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**One Loop Electroweak Corrections
to Polarization Asymmetries on the Toponium Resonance**

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ABSTRACT

We calculate the one loop corrections to toponium production in e^+e^- annihilation. We show that a sufficiently accurate measurement of the longitudinal polarization asymmetry is sensitive to these corrections and could thus provide an independent test of the standard model. Particularly interesting is the possibility to study in this reaction the coupling of the Higgs particle to heavy quarks.

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The Glashow-Salam-Weinberg model has been impressively confirmed in low energy experiments and through the recent discovery of the intermediate bosons. This has lent strong support to the idea that it will not only play the role of an effective theory, but also that the quantum corrections are an important ingredient. These should and will be tested in further experiments, in particular at the electron positron colliders presently under construction. The planned precision measurements will not only serve to test the theory but might eventually also allow to deduce indirectly information on heavy degrees of freedom like the Higgs structure of the theory, additional generations, etc.

A number of calculations of radiative corrections has been published by now [1]. However, these are limited to processes with bosons and light fermions only. Although the Higgs coupling to fermions is important for the structure of the theory, owing to the near masslessness of the fermions in these reactions it is irrelevant for the calculations. However, the situation could be different for a top quark with mass in and above the 40 GeV region as suggested by recent experiments [2]. In particular the study of electroweak interferences on a toponium resonance could be sensitive to such effects. Furthermore only through a variety of measurements one will be able to find and identify clearly potential deviations from the standard model or to pin down additional heavy degrees of freedom through their effects on radiative corrections.

We have therefore evaluated the one loop radiative corrections to the asymmetry in the cross section on a toponium resonance for polarized beams. To keep the discussion simple we consider only final states which can originate from resonance decays but not from the continuum. However, polarization asymmetries for final states which can interfere with the continuum, the muon pair forward-backward asymmetry and the asymmetry in the distribution of leptons from the weak decay of a single quark, are related to the polarization asymmetry in a straightforward way and a measurement of these quantities could in practice serve the same purpose.

The amplitude for the production of toponium in electron positron collisions can be written in terms of reduced helicity amplitudes as

$$A = ie^2 f_v \bar{v} \not{E} (h_R \gamma_R + h_L \gamma_L) u \quad (1)$$

E and M denote the toponium polarization and mass. f_v is defined by the matrix

element of the heavy quark vector current and can be calculated in terms of the boundstate wave function.

The polarization asymmetry is then given by

$$\alpha_{RL} = \frac{h_R^2 - h_L^2}{h_R^2 + h_L^2} \quad (2)$$

In lowest order the helicity amplitudes can be expressed in terms of M_Z and the electroweak couplings as [3]

$$\begin{aligned} h_{R,L} &= Q^e Q^t + \beta_{R,L}^e \frac{\beta_R^t + \beta_L^t}{2} \frac{M^2}{M^2 - M_Z^2} \\ \beta_R^f &= \frac{-s^2 Q^f}{sc}; \quad \beta_L^f = \frac{t_3^f - s^2 Q^f}{sc} \\ s &\equiv \sin^2 \theta_W; \quad c \equiv \cos^2 \theta_W \equiv M_W^2/M_Z^2 \end{aligned} \quad (3)$$

where Q and I_3 denote fermion charge and weak isospin. The asymmetry is then evidently independent of f_v . It has been shown [4] that this result for the asymmetry remains unaffected by leading log corrections, if the Weinberg angle is defined as below. To evaluate the one loop corrections we use the "on shell renormalization scheme" employed already previously [5,6,7]. The poles of the Z and W propagators are fixed at the masses of the intermediate bosons and the Weinberg angle is defined through $M_Z \cos \theta_W = M_W$. This requirement, together with the fine structure constant, the Higgs and the fermion masses fixes the normalization conditions. Since furthermore the muon lifetime has been measured with great precision, it is possible to express the W mass in terms of the remaining quantities and we shall use only masses consistent with this requirement.

The corrections to the lepton vertices and the boson propagators can be found in the literature [6] and we shall make use of these results. The central part of this paper consists in the evaluation of the toponium -A and -Z vertex corrections (Fig. 1a-k) and of the box diagrams (Fig. 1l-n). Corrections from Φ_Z , Higgs and photon exchange (Fig. 1h-k) contribute with equal strenght at the Z and A vertex, are thus irrelevant for the asymmetry and could be dropped – together with the appropriate contributions from the counterterms. We nevertheless give the full weak corrections for completeness, apart from the photon exchange contribution (Fig. 1d), which can be found in the literature [8]. The amplitude which contains a

Higgs -Z coupling (Fig. 1k) is finite by itself and, among others, will be responsible for the Higgs mass dependence of the final answer. Scalar exchange in the s channel does not contribute in the limit of vanishing electron mass.

The evaluation of box diagrams will be tedious but straightforward. We note that also the amplitude from the $Z\gamma$ intermediate state (Fig. 1n) will be finite by itself.

Since we are interested in the production of a vector bound state we have used the appropriate quark antiquark wavefunctions and employed the bound state formalism outlined in [9]. The remaining integrals can be evaluated with standard methods and the details will be given elsewhere [10]. After splitting off an overall factor $ie \frac{\alpha}{4\pi} f_v M^2$ we obtain for the toponium-A and -Z vertex corrections

$$\begin{aligned}
a(W, W) &= \left(\frac{1}{c/s} \right) \frac{2t_3^t}{s^2} \frac{1}{12} \left\{ -1 + 10B_0(P, M_W, M_W) + 2B_1\left(\frac{P}{2}, 0, M_W\right) \right\} \\
a(W, \Phi_W) &= 0 \\
a(\Phi_W, \Phi_W) &= \left(\frac{1}{\frac{c^2-s^2}{2sc}} \right) \frac{2t_3^t}{s^2} \frac{M^2}{48M_W^2} \left\{ 1 + 2B_0(P, M_W, M_W) + B_1\left(\frac{P}{2}, 0, M_W\right) \right\} \\
a(Z, H) &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \frac{\beta_R^t + \beta_L^t}{4s^2c^2} \frac{1}{M_Z^2 + M_H^2 - M^2} \left\{ (-M_Z^2 + M_H^2 - M^2)B_0(P, M_H, M_Z) \right. \\
&\quad \left. + M_Z^2 B_0\left(\frac{P}{2}, \frac{M}{2}, M_Z\right) + (-M_H^2 + M^2)B_0\left(\frac{P}{2}, \frac{M}{2}, M_H\right) \right\} \\
a(W) + a(\Phi_W) &= \begin{pmatrix} -Q^t + 2t_3^t \\ -\beta_L^t + 2t_3^t \frac{c}{s} \end{pmatrix} \frac{1}{6s^2} \left\{ \frac{5}{2} + \frac{M^2}{8M_W^2} \right. \\
&\quad \left. + \left(1 + \frac{M^2}{8M_W^2}\right) \left(2 \frac{M^2 - 4M_W^2}{M^2 + 4M_W^2} B_0\left(\frac{P}{2}, 0, M_W\right) \right. \right. \\
&\quad \left. \left. - \frac{4M^2}{M^2 + 4M_W^2} B_0(P, 0, 0) - B_1\left(\frac{P}{2}, 0, M_W\right) \right) \right\} \\
a(Z) &= \begin{pmatrix} -Q^t(\beta_R^{t^2} + \beta_L^{t^2}) \\ -(\beta_R^{t^3} + \beta_L^{t^3}) \end{pmatrix} \frac{1}{3} \left\{ \frac{5}{2} - B_0\left(\frac{P}{2}, M_Z, M/2\right) + B_1\left(\frac{P}{2}, M_Z, \frac{M}{2}\right) \right\} \\
&\quad + \begin{pmatrix} -Q^t \\ -\frac{\beta_R^t + \beta_L^t}{2} \end{pmatrix} \frac{\beta_R^{t^2} + \beta_L^{t^2}}{2} \frac{M^2}{M_Z^2} \left(B_0\left(\frac{P}{2}, M_Z, \frac{M}{2}\right) - B_0\left(P, \frac{M}{2}, \frac{M}{2}\right) \right) \\
a(\Phi_Z) &= \begin{pmatrix} -Q^t \\ -\frac{\beta_R^t + \beta_L^t}{2} \end{pmatrix} \frac{1}{s^2} \frac{M^2}{48M_W^2} \left\{ 1 - B_0\left(\frac{P}{2}, M_Z, \frac{M}{2}\right) + B_1\left(\frac{P}{2}, M_Z, \frac{M}{2}\right) \right\}
\end{aligned} \tag{4}$$

$$a(H) = \left(-\frac{Q^t}{-\frac{\beta_R^t + \beta_L^t}{2}} \right) \frac{1}{s^2} \frac{M^2}{48M_W^2} \left\{ 1 + \left(3\frac{M^2}{M_H^2} - 1 \right) B_0\left(\frac{P}{2}, M_H, \frac{M}{2}\right) \right. \\ \left. + B_1\left(\frac{P}{2}, M_H, \frac{M}{2}\right) - 3\frac{M^2}{M_H^2} B_0\left(\frac{P}{2}, \frac{M}{2}, \frac{M}{2}\right) \right\}$$

The functions B are defined in the standard manner [11] and the upper and lower entries refer to A and Z respectively. Together with the counterterms, to be derived from

$$\begin{aligned} a_c^A &= ief_v(Q^t(\delta Z_R + \delta Z_L)/2 + t_3^t \delta k) \\ a_c^Z &= ief_v((\beta_R^t \delta Z_R + \beta_L^t \delta Z_L)/2 + \frac{c}{s} t_3^t \delta k) \\ \delta Z_{R,L} &= A_{R,L} + m^2(A'_R + A'_L + 2C') \\ \delta k &= -\frac{\alpha}{4\pi s^2} B_0(0, M_W, M_W) \\ A_R &= \frac{\alpha}{4\pi s^2} \left\{ s^2 \beta_R^{t^2} (1 + 2B_1(p, m, M_Z)) \right. \\ &\quad \left. + \frac{m^2}{4M_W^2} (B_1(p, m, M_Z) + B_1(p, m, M_H) + 2B_1(p, m_b, M_W)) \right\} \\ A_L &= \frac{\alpha}{4\pi s^2} \left\{ (s^2 \beta_L^{t^2} (1 + 2B_1(p, m, M_Z)) + \frac{1}{2} + B_1(p, m_b, M_W)) \right. \\ &\quad \left. + \frac{m^2}{4M_W^2} (B_1(p, m, M_Z) + B_1(p, m, M_H)) + \frac{m_b^2}{2M_W^2} B_1(p, m_b, M_W) \right\} \\ C &= \frac{\alpha}{4\pi s^2} \left\{ 2s^2 \beta_R^t \beta_L^t (1 - 2B_1(p, m, M_Z)) \right. \\ &\quad \left. + \frac{m^2}{4M_W^2} (B_0(p, m, M_H) - B_0(p, m, M_Z)) - \frac{m_b m}{2M_W^2} B_0(p, m_b, M_W) \right\} \end{aligned} \quad (5)$$

$$m = M/2; \quad m_b = 0; \quad p \equiv P/2; \quad A' \equiv \frac{dA}{dp^2}(p^2 = m^2).$$

the result is finite. The box diagrams are ultraviolet finite and, for M different from M_Z , also infrared finite and lead to the following contribution to the helicity amplitudes:

$$\begin{aligned} h_{R,L}^{WW} &= \frac{\alpha}{4\pi} \left(0; \frac{-1}{s^4} \right) \frac{1}{2} F^{WW} \\ h_{R,L}^{ZZ} &= \frac{\alpha}{4\pi} (-\beta_R^{e^2}; \beta_L^{e^2}) (\beta_R^{t^2} - \beta_L^{t^2}) F^{ZZ} \\ h_{R,L}^{Z\gamma} &= \frac{\alpha}{4\pi} (-\beta_R^e; \beta_L^e) (\beta_R^t - \beta_L^t) Q^e Q^t 2F^{Z\gamma} \end{aligned} \quad (6)$$

$$\begin{aligned}
F^{WW} &= \frac{2M^2}{4M_W^2 - M^2} \left\{ B_0(P, M_W, M_W) - B_0\left(\frac{P}{2}, 0, M_W\right) \right. \\
&\quad \left. - M_W^2 C_0\left(\frac{P}{2} + l, -P, 0, M_W, M_W\right) + \frac{M^2}{4} C_0\left(l, \frac{P}{2} - l, 0, 0, M_W\right) \right\} \\
F^{ZZ} &= \frac{1}{\frac{1}{2} - u} \left\{ B_0\left(\frac{P}{2}, \frac{M}{2}, M_Z\right) - B_0(P, M_Z, M_Z) \right. \\
&\quad - \left(\frac{1}{2} - u\right) \left(B_0\left(l, 0, \frac{M}{2}\right) - B_0(P, M_Z, M_Z) \right) \\
&\quad + \left(-u^2 + u - \frac{1}{2}\right) M^2 C_0\left(l, \frac{P}{2}, 0, \frac{M}{2}, M_Z\right) \\
&\quad \left. + (1 - u) M_Z^2 C_0\left(\frac{P}{2} + l, -P, 0, M_Z, M_Z\right) \right\} \\
F^{Z\gamma} &= \frac{1}{u - 1} \left(B_0\left(\frac{P}{2}, \frac{M}{2}, 0\right) + B_0\left(\frac{P}{2}, \frac{M}{2}, M_Z\right) - 2B_0(P, 0, M_Z) \right) \\
&\quad + \left(B_0(P, 0, M_Z) - B_0\left(l, 0, \frac{M}{2}\right) \right) + (u - 1) M^2 C_0\left(l, \frac{P}{2}, 0, \frac{M}{2}, M_Z\right)
\end{aligned}$$

$$\begin{aligned}
M^2 C_0\left(\frac{P}{2} + l, -P, 0, M_Z, M_Z\right) &= -\log^2\left(\frac{\sqrt{1 - 4u} + 1}{\sqrt{1 - 4u} - 1} - i\epsilon\right) \\
M^2 C_0\left(l, \frac{P}{2} - l, 0, 0, M_Z\right) &= 2(\pi^2/4 - Sp(1 - \frac{1}{2u}) + Sp(\frac{1}{2u} - 1 + i\epsilon)) \\
M^2 C_0\left(l, \frac{P}{2}, 0, \frac{M}{2}, M_Z\right) &= \pi^2/6 - Sp(1 - \frac{1}{u}) \\
u &\equiv M_Z^2/M^2; \quad l \equiv (p_{e^-} - p_{e^+})/2.
\end{aligned}$$

Details of the derivation will be given elsewhere [10]. The contribution from the $Z\gamma$ intermediate state (Fig. 1n) exhibits a logarithmic singularity for $M \rightarrow M_Z$ which can be rendered finite, if we take the nonvanishing binding energy and the Z -width into account.

Our numerical results are based on the vertex corrections and box diagrams listed above, together with the corrections from the lepton vertex and the boson propagator from [6]. In Fig. 2a we give the comparison between the asymmetry calculated in lowest order and with one loop corrections for $M_Z = 94\text{GeV}$ and $M_H = 100\text{GeV}$ as a function of the toponium mass. Since we choose the input parameters in such a way that they correctly reproduce the muon lifetime, M_W and thus the Weinberg angle depend mildly on $m_t = M/2$ and on M_H as shown in Fig. 2b.

To study the Higgs mass dependence we show in Fig. 2c the change in the asymmetry, if we vary M_H to 10 or 1000 GeV. If we drop the contribution from the Higgs-fermion coupling $a(Z, H)$ altogether to test the sensitivity of our calculation on this aspect the asymmetry is changed by less than .001 for toponium masses below 160 GeV*.

We find that the changes are rather small, even for extreme choices of the Higgs mass - another example of the Higgs “screening” discussed in [13]. For M_H far larger than M_Z and M one finds only a logarithmic dependence on the Higgs mass.

Finally we should comment on the role of QED and QCD corrections. It is well known that QED corrections to the asymmetry in the continuum are extremely important and partly acceptance dependent. In our case the situation is far simpler. Since the boundstate is neutral, final state radiation can be ignored and the box diagram is infrared finite. The corrections to the photon propagator from the vacuum polarization has to be and was easily accomodated in our numerical evaluation. The momentum which flows through the A and Z propagators is nearly fixed by the resonance mass since the small variation due to the tiny toponium width can safely be ignored and the helicity amplitudes can be treated just as constants. Corrections from initial state bremsstrahlung together with those from the photonic virtual corrections to the electron vertex thus affect only the overall normalization and not the relative importance of h_R versus h_L . This is of course quite different from the situation in the continuum where the invariant mass e. g. of the muon pair is allowed to vary.

Also QCD corrections can be safely ignored. The helicity amplitudes are given as a power series in α and α_s . The correction of order α_s affects only the overall normalization and is absorbed in f_v . Corrections of order $\alpha\alpha_s$ have two qualitatively different sources. Some originate from QCD corrections at the toponium vertex together with electroweak corrections at the lepton vertex or at the boson propagators. These are essentially taken care of by the definition of f_v . The others originate from simultaneous QCD and electroweak corrections at the toponium vertex and have not been calculated. However, these corrections, are not enhanced

*These numbers differ from our unpublished preliminary results [12].

by large logarithms and thus truly of order α_s/π compared to those evaluated in this work.

To summarize: We have calculated the full one loop electroweak corrections to the polarization asymmetry on a toponium resonance and studied their dependence on the Higgs mass. We find that these corrections are small if the Born term has been chosen in the way described above. The remaining QED and QCD corrections can be safely neglected.

REFERENCES

- [1] For a recent review see e. g.: L. Maiani, preprint CERN-TH 4226. To be published in the proceedings of the HEP-85 Conference, Bari, Italy, July 1985.
- [2] G. Arnison et al., *Phys. Lett.* **147 B** (1984) 493.
- [3] S. Güsken, J. H. Kühn and P. Zerwas, *Nucl. Phys. B* **262** (1985) 393 and references therein.
- [4] B. Grzadkowski et al., *Phys. Lett.* **163 B** (1985) 247.
- [5] A. Sirlin, *Phys. Rev. D* **22** (1980) 93.
- [6] B. W. Lynn, R. G. Stuart, *Nucl. Phys. B* **253** (1985) 216;
R. G. Stuart, Oxford University, D. Phil. Thesis (1985); Rutherford and Appleton Lab. Report, RAL T 008.
- [7] M. Böhm, W. Hollik and H. Spiesberger, *Z. Phys. C* **27** (1985) 523.
- [8] R. Karplus and A. Klein, *Phys. Rev.* **87** (1952) 848,
R. Barbieri et al., *Phys. Lett.* **57 B** (1975) 455.
- [9] J. H. Kühn, J. Kaplan and E. G. O. Safiani, *Nucl. Phys. B* **157** (1979) 125.
- [10] B. Grzadkowski, P. Krawczyk, J. H. Kühn and R. G. Stuart,
in preparation.
- [11] G. Passarino and M. Veltman, *Nucl. Phys. B* **160** (1979) 151.
- [12] B. Grzadkowski, P. Krawczyk, J. H. Kühn and R. G. Stuart,
Proc. of the Conf. "Tests of Electroweak Theories", Trieste, Italy, 10.-12. June 1985.
- [13] M. Veltman, *Acta Physica Polonica B* **8** (1977) 475.

FIGURE CAPTIONS

1. Feynman diagrams relevant for the evaluation of one loop corrections. Permutations have been omitted.
2. a) Prediction for the polarization asymmetry as a function of the toponium mass for $M_H = 100\text{GeV}$ and $M_Z = 94\text{GeV}$. Solid line: Full one loop correction. Dashed line: difference between first order and lowest order prediction.
b) Value of $\sin^2 \theta_W$ used both for lowest order and one loop corrected prediction for different values of M_H .
c) Change in the asymmetry $\alpha_{RL}(M_H) - \alpha_{RL}(100\text{GeV})$ for a Higgs mass of 1000 GeV (solid line) and 10 GeV (dashed line).

Fig. 1

Fig. 2