont.} \end{array}$$

with $H_u = M_u M_u^\dagger$, $H_d = M_d M_d^\dagger$, $H'_u = M_u^\dagger M_u$, $H'_d = M_d^\dagger M_d$ and where r is odd, p, q, s are arbitrary integers and n stands for the number of generations. The first of Eqs. (4) represents necessary conditions for CP invariance in the standard model¹¹⁾, with an arbitrary number of generations. For three generations and in the context of the standard model, it has been pointed out¹²⁾ that the condition $\text{tr}[H_u, H_d]^3 = 0$, or equivalently $\det[H_u, H_d] = 0$, is a necessary and sufficient condition for CP invariance. The other Eqs. (4) are the new CP necessary conditions in the LR model. They are valid for an arbitrary number of generations and the violation of any one of them is a sure indication of CP breaking. It is obvious that, for a given number of fermion generations, only a subset of these conditions are independent. For the case of two and three generations, we will derive a subset of invariant conditions which are not only necessary but also sufficient for CP invariance. Before addressing this question, it is useful to understand the significance of Eqs. (3) by analyzing their implications in a conveniently chosen weak basis.

CP RESTRICTIONS IN A SPECIAL WEAK BASIS

In the weak basis where M_u is real, positive and diagonal ($M_u = D_u$), one obtains, from Eqs. (3):

$$\begin{aligned} V_L^\dagger D_u^2 V_L &= D_u^2 \\ V_R^\dagger D_u^2 V_R &= D_u^2 \end{aligned} \quad (5)$$

We will first consider that the quark masses are all non-zero and non-degenerate, and will later consider the case of degenerate or vanishing masses. For non-degenerate, non-vanishing quark masses, Eqs. (5) imply that $V_{L,R}$ are diagonal matrices:

$$V_L = \text{diag.} (\exp i\delta_{Lj}) \quad ; \quad V_R = \text{diag.} (\exp i\delta_{Rj}) \quad (6)$$

From Eqs. (3) and (6), it follows that

$$\begin{aligned} \delta_{Lj} &= \delta_{Rj} \equiv \delta_j \\ \text{phase } (M_d)_{ij} &= \frac{1}{2} (\delta_i - \delta_j) \quad (\text{mod. } \pi) \end{aligned} \quad (7)$$

We conclude from Eq. (7) that in the $SU(2)_L \times SU(2)_R \times U(1)$ model, gauge interactions conserve CP if only if the down quark mass matrix has cyclic phases in the weak basis where M_u is diagonal and real. Since M_d is in general an arbitrary complex matrix, characterized by n^2 independent phases, the requirement of having cyclic phases represents $n^2 - (n-1)$ independent restrictions. This number of CP restrictions coincides with the number of independent KM phases which characterize the quark mixing matrices in $SU(2)_L \times SU(2)_R \times U(1)$ gauge theories.

At this point, it is worthwhile comparing the CP restrictions we have just derived, with those obtained in the standard model. The essential difference between the LR model and the standard model, is that in the LR model CP symmetry constrains an arbitrary matrix to have cyclic phases while in the standard model CP invariance constrains an arbitrary Hermitian matrix (e.g., $M_d M_d^\dagger$ in the weak basis where $M_u M_u^\dagger$ is diagonal) to have cyclic phases. This enables one to understand in a simple way why for two generations it is possible in the LR model to have CP violation by gauge interactions, while in the standard model it is not possible. The point is, of course, that a two-dimensional Hermitian matrix automatically has cyclic phases while the same does not hold true for a two-dimensional arbitrary matrix.

Next we will find a subset of invariant conditions which are both necessary and sufficient for CP invariance in the case of two and three generations.

SUFFICIENT CP CONDITIONS

a) Two generations

We know from our previous discussion that for two generations, CP invariance implies three independent restrictions on the quark mass matrices. Next we will show that for two generations the following three equations are sufficient to imply CP invariance:

$$\begin{aligned}
 \text{tr} (M_d M_u^\dagger) &= \text{tr} (M_u M_d^\dagger) \\
 \text{tr} [(M_u M_d^\dagger)(M_u M_u^\dagger)] &= \text{tr} [(M_u M_u^\dagger)(M_d M_u^\dagger)] \\
 \det (M_d M_u^\dagger) &= \det (M_u M_d^\dagger)
 \end{aligned} \tag{8}$$

Since the above equations are weak-basis independent, one can analyze their implications in a conveniently chosen weak basis. In the weak basis where M_u is diagonal and real, the Eqs. (8) give:

$$\begin{aligned}
 m_1 \text{Im} M_{11} + m_2 \text{Im} M_{22} &= 0 \\
 m_1^3 \text{Im} M_{11} + m_2^3 \text{Im} M_{22} &= 0 \\
 m_1 m_2 [\text{Im} (M_{11} M_{22}) + \text{Im} (M_{12} M_{21})] &= 0
 \end{aligned} \tag{9}$$

where m_i denote up quark masses and $M_{ij} \equiv (M_d)_{ij}$. Since for the moment we are considering non-zero and non-degenerate quark masses, it follows from Eqs. (9) that:

$$\begin{aligned}
 \text{Im} M_{11} &= \text{Im} M_{22} = 0 \\
 \text{phase } M_{12} &= \text{phase } M_{21}^* \pmod{\pi}
 \end{aligned} \tag{10}$$

Note that Eqs. (10) imply that M_d has cyclic phases, as defined by Eq. (7), and therefore it follows that for two generations, Eqs. (9) are necessary and sufficient for CP invariance.

b) Three generations

In this case, the necessary and sufficient conditions for CP invariance are:

$$\begin{aligned} \text{tr}(M_d M_u^\dagger) &= \text{tr}(M_u M_d^\dagger); \quad \text{tr}[(M_u M_d^\dagger) H_u] = \text{tr}[H_u (M_d M_u^\dagger)] \\ \text{tr}[(M_u M_d^\dagger) H_u^2] &= \text{tr}[H_u^2 (M_d M_u^\dagger)] \end{aligned} \quad (11)$$

$$\begin{aligned} \text{tr}(M_d M_u^\dagger)^2 &= \text{tr}(M_u M_d^\dagger)^2; \quad \text{tr}[(M_d M_u^\dagger)^2 H_u] = \text{tr}[H_u (M_u M_d^\dagger)^2] \\ \text{tr}[(M_d M_u^\dagger)^2 H_u^2] &= \text{tr}[H_u^2 (M_u M_d^\dagger)^2] \end{aligned} \quad (12)$$

$$\begin{aligned} \Delta \det(M_u M_d^\dagger) + \det[(M_u M_d^\dagger), H_u] &= \Delta \det(M_d M_u^\dagger) - \\ &- \det[(M_d M_u^\dagger), H_u] \end{aligned} \quad (13)$$

where Δ can be expressed in terms of the coefficients of the characteristic equation of H_u . If one considers the characteristic equation $\det(H_u - \lambda I) = 0$ and makes the standard substitution $\lambda = \lambda' + 1/3 \text{tr } H_u$, the characteristic equation assumes the form $\lambda'^3 + p\lambda' + q = 0$. Then, the coefficient Δ is given by:

$$\Delta = [4p^3 - 27q^2]^{1/2} \quad (14)$$

Alternatively, Δ can be expressed as: $\Delta = (m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_3^2 - m_1^2)$. One can easily verify that Eqs. (11), (12) and (13) indeed constitute a set of necessary and sufficient conditions for CP invariance, for three generations. We will consider again the weak basis where M_u is diagonal. In this basis, Eqs. (11) imply:

$$\text{Im } M_{11} = \text{Im } M_{22} = \text{Im } M_{33} = 0 \quad (15)$$

while Eqs. (12), together with Eq. (15) lead to:

$$\text{Im } M_{12} M_{21} = \text{Im } M_{13} M_{31} = \text{Im } M_{23} M_{32} = 0 \quad (16)$$

Finally, Eq. (13) combined with Eqs. (15) and (16) implies:

$$\text{Im } (M_{21} M_{13} M_{32}) = 0 \quad (17)$$

It is straightforward to see that Eqs. (15)-(17) do indeed imply that M_d has cyclic phases in the basis where M_u is diagonal. We have seen that this is necessary and sufficient for CP invariance.

At this point, it is worthwhile reminding ourselves of the range of applicability of the various CP conditions we have derived in the context of the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory. On the one hand, the necessary and sufficient conditions of Eqs. (3), as well as the invariant necessary conditions of Eqs. (4) apply to an arbitrary number of generations, without any a priori assumption about the spectrum of quark masses. On the other hand, Eqs. (8) and Eqs. (11)-(13) are a set of sufficient conditions for CP invariance only if the quark masses are non-vanishing and non-degenerate.

Next we will consider the case of degenerate or vanishing masses which, as we have previously mentioned, is relevant for the leptonic sector, as well as for the study of CP violation at high energies.

VANISHING MASSES

For definiteness, let us consider that there are l massless up quarks and that the down quark masses are all non-vanishing and non-degenerate. We will work in a weak basis where the up-quark mass matrix is diagonal:

$$M_u = D_u = \begin{pmatrix} 0 & 0 \\ 0 & D'_u \end{pmatrix} \quad (18)$$

where D'_u is a diagonal $(n-l)$ dimensional matrix. Then, it can be easily seen that the unitary matrices V_L and V_R which satisfy Eqs. (5), in this special weak basis, will have the form:

$$V_L = \left(\begin{array}{c|c} X_L & O \\ \hline O & Y_L \end{array} \right) ; V_R = \left(\begin{array}{c|c} X_R & O \\ \hline O & Y_R \end{array} \right) \quad (19)$$

with Y_L, Y_R diagonal unitary matrices, but with X_L, X_R still arbitrary unitary matrices, before imposing that V_L, V_R satisfy Eqs. (3). Furthermore, one can take advantage of having l massless up quarks to choose a weak basis where M_u is kept diagonal and M_d is:

$$M_d = \left(\begin{array}{c|c} D_d & J \\ \hline L & K \end{array} \right) \quad (20)$$

where D_d is a diagonal l dimensional real matrix, while J, L, K are arbitrary complex matrices. If we now substitute Eqs. (18)-(20) in Eq. (3), we obtain:

$$Y_L^\dagger D'_u Y_R = D'_u ; X_L^\dagger D_d X_R = D_d \quad (21a)$$

$$Y_L^\dagger K Y_R = K^* ; X_L^\dagger J Y_L = J^* ; Y_L^\dagger L X_R = L^* \quad (21b)$$

From Eqs. (21a) it follows that

$$\begin{aligned} X_L = X_R &= \text{diag.} [\exp(i\alpha_p)] \quad (p = 1, \dots, l) \\ Y_L = Y_R &= \text{diag.} [\exp(i\beta_q)] \quad (q = 1, \dots, (n-l)) \end{aligned} \quad (22)$$

which, in view of Eqs. (21b), in turn implies:

$$\arg K_{rs} = \frac{1}{2}i(\beta_r - \beta_s) ; \arg J_{mn} = \frac{1}{2}i(\alpha_m - \beta_n) ; \arg L_{fg} = \frac{1}{2}i(\beta_f - \alpha_g) \quad (23)$$

(mod. π)

It is straightforward to convince oneself that the number N_l of CP restrictions

implied by Eqs. (23) is given by:

$$N_\ell = n^2 - (n-1) - \ell^2 \quad (\ell \leq n-1) \quad (24)$$

This number of constraints coincides with the number of KM phases that appear in the charged LH and RH currents. Indeed, it can be shown that in the LR model with ℓ massless quarks of the same charge, if one chooses to eliminate a maximum number of unphysical phases in the LH sector, one obtains:

$$\phi(K_L) = \frac{1}{2}(n-1)(n-2) - \frac{1}{2}\ell(\ell-1) ; \quad \phi(K_R) = \frac{1}{2}n(n+1) - \frac{1}{2}\ell(\ell+1) \quad (25)$$

where $\phi(K_L)$, $\phi(K_R)$ designate the number of physical phases in the LH and RH charged currents. The result of Eqs. (25) is a generalization of the analysis done in Ref. 8) and can be easily obtained by using an appropriate parametrization¹³⁾ of the unitary matrices. Note that $\phi(K_L) + \phi(K_R) = N_\ell$ and furthermore $\phi(K_L)$ coincides with the number of CP violating phases in the standard model with n generations and ℓ massless quarks of the same charge. As previously mentioned, there will be CP violation for three fermion generations even if there are two massless quarks of same charge. This is specially relevant for the leptonic sector in the limit $m_{\nu_e} = m_{\nu_\mu} = 0$, but $m_{\nu_\tau} \neq 0$.

DEGENERATE MASSES

Let us now consider that there are ℓ degenerate up quarks and choose the weak basis where M_u is diagonal:

$$M_u = D_u = \left(\begin{array}{c|c} m \mathbb{1}_\ell & 0 \\ \hline 0 & D'_u \end{array} \right) \quad (26)$$

where $\mathbb{1}_\ell$ denote the ℓ -dimensional unit matrix, m stands for the degenerate up quark masses and D'_u is a non-degenerate $(n-\ell)$ dimensional diagonal matrix. In the weak basis where M_u is diagonal, Eq. (3a) constrains V_L , V_R to be of the form of Eq. (19) with:

$$Y_L = Y_R = \text{diag.} [\exp(i\beta_q)] ; \quad X_L = X_R \equiv X \quad (27)$$

Note that due to the degeneracy of M_u , at this point the matrix X is an arbitrary unitary matrix. Let us write M_d in block form:

$$M_d = \left(\begin{array}{c|c} F & J \\ \hline L & K \end{array} \right) \quad (28)$$

and separate F in its Hermitian and anti-Hermitian parts $F = F_h + F_a$, with $F_h^\dagger = F_h$, $F_a^\dagger = -F_a$. Note that due to the degeneracy of M_u , it is possible to make a weak basis transformation which keeps M_u diagonal and diagonalizes F_h . Therefore, we write without loss of generality $F = D'_d + F_a$, where D'_d is a real diagonal matrix. We can then use Eqs. (3b), (27) and (28) to obtain

$$X^\dagger [D'_d + F_a] X = D'_d + F_a^* \quad (29)$$

Adding Eq. (29) to its Hermitian conjugate, one obtains $X^\dagger D'_d X = D'_d$, which constrains X to be of the form $X = \text{diag}[\exp(i\alpha_p)]$. Taking into account the diagonal form of X , one then obtains from Eq. (3b):

$$\arg F_{ij} = \frac{1}{2} i (\alpha_i - \alpha_j) \quad (30)$$

while the phases of K , J , L are again restricted to be as in Eq. (23). In order to evaluate the number of CP restrictions, one should take into account that in the weak basis we considered, where the Hermitian part of F is diagonal, M_d contains only $n^2 - \ell(\ell-1)/2$ phases. It follows then that the number of CP restrictions in the present case is:

$$N'_\ell = n^2 - (n-1) - \frac{\ell(\ell-1)}{2} \quad (31)$$

Again the number of CP restrictions coincides with the number of KM phases that appear in the charged weak currents:

$$\begin{aligned}\phi(k_L) &= \frac{1}{2}(n-1)(n-2) - \frac{1}{2}\ell(\ell-1) \\ \phi(k_R) &= \frac{1}{2}n(n+1)\end{aligned}\tag{32}$$

where we have again chosen to eliminate a maximum number of unphysical phases in the LH sector.

We have just derived the CP restrictions on fermion mass matrices in the case of vanishing or degenerate fermion masses. One is then naturally led to the question of how to translate these restrictions in terms of invariant sufficient CP conditions. The problem consists again in selecting from the invariant necessary CP conditions of Eq. (4), a subset which would constitute sufficient CP conditions, in the case of degenerate or vanishing masses. The task becomes more involved due to the fact that in this case some of the conditions of Eq. (4) are trivially satisfied and therefore do not provide any useful information. For illustration, we will construct the necessary and sufficient conditions for a simple case. Let us consider two generations and one massless quark (e.g., $m_u = 0$). The number of independent CP restrictions is given by Eq. (24), with $n = 2$, $\ell = 1$. It turns out that in this case the following two conditions are necessary and sufficient for CP invariance:

$$\text{tr}(M_d M_u^\dagger) = \text{tr}(M_u M_d^\dagger); \quad \text{tr}\left[(M_u M_d^\dagger)(M_d M_d^\dagger)\right] = \text{tr}\left[(M_d M_d^\dagger)(M_d M_u^\dagger)\right]\tag{33}$$

Note that in the weak basis where M_u is real diagonal, one still has the freedom to put $(M_d)_{11}$ real. The Eqs. (33) lead to $\text{Im}(M_d)_{22} = \text{Im}[(M_d)_{12}(M_d)_{21}] = 0$, thus restricting M_d to have cyclic phases in the weak basis where M_u is real. We have seen that this is sufficient to guarantee CP invariance.

In conclusion, we have derived weak-basis independent necessary conditions for CP invariance in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge model, for an arbitrary number of generations. These conditions have the advantage of being directly applicable to the fermion mass matrices in an arbitrary weak basis, without requiring their diagonalization. In the case of two and three generations, we have selected a set of necessary conditions which are also sufficient for CP invariance. The case of degenerate or vanishing fermion masses was also considered, with emphasis on some novel features which arise in the LR model.

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