On the role of antiparticles in Dirac phenomenology

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Abstract

We show that the primary role of the antiparticles in Dirac phenomenology is to cancel short-range structure from the particle propagator. The elimination of this short-range structure leads to the success of the Dirac approach.

The use of the Dirac equation for intermediate-energy proton scattering has become very popular.^{1,2} The main reason for this is that it not only allows a simple phenomenology for fitting the proton elastic spin observables simultaneously³ but also because this phenomenology can be derived in a reasonably parameter-free way through the Dirac impulse prescription.⁴

Hynes et al.⁵ have argued, and confirmed numerically, that elastic scattering observables for positive-energy-only (particle) Dirac scattering and for Schrödinger scattering are similar. This leads to the conclusion that it is the antiparticles that are responsible for the success of the Dirac approach. This importance of the antiparticles in Dirac scattering is one of the main sources of uneasiness with this approach. It is hard to believe that contributions of such short range as the antineutrons, or equivalently the pair terms, can be so important since they correspond to propagation over distances smaller than the size of the nucleon. This has been emphasized especially by Brodsky,⁶ who claims that the antineutron contributions should be suppressed by form factors. Also other contributions of longer range than the antiparticles such as those from the excited baryon states are neglected. In this letter we will show that the real role of the antiparticles is to suppress short-range contributions from the Dirac particle propagator, i.e. the antiparticles are just providing a Lorentz invariant smoothing of the particle propagator. In fact we find that the Schrödinger propagator can give more short-range structure than the Dirac. We also shed new light on the relation between the Dirac approach with particles only and the Schrödinger approach. (They are not identical.)

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It is a well-known result from pion scattering⁷ that, in the presence of momentum-dependent potentials, iterating the Lippmann-Schwinger equation can generate δ -function terms that correspond to two successive scatterings occurring at the same point. These contributions are suppressed by short-range correlations (the LLEE effect). The relevance of this to proton-nucleus scattering has recently been pointed out by Thies,⁸ who showed that the suppression of zero-range propagation in the Schrödinger picture with momentum-dependent potentials reproduces the results of the Dirac picture. Starting with the Feynman (Dirac) propagator we show explicitly that the antiparticle propagator removes short-range structure (approximately a δ function) that is present in the particle propagator. The removal of this δ function is responsible for both the enhanced spin orbit and the 'wine bottle bottom' shape of the potential.

We start from the Lippmann-Schwinger equation for the scattering of Dirac particles. Equation (1) gives the Lippmann-Schwinger equation in coordinate space:

$$\psi(\vec{r}) = u(k) \exp(i\vec{k} \cdot \vec{r}) - \int G_E(\vec{r} - \vec{r}') \gamma_0 V(r') \psi(\vec{r}') d^3r'. \quad (1)$$

Here $u(k)$ is the Dirac particle spinor, $\psi(\vec{r})$ is the four-component Dirac wave function, $V(r)$ the potential matrix, and $G_E(\vec{r} - \vec{r}')$ is the full Feynman propagator in a mixed energy/space representation. It is given by

$$G_E(r-r') = \int \frac{d^3p}{(2\pi)^3} \frac{\exp i\vec{p} \cdot (\vec{r} - \vec{r}')}{p^2 - k^2 - i\epsilon} \begin{pmatrix} E+m & -\sigma \cdot p \\ \sigma \cdot p & m-E \end{pmatrix}. \quad (2)$$

The E with no subscript refers to the on-shell energy $E = \sqrt{m^2 + k^2}$. Note the simple structure in the complex momentum space. The only singularity

is at $p^2 = k^2$. We now decompose the propagator into particle (positive energy) and antiparticle (negative energy) contributions, respectively.

$$G_E(r-r') = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} \exp(i\vec{p} \cdot (\vec{r} - \vec{r}')) \times \\ \times \left[\begin{pmatrix} m+E_p & -\vec{\sigma} \cdot \vec{p} \\ \vec{\sigma} \cdot \vec{p} & m-E_p \end{pmatrix} \frac{1}{E_p - E - i\epsilon} + \frac{1}{E_p + E - i\epsilon} \begin{pmatrix} m-E_p & -\vec{\sigma} \cdot \vec{p} \\ \vec{\sigma} \cdot \vec{p} & m+E_p \end{pmatrix} \right]. \quad (3)$$

Note the appearance of branch points at $p = \pm im$ in both the particle and antiparticle propagators. In coordinate space these cuts will generate structure that falls off like $\exp(-mr)$. Both the particle and antiparticle propagator have this short-range structure which is not present in the full propagator. If we take just the particle propagator, do the momentum space integral as a contour integral and keep just the contribution from the pole at $p^2 = k^2$ (i.e. neglect the short-ranged contribution from the branch cuts), we are led back to the full Feynman propagator.

Thus we see that the role of the antiparticles is to exactly cancel terms in the particle propagator which correspond to propagation over the short distances ($\sim 1/m$) and hence by the very same arguments we used to condemn the antiparticles are spurious anyway. Note also that the Feynman propagator falls off more rapidly at infinity than either the positive or negative energy pieces separately; in most cases this will have no effect. It is worth emphasizing that the structure we are talking about has a range of order $1/m$ in coordinate space and arises from the kinematic singularity at $p = \pm im$. It does not have a range of $1/(2m)$ as suggested by the mass of the pair term.

To demonstrate the effect of short-range propagation, we start from the particle propagator and make a gradient expansion

$$G_E^{(+)}(r-r') = \frac{\hat{E} + E}{2\hat{E}} \left(\frac{m\hat{E} + i\hat{\sigma} \cdot \nabla}{-i\hat{\sigma} \cdot \nabla - m\hat{E}} \right) \frac{e^{ik|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|}, \quad (4)$$

where $\hat{E}$ is the differential operator $\sqrt{m^2 - \nabla^2}$. Now, if we consider where this propagator is to be used, specifically Eq. (1), we see that the gradient operators can be turned around to act on the wave function.

These gradients are then going to generate contributions close in "value" to the local wave vector of the particle, which for the case of fairly high-energy scattering is close to its asymptotic value of k . For a 500 Mev proton, k is approximately the nucleon mass (5 fm⁻¹). Thus if we treat $\hat{E}$ by expanding in powers of ∇/m , this series will converge slowly or not at all. However, the route to approximate evaluation of Eq. (3) is now clear; we write:

$$\hat{E} = \sqrt{m^2 + k^2 - (\nabla^2 + k^2)} = E^2(1 - D^2/2 - \dots), \quad (5)$$

where
$$D = \frac{(\nabla^2 + k^2)}{2E^2}. \quad (6)$$

The series as shown in Eq. (5) is rapidly convergent, as may be ascertained by putting in values for the parameters pertinent to protons of 500 Mev scattering from a nucleus. Replacing all the $\hat{E}$ operators in Eq. (4) with the series in Eq. (5), we get a rapidly convergent series in D for the particle propagator. Note that the first term of this series is obtained by the replacement $\hat{E} \rightarrow E$. This term is exactly the Feynman propagator again!! Thus once more we have seen that the positive energy propagator gives the full Feynman propagator once it is smoothed in some sense. In this particular case, the smoothing corresponds (in momentum space) to expanding the square root function in a power series about the

on-shell value of the momentum. The second term in the series is going to entail the operator D operating on the "Schrödinger propagator" of Eq. (4). Clearly this is going to give us a δ -function contribution. Specifically keeping the first two terms in the series we get

$$G_E^{(+)}(\vec{r}-\vec{r}') = G_E(\vec{r}-\vec{r}') - \frac{(H-E)\gamma_0 \delta(r-r')}{4E^2}, \quad (7)$$

where H is the free Dirac Hamiltonian. Since the sum of the particle and antiparticle propagators is the full Feynman propagator, the second term in Eq. (7) can be identified with the antiparticle propagator. It is, in fact, exactly the first term in the series obtained by expanding the antiparticle propagator in the gradient expansion of Eq. (5). Since the antiparticle propagator is short ranged it is not surprising that it can be approximated by a δ function. We now have a very neat approximate expression for the antiparticle piece of the Feynman propagator. For later convenience, we note that since the antiparticle propagator is short ranged, so is the expression given in Eq. (8) which therefore also has a δ -function expansion:

$$(H-E) G_E^{(-)}(\vec{r}-\vec{r}') = - \frac{(H-E)\gamma_0 \delta(r-r')}{2E}. \quad (8)$$

Now Eq. (8) is especially useful, since we can use it to examine the effects of turning on and off the antiparticle propagator. We now introduce a modified Feynman propagator with a parameter t so we can easily keep track of the antiparticle contribution. We have:

$$G_E^{(+)}(\vec{r}-\vec{r}') = G_E^{(+)}(\vec{r}-\vec{r}') + t G_E^{(-)}(\vec{r}-\vec{r}'), \quad (9)$$

where $t=0$ corresponds to propagation through positive energy states only

and $t=1$ corresponds to propagation through both positive and negative energy states. Inserting Eq. (9) into Eq. (1) and then operating on both sides with the differential operator $(H-E)$, we get, with the help of Eq. (8), the differential equation

$$(H-E)\psi(\vec{r}) = -V(r)\psi(\vec{r}) - (1-t)(H-E)V(r)\psi(\vec{r}). \quad (10)$$

As it stands Eq. (10) is not very illuminating; it does not look much like a Dirac or Schrödinger equation. However, if we make a phase-shift-equivalent transformation on the wave function, by defining

$$\xi(\vec{r}) = \left(1 + \frac{(1-t)V(r)}{2E}\right) \psi(\vec{r}), \quad (11)$$

we get the Dirac-like equation

$$(H + U(r) - E)\xi(\vec{r}) = 0, \quad (12)$$

where $U(r)$ is the new (Dirac) potential matrix given by:

$$U(r) = \begin{pmatrix} \frac{V+S}{1 + (1-t)(V+S)/2E} & 0 \\ 0 & \frac{V-S}{1 + (1-t)(V-S)/2E} \end{pmatrix}, \quad (13)$$

where V and S are the vector and scalar potentials felt by the incident nucleon. Now we can see the effect of turning on and off the antinucleon piece of the propagator rather clearly. If $t=1$ we obviously get back the usual expression for the Dirac equation. This equation, if turned into a two-component equation, yields the well-known wine-bottle bottom shape for the central potential and the enhancement of the spin-orbit potential so necessary for the spin observables.

If $t=m/(E+m)$ then clearly we get a suppression of $V-S$ in the nuclear interior ($V-S$ is positive). We also get an enhancement of $V+S$ in the

nuclear interior but this is much smaller since $V-S$ is about seven times larger than $V+S$. The suppression of $V-S$ cancels almost exactly the usual enhancement of the spin-orbit force which was found for $t=1$. Also the central potential now is mostly just $V+S$ with some small nonlinear terms. Thus we can identify taking $t=m/(E+m)$ as doing a conventional Schrödinger calculation with a Woods-Saxon potential and a derivative of a Woods-Saxon potential for the central and spin-orbit potentials, respectively.

If $t=0$, then we get approximately twice the suppression of $V-S$ in the nuclear interior as for $t=m/(E+m)$. This leads to a net suppression of the spin-orbit force in the nuclear interior, and gives nonlinear terms in the central potential which deepen it rather than making it look like a wine-bottle bottom. The central potential still does resemble a Woods-Saxon but with slightly different geometry. The Schrödinger equivalent potentials for $t=0$ and $t=m/(E+m)$ are much closer to each other than $t=m/(E+m)$ is to $t=1$. Thus we find the result of Hynes et al.⁵ that Dirac calculations without antinucleons resemble Schrödinger calculations. Presumably if Hynes et al.⁵ had multiplied their antiparticle propagator by $m/(E+m)$ instead of zero they would have come even closer to the Schrödinger result, although we emphasize that the big differences come about in going from $t=m/(E+m)$ to $t=1$ not from $t=0$ to $t=m/(E+m)$.

It has recently been suggested by Thies⁸ that the antiparticles are not necessary because the propagation over short distances (between collisions) can be suppressed equally well by the short-range correlations. These correlations in a Schrödinger calculation thus serve the same purpose as the antinucleons in the Dirac picture, namely they suppress the effects of the short-ranged piece of the particle propagator. This argument is rather pretty since if correct it provides a

direct connection between the Dirac phenomenology world and that of Schrödinger physics. It explains, for instance, why correlations and other medium effects are important for Schrödinger physics,⁹ but do not appear to be needed in Dirac physics.¹⁰ The explanation is simply that the medium effects prevent short-ranged propagation between scatterings. In the Dirac picture this short-ranged propagation is already suppressed by the antinucleons, and so further suppression has little effect.

Unfortunately Thies's argument is not correct in detail. Whilst the correlations in nuclei are responsible for suppressing some of the short-ranged propagation, they leave the bulk untouched. This is because the range of the nucleon-nucleon force is comparable to the correlation length in a nucleus, so a nucleon passing through a nucleus can indeed interact with two different nucleons without changing its position. To see this, we take the short-ranged propagation we wish to suppress to have a range of order $1/m$ as suggested by our discussion above, and the nucleon-nucleon force to be mediated by a meson with mass $\mu = 600$ MeV, i.e. the σ -meson mass. We consider the double scattering term of the multiple scattering series,

$$T_2 \sim \bar{u}(k_f) e^{ik_f \cdot r} V_{nn}(\vec{r}-\vec{r}') \rho(\vec{r}') \rho(\vec{r}'-\vec{r}'') m^2 \frac{e^{-m|\vec{r}-\vec{r}''|}}{4\pi|\vec{r}-\vec{r}''|} \times \\ \times V_{nn}(\vec{r}''-\vec{r}) \rho(\vec{r}'') e^{ik_i \cdot r''} u(k_i) , \quad (14)$$

where $\rho(r)$ is the nuclear density and V_{nn} is the nucleon-nucleon potential. Now we choose our correlation function to be

$$C(\vec{r}-\vec{r}') = (1 - \exp(-\beta|\vec{r}-\vec{r}'|^2)) . \quad (15)$$

This correlation function approximates quite well that coming from the

Reid potential if we take $\beta = 2.5 \text{ fm}^{-2}$.¹¹ The integrals in Eq. (14) may now be done, and the net result is that only a fifth of the short-ranged structure is suppressed. In other words, Schrödinger calculations with correlations would be comparable to a Dirac calculation with the t defined in Eq. (8) taken to be 0.2 instead of the 1 required by the full Dirac calculation. If we had used a zero-range nucleon-nucleon interaction as well as zero-range propagation (i.e. a δ function instead of a function with range $1/m$), we would, of course, have got full suppression of the short-range structure (corresponding to $t=1$). If a more realistic nucleon-nucleon interaction, consisting of an attractive long-ranged ($\mu = 600$ MeV) potential and a repulsive short-ranged ($\mu = 800$ MeV) potential, is used in Eq. (14), the amount of suppression increases to $t=0.25$, but still falls short of $t=1$. As previously mentioned the wine-bottle-bottom potential shape and enhanced spin orbit do not start to appear until t is greater than $m/(E+m)$.

In conclusion we have demonstrated that there is present in the particle propagator some short-ranged structure. Once this structure is removed, the particle propagator then becomes the full Feynman propagator. Thus the antiparticles are not adding short-ranged (and therefore probably spurious) behaviour, but rather cancelling short-ranged propagation which is present in the particle-only propagator. The short-ranged correlations in nuclei suppress this behaviour a little but nowhere near enough to reproduce the Dirac results.

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