CONSTITUENT GLUONS IN THE PRESENCE OF HEAVY QUARKS

A.T.M. Aerts^{*)}, T.H. Hansson^{**)} and J. Wroldsen

CERN -- Geneva

and

Richard J. Hughes
Los Alamos National Laboratory,
Theoretical Division, T-8, Los Alamos, NM 87545, U.S.A.

A B S T R A C T

The formalism necessary to describe bag model constituent gluons in the presence of heavy sources is presented. Numerical values for energies and magnetic moments of low-lying modes are given, as well as some applications relevant to the phenomenology of hybrid $\bar{Q}qG$ states.

^{*)} Permanent address : Department of Mathematics and Computing Science, University of Technology, Eindhoven, The Netherlands

^{**)} Permanent address : Department of Physics, State University of New York, Stony Brook, NY 11794, U.S.A.

1 INTRODUCTION

Because of confinement and chiral symmetry breaking, the QCD vacuum is very complicated and there is a priori no connection between the elementary field degrees of freedom in the Lagrangian - quarks and gluons - and the low-lying physical excitations - the hadrons. Nevertheless, the great success of the simple quark model^[1] suggests that at least the lowest states in the hadron spectrum can indeed be thought of as built from "constituent" quarks with the same quantum numbers as the field or "current" quarks but with a mass $m_{\text{const}} \approx 300$ MeV. This picture was later put on a somewhat firmer theoretical basis with the introduction of the QCD based MIT bag^[2]. In this model the constituent "mass" is identified as a kinetic energy due to the quarks being confined to a volume of size $R \sim B^{-1/4} \approx 1$ fm, where the bag constant B is the energy density of the perturbative vacuum. The hadrons are believed to be small enough to invoke asymptotic freedom and use lowest-order perturbation theory to compute multiplet splittings.

So far, it has been possible to explain most of the low-lying hadron spectrum using the quark model^[3,4]. However, from the study of the hadron excitation spectrum there are strong suggestions, based on the success of the string-type descriptions^[5,6], that gluonic contributions to the masses is a necessary ingredient. In addition, there are several theoretical reasons to expect extra states related to the gluon degrees of freedom. The strongest indication is based on lattice Monte Carlo calculations, which show that QCD without quarks is a confining theory with a mass gap, or scalar glueball mass, of ≈ 1 GeV^[7,8].

Accepting the likely presence of gluon excitations, can they also be understood in a simple picture, similar to the naive quark model? Although there are no very strong reasons for this to be true (after all, gluons are bosons and the QCD vacuum contains a rather thick "gluon soup"), attempts have been made to describe gluonic hadrons as built from "constituent gluons"^[9-14]. However, since such a constituent gluon cannot be thought of simply as a massive coloured vector particle - that would violate gauge invariance - a description in the style of the non-relativistic quark model is not very appealing^[15]. In the bag model, on the other hand, constituent gluons enter on the same footing as constituent quarks, i.e., as cavity modes satisfying confining boundary conditions^[16]. To the extent that lowest-order cavity perturbation theory is good, states can be classified according to the number of constituent gluons.

For a spherical bag, constituent gluons come in two families, TE (transverse electric) and TM (transverse magnetic) modes with the following quantum numbers:

$$\text{TE} : J^{PC} = 1^{+-}, 2^{--}, 3^{+-}, \dots$$

$$\text{TM} : J^{PC} = 1^{--}, 2^{+-}, 3^{--}, \dots$$

The lowest TE mode has an energy $\omega_{\text{TE}}^1 = 2.74/R \approx 0.7 \text{ GeV}$ and the lowest TM mode $\omega_{\text{TM}}^1 = 4.49/R \approx 1.1 \text{ GeV}$, where the GeV values are obtained using a typical bag radius of 4 GeV^{-1} .

The masses of states containing constituent gluons and/or light quarks can be obtained in the static cavity approximation using standard bag model methods [9,11]. For states containing heavy quarks, a Born-Oppenheimer approximation is more suitable [17-19]. This consists in first solving the light degrees of freedom problem (quarks and/or gluons and the bag surface) in the presence of heavy sources at fixed positions. Concerning the light constituent wave functions, this can be done in two different ways. The simplest approach is to take the usual MIT cavity solutions, which for gluons means the TE and TM modes discussed above. However, since in the Born-Oppenheimer approach the heavy particles, from the point of view of the light constituents, act like classical sources, it might be more reasonable to use background field modes. For example, in the case of a Qqq system, this implies the use of confined spinor solutions in a Coulomb field rather than the usual MIT modes to describe the light quark [20-22]. In the language of cavity perturbation theory this amounts to a summation to all orders in the heavy-light interaction.

Although the latter method using C(oulomb)-modes would seem to be an improvement over the F(ree) mode approach, this is by no means certain. One must realise that to the extent that higher order effects in the q-Q interaction are important, also higher order q-q interactions might be sizeable. We can at present not say much about this question, but take the attitude that both possibilities should be explored.

The aim of this paper is to extend the C-mode formalism in a consistent fashion to include gluons, i.e., to find the constituent gluon modes in the presence of heavy sources. Because of gauge-invariance this problem cannot be solved by a trivial change of the radial wave equation, as in the case for the quark C-modes [20-22]. Still, by choosing an appropriate (background field [23,24]) gauge, we can identify the physical degrees of freedom and obtain explicit solutions in terms of Whittaker's functions.

In the next section we present the necessary formalism and give numerical values for the constituent gluon energies and magnetic moments. Section 3 contains some simple phenomenological applications. The matrix elements necessary for the calculation of the colour magnetic interactions of light and heavy quarks and constituent gluons are given, together with a discussion of the low-lying part of the $\bar{Q}qG$ mass spectrum. Some technicalities have been collected in three appendices.

2 FORMALISM AND NUMERICAL RESULTS

We will now study the colour field of the heavy constituents which we will treat as localized at the center of the bag. The Yang-Mills field equations read :

$$D_\mu G^{\mu\nu} = J^\nu \quad (2.1)$$

where $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$, $D_\mu = \partial_\mu - ig[A_\mu, \cdot]$ and J_ν is the heavy source distribution. We use matrix notation for colour, so $G_{\mu\nu} = G_{\mu\nu}^a T^a$ and $[A_\mu, A_\nu]^a = if^{abc} A_\mu^b A_\nu^c$, etc. Now split the potentials and fields in a quantum field $G_{\mu\nu}$ and a classical background field $\tilde{G}_{\mu\nu}$, in such a way that :

$$\tilde{D}_\mu \tilde{G}^{\mu\nu} = J^\nu \quad (2.2)$$

where $\tilde{D}_\mu$ is the covariant derivative w.r.t. the background potential $\tilde{A}_\mu$. Using the background field gauge :

$$\tilde{D}_\mu A^\mu = 0 \quad (2.3)$$

we get the following linearized equations of motion [25,24]:

$$\tilde{D}^2 A_\mu - 2ig[\tilde{G}_{\mu\nu}, A^\nu] = 0 \quad (2.4)$$

For a derivation of Eq.(2.4) together with a consistent set of confining boundary conditions, see Appendix A. If the total system, consisting of both the gluon and the static source, is in some fixed colour representation r , the colour structure simplifies and one gets an equivalent Abelian problem (see Appendix B for a derivation of this result):

$$\{\hat{D}^2 g_{\mu\nu} - 2gc_r \hat{G}_{\mu\nu}\} A^\nu = 0 \quad (2.5)$$

Here $c_r = 1/2 [C^r - 3C^{\text{source}}]$, where C^r is the value of the quadratic Casimir operator in the representation r , so, e.g., $c_r = -3$ for an overall colour singlet and a colour octet source. $\hat{D}$ in Eq.(2.5) is an Abelian covariant derivative :

$$\hat{D}_\mu = \partial_\mu - igc_r \hat{A}_\mu \quad (2.6)$$

and A_μ is now the Abelian field representing the space part of the constituent gluon wave function. In our case the (Abelian) background potential $\hat{A}_\mu$ is calculated from the source distribution corresponding to an infinitely heavy point particle :

$$\hat{J}^\mu = -g^{\mu 0} g \delta^3(x) \quad (2.7)$$

giving in Coulomb gauge:

$$\hat{A}^\mu = -g^{\mu 0} (g/4\pi r) \quad (2.8)$$

and the corresponding field strengths are $\hat{G}^{i0} = -(g/4\pi) \partial^i (1/r)$ and $\hat{G}^{ij} = 0$. Substituting (2.8) in (2.5) and (2.3) gives :

$$\hat{D}^2 \vec{A} + 2ia(\vec{\nabla} \cdot 1/r) A^0 = 0 \quad (2.9.a)$$

$$\hat{D}^2 A^0 + 2ia(\vec{\nabla} \cdot 1/r) \cdot \vec{A} = 0 \quad (2.9.b)$$

$$\vec{\nabla} \cdot \vec{A} - i(\omega + a/r) A^0 = 0 \quad (2.9.c)$$

where $a = -c_r \alpha_s = -c_r g^2/4\pi$ and $A^\mu(x) = A^\mu(\vec{r}) e^{-i\omega t}$ has been assumed. In addition to this system of equations, there is a self-consistency relation

$$A^0(0) = 0 \quad (2.10)$$

which follows from the requirement of covariant current conservation $D_\mu J^\mu = 0$.

As pointed out by Mandula^[25], in a slightly different context, Eqs.(2.9) are not independent. In fact (2.9.b) can be derived by taking the divergence of (2.9.a) and using (2.9.c).

The solutions of Eqs.(2.9) under subsidiary condition Eq.(2.10) will give the constituent gluon modes. In order to have a properly defined normalization of these modes, we use the following scalar product^[24]:

$$\langle A_\mu \cdot A'^\mu \rangle = i \int d^3x A_\mu^* (\overleftarrow{D}^0 - \overrightarrow{D}^0) A'^\mu = 2\omega \int d^3x (\vec{A}^* \cdot \vec{A}' - B^* B') \quad (2.11)$$

in the notation

$$A^\mu = (iB, \vec{A}) . \quad (2.12)$$

Eq.(2.11) implies for stationary, physical modes the normalization

$$\int d^3x (|\vec{A}|^2 - |B|^2) = 1/2\omega . \quad (2.13)$$

In the $a=0$ case the gauge condition Eq.(2.9.c) can be realized in the radiation gauge $A^0=0, \vec{\nabla} \cdot \vec{A}=0$. One can construct the vector potentials, $\vec{A}$, for the TE and TM modes by letting the operators $\vec{L}$ and $\vec{\nabla} \times \vec{L}$ act on the solutions $\phi_{\ell m}$ of the scalar equation:

$$\hat{D}^2 \phi_{\ell m} = 0 \quad (2.14)$$

since in this case $[\vec{L}, \hat{D}^2]=0$ and $[\vec{\nabla} \times \vec{L}, \hat{D}^2]=0$. A third, independent vector potential can be constructed by letting the operator $\vec{\nabla}$ act on $\phi_{\ell m}$. This L(ongitudinal) vector potential does not satisfy the gauge condition Eq.(2.9.c) for $a=0$.

Since $[\vec{L}, \vec{\nabla}^2 + f(r)]=0$ for any $f(r)$ (we have $f(r)=(\omega+a/r)^2$), the TE solution can, also for $a \neq 0$, be written as :

$$A_{\ell TE}^\mu = N_\ell^{TE}(0, \vec{L} \phi_{\ell m}) \quad (2.15)$$

In the interacting case both $[\vec{\nabla} \times \vec{L}, f(r)] \neq 0$ and $[\vec{\nabla}, f(r)] \neq 0$ and we no longer can have $A^0=0$ for the TM and L modes. (Although not appropriate for $a \neq 0$ we shall for simplicity continue to use the terms "TM" and "L" for the physical and the unphysical mode respectively.) Eqs.(2.9) become a system of coupled second-order differential equations which can, however, be diagonalized and solved using the methods of Corben and Schwinger^[26].

The solution for the TM mode can be expressed as follows^[27]:

$$\begin{aligned} A^0 &= -iN_\ell^{TM} \left[a \left(\frac{\partial}{\partial r} + \frac{1}{r} \right) + \ell(\ell+1) \left(\omega + \frac{a}{r} \right) \right] \phi_{\ell m} \\ \vec{A} &= N_\ell^{TM} \left\{ i \vec{\nabla} \times \vec{L} + a(\omega + a/r) \hat{r} - a\omega \frac{i \vec{r} \times \vec{L}}{\ell(\ell+1)} - \vec{\nabla} L^2 \right\} \phi_{\ell m} \end{aligned} \quad (2.16)$$

The L(ongitudinal) mode is a pure gauge mode of the form:

$$A_g^\mu = i\hat{D}^\mu \phi_{\ell m} \quad (2.17)$$

where $\phi_{\ell m}$ satisfies Eq.(2.14). Consequently, A^μ satisfies Eq.(2.9.c). Explicitly we have:

$$\begin{aligned} A^0 &= (\omega + a/r)\phi_{\ell m} \\ \vec{A} &= -i\vec{\nabla}\phi_{\ell m} \end{aligned} \quad (2.18)$$

This gauge mode is unphysical in the sense that it has zero norm w.r.t. the scalar product defined in Eq.(2.11).

The function $\phi_{\ell m}$ is found to be :

$$\phi_{\ell m} = F_0^\ell(r) Y_\ell^m(r) \quad (2.19)$$

in terms of the regular solution to Whittaker's equation^[28]:

$$F_0(r) = (1/r)M_{-ia,s}(2i\omega r) \quad (2.20)$$

Here $s = [(\ell + 1/2)^2 - a^2]^{1/2}$. In order to facilitate the comparison with the $a=0$ case^[29,30], we have given the expressions for the gluon wavefunctions in terms of vector harmonics in Appendix B.

Next we must impose the confining bag boundary conditions:

$$\hat{r} \cdot \vec{E} = 0 \quad (2.21.a)$$

$$\hat{r} \times \vec{B} = 0 \quad (2.21.b)$$

For the TE modes this implies

$$d/dz\{ zF_0(z) \} = 0 \text{ at } z=2i\omega R \quad (2.22)$$

which is easily solved numerically. In the limit $a \rightarrow 0$, we recover the usual result: $d/dr[rj_\ell(\omega r)] = 0$. Given ω_{TE}^j , N_{TE}^j is obtained from Eq.(2.13). For phenomenological purposes, the lowest TE-mode (with $\ell=1$) is the most interesting one, since it has the lowest energy-eigenvalue. In Fig.1 we show the lowest ω_{TE} as a function of a in the interval $0 \leq a \leq 3/2$.

For the TM mode Eqs.(2.21.a) and (2.21.b) imply:

$$a(d/dr)F_0 + (\omega\ell(\ell+1)+a(1+\ell(\ell+1))/r)F_0=0 \quad (2.23)$$

which has the usual limit: $j_\ell(\omega R)=0$, for $a \rightarrow 0$. In Fig.1 we display the results for ω_{TM} as found from Eq.(2.23) for the lowest $\ell=1$ mode.

The magnetic field for the L mode vanishes identically. The energy eigenvalues, therefore, are determined by Eq.(2.21.a), which can be written as:

$$F_0(2i\omega R)=0 \quad (2.24)$$

In the limit $a \rightarrow 0$, this expression becomes: $j_\ell(\omega R)=0$, just as for the TM mode. This result differs from previous work, see, e.g., Refs.29 and 30, where:

$$n^\mu A_\mu^a = 0 \quad (2.25)$$

was imposed with $n^\mu=(0,\hat{r})$ for a static cavity. This yields $d/dr[j_\ell(\omega r)]=0$. In the presence of a background field Eq.(2.25) is not consistent with Eqs.(2.21). On the other hand, in Refs.29 and 30 one could equally well have imposed Eq.(2.24) and still have obtained a consistent description. This ambiguity, which for $a=0$ arises in the presence of a bag, is discussed in detail in Appendix A. The results for ω_L are also shown in Fig.1. It shows that, for finite values of a , ω_L and ω_{TM} are no longer degenerate.

So far we did not worry about the consistency condition Eq.(2.10). For the TE modes it is satisfied trivially, since $A_{TE}^0=0$. For the TM and L modes we have that

$$A^0 \sim F_0(r)/r \sim r^{s-3/2} \quad \text{for } r \rightarrow 0 \quad (2.26)$$

We see that $s \geq 3/2$ is needed, which implies that the first allowed mode has $\ell=1$ for $a=0$ only, or $\ell=2$ for $|a| \leq 2$. Note that Eq.(2.10) is a direct consequence of our usage of a point charge density for the heavy quark density. Less singular distributions will put less severe restrictions on the value of scalar potential A_0 at the origin and then all non-zero values of ℓ will be allowed for a finite range of a . In Section three we give an example of such a "smearing" of the charge distribution. Since our application will be restricted to the lowest TE gluon mode, however, we will not attempt any explicit construction of the $\ell=1$ TM and L modes.

In order to give a qualitative idea about the behavior of the strength of the constituent gluon interactions as a function of a we also compute the colour magnetic moment of the lowest TE mode. This quantity is determined from the gluon current^[31]:

$$J_G^{a\mu} = ig[G^{\sigma\mu}, A_\sigma]^a \quad (2.27)$$

by the equation:

$$\vec{\mu}^a = \mu(R) \vec{S} T^a = 1/2 \int d^3x (\vec{r} \times \vec{j})^a \quad (2.28)$$

where A_μ is the full non-Abelian field, and $\vec{S}$ is the spin. The result is

$$\mu(R) = (g/2\omega) \quad (2.29)$$

In Fig.2 we display $1/\omega R$ as a function of a . For $a \rightarrow 3/2$, $\mu(R, a)$ rises sharply, but stays finite, as $\omega R \rightarrow 0.129$. Outside this region, i.e., for $a < 3/2$, μ is a rather flat function of a , and particularly for small a 's one does not expect big changes in the matrix elements involving constituent gluons.

3 APPLICATIONS

In this section we will calculate the hyperfine splitting of $\bar{Q}qG$ hybrids due to one gluon exchange. The light quark q is taken to be in its lowest state $S_{1/2}$ and the gluon in its lowest TE mode. We consider the vertices, shown in Fig. 3., that allow us to calculate the hyperfine splittings of $\bar{Q}qG$ states. We will calculate to lowest order in cavity perturbation theory, employing the method of Barnes et al.^[9], using Coulomb wavefunctions both for the gluon and the light quark^[21,22] (C-type approach) or free MIT wave functions (F-type approach).

The vertex for three TE gluons (Fig. 3a) can be written, following the notation of Chanowitz and Sharp^[12], as

$$H^{EEE} = (\sqrt{\alpha_s}/R) \vec{S}_{m_c, m_b} \cdot \vec{e}_{m_a}^* (-f^{acb} L^{EEE}) \quad (3.1)$$

L^{EEE} is an integral over the spaceparts of the three TE gluon wave functions. For C-type gluon modes, L^{EEE} will be a function of α_s (or

the parameter a of section 2). The L^{EEE} 's corresponding to a heavy quark-gluon system in a colour 3 representation are given in Table 1 (L^{EEE} is of course independent of the bag radius).

The quark-gluon vertex (Fig.3b) are given by

$$H^{qqE} = -\int d^3x \vec{j}_{qq}^a \cdot \vec{A}_{TE}^{a*} \quad (3.2)$$

For the light quark in the lowest $S_{1/2}$ mode we write this vertex in terms of the constants L^{ssE} [12]:

$$H^{ssE} = (\sqrt{\alpha_s}/R) \vec{S}_{fi} \cdot \vec{e}_{m_a}^* T_{fi}^a (-iL^{EEE}) . \quad (3.3)$$

For $m_q=0$, L^{ssE} is independent of the bag radius. The numbers are given in Table 1, and they agree with Chanowitz and Sharp^[12] in the limit $a=0$.

For the heavy quark it turns out that Eq.(3.2) with the conventional current ($\vec{\mu}=\vec{\sigma}/2m_Q$)

$$\vec{j}_{QQ} = -\vec{\mu} \times \vec{\nabla} \delta^{(3)}(\vec{r}) \quad (3.4)$$

becomes singular for C-mode gluon wave functions with $a \neq 0$ (see Section 2). We therefore introduce a cut-off λ , by smearing out the heavy quark colour charge over a small region of space with $r \leq \lambda \ll R$. The simplest way of smearing is to take

$$\vec{j}_{QQ} = \begin{cases} (3/\pi\lambda^4) \cdot (\vec{\mu} \times \vec{r}/r) & , \quad r \leq \lambda < R \\ 0 & , \quad r > \lambda \end{cases} \quad (3.5)$$

This current satisfies $\vec{\mu} = 1/2 \int d^3x \vec{r} \times \vec{j}_{QQ}$. In our numerical calculations we take λ proportional to the Compton wavelength of the heavy quark,

$$\lambda = q/m_Q , \quad (3.6)$$

using $q=1/2, 1$ and 2 . Parametrizing as in Eq.(3.3) (but now in terms of L^{QQE}), we find the numbers given in Table 2. L^{QQE} is a function of the cut-off parameter q , the bag radius R , the heavy quark mass m_Q and α_s .

From the tables we see that L^{EEE} , L^{ssE} and L^{QQE} are increasing with increasing α_s . Furthermore, L^{QQE} decreases if we increase q , R or m_Q . L^{QQE} is more stable against variations in q for high m_Q than for low m_Q . When $\alpha_s \rightarrow 1.0$, we see that L^{EEE} , L^{ssE} and L^{QQE} all increase quickly giving rise to unrealistically big hyperfine splittings. For $\alpha_s=0$, L^{QQE} depends on m_Q and R in the following simple way,

$$L^{QQE} = 4.073/m_Q R \quad (3.7)$$

This can be found from Eq.(3.4) (or Eq.(3.5) with $\lambda=0$).

With the results in Tables 1 and 2 at hand, it is an easy task to compute the hyperfine splittings of the hybrids, by using the formula from Chanowitz and Sharp^[12];

$$\begin{aligned} \Delta E_{hfs} = & (\alpha_s/Rx_{TE}) \cdot (-1/3) \langle \vec{S}_q \cdot \vec{S}_{\bar{Q}} \rangle \text{Re}(L^{ssE} L^{QQE*}) \\ & + 6 \langle \vec{S}_q \cdot \vec{S}_G \rangle \text{Re}(L^{ssE} L^{EEE*}) \\ & + 6 \langle \vec{S}_{\bar{Q}} \cdot \vec{S}_G \rangle \text{Re}(L^{QQE} L^{EEE*}) \end{aligned} \quad (3.8)$$

where x_{TE} is the mode number of the TE-gluon (Fig.1). The different terms in Eq.(3.8) are illustrated in Fig.4. By using $\alpha_s=0.4$ and $R=4.0\text{GeV}^{-1}$ ^[22], we tentatively calculate the hyperfine splitting in the lowest "Coulomb"-hybrids for $q=1$ and $m_q=0$ (Table 3). These numbers turn out to depend weakly on q . For $q=0.5$ (or 2) the absolute values of the numbers for $m_Q=1.35\text{GeV}$ would go up (or down) $\approx 10\%$, for $m_Q=4.75\text{GeV}$ $\approx 4\%$ and for $m_Q=40\text{GeV}$ $\approx 1\%$.

The hyperfine splitting for $\bar{Q}qG$ hybrids with F-type cavity modes can, by using Table 1 (for $\alpha_s=0$) and Eq.(3.7), be written simply as

$$\begin{aligned} \Delta E_{hfs} = & (\alpha_s/Rx_{TE}) \cdot [(\langle \vec{S}_{\bar{Q}} \cdot \vec{S}_G \rangle 8.31 - \langle \vec{S}_{\bar{Q}} \cdot \vec{S}_q \rangle 1.34)/m_Q R \\ & + \langle \vec{S}_q \cdot \vec{S}_G \rangle 2.01] \quad . \end{aligned} \quad (3.9)$$

In Table 3 we also give these values (in parentheses), for the set of parameters as used for the C-type modes. It is clear from this table that the wave function modifications for the C-type modes strongly enhance the hyperfine splitting in the $\bar{Q}qG$ hybrids. It is seen from Eq.(3.9) and Table 3 that for the F-type splitting to be as big as the C-type splitting (for $\alpha_s=0.40$), one has to increase α_s approximately by a factor of four.

To get an estimate of C-mode hybrid masses, we will now use the formula

$$E = m_Q + (4/3)\pi R^3 B + (x_q + x_G)/R + Z_0/R + \Delta E_{hfs} , \quad (3.10)$$

which, for $\alpha_s=0.40$, $B^{1/4}=0.145\text{GeV}$, $Z_0=0.962^{[22]}$ and $R=4.0\text{GeV}^{-1}$, gives for $m_Q=m_c=1.35\text{GeV}$

$$E(0^{++}) = 2.18\text{GeV} , \quad E(1^{++}) = 2.45\text{GeV} , \quad (3.11)$$

$$E(1^{+-}) = 2.74\text{GeV} , \quad E(2^{++}) = 2.98\text{GeV} .$$

For the gluon kinetic energy (x_G of Eq.(3.10)), we have used the TE gluon mode as found from Fig.1 (remembering that $a=3\alpha_s/2$ in our problem). For the light quark's kinetic energy (x_q of Eq.(3.10)) in our $\bar{Q}qG$ system, we have

$$x_q = 2.04 + 0.28 \alpha_s , \quad \text{for } \alpha_s \leq 0.40 . \quad (3.12)$$

This is found by using the repulsive colour octet $\bar{Q}q$ -potential $V(r)=(1/6) \alpha_s/r$ in a C-mode calculation.

Compare now Eq.(3.11) with the results from a bag model calculation using F-type cavity modes. Use now also the parameters from ref.22, i.e. $\alpha_s=0.80$, $Z_0=0.026$ and B and R unchanged. This gives

$$E(0^{++}) = 2.46\text{GeV} , \quad E(1^{++}) = 2.52\text{GeV} , \quad (3.13)$$

$$E(1^{+-}) = 2.60\text{GeV} , \quad E(2^{++}) = 2.65\text{GeV} .$$

However, it would perhaps be a more appropriate comparison if in the F-mode calculation we also include the Coulomb potential from the heavy quark perturbatively, by calculating the colour electric energy shift between the $\bar{Q}q$ and $\bar{Q}G$ systems from

$$H_{\bar{Q}}^{\text{Coul}} = gT_{\bar{Q}}^a/8\pi \int d^3x \rho^a(r)/r \quad . \quad (3.14)$$

Here $\rho^a(r)$ is the light quark and gluon colour charge density. These are given by

$$\rho_q^a(\vec{r}) = gT_q^a : \psi^\dagger \psi : \quad (3.15)$$

and

$$\rho_G^a(\vec{r}) = -gf^{abc} : \vec{A}_b \cdot \vec{A}_c : \quad . \quad (3.16)$$

respectively. Thus,

$$\begin{aligned} H_{\bar{Q}q}^{\text{Coul}} &= (0.831\alpha_s/R) T_{\bar{Q}}^a T_q^a \quad , \\ H_{\bar{Q}G}^{\text{Coul}} &= (0.689\alpha_s/R) T_{\bar{Q}}^a T_G^a \quad , \end{aligned} \quad (3.17)$$

which for a $\bar{Q}qG$ hybrid with F-type modes finally gives ($\alpha_s=0.4$ and $R=4.0\text{GeV}^{-1}$)

$$H_{\bar{Q}q}^{\text{Coul}} + H_{\bar{Q}G}^{\text{Coul}} = -\alpha_s 0.895/R = -0.09\text{GeV} \quad , \quad (3.18)$$

which should be subtracted from the numbers of Eq.(3.13).

The above colour electric corrections can also be seen as first order corrections to the mode numbers x_q and x_G of the lowest $S_{1/2}$ quark and TE gluon respectively. However, as seen from Eq.(3.12) and Fig. 1, the corrections due to a proper resummation of the Coulomb graph as done in the C-type calculation give a much stronger variation with α_s than this first order calculation.

Using the results of this and the previous section, our analysis can be straightforwardly extended to more complicated states like $QqqG$.

One interesting state which we cannot treat is $\bar{Q}QG^{[11]}$, where the $\bar{Q}Q$ pair forms an octet and is thus not bound by the colour Coulomb force. Consequently it is presumably a bad approximation to consider the heavy colour octet $\bar{Q}Q$ subsystem as a point-like source.

4 SUMMARY AND CONCLUSIONS

In this paper we have developed the necessary formalism for describing constituent gluons in a colour Coulomb field. We have applied our techniques to the phenomenologically interesting case of $\bar{Q}qG$ states, where we conclude that the hyperfine splitting is strongly enhanced in these systems due to wave function modifications.

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Appendix A

The equations of motion and the boundary conditions of Section 2 can be derived from the following Lagrangian :

$$\mathcal{L}_{cl} = \mathcal{L}_0 + \mathcal{L}_{GF} \quad (A.1)$$

where

$$\mathcal{L}_0 = -1/4 G^{\mu\nu a} G_{\mu\nu}^a - J^{\mu a} A_\mu^a \quad (A.2)$$

in terms of the Yang-Mills field strengths $G^{\mu\nu a}$, the vector fields $A^{\mu a}$, and the heavy quark current $J^{\mu a}$. Varying $\mathcal{L}_0$ with respect to $A^{\mu a}$ gives the classical equation of motion Eq.(2.1). In the presence^[24] of a classical background field $\tilde{A}^{\mu a}$, the gauge field $A^{\mu a}$ is replaced by:

$$A_\mu^a \Rightarrow \tilde{A}_\mu^a + A_\mu^a \quad (A.3)$$

where A_μ^a now describes the fluctuations around the classical field. We fix the gauge for the gluon fields $A^{\mu a}$ by means of the gauge-fixing Lagrangian:

$$\mathcal{L}_{GF} = -B^a (\tilde{D}_\mu^{ab} A^{\mu b}) + \alpha/2 B^a B^a \quad (A.4)$$

in terms of $\tilde{D}_\mu$, the covariant derivative with respect to the background potential $\tilde{A}_\mu$, and the Lagrange multiplier field B^a . The background Lorentz gauge used in the text (eq.(2.3)) corresponds to $\alpha=0$.

The classical Lagrangian $\mathcal{L}_{cl}$ is invariant under the background gauge transformations:

$$A_\mu^a \rightarrow A_\mu^a + \tilde{D}_\mu^{ab} \lambda^b \quad (A.5)$$

$$A_\mu^a \rightarrow A_\mu^a - g f^{abc} A_\mu^c \lambda^b \quad (A.6)$$

$$B^a \rightarrow B^a - g f^{abc} B^c \lambda^b \quad (A.7)$$

where λ is a smooth c-number function of space and time.

Varying $\mathcal{L}_{cl}$ with respect to A_μ^a and B^a gives Eqs.(2.2)-(2.4). We furthermore obtain the boundary conditions :

$$n^\mu G_{\mu\nu}^a = 0 \quad (A.8)$$

and

$$B = 0 \quad (A.9)$$

with $G_{\mu\nu}^a$ in terms of A_μ^a , and using Eq.(2.3).

We end this appendix by two comments about the form of the gauge-fixing term eq.(A.4) and the form of the full quantum Lagrangian. Note that had we instead chosen :

$$\mathcal{L}'_{GF} = -(D_\mu^{ab} B^b) A^{\mu a} + \alpha/2 B^a B^a \quad (A.10)$$

which is equivalent to Eq.(A.4) up to a surface term (!), the boundary condition :

$$n^\mu A_\mu^a = 0 \quad (A.11)$$

would have arisen, instead of that of Eq.(A.9). This choice leads to an inconsistent set of boundary conditions for states satisfying the background gauge condition Eq.(2.3). For instance, imposing Eqs.(A.8) on the L-mode : $A^\mu = \tilde{D}^\mu \phi$, gives :

$$\phi = 0 \text{ on the bag surface,} \quad (A.12)$$

while imposing Eq.(A.10) yields :

$$(\partial/\partial r) \phi = 0 \text{ on the bag surface} \quad (A.13)$$

Eqs.(A.12) and (A.13) clearly are incompatible. This situation is quite different from that encountered in the non-interacting case (see Refs 29 and 30), where Eq.(A.8) vanishes identically for the L-mode, and Eq.(2.17) has to be imposed to fix the L-mode spectrum.

The full quantum Lagrangian $\mathcal{L}$ is found by treating $A^{\mu a}$ and B^a as quantum fields and adding the Fadeev-Popov ghost term to $\mathcal{L}_{cl}$ [24]:

$$\mathcal{L}_{FP} = -i(\tilde{D}_\mu \tilde{C})^a (D^\mu C)^a \quad (A.14)$$

where D^μ now is the covariant derivative w.r.t. $\tilde{A}^\mu + A^\mu$. As before [30] the ghost fields satisfy:

$$\hat{r} \cdot \vec{\nabla} C = 0 \quad (A.15)$$

and

$$\hat{n} \cdot \vec{\nabla} \bar{C} = 0 \quad (A.16)$$

on the bag surface.

We further note that the part of $\mathcal{L}$ which is linear in the fluctuations :

$$\mathcal{L}_1 = -\tilde{G}^{\mu\nu a} (\tilde{D}_\mu A_\nu)^a - J^{\mu a} A_\mu^a \quad (A.17)$$

can be written as a total divergence, using the equation of motion Eq.(2.2). The unwanted contributions this term would give on the bag surface are cancelled by adding the appropriate surface term.

The part of $\mathcal{L}$ which is at least quadratic in the quantum fields is given by:

$$\mathcal{L}_Q = \mathcal{L} - \mathcal{L}_0(A^\mu) - \mathcal{L}_1 \quad (A.18)$$

It is easy to show that the addition of surface terms does not change the invariance of $\mathcal{L}_Q$ under the following BRS transformations:

$$\begin{aligned} \delta \tilde{A}_\mu^a &= 0 \\ \delta A_\mu^a &= \lambda (D_\mu C)^a \\ \delta C^a &= -1/2 \lambda g f^{abc} C^b C^c \\ \delta \bar{C}^a &= -i\lambda B^a \\ \delta B^a &= 0 \end{aligned} \quad (A.19)$$

where λ is now an anticommuting parameter, and Eq.(A.15) has been used.

Appendix B

In this appendix we will derive the equivalent Abelian equation of motion Eq.(2.5). Our goal is to separate the valence gluon wave-function into a product of colour and space-time parts.

We start by introducing a basis in which to describe the n_r dimensional colour source. This will be a set of n_r unit vectors, $\{|\alpha\rangle_s, \alpha=1, \dots, n_r\}$. Any source vector, $|\phi\rangle_s$, can be written as a linear combination of the $|\alpha\rangle_s$. The colour current of the source in the state $|\phi\rangle_s$ is then

$$J_0^a = \langle \phi | T_{(s)}^a | \phi \rangle_s \quad (B.1)$$

where the eight (hermitian) matrices $T_{(s)}^a$ generate an n_r -dimensional representation of the Lie algebra of $SU(3)$. This colour current is the source of the classical colour Coulomb field, $\tilde{A}_\mu^a$, of section 2.

Similarly, we introduce another independent set of eight unit vectors, $\{|a\rangle_G, a=1, \dots, 8\}$, to describe the colour orientations of the (valence) gluon bound to the source by the colour Coulomb field, $\tilde{A}_\mu$.

The colour wave-function of the combined system of source and bound gluon can be written as a linear combination of the direct product states, $|\alpha\rangle_s |b\rangle_G$.

In component notation the background Lorentz gauge condition on the valence gluon wave-function may be written,

$$\partial_\mu A^{\mu a} - g f^{abc} \tilde{A}_\mu^b A^{\mu c} = 0 \quad (B.2)$$

where, $\tilde{A}_\mu^b = \hat{A}_\mu T_{(s)}^b$

We now introduce the operator,

$$A^\mu \equiv \sum_{a=1}^8 |a\rangle_G A^{\mu a} \langle a|_G \quad (B.3)$$

and the hermitian operator,

$$T_{(G)}^b \equiv -i \sum_{a,c} |a\rangle_G f^{abc} G^c | \quad (B.4)$$

which acts on the $|a\rangle_G$ basis. The gauge condition, Eq.(B.2), then implies

$$(\partial_\mu A^\mu - ig \hat{A}_\mu T_{(s)}^b T_{(G)}^b A^\mu) |\psi\rangle = 0 \quad (B.5)$$

where $|\psi\rangle$ is a (normalized) combined source-gluon wavefunction having a definite total colour, and A_μ is the (Abelian) background Coulomb field, Eq.(2.8). $|\psi\rangle$ is easily constructed for any representation using the SU(3) Clebsch-Gordan coefficients, but we will not need the explicit form here. Since,

$$T_{(s)}^b T_{(G)}^b = 1/2 [(T_{(s)}^b + T_{(G)}^b)^2 - T_{(s)}^b T_{(s)}^b - T_{(G)}^b T_{(G)}^b] \quad (B.6)$$

and,

$$(T_{(s)}^b + T_{(G)}^b)^2 |\psi\rangle = C^r |\psi\rangle \quad (B.7)$$

$$T_{(s)}^b T_{(s)}^b |\psi\rangle = C^{\text{source}} |\psi\rangle \quad (B.8)$$

$$T_{(G)}^b T_{(G)}^b |\psi\rangle = C^{\text{gluon}} |\psi\rangle \quad (B.9)$$

we may deduce from Eq.(B.5), the effective Abelian equation,

$$\hat{D}_\mu a^\mu = 0 \quad (B.10)$$

where, a^μ is a complex function of space and time (gluon wavefunction), and,

$$\hat{D}_\mu \equiv \partial_\mu - ig c_r \hat{A}_\mu \quad (B.11)$$

is an Abelian covariant derivative, with

$$c_r \equiv 1/2 [C^r - C^{\text{gluon}} - C^{\text{source}}] \quad (B.12)$$

Similarly, the linearized form of the valence gluon wave equation in component notation is

$$\begin{aligned}
 & (\partial_\mu \delta^{ac} - g f^{abc} \tilde{A}_\mu^b) (\partial^\mu \delta^{ce} - g f^{cde} \tilde{A}^{\mu d}) A^\nu \\
 & - 2 g f^{abc} \tilde{G}^{bv\mu} A_\mu^c = 0
 \end{aligned} \tag{B.13}$$

which, after similar reasoning may be re-written as,

$$\begin{aligned}
 & [(\partial_\mu - i g \hat{A}_\mu^b T_{(s)}^b) T_{(G)}^b] (\partial^\mu - i g \hat{A}^\mu T_{(s)}^c T_{(G)}^c) A^\nu \\
 & - 2 g c_r \hat{G}^{\nu\mu} T_{(s)}^b T_{(G)}^b A_\mu] |\psi\rangle = 0
 \end{aligned} \tag{B.14}$$

leading to the effective Abelian equation of motion for the space-time part of the gluon wave-function,

$$[\hat{D}_\mu \hat{D}^\mu g^{\nu\alpha} - 2 g c_r \hat{G}^{\nu\alpha}] a_\alpha = 0 \quad . \tag{B.15}$$

Appendix C

Here we give the wavefunctions of the TE, TM and L modes expressed in terms of spherical and vector harmonics^[27]. We use the notation of Eq.(2.12) with:

$$B(\vec{r}) = N_\ell G(r) Y_m^\ell(\hat{r}) \quad (C.1)$$

and

$$\vec{A}(\vec{r}) = N_\ell \sum_{\alpha} A_{\alpha}(r) \vec{Y}_{\ell\alpha}^m(\hat{r}) \quad (C.2)$$

where $\alpha \in (\ell-1, \ell, \ell+1)$ and N_ℓ is the normalization constant. (For L modes we have $N_\ell \equiv 1$). The vector harmonics are defined as^[27]:

$$\vec{Y}_{\ell J}^m(\hat{r}) = \vec{Y}_{\ell J m}(\hat{r}) = C_{\mu q m}^J \frac{1}{\ell} Y_{\mu}^J(\hat{r}) \vec{e}_q \quad (C.3)$$

in terms of the spherical harmonics $Y_{\ell}^m(r)$ and the spherical unit vectors $\vec{e}_q$ (we have $\vec{e}_q^* \cdot \vec{e}_q = (-)^q \vec{e}_{-q} \cdot \vec{e}_q = \delta_{qq}$). Only the non-zero $A_{\alpha}(r)$ will be listed.

TE mode

$$\begin{aligned} G(r) &= 0 \\ A_{\ell}(r) &= \sqrt{\ell(\ell+1)} F_0(r) \end{aligned} \quad (C.4)$$

TM mode

$$\begin{aligned} G(r) &= -[a(\partial/\partial r + 1/r) + \ell(\ell+1)(\omega + a/r)] F_0(r) \\ A_{\ell+1}(r) &= a_+ \{-\ell^2(\partial/\partial r - \ell/r) + a[a/r - \omega/(\ell+1)]\} F_0(r) \\ A_{\ell-1}(r) &= a_- \{-(\ell+1)^2[\partial/\partial r + (\ell+1)/r] + a(a/r + \omega/\ell)\} F_0(r) \end{aligned} \quad (C.5)$$

L mode

$$G(r) = -i(\omega + a/r) F_0(r)$$

$$A_{\ell+1}(r) = -ia_+(\partial/\partial r - \ell/r) F_0(r) \quad (C.6)$$

$$A_{\ell-1}(r) = -ia_-(\partial/\partial r + (\ell+1)/r) F_0(r)$$

In Eqs. (C.5)-(C.6) we use the coefficients:

$$\begin{aligned} a_+ &= -[(\ell+1)/(2\ell+1)]^{1/2} \\ a_- &= [\ell/(2\ell+1)]^{1/2} \end{aligned} \quad (C.7)$$

The function $F_0(r)$ is as in Eq. (2.20). In the limit $a \rightarrow 0$, we have:

$$F_0(r) \rightarrow (2i)^{\ell+1} (2\ell+1)!! \omega j_\ell(\omega r) \quad (C.8)$$

The constant in front of the spherical bessel function j_ℓ can be absorbed in the normalization constant. One can therefore put a to zero in Eqs. (C.4)-(C.6) and read $j_\ell(\omega r)$ in place of $F_0(r)$ to recover the $a=0$ result for the gluon wavefunctions above.

To lowest order in A^μ and $\hat{A}^\mu$ the colour electric and colour magnetic fields are given by the following expressions:

$$\begin{aligned} \vec{E}(\vec{r}) &= i\{(\omega + a/r)\vec{A} - \vec{\nabla}B\} = N_\ell \sum_\alpha E_\alpha \vec{Y}_{\ell\alpha}^m \\ \vec{B}(\vec{r}) &= \vec{\nabla} \times \vec{A} = N_\ell \sum_\alpha B_\alpha \vec{Y}_{\ell\alpha}^m \end{aligned} \quad (C.9)$$

in the basis of vector harmonics. This gives (only non-zero contributions are listed)

TE mode

$$\begin{aligned} E_\ell &= i(\omega + a/r) \sqrt{\ell(\ell+1)} F_0(r) \\ B_{\ell+1}(r) &= ia_- \sqrt{\ell(\ell+1)} (\partial/\partial r - \ell/r) F_0(r) \\ B_{\ell-1}(r) &= -ia_+ \sqrt{\ell(\ell+1)} (\partial/\partial r + (\ell+1)/r) F_0(r) \end{aligned} \quad (C.10)$$

TM mode

$$\begin{aligned}
 E_{\ell+1}(r) &= ia_+ \{ [\ell\omega - a/r] [\partial/\partial r - \ell/r] + \\
 &\quad - [a\omega/(\ell+1)] (\omega + a/r) - (\ell+1)^2 a/r^2 \} F_0(r) \\
 E_{\ell-1}(r) &= ia_- \{ - [(\ell+1)\omega + a/r] [\partial/\partial r + (\ell+1)/r] + \\
 &\quad + (a\omega/\ell) (\omega + a/r) + \ell^2 a/r^2 \} F_0(r) \\
 B_\ell(r) &= \frac{i\omega}{\sqrt{\ell(\ell+1)}} \{ a(\partial/\partial r + 1/r) + \ell(\ell+1)(\omega + a/r) \} F_0(r)
 \end{aligned} \tag{C.11}$$

L mode

$$\begin{aligned}
 E_{\ell+1}(r) &= a_+ a/r^2 F_0(r) \\
 E_{\ell-1}(r) &= a_- a/r^2 F_0(r)
 \end{aligned} \tag{C.12}$$

The colour magnetic field of the L modes vanishes identically.

Table 1. Values of L^{EEE} and L^{ssE} for different values of α_s (for light quark mass $m_q=0$).

α_s	L^{EEE}	L^{ssE}
0.0	0.340	0.985
0.1	0.385+i0.005	1.022+i0.012
0.2	0.444+i0.021	1.066+i0.051
0.3	0.525+i0.057	1.116+i0.122
0.4	0.646+i0.129	1.177+i0.234
0.5	0.849+i0.277	1.250+i0.408
0.6	1.246+i0.635	1.343+i0.684
0.7	2.231+i1.781	1.467+i1.171
0.8	6.111+i8.411	1.650+i2.271
0.9	74.4+i301.9	1.976+i8.017

Table 2a: Values of L^{QQE} for different values of the bagradius R (in GeV^{-1}), cut-off parameter q and α_s for heavy quark mass $m_Q=1.35\text{GeV}$.

α_s	R	$q=0.5$	$q=1.0$	$q=2.0$
0.0	3	1.00	0.98	0.89
	4	0.75	0.74	0.70
0.2	3	1.31+i0.06	1.22+i0.06	1.04+i0.05
	4	1.01+i0.05	0.95+i0.05	0.85+i0.04
0.4	3	1.91+i0.38	1.63+i0.33	1.26+i0.25
	4	1.51+i0.30	1.32+i0.26	1.07+i0.21
0.6	3	3.21+i1.63	2.40+i1.22	1.62+i0.83
	4	2.67+i1.36	2.05+i1.04	1.46+i0.74

Table 2b: Same as table 2a, but with $m_Q=4.75\text{GeV}$.

α_s	R	q=0.5	q=1.0	q=2.0
0.0	3	0.29	0.29	0.29
	4	0.21	0.21	0.21
0.2	3	0.40+i0.02	0.39++i0.02	0.37+i0.02
	4	0.30+i0.01	0.29+i0.01	0.28+i0.01
0.4	3	0.67+i0.13	0.60+i0.12	0.53+i0.11
	4	0.52+i0.10	0.47+i0.09	0.42+i0.08
0.6	3	1.41+i0.72	1.12+i0.57	0.87+i0.44
	4	1.15+i0.59	0.92+i0.47	0.73+i0.37

Table 2c: Same as table 2a, but with $m_Q=40.0\text{GeV}$.

α_s	R	q=0.5	q=1.0	q=2.0
0.0	3	0.034	0.034	0.034
	4	0.025	0.025	0.025
0.2	3	0.051+i0.002	0.050+i0.002	0.049+i0.002
	4	0.039+i0.002	0.038+i0.002	0.037+i0.002
0.4	3	0.105+i0.021	0.096+i0.019	0.088+i0.017
	4	0.082+i0.016	0.075+i0.015	0.068+i0.014
0.6	3	0.322+i0.164	0.261+i0.133	0.211+i0.107
	4	0.264+i0.134	0.214+i0.109	0.173+i0.088

Table 3: Hyperfine splitting (in GeV) of the lowest $\bar{Q}qG$ hybrids with C-type cavity modes for $\alpha_s=0.40$, $R=3.4\text{GeV}^{-1}$, $q=1$ and $m_q=0$. The splitting for the four different J^{PC} states are given for three different m_Q masses (in GeV). The corresponding splittings for F-type cavity modes are given in parenthesis.

m_Q			
J^{PC}	1.35	4.75	40.0
0^{-+}	-0.54 (-0.13)	-0.35 (-0.09)	-0.27 (-0.08)
1^{-+}	-0.27 (-0.07)	-0.18 (-0.05)	-0.13 (-0.04)
1^{--}	0.02 (0.01)	0.01 (0.00)	0.00 (0.00)
2^{-+}	0.26 (0.06)	0.17 (0.04)	0.13 (0.04)

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FIGURE CAPTIONS

- Figure 1: The energy eigenvalues ωR ($=x_G$) as a function of $a=-c_r\alpha_s$ for the lowest TE, TM and L gluon modes with $\ell=1$.
- Figure 2: The magnetic moment of a gluon in the lowest TE mode with $\ell=1$ as a function of a .
- Figure 3: The vertices used in our phenomenological application.
- Figure 4: Contributions to the hyperfine splitting in $\bar{Q}qG$ hybrids.

FIG. 1

FIG. 2

(a)

(b)

FIG. 3

FIG. 4