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A NEW IMPROVED TEMPORAL GAUGE: THE SOFT TEMPORAL GAUGE

F. Steiner^{*)}

CERN -- Geneva

ABSTRACT

A new gauge called the "soft temporal gauge" is considered, which automatically leads to a "softening" of the gauge singularities at $k_0 = 0$, while preserving the nice features of the traditional temporal gauge. In the physical sector, we obtain Landshoff's α -prescription. Using the Slavnov-Taylor identities (derived from BRS invariance), we give a simple proof that, for non-anomalous Yang-Mills theories, Gauss' law exactly holds.

^{*)} Permanent address: II. Institut für Theoretische Physik, University of Hamburg, Luruper Chaussee 149 D-2000 Hamburg 50, Fed. Rep. Germany.

Although the temporal gauge has long been proposed¹⁾⁻³⁾ as a natural one to be used in the quantization of gauge theories, we still do not have a simple set of Feynman rules for Yang-Mills theories in this gauge. To be specific, let us consider non-Abelian gauge theories with gauge group $SU(N)$, for which the temporal gauge is traditionally defined by $A_0^a(x) = 0$, $a = 1, \dots, N^2-1$. Since the time component of the vector potential is set equal to zero, one has lost Gauss' law arising from the variation of A_0^a . But the implementation of Gauss' law meets with difficulties, since it cannot be achieved in the usual Fock space, and therefore requires the construction of a non-trivial "dressed" vacuum state⁴⁾. As a result, the gauge field propagator is rather complicated and is not translationally invariant in time. Even though this lack of time-translation invariance cancels in gauge-invariant quantities⁵⁾, it is required by the regularization of the gauge field propagator. Indeed, if one tries to construct a translationally-invariant Feynman propagator by standard methods, one finds a double pole at $k_0 = 0$ in the longitudinal part^{*}). This gauge singularity is closely related to the fact that in the traditional temporal gauge the gauge has not been fixed completely, because there is still the freedom to perform arbitrary time-independent gauge transformations.

It has been argued by Kummer and others⁶⁾ that a principal-value prescription of the pole at $k_0 = 0$ would lead to a consistent theory with unitary S matrix. For the light-cone gauge, T.T. Wu⁷⁾ has presented general arguments which show that a principal-value prescription leads to inconsistencies in perturbation theory^{**)}. Recently it has been found⁵⁾ in calculations of the Wilson loop in quantum chromodynamics that the principal-value prescription for the gluon propagator in the temporal gauge does not indeed reproduce the order g^4 results obtained in the Feynman and Coulomb gauges. The known results can only be obtained if an additional term is added by hand to the gluon propagator which, however, violates time-translation invariance⁺).

*) This is a common feature of all axial gauges, where one finds poles at $n \cdot k = 0$ if n is the vector that defines the gauge.

***) For a recent discussion of the light-cone gauge, see Ref. 8).

+) Actually, this observation was the starting point for the general investigations presented in Refs. 4) and 9).

Recently, Landshoff^{10)*)} proposed an extremely simple prescription (depending on a parameter α) for dealing with the singularities at $n \cdot k = 0$ in axial gauges. Most remarkably, his so-called alpha-prescription retains translation invariance for the propagator. Furthermore, Landshoff showed that the Wilson loop is correctly reproduced in perturbation theory up to order g^4 when $\alpha \rightarrow 0$. He did not, however, offer "any fundamental justification for the α -prescription".

It is the purpose of this note to show that Landshoff's α -prescription can be derived from a new improved temporal gauge. Since the gauge singularities are "softened" in the new gauge, I call this gauge the "Soft Temporal gauge" (ST gauge). Although this gauge requires the introduction of Faddeev-Popov ghosts to start with, the ghosts finally decouple. This is consistent with Landshoff's findings, since ghosts have not been taken into account in his calculation. To demonstrate the consistency of the soft temporal gauge, it is nevertheless convenient to keep the ghosts, since they allow the construction of a Becchi-Rouet-Stora transformation¹¹⁾. From BRS invariance I derive the Slavnov-Taylor identities which are crucial for the derivation of Gauss' law.

Let us decompose the spatial components of the gauge field into (divergenceless) transverse fields (T) and (irrotational) longitudinal fields (L). We can then define a "scalar potential" $\phi^a(x)$ by^{**)}

$$\partial^i A_i^{aT}(x) = 0, \quad A_i^{aL}(x) = -\partial_i \phi^a(x)$$

$$\phi^a(x) = \phi^a[A(x)] \equiv \Delta^{-1} \partial^i A_i^a(x) \quad (1)$$

In terms of A_0 and ϕ , the new improved temporal gauge is defined by

$$A_0^a(x) = \alpha \phi^a(x) \quad (2)$$

*) I am grateful to H. Hüffel for informing me of Ref. 10).

**) As indicated, ϕ^a will be considered as a functional of $A_i^a(x)$.

where $\alpha > 0$ is a real parameter. For $\alpha \equiv 0$ we get back the ill-defined traditional temporal gauge. The soft temporal gauge will be defined by the limit $\alpha \rightarrow 0$.

It is not difficult to see that (2) fixes the gauge uniquely. Consider the infinitesimal gauge transformation

$$\delta A_\mu^a(x) = D_\mu^{ab}[A(x)] \theta^b(x) \equiv \left(\delta^{ab} \partial_\mu + g f^{abc} A_\mu^c(x) \right) \theta^b(x)$$

$$\theta^a(x) = \beta^a(x) + g \gamma^a(x) + O(g^2) \quad (3)$$

and ask for the solution which leaves (2) invariant. We then obtain for the lowest order term $\beta^a(t, \vec{x}) = \beta^a(\vec{x}) e^{-\alpha t}$, where $\beta^a(\vec{x})$ is an arbitrary function of $\vec{x}$. But this gauge function blows up for $t \rightarrow -\infty$, which is not acceptable if we allow only finite incoming field configurations. Thus $\beta^a(\vec{x})$ has to vanish everywhere. Repeating the analysis for the higher-order terms finally leads to $\theta^a(x) \equiv 0$, which implies that no residual gauge freedom has been left over in contrast to the situation encountered in the traditional temporal gauge.

To quantize the pure Yang-Mills theory, we start with the generating functional^{*)}

$$Z'[J] = \int \mathcal{D}A_\mu^a \det M e^{iS'[J]} \quad (4a)$$

$$S'[J] = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} - \frac{\lambda}{2} \left(A_0^a - \alpha \phi^a[A] \right)^2 + J^\mu A_\mu^a \right] \quad (4b)$$

The second term in the action (4b) represents the gauge-fixing Lagrangian and depends on a gauge parameter λ . The ST gauge will be defined by taking the limits $\lambda \rightarrow \infty$, $\alpha \rightarrow 0$ (in this order). For $\lambda \rightarrow \infty$, the gauge-fixing term acts like a delta function enforcing the constraint (2) under the path integral (4a). As a result, the "unphysical" field A_0^a decouples in the limit $\alpha \rightarrow 0$. In the ST gauge, the

*) In the following, quantities with a prime always stand for the full interacting theory ($g \neq 0$), while corresponding quantities without a prime refer to the free theory ($g = 0$).

generating functional will therefore no longer depend on the time-component of the c-number source J_μ^a which, in principle, could be omitted from the very beginning. It turns out to be useful, however, to keep the full source since it allows a simple derivation of the Slavnov-Taylor identities.

In Eq. (4a), $\det M$ denotes the Faddeev-Popov determinant where M is given by

$$M^{ab}(x, y) = \left\{ \delta^{ab} (\partial_0 + \alpha) + g f^{abc} [A_0^c(x) - \alpha \phi^c[A(x)]] \right\} \delta^4(x - y) \quad (5)$$

Replacing $\det M$ in the usual way by the Faddeev-Popov ghosts yields the ghost field Lagrangian

$$\mathcal{L}_G = \bar{\eta}^a(x) [\partial_0 + \alpha] \eta^a(x) + g f^{abc} \bar{\eta}^a(x) [A_0^c(x) - \alpha \phi^c[A(x)]] \eta^b(x) \quad (6)$$

which leads to the following free ghost propagator

$$\delta^{ab} G(x) \equiv \int \frac{d^4 k}{(2\pi)^4} \frac{-i \delta^{ab}}{k_0 - i\alpha} e^{ikx} = \delta^{ab} e^{-\alpha t} \Theta(t) \delta^3(\vec{x}) \quad (7a)$$

$$(\partial_0 + \alpha) G(x) = \delta^4(x) \quad (7b)$$

Obviously, the propagator (7a) is well defined (even for $\alpha \rightarrow 0$) and is not given by a principal-value prescription. In fact, in the limit $\alpha \rightarrow 0$ the expression (7a) corresponds to a retarded boundary condition. This property is of great importance, since it is closely related to the fact that the Faddeev-Popov ghosts completely decouple in the gauge (2). In fact, in the limit $\lambda \rightarrow \infty$ the ghost interaction with the gauge fields becomes proportional to $\alpha \cdot g$, and the ghosts decouple when $\alpha \rightarrow 0$ ^{*)}.

*) The decoupling of ghosts actually holds for any value of α . This is shown by an explicit evaluation of $\det M$ using the coherent-state fermion path integral¹²⁾.

To derive the free gauge field propagator, we have to evaluate the path integral (4a) which in this case is a simple Gaussian one with a kernel $[n^\mu = (1, 0, 0, 0)]$

$$K_{ab}^{\mu\nu}(k) = \delta_{ab} \left[-k^2 g^{\mu\nu} + K_1 k^\mu k^\nu - K_2 n^\mu n^\nu - K_3 n^\mu k^\nu - K_3^* k^\mu n^\nu \right]$$

$$K_1 = 1 - \frac{\lambda \alpha^2}{(\vec{k}^2)^2}, \quad K_2 = \lambda \left[1 + \frac{\alpha^2 (n \cdot k)^2}{(\vec{k}^2)^2} \right], \quad K_3 = \frac{\lambda \alpha}{\vec{k}^2} \left[\frac{\alpha (n \cdot k)}{\vec{k}^2} + i \right] \quad (8)$$

From (8) we get, for the free gauge field propagator,

$$\begin{aligned} D_{ab}^{\mu\nu}(k) = & \delta_{ab} \left[D_1^{\mu\nu}(k) + \alpha^2 D_2^{\mu\nu}(k) + i\alpha D_3^{\mu\nu}(k) \right] \\ & - \frac{\delta_{ab}}{\lambda} \frac{k^\mu k^\nu}{(n \cdot k)^2 + \alpha^2} \end{aligned} \quad (9a)$$

$$D_1^{\mu\nu}(k) = \frac{1}{k^2 + i\varepsilon} \left[-g^{\mu\nu} + \frac{(n^\mu k^\nu + k^\mu n^\nu)(n \cdot k) - k^\mu k^\nu + \alpha^2 n^\mu n^\nu}{(n \cdot k)^2 + \alpha^2} \right] \quad (9b)$$

$$D_2^{\mu\nu}(k) = \frac{(n^\mu k^\nu + k^\mu n^\nu)(n \cdot k) - k^\mu k^\nu - n^\mu n^\nu \vec{k}^2}{(k^2 + i\varepsilon) \vec{k}^2 [(n \cdot k)^2 + \alpha^2]} \quad (9c)$$

$$D_3^{\mu\nu}(k) = \frac{n^\mu k^\nu - k^\mu n^\nu}{\vec{k}^2 [(n \cdot k)^2 + \alpha^2]} \quad (9d)$$

Since the tensors $D_i^{\mu\nu}$, $i = 1, 2, 3$ do not depend on the gauge parameter λ , and the last term in (9a) vanishes for $\lambda \rightarrow \infty$, it follows that the gauge field propagator has a well-defined non-singular limit for $\lambda \rightarrow \infty$ and any $\alpha > 0$.

Let us first consider the general case, where α is not assumed to go to zero. In this case, A_0 does not decouple, and one must use the complete propagator (9a), which has the following structure when $\lambda \rightarrow \infty$ (a common factor δ^{ab} will be omitted)

$$\mathcal{D}^{ij}(k) = \mathcal{D}_1^{ij}(k) + \alpha^2 \mathcal{D}_2^{ij}(k) \quad (10a)$$

$$= \mathcal{D}_T^{ij}(k) + \mathcal{D}_L^{ij}(k) \quad (10b)$$

$$\mathcal{D}^{00}(k) = \alpha^2 \mathcal{D}_2^{00}(k) = \frac{\alpha^2}{k_0^2 + \alpha^2} \cdot \frac{1}{\vec{k}^2} \quad (10c)$$

$$\mathcal{D}^{0j}(k) = \mathcal{D}^{j0}(k)^* = i\alpha \mathcal{D}_3^{0j}(k) = \frac{i\alpha}{k_0^2 + \alpha^2} \cdot \frac{k^j}{\vec{k}^2} \quad (10d)$$

In (10b) the spatial propagator has been decomposed in a (standard) transverse and a (regularized) longitudinal part

$$\mathcal{D}_T^{ij}(k) = \frac{1}{k^2 + i\varepsilon} \left[\delta^{ij} - \frac{k^i k^j}{\vec{k}^2} \right] \quad (11a)$$

$$\mathcal{D}_L^{ij}(k) = \frac{1}{k_0^2 + \alpha^2} \cdot \frac{k^i k^j}{\vec{k}^2} \quad (11b)$$

In the spatial propagator (10b), the α -dependence has been isolated in the longitudinal propagator which, in space-time, factorizes as follows

$$\mathcal{D}_L^{ij}(x) = \mathcal{D}(t) \partial^i \partial^j \Delta^{-1} \delta^3(\vec{x}) \quad (12a)$$

$$\mathcal{D}(t) \equiv \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \frac{e^{ik_0 t}}{k_0^2 + \alpha^2} = \frac{1}{2\alpha} e^{-\alpha|t|} \quad (12b)$$

We have thus obtained a well-defined regularization of the gauge field propagator, which differs from the principal-value prescription⁶⁾. [The latter corresponds to the term $-|t|/2$ in the expansion of (12b) for $\alpha \rightarrow 0$.] There is no breaking of time-translation invariance as in Refs. 4), 5) and 9). Below we shall show that (10) exactly satisfies the Slavnov-Taylor identity¹³⁾.

Let us now consider the soft temporal gauge defined by the limits $\lambda \rightarrow \infty$, $\alpha \rightarrow 0$. In this case, A_0 decouples in the path integral (4a), and the physical sector involves only the spatial components A_i . Thus the propagator in the ST gauge is uniquely given by the spatial projection of (9a), i.e., $PD_{ST} = D_{ST}$ where $p^{\mu\nu} \equiv g^{\mu\nu} - n^\mu n^\nu$ is the spatial projection operator. We are then led to the following free gauge field propagator in the ST gauge

$$\mathcal{D}_{ST}^{ij}(k) \equiv \mathcal{D}_1^{ij}(k) = \frac{1}{k^2 + i\epsilon} \left[\delta^{ij} - \frac{k^i k^j}{k_0^2 + \alpha^2} \right] \quad (13)$$

where α has to be considered as a small positive parameter which must be set equal to zero at the end of a calculation. Equation (13) is exactly the propagator proposed by Landshoff¹⁰⁾. The soft temporal gauge therefore provides a justification for Landshoff's α -prescription.

Using the propagator (13), Landshoff has checked that the Wilson loop in quantum chromodynamics is correctly reproduced in perturbation theory up to order g^4 when $\alpha \rightarrow 0$. Following Caracciolo et al. [see Ref. 5)], one analyzes the relevant diagrams in terms of two propagators: the complete propagator (13) and the $k^i k^j$ term in (13). Since the large time behaviour of the Wilson loop arises from the infra-red behaviour of the $k^i k^j$ term in (13), i.e., from small k_0 , one can set $k_0 = 0$ in the k^2 denominator

$$- \frac{k^i k^j}{(k^2 + i\varepsilon)(k_0^2 + \alpha^2)} \underset{(k_0 \rightarrow 0)}{\sim} + \frac{1}{k_0^2 + \alpha^2} \cdot \frac{k^i k^j}{k^2} \quad (14)$$

which exactly yields the longitudinal propagator (11b) with the nice factorization property (12a).

At this point, I should like to make a remark on the rôle played by the "unphysical" time component of the gauge field. In this paper, our main interest is in the limit $\alpha \rightarrow 0$ in order to approach as closely as possible the traditional temporal gauge. The gauge condition (2) leads, however, for any finite value of α to a consistent gauge fixing of the theory. But for arbitrary, non-vanishing α the field A_0 can no longer be omitted, and thus the complete propagator (9a) has to be used. The crucial point then is that gauge-invariant quantities must be independent of α . It is thus precisely the contribution from the A_0 field which must lead to a complete cancellation of the non-singular α -terms still present in the physical sector of the ST gauge. That such a cancellation really takes place has been verified by an explicit calculation¹⁴⁾ of the Wilson loop in quantum electrodynamics to all orders in g .

For the consistency of the proposed quantization scheme, it is important to check the Slavnov-Taylor identities and, furthermore, to show that the Gauss law constraint is fulfilled. To derive the Slavnov-Taylor identities, it is most convenient to keep the Faddeev-Popov ghosts, even though they ultimately decouple. If we define an effective action by

$$S'_{\text{eff}}[J] \equiv S'[J] + \int d^4x \mathcal{L}_G \quad (15)$$

where $S'[J]$ is given by (4b), and $\mathcal{L}_G$ is the ghost Lagrangian (6), it is easy to see that $S'_{\text{eff}}[0]$ is invariant under the following BRS transformation¹¹⁾ ($\bar{\varepsilon}$ is a constant Grassmann number)

$$\begin{aligned}
 \delta A_\mu^a(x) &= -D_\mu^{ab}[A(x)] \eta^b(x) \bar{\epsilon} \\
 \delta \eta^a(x) &= -\frac{g}{2} f^{abc} \eta^b(x) \eta^c(x) \bar{\epsilon} \\
 \delta \bar{\eta}^a(x) &= \lambda [A_0^a(x) - \alpha \Delta^{-1} \partial^i A_i^a(x)] \bar{\epsilon}
 \end{aligned} \tag{16}$$

From the BRS invariance of the theory, we derive the following Slavnov-Taylor identity¹³⁾

$$\begin{aligned}
 &\lambda \left[\frac{1}{i} \frac{\delta}{\delta J_0^b(x)} - \alpha \Delta^{-1} \partial_j \frac{1}{i} \frac{\delta}{\delta J_j^b(x)} \right] T'[J] \\
 &= \int d^4y J^{d\mu}(y) D_\mu^{dc} \left[\frac{1}{i} \frac{\delta}{\delta J(y)} \right] G^{'cb}(y, x | J)
 \end{aligned} \tag{17}$$

where the ghost propagator, in the presence of the source J, satisfies the following equation of motion

$$\begin{aligned}
 &[\delta^{ac}(\partial_0^* + \alpha) - i g f^{acd} \left(\frac{\delta}{\delta J_0^d(y)} - \alpha \Delta_j^{-1} \partial_j^* \frac{\delta}{\delta J_j^d(y)} \right)] G^{'cb}(y, x | J) \\
 &= \delta^{ab} \delta^4(y-x) T'[J]
 \end{aligned} \tag{18}$$

Here we have introduced $T'[J]$, the generating functional for Green's functions

$$\begin{aligned}
 T'[J] &= Z'_{eff}[J] / Z'_{eff}[0] \\
 Z'_{eff}[J] &= \int \mathcal{D}A_\mu^a \mathcal{D}\bar{\eta}^a \mathcal{D}\eta^a e^{i S'_{eff}[J]}
 \end{aligned} \tag{19}$$

In the free case ($g = J = 0$), Eq. (18) gives for the ghost propagator $G^{ab}(y, x|0) = \delta^{ab} G(y-x)$, in agreement with Eqs. (7a) and (7b).

Equation (17) can easily be derived from $\langle \bar{\eta}^a(x) \rangle^J = 0$ and the BRS transformation (16). Here the expectation value of an operator $O(x)$ in the presence of the source J is defined by

$$\langle O(x) \rangle^J \equiv \frac{1}{Z'[J]} \int \mathcal{D}A_\mu^a \mathcal{D}\bar{\eta}^a \mathcal{D}\eta^a O(x) e^{iS'_{eff}[J]} \quad (20)$$

Acting on (17) with the operator $\Delta_z^{-1} \partial_i^z \delta / \delta J_i^a(z)$, we obtain the Slavnov-Taylor identity

$$\begin{aligned} \Delta_z^{-1} \partial_i^z \left[D_{ab}^{io}(z-x) - \alpha \Delta_x^{-1} \partial_j^x D_{ab}^{ij}(z-x) \right] \\ = - \frac{\delta_{ab}}{\lambda} G(z-x) + i \frac{g}{\lambda} f_{acd} \Delta_z^{-1} \partial_j^z \frac{\delta}{\delta J_j^d(z)} G_{cb}(z, x | J) \Big|_{J=0} \end{aligned} \quad (21)$$

for the full gauge field propagator defined by

$$D_{ab}^{\mu\nu}(x-y) \equiv -i \frac{\delta^2 T'[J]}{\delta J_\mu^a(x) \delta J_\nu^b(y)} \Big|_{J=0} \quad (22)$$

Using (9a) we deduce from (21)

$$k^i k^j D_{ab}^{ij}(k) = \delta_{ab} \left[\vec{k}^2 (k^i D_3^{oi}(k)) - \frac{(\vec{k}^2)^2}{\alpha \lambda} G(k) \right] \quad (23)$$

which for $\lambda \rightarrow \infty$ reduces to the following Slavnov-Taylor identity

$$k^i k^j D_{ab}^{ij}(k) = \frac{\delta_{ab} \vec{k}^2}{k_0^2 + \alpha^2} \quad (24)$$

The non-vanishing right-hand side of (24) imposes a non-trivial constraint on the propagator. While the ghosts decouple in the limit $\lambda \rightarrow \infty$, as expected, there remains a contribution from the $(A^0 A^i)$ propagator. It is easy to check that the

propagators (9a) (for all $\alpha, \lambda \neq 0$) and (10a) (for all $\alpha \neq 0$) exactly satisfy the identities (23) and (24) respectively. For the ST propagator (13), one obtains

$$k^i k^j D_{ab}^{ij}(k) = \frac{\delta_{ab} \vec{k}^2}{k_0^2 + \alpha^2} \left[1 + \frac{\alpha^2}{k^2 + i\varepsilon} \right] \quad (25)$$

which agrees with the Slavnov-Taylor identity (24) when $\alpha \rightarrow 0$.

Finally, let us show that Gauss' law holds in the ST gauge^{*)}. Consider a non-anomalous gauge theory whose action $S'_{\text{tot}}[J]$ is obtained by adding to (15) the fermion action

$$\int d^4x \mathcal{L}_F = \int d^4x [\bar{\psi}(x) i \gamma^\mu (\partial_\mu - ig T^a A_\mu^a(x)) \psi(x)] \quad (26)$$

[T^a are the generators of $SU(N)$ in the fundamental representation]. The action $S'_{\text{tot}}[0]$ is invariant under the BRS transformation (16) supplemented by

$$\delta \psi(x) = -ig T^a \psi(x) \eta^a(x) \bar{\varepsilon}$$

$$\delta \bar{\psi}(x) = ig \bar{\psi}(x) T^a \eta^a(x) \bar{\varepsilon}$$

(27)

Gauss' law is derived from the identity

*) See also the recent paper by Fujikawa¹⁵⁾ where, however, the principal-value prescription is used.

$$\begin{aligned}
0 &= \left\langle \frac{\delta}{\delta A_0^a(x)} S'_{\text{tot}}[J] \right\rangle^J \\
&= \left\langle \mathcal{D}_\mu^{ab}[A(x)] F_{(x)}^{b\ 0\mu} + g \bar{\psi}(x) \gamma^0 T^a \psi(x) \right\rangle^J \\
&\quad + g f^{abc} \langle \bar{\eta}^b(x) \eta^a(x) \rangle^J - \lambda \langle A_0^a(x) - \alpha \Delta^{-1} \partial^i A_i^a(x) \rangle^J
\end{aligned}
\tag{28}$$

where the expectation values are defined by (20) but with the action $S'_{\text{tot}}[J]$ with $J_0^a \equiv 0$. In the ST limit, A_0^a decouples under the path integral, and we obtain from (28) the expected value of the Gauss-law operator

$$\left\langle \mathcal{D}_i^{ab}[A] E^{bi}(x) + g \bar{\psi}(x) \gamma^0 T^a \psi(x) \right\rangle_{\text{ST}}^{J=0} = 0 \tag{29}$$

(In the ST limit, the non-Abelian electric field is defined by $E_i^a = -\partial_0 A_i^a$.) In deriving (29), we have used the fact that in the ST limit and for $J \equiv 0$, the ghost term in (28) becomes the free ghost propagator (7a), and thus this term vanishes because of the colour structure.

The crucial point in the derivation of (29) concerns the last term in Eq. (28). Since this expectation value behaves like $\lambda \cdot \alpha$ in the ST limit, it seems to be a very delicate one. Fortunately, this term is nothing but the left-hand side of the Slavnov-Taylor identity (17), written in a different form, and therefore vanishes identically for $J \equiv 0$, since the right-hand side of (17) is proportional to J^* .

It is just the last term in Eq. (28) which takes a non-zero value for a chiral non-Abelian gauge theory. In fact, in this case this term can be expressed by the non-Abelian anomaly and the free ghost propagator. A study of anomalous

*) Actually, this holds for any finite value of the gauge parameters α, λ .

gauge theories in the ST gauge and a derivation of the anomalous commutator for the Gauss-law operator, recently found by Faddeev¹⁶⁾, will be presented in a subsequent paper.

To summarize, in this note I have proposed a new gauge, called the "soft temporal gauge", which automatically leads to a consistent "softening" of the gauge singularities. In the physical sector, we exactly obtained Landshoff's¹⁰⁾ α -prescription. On using the Slavnov-Taylor identity, we showed that Gauss' law exactly holds. The Wilson loop is correctly reproduced in perturbation theory up to order g^4 ¹⁰⁾.

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REFERENCES

- 1) W. Heisenberg and W. Pauli, Z. Phys. 59 (1930) 168.
- 2) H. Weyl, "Gruppentheorie und Quantenmechanik", Hirzel Verlag, Leipzig, 2nd ed. (1931), p. 228.
- 3) P.A.M. Dirac, Can. Journ. Phys. 33 (1955) 650.
- 4) H.D. Dahmen, B. Scholz and F. Steiner, Phys. Lett. 117B (1982) 339;
B. Scholz and F. Steiner, DESY preprint 83-055 (1983);
D. Zeppenfeld, Nucl. Phys. B247 (1984) 125;
H.O. Girotti and H.J. Rothe, Heidelberg preprint HD-THEP-85-14 (1985).
- 5) V.F. Müller and W. Rühl, Ann. Phys. (NY) 133 (1981) 240;
S. Caracciolo, G. Curci and P. Menotti, Phys. Lett. 113B (1982) 311.
- 6) W. Kummer, Acta Phys. Austr. 41 (1975) 315;
W. Konetschny and W. Kummer, Nucl. Phys. B100 (1975) 106; B108 (1976) 397;
B124 (1977) 145;
J. Frenkel and J.C. Taylor, Nucl. Phys. B109 (1976) 439;
J. Frenkel, Phys. Lett. 85B (1979) 63.
- 7) T.T. Wu, Phys. Lett. 71B (1977) 142; Phys. Reports 49 (1979) 245.
- 8) S. Mandelstam, Nucl. Phys. B213 (1983) 149;
G. Leibbrandt, Phys. Rev. D29 (1984) 1699;
H.C. Lee and M.G. Milgram, Phys. Rev. Lett. 55 (1985) 2122.
- 9) J.P. Leroy, J. Micheli and G.C. Rossi, Nucl. Phys. B232 (1984) 511.
- 10) P.V. Landshoff, Cambridge preprint DAMTP 85/36 (December 1985).
- 11) C. Becchi, A. Rouet and R. Stora, Phys. Lett. 52B (1974) 344; Comm. Math. Phys. 42 (1975) 127; Ann. Phys. (NY) 98 (1976) 287.
- 12) L.P. Singh and F. Steiner, Phys. Lett. 166B (1986) 155.
- 13) J.C. Taylor, Nucl. Phys. B33 (1971) 436;
A.A. Slavnov, Theor. and Math. Phys. 10 (1972) 99.
- 14) F. Steiner, to be published.
- 15) K. Fujikawa, Hiroshima preprint RRK 86-1 (January 1986).
- 16) L.D. Faddeev, Phys. Lett. 145B (1984) 81.