CERN-TH.4394/86
PITHA 86/04

ρ , π MASSES AND CHIRAL SYMMETRY BREAKING:
SU(2) COLOUR GAUGE THEORY WITH DYNAMICAL FERMIONS^{*)}

E. Laermann, F. Langhammer

Inst.Theor.Physik, RWTH Aachen, D-5100 Aachen

I. Schmitt

Inst.Theor.Physik, Bergische Universität, D-5600 Wuppertal

and

P.M. Zerwas

CERN - Geneva
and

Inst.f.Theor.Physics, University of California, Santa Barbara CA 93106

A B S T R A C T

The masses of ρ , π mesons and the quark condensate $\langle \bar{q}q \rangle$ are investigated in a Monte Carlo simulation of the SU(2) colour gauge theory including dynamical Kogut-Susskind fermions on a 16×8^3 lattice. When extrapolated to zero quark mass, we find a finite ρ -mass value, but a π -mass compatible with zero, and a non-zero value of the quark condensate, signalling spontaneous breaking of chiral symmetry in non-Abelian gauge theories.

^{*)} Supported in part by the W. German Bundesministerium für Forschung und Technologie, Studienstiftung des deutschen Volkes and the (U.S.) National Science Foundation under Grant No. PHY 82-17853

The determination of the spectra of the lowest-lying hadrons and the observation that chiral symmetry is spontaneously broken, implying the existence of Goldstone particles, are among the most remarkable results achieved so far in numerical solutions of quantum chromodynamics. These calculations are based on integrations of QCD over discrete space-time lattices in the valence quark approximation /F1/. Guided by the experience accumulated over a long time of successful hadron phenomenology in the quark picture, an upper limit of 10 to 15 % for the error can reasonably be expected in such an approach. The lattice formulation, on the other hand, allows us in principle to solve QCD completely. However, since this final goal requires a huge computational effort the natural first step should be to check whether quark-loop effects change the quenched results of the lattice approach dramatically or not. A few attempts /2-7/ in this direction have been reported so far, most of the papers focusing on the problems of deconfinement phase transitions and chiral symmetry restoration in QCD at non-zero temperatures.

Extending previous work on the inter-quark potential /8/ we report in this note on the analysis of the ρ and π masses, and the measurement of the quark condensate in the vacuum state. The calculation has been carried out for color SU(2) including four degenerate Kogut-Susskind fermions on a 16×8^3 lattice. Details on the construction of gauge field configurations by means of the pseudofermion method /9/ have been presented in ref. /8/ and will not be repeated here. The measurements have been performed mainly at $\beta = 4/g^2 = 1.95$ and 1.85 . The quark masses were chosen between 0.035 and 0.20 in lattice units. For these parameters the lattice spacing is expected to be small enough /8/ and the spatial volume just large enough to accommodate hadrons of the size $\approx 1 \text{ fm}^3$. Since approximate scaling has been observed in ref. /4/ for the range $\beta > 1.85$ and 4 flavors, in agreement with the data to be presented later in this note, first estimates of hadron properties may be seriously considered [though larger lattices, smaller quark mass values, etc., are certainly needed to refine the analyses].

ρ, π MASSES

In the Kogut-Susskind version of the fermion theory, quark fields are smeared out over the unit cells of the lattice /10/,

$$(1) \quad q^{\alpha f}(y) = 1/8 \sum_{\eta} \Gamma_{\eta}^{\alpha f} \hat{U}_{\eta}(2y) \chi(2y+\eta)$$

The χ fields are single-component Grassmann fields for each color index defined on the sites of the lattice with spacing a , y is the variable that characterizes the position of the unit cell. Γ_{η} is the product of Dirac matrices, $\hat{U}_{\eta}(2y)$ the product of link variables along a path connecting the sites $2y$ and $2y+\eta$. α is interpreted as Dirac index, f as flavor index for four degenerate quarks. Meson operators can be constructed from the quark fields, carrying the correct quantum numbers in the continuum limit /10,11/,

$$(2) \quad \begin{aligned} M_{\rho}(y) &= \bar{q}(y) (\gamma_i \otimes \gamma_i^*) q(y) \quad ; i = 1, 2, 3 \\ M_{\pi}(y) &= \bar{q}(y) (\gamma_5 \otimes \gamma_5^*) q(y) \end{aligned}$$

where the first γ matrix acts on Dirac and the second on flavor indices. Expressed in terms of χ fields the interpolating fields (2) are sums of local operators,

$$(3) \quad M(y) = 1/16 \sum_{\eta} (-)^M \bar{\chi}(2y+\eta) \chi(2y+\eta)$$

with phase factors given by $(-)^{M_{\rho}} = (-)^{\sum_{\mu \neq i} \eta_{\mu}}$ and $(-)^{M_{\pi}} = (-)^{\sum_{\mu} \eta_{\mu}}$.

The propagators of these fields which determine the mass values of the mesons, can be expressed in terms of correlation functions for even and odd time values which are preferable for Monte Carlo calculations,

$$(4a) \quad \begin{aligned} & 1/32 \sum_{\vec{x}} \sum_{A,B} \langle |\bar{\chi}_A(\vec{x}, t) \chi_B(0)|^2 \rangle \\ &= K_{\pi} [e^{-M_{\pi} t} + e^{-M_{\pi}(T-t)}] + (-)^t K' [e^{-M' t} + e^{-M'(T-t)}] + \dots \end{aligned}$$

$$\begin{aligned}
 (4b) \quad & \frac{1}{32} \sum_{\vec{x}} \sum_{A,B} [(-)^{x_1} + (-)^{x_2} + (-)^{x_3}] \langle |\bar{\chi}_A(\vec{x}, t) \chi_B(0)|^2 \rangle \\
 & = K_\rho [e^{-M_\rho t} + e^{-M_\rho (T-t)}] + (-)^t K_B [e^{-M_B t} + e^{-M_B (T-t)}] + \dots
 \end{aligned}$$

T is the periodicity of the lattice, and A, B are color indices. The disconnected part of the propagator does not contribute for flavor-nonsinglets in the continuum limit and has been suppressed. The oscillatory terms are the opposite parity admixtures 1^{+-} and 0^{+-} to $\rho(1^{--})$ and $\pi(0^{-+})$, respectively, an artifact of the Kogut-Susskind formulation of lattice fermions. The dots indicate possible contributions from higher recurrences. The ρ can (unfortunately) be treated as a stable particle in our analysis since at the quark mass values where we have measured the propagators - before the extrapolation to zero quark mass is carried out - the ρ mass is still slightly below twice the π mass.

The propagators have been calculated by means of the conjugate gradient method. The program was implemented on a CYBER 205. Using half-precision arithmetics we stopped the iteration if the length of the rest vector fell below 5×10^{-7} . At $\beta = 1.95$, the number of CG iterations to achieve this accuracy was between 80 for $m = 0.2$ and 270 for $m = 0.05$. For each quark mass value we analyzed ≈ 40 configurations separated by 50 gauge sweeps. Within each configuration ≈ 35 propagator source points were selected. The fluctuations of the propagators among different source points were comparable with those among different configurations. The statistical error bars result from a blocking analysis. However, considering each configuration as statistically independent did not affect the errors significantly. Non-gauge invariant quantities as the single quark propagator were checked to vanish within statistical errors.

Fig. 1 shows the data points of the propagators for the quark mass value 0.07 at $\beta = 1.95$. Measurements of other propagators are of comparable quality. While the B signal is clearly visible besides the ρ in the spin 1 propagator, there is no 0^{+-} parity partner to the π in the spin 0 propagator [in accord with the rules of the naive quark model].

Over the whole t-range the propagators can well be described by allowing for one excited state besides the ground state plus, for the spin 1 channel, one state accounting for the oscillating part. Using only time slices

$4 \leq t \leq 12$ single mass fits lead to the same results for π - and ρ -masses. Alternatively it has been checked that fits to the meson propagators defined in terms of the quark fields q (eq. 2) only on even time slices return the same masses for the lowest lying states.

Masses of higher recurrences cannot be determined, but for the pion they are bounded from below by at least twice the measured mass parameter, their amplitude being small.

The ρ -mass values extracted from the propagators are displayed in Fig. 2a. To reduce lattice artifacts, we plot $am_\rho = 2 \sinh(am_\rho/2) = am_\rho + O(a^2)$ as a function of the quark mass $m_q a$. The distribution of the data points allows for a linear fit in a natural way. For $m_q a > 0.1$, deviations from the linear behavior were observed. [We have checked that our conclusions are reproduced within an error margin of 20 % by a linear extrapolation of $(am_\rho)^2$, often employed in the quenched approximation. The lattice spacing drops correspondingly in this case.] At $\beta = 1.95$, the ρ -mass value extrapolated to zero quark mass is given as $am_\rho = 0.74 \pm 0.05$. With $m_\rho = 770$ MeV, this corresponds to a lattice spacing $a(1.95) = 0.19 \pm 0.02$ fm, leading also to a consistent picture of the screened heavy quark-antiquark potential /8/. In Fig. 2b we compare the mass values at $\beta = 1.95$ and 1.85 with the prediction from asymptotic scaling which appears to be reasonably well fulfilled. Also shown are our mass values in the quenched approximation obtained by an extrapolation of $(am_\rho)^2$; they are in good agreement with the corresponding results of ref. /11/. The B-meson mass turns out to be substantially larger than the ρ -mass, $m_B = 1.1 \pm 0.3$ GeV $\approx 1.5 m_\rho$. A more complete description of the other local mesons /12/ will be the subject of a forthcoming publication.

Relating the lattice spacing to $\Lambda_{\overline{MS}}$ [the relevant formulae are recorded in ref. /8/], we find that this scale parameter is for 4 flavors reduced by almost a factor 2 compared with the pure glue case, $\Lambda_{\overline{MS}}(N_F=4)/\Lambda_{\overline{MS}}(N_F=0) \approx 0.5$, i.e. $\Lambda_{\overline{MS}}(N_F=4) \approx 60$ MeV. Of course, a high precision measurement of absolute numbers on a lattice as small as 16×8^3 is not possible as quenched computations have shown all too well in the past. However, similar observations have been made in other analyses /7/, and one may be tempted to cautiously conclude that lattice gauge theories including

fermions prefer $\Lambda_{\overline{MS}}$ values in the lower range of fits based on perturbative QCD processes /13/.

The pion mass squared extrapolates down to zero at vanishing quark mass within 2 standard deviations. This behavior, suggested by current algebra, is demonstrated in Fig. 2a. The result is consistent with the expectation that a Goldstone particle exists in non-abelian gauge theories due to spontaneous breaking of chiral symmetry. In the Kogut-Susskind version of the lattice action which is invariant under a continuous flavor nonsinglet chiral transformation, this sector is directly accessible even at non-zero lattice spacing /F2/.

Forcing the straight line for $\beta = 1.95$ in Fig. 2a. to cross the origin we can extract the quark mass in physical units from the slope $(am_\pi)^2/(am_q) \approx 7$ combined with the measurement of the lattice spacing. Relating [see ref. /15/] the quark mass $m_q(a)$ associated with the lattice spacing a , to the renormalization group invariant mass as defined in ref. /16/, we derive

$$m_q^{RGI} = 3.1 m_q(a[\beta=1.95]) = 7 \pm 1 \text{ MeV}$$

from $m_\pi = 135 \text{ MeV}$. This number is (surprisingly) close to the value of $\approx 10 \text{ MeV}$ extracted from current algebra analyses /16/. Note, however, that by going from $\beta = 1.95$ down to 1.85 the slope in Fig. 2a. does not change which implies a lack of scaling for this quantity in the parameter range considered here. Similar problems had been reported in the quenched approximation for $\beta < 2.4$ /11/. However, for $\beta \geq 2.4$, the pion-slope was found to be consistent with asymptotic scaling in the quenched case /17/, indicating that $\beta = 1.95$ might be at the boundary of the scaling region for the pion mass in the unquenched theory.

QUARK CONDENSATE

A non-zero value of $\langle \bar{q}q \rangle_{2f} = 1/2 \sum_i \langle \bar{\chi}_i \chi_i \rangle$ [2 flavors] for vanishing quark masses signals spontaneous breaking of chiral symmetry. This order parameter is evaluated by taking the trace of the inverse Dirac operator,

$$(6) \quad \langle \bar{q}q \rangle_{2f} a^3 = 1/2V \langle \text{tr}(D+am_q)^{-1} \rangle \quad ,$$

V being the lattice volume. Since large eigenvalues $|\lambda|$ of D get emphasized by the determinantal part of the action $\sim \prod [|\lambda|^2 + (am_q)^2]$, the quark condensate is expected to drop in lattice units if fermions are switched on [this is not necessarily so in physical units].

We have measured the condensate at $\beta = 1.85, 1.9$ and 1.95 for various quark mass values as shown in Fig. 4a. Extrapolating these values linearly down to zero-mass quarks as indicated in the figure, we find non-vanishing values for the condensate in accord with the spontaneous breaking of chiral symmetry. In Fig. 3b. we compare $-1/3 \log \langle \bar{q}q \rangle_{2f} a^3$ with the quenched results for the same quantity, adopted (after verification) from ref. /11/. The large β shift of the condensate in lattice units is apparent. We also show 4 points of the analysis presented in ref. /4/ which are in remarkable agreement with the results found here /F3/. Note that the numbers are compatible with asymptotic scaling. In analogy to eq. (5), we might try to extract the renormalization group invariant value of the quark condensate (for one flavor) from the data. Multiplying the result with a suggestive factor $^3\sqrt{3/2}$ to correct for the transition from $SU(2)$ to $SU(3)$, we find

$$(7) \quad \langle \bar{q}q \rangle_{1f, SU_3}^{1/3 \text{ RGI}} = 185 \pm 20 \text{ MeV}$$

again very close to the current algebra result of $190 \pm 20 \text{ MeV}$ following from ref. /16/.

Encouraged by these results we exploit the current algebra relation

$$(8) \quad f_\pi^2 = m_q \langle \bar{q}q \rangle_{2f, SU_3} / m_\pi^2$$

to calculate the leptonic (charged) pion decay constant. Utilizing the values for $(am_\pi)^2/(am_q)$, $\langle \bar{q}q \rangle a^3$ and a at $\beta = 1.95$ we find $f_\pi = 83 \pm 7 \text{ MeV}$ as compared to the experimental value $f_\pi^{\text{exp}} = 94 \text{ MeV}$, still within the error margin of $\approx 15 \%$ which can reasonably be expected in our experiment. [At $\beta = 1.85$ the value rises to $122 \pm 6 \text{ MeV}$.]

To summarize. In this note we have shown that lattice gauge theories including light quarks provide a consistent picture for some of the essential properties of low lying mesons we have probed [even though this could

only be demonstrated in color SU(2) due to the limitations by present computer resources].

(i) There are clear indications for spontaneous breaking of chiral symmetry, a non-vanishing quark condensate together with a zero pion mass, when these quantities are extrapolated down to zero quark mass. Physical parameters like the RGI quark mass and the condensate match current algebra values surprisingly well. The π_{12} constant f_π turned out to be close to the experimental number. $\Lambda_{\overline{\text{MS}}}$ appeared to be somewhat reduced compared with the quenched value. (ii) The agreement with other, totally different approaches to the fermion problem gives us confidence that systematic errors induced through technical approximations in the pseudofermion method, are under control for lattices of (semi-) realistic size.

ACKNOWLEDGEMENTS

We thank the Computer Centers at RWTH Aachen and Bochum University for their support. P.Z. is grateful to S. Brodsky and J.R. Schrieffer for the warm hospitality extended to him at the QCD Workshop in Santa Barbara, likewise to the CERN Theory Division. We have greatly benefited from discussions with A.N. Burkitt and T. Neuhaus.

FOOTNOTES

/F1/ See ref. /1/ for recent analyses and the earlier pioneering papers quoted therein.

/F2/ Since the fermion fields χ and $\bar{\chi}$ in the fundamental representation of the SU(2) color gauge group belong to equivalent representations, the action for $m = 0$ is not only $U(1) \times U(1)$ invariant as in SU(3) color but U(2) invariant. A hard mass term [defining the direction of the symmetry breaking also in the continuum] breaks U(2) but is still left invariant under O(2). There are thus 1 Goldstone meson $\bar{q}q$ and 2 Goldstone "baryons" qq , $\bar{q}\bar{q}$ which we do not discuss as SU(2) artifacts in the following. More details are presented in ref. /14/. We thank O. Napoly, B. and F. Schrempp for a discussion of this point and related problems.

/F3/ The microcanonical method has been used in ref. /4/ while our results have been obtained by means of the pseudofermion method. Both methods rely on different assumptions and incorporate different approximations. The agreement of the results is thus highly nontrivial and gives mutual support to both these methods. [The same agreement has been observed for plaquette values and Polyakov-Wilson lines, see /8/.]

REFERENCES

- /1/ D. Barkai, K.J.M. Moriarty and C. Rebbi, Phys. Lett. 156 B (1985) 385;
A. Billoire, E. Marinari and R. Petronzio, Nucl. Phys. B 251 (1985) 141;
J. Gilchrist, G. Schierholz, H. Schneider and M. Teper, Nucl. Phys. B 248 (1984) 29.
- /2/ H. W. Hamber, E. Marinari, G. Parisi and C. Rebbi, Phys. Lett. 124 B (1983) 99.
- /3/ R. Gavai, M. Lev and B. Petersson, Phys. Lett. 140 B (1984) 397;
F. Fucito and S. Solomon, Phys. Rev. Lett. 55 (1985) 2641;
F. Fucito, C. Rebbi and S. Solomon, Phys. Rev. D 31 (1985) 1460;
U. Heller and F. Karsch, Nucl. Phys. B 258 (1985) 29.
- /4/ J. Kogut, J. Polonyi, H.W. Wyld and D.K. Sinclair, Nucl. Phys. B265[FS15] (1986) 293.
- /5/ U. Heller and F. Karsch, Phys. Rev. Lett. 54 (1985) 1765.
- /6/ H. Joos and I. Montvay, Nucl. Phys. B 225 (1983) 565;
W. Langguth and I. Montvay, Phys. Lett. 145 B (1984) 261.
- /7/ I. Montvay, DESY 85-072 (1985);
F. Fucito and S. Solomon, Proceedings, Nuclear QCD Workshop, Santa Barbara, 1985.
- /8/ E. Laermann, F. Langhammer, I. Schmitt and P.M. Zerwas, Aachen preprint PITHA 86/03; CERN-TH.4393/86 (1986).
- /9/ F. Fucito, E. Marinari, G. Parisi and C. Rebbi, Nucl. Phys. B 180 (1981) 369.
- /10/ H. Kluberg-Stern, A. Morel, O. Napoly and B. Petersson, Nucl. Phys. B 220 (1983) 447;
J. Kogut, M. Stone, H.W. Wyld, S.H. Shenker, J. Shigemitsu and D.K. Sinclair, Nucl. Phys. B 225 (1983) 326.
- /11/ A. Billoire, R. Lacaze, E. Marinari and A. Morel, Nucl. Phys. B 251 (1985) 581.
- /12/ F. Langhammer, Diplomarbeit Physik, RWTH Aachen (1986).
- /13/ G. Peter Lepage, Proceedings, Lepton-Photon Symposium, Cornell 1983.
- /14/ H. Kluberg-Stern, A. Morel and B. Petersson, Nucl. Phys. B 215 (1983) 527.

- /15/ M. Göckeler, Phys. Lett. 142 B (1984) 197.
- /16/ J. Gasser and H. Leutwyler, Phys. Rep. C 87 (1982) 77.
- /17/ A. Billoire, R. Lacaze, E. Marinari and A. Morel, Saclay PhT 85-180.

FIGURE CAPTIONS

Fig. 1 The pion and ρ -propagators as defined in eq. (4) at $\beta = 1.95$ with dynamical quarks of mass $m_q a = 0.07$. If not shown, the error bars are smaller in size than the data points.

Fig. 2 (a) Linear extrapolation of $(m_\pi a)^2$ and $m_\rho a$ to zero quark mass.

(b) Scaling properties of the ρ -mass. Our results in the quenched approximation are shown for comparison, together with the values reported in ref. /11/.

Fig. 3 (a) Linear extrapolation of the quark condensate for two flavors to zero quark mass at $\beta = 1.85$ and 1.95 .

(b) Scaling properties of the quark condensate for two flavors with dynamical quarks (dots). The unquenched results of ref. /4/ (crosses) and the quenched results of ref. /11/ (open circles) are included for comparison.

Fig. 1

Fig. 2a

Fig. 2b

Fig. 3a

Fig. 3b